The Effect of Options on Liquidity and Asset Returns*

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Abstract

This paper examines how introducing an options market affects the liquidity and expected returns of underlying assets when the economy features asymmetric information. Options provide hedging benefits and increase the risk sharing between liquidity demanders and liquidity suppliers, although these two agent types experience unequal welfare improvement. I show that introducing financial derivatives can have opposite effects on underlying asset prices: doing so increases (resp., reduces) those prices when the market has relatively more liquidity suppliers (resp., liquidity demanders). Thus the non-monotonic effects of derivatives on underlying assets could reconcile the mixed empirical evidence on options listing effects. Introducing derivatives reduces the price impact of liquidity demanders' trades on the underlying risky asset but has no effect on its price reversal dynamics. The mechanism proposed here is robust to endogenous participation and to derivatives with a general payoff structure. Finally, I provide new implications that can be tested empirically.

Keywords: derivative assets, liquidity, asymmetric information, rational expectations equilibrium

JEL Classifications: G12, G13, G14

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1 Introduction

Recent years have seen rapid development of the derivatives market.[1] The effect of these new assets on the whole financial market has led to heated discussion, and the topic has been of extreme importance to practitioners and regulators since the recent financial crisis.[2] Motivated by policy concerns, a large body of empirical work examines how the introduction of derivative assets, such as options, affects the underlying asset.

Most of this research has focused on how derivative securities affect the price level and volatility of underlying assets, and little is known about how they affect market liquidity.[3] Although a few theoretical studies have investigated the effect of derivatives on their underlying assets through the price discovery channel (Cao, 1999; Huang, 2015), the theoretical framework for these empirical findings is incomplete—especially with regard to the effect of derivatives on liquidity of the underlying securities.

This paper examines how introducing financial derivatives affects the underlying asset's price level and liquidity. I find that the advent of options trading improves that liquidity and also the welfare of market participants, but it has a surprisingly non-monotonic effect on the asset's price. Further analysis reveals that the reported effects of derivatives on the asset's liquidity and price are dependent on the measures of illiquidity used and the factors driving asset-specific characteristics. In this regard, the paper offers new and comprehensive guidelines for empirical studies designed to analyze—via the liquidity channel—how securities are affected by the introduction of derivatives.

This study of how derivatives affect liquidity employs a rational expectations equilibrium (REE) model.[4] I start by considering an economy with three dates and two assets as the benchmark (Vayanos and Wang, 2012a,b).[5] Specifically, at dates 0 and 1, agents can trade a risk-free asset and a risky asset that pay off at date 2. At date 0, agents are identical and therefore no trade occurs. At date 1, agents can be one of two types: a *liquidity demander*, who

[1] In the United States, the trading volume of individual stock options has grown exponentially from 5 million contracts in 1974 to more than 3,845 million contracts in 2014 (http://www.optionsclearing.com/webapps/historical-volume-query).

[2] Some regulators have argued that the complicated new derivatives products, which are designed to provide risk management and liquidity benefits to the financial system, produced exactly the opposite effect during the financial crisis of 2007 to 2010. In response to the 2007 credit crunch and the ensuing liquidity crisis, the Dodd-Frank Wall Street Reform and Consumer Protection Act became federal law, bringing significant changes to financial regulation of the derivative markets.

[3] See, for example, Damodaran and Lim (1991), Fedenia and Grammatikos (1992), and Kumar, Sabin, and Shastri (1998).

[4] Because options have *no* effect on the underlying assets if the market is complete, competitive, and frictionless (Black and Scholes, 1973), the focus here is on the case of asymmetric information between liquidity demanders and suppliers.

[5] This setting is similar to that described by Grossman and Stiglitz (1980). The only difference is that Grossman and Stiglitz introduce exogenous noise traders whereas Vayanos and Wang (2012a,b) replace the noise traders with rational hedgers who are influenced by a liquidity shock.

at date 2 will receive a random endowment whose payoff is correlated with the risky asset's payoff; or a *liquidity supplier*, who will receive no such endowment. Only those agents who receive the random endowment observe the covariance between that endowment and the risky asset's payoff. Liquidity demanders can hedge the liquidity shock, which is modeled here as a random endowment, by trading with liquidity suppliers. Therefore, the existence of two agent types results in trade at date 1.[6] In addition, liquidity demanders receive private information about the risky asset's payoff before trade begins at date 1 whereas liquidity suppliers cannot distinguish the private signal from the demanders' liquidity shock.[7]

Next I introduce an options market into the economy. As in Cao and Ou-Yang (2009), the options market consists of a complete set of European call and put options written on the risky asset. At date 0, all agents know that options will be introduced into trade at date 1. Under this setup, I compare the ex ante price and measures of illiquidity of the risky asset before and after options are introduced—a comparison that quantifies the effects of this options market. To examine market liquidity, I follow Vayanos and Wang (2012a,b) and consider two widely used empirical measures of illiquidity: price impact λ and price reversal γ.

I find that liquidity demanders who observe the private signal take short positions in the introduced options, whereas liquidity suppliers who learn information through asset prices take long positions in those options. The payoff structure is such that options may yield hedging benefits for the second moment of the risky asset's payoff. Since liquidity demanders have more precise information, they always have less incentive than suppliers to hedge against such volatility. Given that options are in zero net supply, liquidity demanders (resp. suppliers) take short (resp. long) positions in options. Moreover, the trading volume of options increases with the information dispersion across agents because widely dispersed information is followed by high demand for options.

In this paper I establish that options provide hedging benefits and increase risk sharing between liquidity demanders and liquidity suppliers. In my model, liquidity demanders trade to hedge and/or exploit their information. Before options are introduced, uninformed liquidity suppliers are unable to distinguish between these two motives, which reduces the incentive to trade with liquidity demanders. Yet when financial derivatives are available, liquidity suppliers can hedge against uncertainty in the risky asset's payoff and so are more willing to accommodate the trades of informed demanders. Because of this options-enabled increase in risk sharing, their introduction increases all agents' utilities. In other words, each market participant's welfare has improved at date 0.[8]

[6]Endowment shocks have been modeled as a non-informational trading motive in different forms. See, for example, Diamond and Verrecchia (1981), Wang (1994), O'Hara (2003), and Vayanos and Wang (2012a,b).

[7]Qiu and Wang (2010) employ a more general framework to analyze the asset pricing implications of asymmetric information and endowment shocks.

[8]The so-called noisy REE models usually introduce exogenous noise trading into the economy. That approach

However, the effects of options on liquidity demanders and suppliers are not symmetric. Intuitively, the relative benefit from more risk-sharing opportunities is determined by the competition within each group. For example, if the market consists mostly of liquidity suppliers then competition within that group is intense; hence the welfare improvement is greater for liquidity demanders than for liquidity suppliers. These results are reversed when the market is dominated by liquidity demanders. Although both agent types benefit when options are introduced, the magnitude of that benefit differs by type and this difference affects the trading incentives of agents at date 0.

More importantly, I show that introducing derivatives has (surprisingly) non-monotonic effects on the underlying asset prices—a finding that could reconcile, at least in part, conflicting evidence on options listing and its effect on the underlying asset.[9] When the market has many more liquidity suppliers than demanders, the benefits stemming from options are greater for the latter than the former. At date 0, the identical investor who can anticipate the effects of options is thus less worried about liquidity shock. Hence this investor is more willing to hold the risky asset at date 0, which increases that asset's ex ante price. The opposite effect is observed when the agent population consists mostly of liquidity demanders. The mechanism is robust to derivatives with general payoff structures.

Further analysis shows that the effects of derivatives on the price of their underlying asset are sensitive to the illiquidity measures used and to the particular factors that drive the asset-specific characteristics. For example, if illiquidity is measured by price impact λ and if the cross-sectional variation in λ is driven by the private signal's precision, then introducing options will lower (resp. raise) the prices of stocks that are relatively more (resp. less) liquid. Yet if illiquidity is measured by price reversal γ then the opposite dynamic is observed. When the cross-sectional variation in different illiquidity measures is due to the liquidity shock's precision, one observes different effects of options on the underlying asset's price. These novel implications, which concern how options affect their underlying assets, can be tested empirically.

I also find that introducing an options market reduces the price impact of liquidity demanders' trades on the underlying risky asset—but that it has no effect on price reversal. Options expand the scope of risk sharing between liquidity demanders and suppliers and reduce the price effect per trade captured by λ. Because the introduction of options increases the trade size, the effect of options on overall trade, which is measured by γ, does not change.

complicates welfare analysis because one cannot compute the welfare of noisy traders. My model replaces noisy traders with rational hedgers whose utility can be calculated, which enables a welfare analysis of the effects of introducing derivatives.

[9] Several empirical studies document that an options listing reduces the underlying asset's price (e.g., Conrad, 1989; Detemple and Jorion, 1990; Skinner, 1989), although a few find just the opposite (e.g., Sorescu, 2000; Mayhew and Mihov, 2000).

Furthermore, the liquidity improvement of more liquid (low-λ) stocks is less than that of less liquid (high-λ) stocks. When information becomes more asymmetric, the consequent adverse selection is more severe and so the price impact λ is greater. In other words, high-λ stocks are more subject to information asymmetry. The hedging benefit provided by options is greater for stocks that are more likely to be subject to asymmetric information. That is, the improvement in liquidity is greater for stocks with high λ than for stocks with low λ. The converse effect is observed when illiquidity is instead measured by γ because asymmetric information can *reduce* γ.[10]

In order to endogenize further the participation decisions of agents, I examine a scenario in which they must pay a participation fee to enter the market at date 1; this fee can be interpreted as learning costs or opportunity costs, much as in Huang and Wang (2009, 2010). A participation fee reduces the participation of liquidity suppliers, which changes the proportion of liquidity suppliers in the population as a whole.[11] I find that the introduction of an options market always reduces the illiquidity discount in the ex ante price and also reduces the expected return of the underlying asset—that is, irrespective of the proportion of liquidity demanders. When options are introduced, more liquidity suppliers are willing to enter the market and to accommodate the trades of liquidity demanders; the result is a decline in the illiquidity discount that a demander would normally require. Moreover, both illiquidity measures decrease after derivatives are introduced, which contrasts to the case *without* costly participation. The reason is that an increased proportion of liquidity suppliers facilitates risk sharing and thus lowers both price impact λ and price reversal γ. Finally, the model predicts that options trading volume is not a monotonic function of the participation cost; rather, it should exhibit an inverse U shape with respect to that cost.

Related Literature. My study is related to the literature that addresses market frictions and liquidity.[12] Most studies on REE with asymmetric information focus on price informativeness. In contrast, market liquidity is the focus of most literature on strategic trading and sequential trading. For example, Biais and Hillion (1994) investigate how derivatives affect market liquidity in a strategic trading model. Vayanos and Wang (2012a,b) take a first step in analyzing how imperfections such as asymmetric information affect ex ante prices and market liquidity in REE models.[13] Inspired by Vayanos and Wang (2012a,b), this paper is the first

[10] In the model, price reversal γ captures the importance to price of liquidity shocks. When the private signal is extremely precise (i.e., when information asymmetry is severe), the effect of the liquidity shock captured by γ is rather limited.

[11] Because it is the liquidity demanders who face the risk of liquidity shock, they stand to benefit more from participation than do suppliers. Recall also that it is the *relative* measure of participating suppliers and demanders that matters in this model. Thus I limit the analysis to an equilibrium under which liquidity demanders fully participate and liquidity suppliers partially participate (cf. Vayanos and Wang, 2012b).

[12] The literature is vast; Vayanos and Wang (2012b) provide a comprehensive review.

[13] Several recent papers examine the effect of asymmetric information on expected returns (see e.g. O'Hara,

study of how—in a competitive market—the introduction of an options market affects both the returns and market liquidity of the underlying assets.

This paper contributes also to the theoretical literature on the effects of derivative securities on underlying assets (Grossman, 1988; Back, 1993; Biais and Hillion, 1994; Brenna and Cao, 1996; Huang and Wang, 1997; Cao, 1999; Chabakauri, Yuan, and Zachariadis, 2014; Huang, 2015; Malamud, 2015). The most closely related to mine is the work of Biais and Hillion (1994), who show that options can help prevent breakdowns caused by the adverse selection problem in a noncompetitive market and thereby make the market more liquid. In Biais and Hillion (1994), market makers are risk neutral and the expected returns on assets are always equal to the risk-free rate. However, the analysis here is conducted in a competitive market and yields empirical implications for how the expected returns of underlying assets are affected by options listing. Brenna and Cao (1996) incorporate a quadratic option into a noisy rational expectations model and find that the derivative allows agents to achieve a Pareto-efficient allocation. However, they find that the underlying asset's price is *not* affected by the option. There are, in addition, several other studies that investigate the effects of derivatives on the underlying asset when the acquisition of information is endogenous. Following Grossman and Stiglitz (1980) and Hellwig (1980), Cao (1999) and Huang (2015) focus on the information acquisition channel and on how introducing options affects the price of risky assets through price informativeness. In contrast, I focus on market liquidity and show that the ex ante price of the risky assets is indeed affected by options—even in the absence of an information acquisition channel (i.e., even if the price informativeness remains unchanged). Furthermore, this paper is one of only a few studies that introduce a set of explicit options into an economy with asymmetric information. Those works include Chabakauri, Yuan, and Zachariadis (2014), who generalize the distribution of asset payoffs, Huang (2015), who focuses on endogenous information acquisition, and Malamud (2015), who examines price discovery under general preferences. My paper is also associated with the strand of literature on financial innovation (Allen and Gale, 1994; Duffie and Rohi, 1995; Dow, 1998; Brock, Hommes, and Wagener, 2009; Dieckmann, 2011; Simsek, 2013a,b; Chabakauri, Yuan, and Zachariadis, 2014).

The rest of the paper is organized as follows. Section 2 sets up the model and presents the benchmark case with asymmetric information but without options, and Section 3 examines what happens when an options market is introduced into the economy. Section 4 extends the model further by considering costly participation, after which Section 5 discusses some more general derivatives. Section 6 concludes. All proofs are given in the appendices.

2003; Easley and O'Hara, 2004; Garleanu and Pedersen, 2004; Qiu and Wang, 2010).

2 Model

2.1 Economy

Given the unified framework of liquidity with one risk-free asset and one risky asset as proposed by Vayanos and Wang (2012a,b), I introduce an options market into the economy. To examine the subsequent effects, I compare the price and two measures of the risky asset's illiquidity before and after options are introduced. Before solving in Section 3 the equilibrium for the economy with options, I solve the equilibrium in the absence of an options market.

Timeline and Assets. As the benchmark, I consider an economy with three dates, $t = 0, 1, 2$, and two assets. The risk-free asset is in a supply of b shares and pays off one unit of a consumption good at date 2. The supply of the risky asset is $\bar{X} > 0$ shares, each of which pays off D units of the consumption good, where $D \sim N(\bar{D}, 1/h)$. The price of the risky asset is denoted by P_t at date t. With the risk-free asset as numéraire, the price of the risky asset at date 2 is equal to D; that is, $P_2 = D$.

There is a measure one of investors whose utility function over consumption follows a negative exponential utility function with absolute risk aversion coefficient α:

$$-\exp(-\alpha C_2), \tag{2.1}$$

where C_2 is the consumption at date 2. All investors are identical at date 0 and are endowed with the per capita supply of both the risk-free asset and the risky asset. Then they become heterogeneous, and that heterogeneity generates trade at date 1. At date 2, all asset payoffs are realized and all investors consume their total wealth.

Investors and Asymmetric Information. At date $\frac{1}{2}$, investors' types are realized. At this interim date, investors learn whether or not they will receive an extra endowment at date 2. A proportion π of these agents encounter the liquidity shock of receiving an additional endowment $z(D - \bar{D})$ of the consumption good at date 2; the remaining proportion $1 - \pi$ of agents receive no extra endowment. Only those who receive the endowment observe the liquidity shock z. Because the endowment received by the proportion π of agents is correlated with D, the agents have an incentive to hedge against the risk exposure induced by that liquidity shock.[14] These agents are referred to as liquidity demanders because trades are initiated by their hedging against the liquidity shock. When, for example, a positive endowment shock occurs (i.e., z is positive), liquidity demanders are willing to sell the risky asset to hedge against the risk that the payoff D will be low. The remaining agents accommodate this hedging

[14] Like Vayanos and Wang (2012a), for simplicity I assume that the endowment is perfectly correlated with the payoff D. If the correlation is not perfect then the results remain qualitatively similar, since the crux of this study is a nonzero correlation.

demand and are referred to as liquidity suppliers. The liquidity shock z is normally distributed with mean zero and precision n—that is, $z \sim N(0, 1/n)$—and is independent of D.[15] In addition to the liquidity shock, liquidity demanders receive a private signal s about the risky asset payoff D before trading at date 1.[16] The signal is

$$s = D + \epsilon, \tag{2.2}$$

where ϵ is normally distributed with mean zero and precision m (i.e., $\epsilon \sim N(0, 1/m)$) and is independent of (D, z). To simplify the analysis, I assume that all liquidity suppliers are uninformed about the private signal s; the consequent information asymmetry serves as a benchmark for the following studies.

At date 1, liquidity demanders and liquidity suppliers trade with each other on the basis of their information and the liquidity shock. In this context, liquidity demanders have two trading motives: speculating (informational) and hedging (non-informational). Specifically, demanders can extract profit from the private signal they observe and also wish to hedge against the random endowment they will encounter at date 2. The inability of liquidity suppliers to distinguish between these two motives of demanders reduces the former's incentive to trade with the latter. As shown by Vayanos and Wang (2012a,b), an illiquidity discount in the price of the risky asset at date 0 (i.e., P_0) arises in response to a liquidity shock and is magnified when the economy features asymmetric information.

2.2 Asymmetric Information Benchmark without an Options Market

Here I recall the results of Vayanos and Wang (2012a,b) obtained under asymmetric information. It is the benchmark to be used for comparisons with the setting in which an options market has been introduced.

I follow Vayanos and Wang (2012a,b) in first solving the equilibrium at date 1 and then working backward to obtain the equilibrium at date 0. Following the model setup, liquidity demanders observe the signal s and know their liquidity status z. As a result, their information comprises the private signal s, the liquidity shock z, and the prices P_0 and P_1; formally, $\mathcal{F}_d = \{s, z, P_0, P_1\}$. At the same time, liquidity suppliers cannot observe the private signal and so their only information is the prices themselves. Hence the information set of these suppliers is $\mathcal{F}_s = \{P_0, P_1\}$. All investors formulate their demand functions conditional on their respective information sets, and the equilibrium price clears the market. I shall denote by X_d

[15] The endowment $z(D - \bar{D})$ can take large negative values under the normal distribution, which could yield an infinitely negative expected utility. Similarly to Vayanos and Wang (2012a), I assume that the precisions of D and z satisfy $\alpha^2 < nh$ to ensure that utility is finite.

[16] Without loss of generality, I assume that all liquidity demanders are informed. Even if they do not receive the private signal, they can perfectly infer it from the price because they observe the liquidity shock.

and X_s the demand of liquidity demanders and liquidity suppliers (respectively) for the risky underlying asset.

In this study, I assume a linear price function and conjecture that the risky asset's price is a linear function of the signal s and the liquidity shock z:

$$P_1 = A + B(s - \bar{D} - Cz), \tag{2.3}$$

where A, B, C are constants. The expectations of liquidity demanders are such that the conditional mean and variance of the risky asset payoff D are[17]

$$\mathrm{E}[D|\mathcal{F}_d] = \bar{D} + \beta_s(s - \bar{D}) \quad \text{and} \quad \mathrm{Var}[D|\mathcal{F}_d] = \frac{1}{h+m}, \tag{2.4}$$

where $\beta_s = \frac{m}{h+m}$. At date 1, liquidity demanders maximize their expected utility over the wealth at date 2, $W_{d2} = W_1 + X_d(D - P_1) + z(D - \bar{D})$,[18] and then submit a demand schedule for the risky asset as follows:

$$X_d = \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\alpha \,\mathrm{Var}[D|\mathcal{F}_d]} - z. \tag{2.5}$$

As this equation shows, the demand schedule of liquidity demanders is affected by the liquidity shock z. For instance, if a positive liquidity shock occurs then demanders will attempt to sell the risky asset because they are overly exposed to the risk that the payoff D will be low.

In contrast to liquidity demanders, liquidity suppliers are unable to observe the private signal; for this reason, they can learn about D only by observing the price of the risky asset. Conditional on their information set $\mathcal{F}_s$, the payoff D is normal with mean and variance

$$\mathrm{E}[D|\mathcal{F}_s] = \bar{D} + \beta_P(s - \bar{D} - Cz) \quad \text{and} \quad \mathrm{Var}[D|\mathcal{F}_s] = \frac{1}{h+q}, \tag{2.6}$$

respectively; here $\beta_P = \frac{q}{h+q}$ and $q = \left(\frac{1}{m} + C^2\frac{1}{n}\right)^{-1}$. Similarly, liquidity suppliers maximize their expected utility and submit a demand schedule for the risky asset as follows:

$$X_s = \frac{\mathrm{E}[D|\mathcal{F}_s] - P_1}{\alpha \,\mathrm{Var}[D|\mathcal{F}_s]}. \tag{2.7}$$

After both groups of investors submit their demand schedules, the equilibrium price clears the

[17]Given that the price P_1 is a coarser indicator than the signal s, no additional information is conveyed by the former than the latter. Furthermore, the liquidity shock z is independent of D and so is not used to compute the conditional expectation and variance of D. It follows that, in the setting without options, we have $\mathrm{E}[D|\mathcal{F}_d] = \mathrm{E}[D|s]$, $\mathrm{Var}[D|\mathcal{F}_d] = \mathrm{Var}[D|s]$, $\mathrm{E}[D|\mathcal{F}_s] = \mathrm{E}[D|P_1]$, and $\mathrm{Var}[D|\mathcal{F}_s] = \mathrm{Var}[D|P_1]$.

[18]Since investors are identical at date 0, it follows that the wealth of a liquidity demander and of a liquidity supplier are the same at date 1: $W_1 = W_{d1} = W_{s1} = W_0 + X_0(P_1 - P_0)$. The wealth of a liquidity supplier at date 2 is $W_{s2} = W_1 + X_s(D - P_1)$, which can be derived from W_{d2} by setting $z = 0$.

market and thereby equates investors' aggregate demands and the asset supply $\bar{X}$:

$$\pi X_d + (1-\pi)X_s = \bar{X}. \tag{2.8}$$

The equilibrium price of the risky asset is affine in both the private signal and the liquidity shock, per (2.3), and the coefficients A, B, C can be written as

$$A = \bar{D} - \frac{\alpha}{h + \pi m + (1-\pi)q}\bar{X}, \quad B = \frac{\pi m + (1-\pi)q}{h + \pi m + (1-\pi)q}, \quad C = \frac{\alpha}{m}. \tag{2.9}$$

Substituting the agents' demands into the exponential utility function yields their expected utilities at date 1. The expected utility of a liquidity supplier can be calculated as

$$-\exp\left\{-\alpha\left[W_0 + X_0(P_1 - P_0) + \frac{[\bar{D} + \beta_P(s - \bar{D} - Cz) - P_1]^2}{2\alpha \operatorname{Var}[D|\mathcal{F}_s]}\right]\right\}. \tag{2.10}$$

Similarly, one can calculate the expected utility of a liquidity demander as

$$-\exp\left\{-\alpha\left[W_0 + X_0(P_1 - P_0) + z(P_1 - \bar{D}) + \frac{[\bar{D} + \beta_s(s - \bar{D}) - P_1]^2}{2\alpha \operatorname{Var}[D|\mathcal{F}_d]}\right]\right\}. \tag{2.11}$$

The expected utility at date 1 is affected not only by the private signal s but also by the liquidity shock z—given that the price P_1 depends on both s and z. I denote by U_s and U_d the expected utilities of (respectively) liquidity suppliers and demanders at the interim date $\frac{1}{2}$, which are the expectations of (2.10) and (2.11) over (s, z). These interim utilities can be used to derive the identical investor's expected utility at date 0 as the weighted average of suppliers' and demanders' utilities:

$$U \equiv \pi U_d + (1-\pi)U_s. \tag{2.12}$$

At date 0, the identical investor chooses X_0 to maximize utility. Then the ex ante price P_0 clears the market and we have $X_0 = \bar{X}$ in equilibrium. As shown in Vayanos and Wang (2012a,b), the following linear equilibrium price exists at date 0:

$$P_0 = \bar{D} - \alpha\frac{1}{h}\bar{X} - \frac{\pi M}{1 - \pi + \pi M}\Delta_1\bar{X}, \tag{2.13}$$

where

$$\Delta_0 = \frac{(B-\beta_P)^2\left(\frac{1}{h}+\frac{1}{q}\right)}{\pi^2\,\mathrm{Var}[D|\mathcal{F}_s]},$$

$$\Delta_1 = \frac{\alpha^3 B \frac{1}{nh}\left(\frac{1}{h}+\frac{1}{m}\right)}{1+\Delta_0(1-\pi)^2-\alpha^2\frac{1}{nh}},$$

$$\Delta_2 = \frac{\alpha^3\frac{1}{nh^2}\left[1+\frac{(B-\beta_s)^2(\frac{1}{h}+\frac{1}{m})}{\mathrm{Var}[D|\mathcal{F}_d]}\right]}{1+\Delta_0(1-\pi)^2-\alpha^2\frac{1}{nh}},$$

and

$$M = \exp\left(\frac{1}{2}\alpha\Delta_2\bar{X}^2\right)\sqrt{\frac{1+\pi^2\Delta_0}{1+\Delta_0(1-\pi)^2-\alpha^2\frac{1}{nh}}}. \tag{2.14}$$

The ex ante price P_0 consists of three terms: the expected payoff $\bar{D}$; the risk premium, which compensates investors for the risk they bear; and the illiquidity discount. That discount is itself the product of two other terms. The first of these, $\frac{\pi M}{1-\pi+\pi M}$, can be interpreted as the risk-neutral probability of being a liquidity demander; π is the true probability and M is the ratio of marginal utilities of demanders to those of suppliers. The illiquidity discount's second term, $\Delta_1\bar{X}$, is the discount required if one is certain to be a liquidity demander.

To examine market liquidity, I consider two widely used empirical measures of illiquidity. The first one is based on the price impact of trades by liquidity demanders. More specifically, it is the coefficient derived by regressing the price change (between date 0 and date 1) on the signed volume of liquidity demanders at date 1:

$$\lambda \equiv \frac{\mathrm{Cov}[P_1-P_0,\pi(X_d-\bar{X})]}{\mathrm{Var}[\pi(X_d-\bar{X})]}. \tag{2.15}$$

A large λ indicates that trades have a strong price impact, which implies that the market is illiquid. The second measure is based on the autocovariance of price changes between two periods:

$$\gamma \equiv -\mathrm{Cov}[P_2-P_1, P_1-P_0], \tag{2.16}$$

which is referred to as price reversal. The price reversal measure γ captures the price deviation from fundamental value that a liquidity supplier requires to absorb a liquidity shock. When γ is high, trades generate substantial price deviation and the market is illiquid. In the case of asymmetric information *without* options, these two illiquidity measures are calculated as follows:

$$\lambda = \frac{\alpha \operatorname{Var}[D|\mathcal{F}_s]}{(1-\pi)(1-\frac{\beta_P}{B})}; \tag{2.17}$$

$$\gamma = B(B-\beta_P)\left(\frac{1}{h}+\frac{1}{q}\right). \tag{2.18}$$

The foregoing results on the risky asset's price, the price impact λ, and the price reversal γ follow Vayanos and Wang (2012a,b). These results will be the benchmark for subsequent analysis for the effects of derivatives. In the next section I explore how the introduction of an options market affects the underlying asset price and two illiquidity measures. Then, in Section 4, I account for costly participation and investigate how derivatives affect the underlying asset in an economy with both asymmetric information and participation costs.

3 Introduction of an Options Market

In this section, an options market is introduced into the economy. I examine the effects of the options market on the prices and liquidity of the underlying asset in the presence of asymmetric information. Of particular interest are the illiquidity discount (as reflected in the ex ante price of the risky asset) and two distinct measures of illiquidity.

3.1 Equilibrium

To model an options market in a static setting, I follow Cao and Ou-Yang (2009) and define this market as a collection of call and put options. At date 0, all agents know that options will be introduced to trade at date 1 and will expire at date 2. Let K denote the strike price. A call option with strike price K pays off $(D-K)^+$, and its price at date 1 is denoted by P_{CK}. Similarly, the payoff structure of a put option is $(K-D)^+$ and its price at date 1 is denoted by P_{PK}. The net supply of each option is zero. In light of put-call parity, the analysis can be simplified (without loss of generality) by considering only call options with positive strike prices and put options with negative strike prices.[19] I assume that the demand of a liquidity demander for call options with strike prices ranging from K to $K+dK$ is $X_{d,CK}$ and that her demand for put options with strike prices ranging from K to $K+dK$ is $X_{d,PK}$. Likewise, the demand of a liquidity supplier for call options with strike prices ranging from K to $K+dK$ is $X_{s,CK}$ and his demand for put options with strike prices ranging from K to $K+dK$ is $X_{s,PK}$. This paper differs from Cao and Ou-Yang (2009) in that I model asymmetric information and

[19]The results are robust to introducing a complete set of calls and puts because call options with negative strike prices can be replicated by a combination of stocks and put options with negative strike prices.

they focus on heterogeneous beliefs. The equilibrium at date 1 with options is closely related to Huang (2015).

Liquidity demanders and suppliers submit their demand schedules conditional on their new information sets after options are introduced. Specifically, the information set of liquidity demanders at date 1 is $\mathcal{F}_d = \{s, z, P_0, P_1, P_{CK}, P_{PK}\}$ whereas that of liquidity suppliers is $\mathcal{F}_s = \{P_0, P_1, P_{CK}, P_{PK}\}$. If there is an options market, then the wealth of liquidity demanders at date 2 is given by

$$W_{d2} = W_1 + X_d(D - P_1) + z(D - \bar{D}) + \int_0^{+\infty} X_{d,CK}[(D-K)^+ - P_{CK}]\, dK + \int_{-\infty}^{0} X_{d,PK}[(K-D)^+ - P_{PK}]\, dK \tag{3.1}$$

and the wealth of liquidity suppliers is given by

$$W_{s2} = W_1 + X_s(D - P_1) + \int_0^{+\infty} X_{s,CK}[(D-K)^+ - P_{CK}]\, dK + \int_{-\infty}^{0} X_{s,PK}[(K-D)^+ - P_{PK}]\, dK. \tag{3.2}$$

If options are redundant and are not traded by investors, then the wealth calculated by (3.1) and (3.2) is the same as that in the absence of options. Once an options market is introduced, there is a partially revealing rational expectations equilibrium—in the prices P_1, P_{CK}, and P_{PK} of different assets and the corresponding demands of each agent type at date 1—as follows.

Proposition 3.1. *At date 1, there exists one equilibrium. The price P_1 of the risky asset is given by*

$$P_1 = A + B(s - \bar{D} - Cz), \tag{3.3}$$

where

$$A = \bar{D} - \frac{\alpha}{h + \pi m + (1-\pi)q}\bar{X},$$
$$B = \frac{\pi m + (1-\pi)q}{h + \pi m + (1-\pi)q},$$
$$C = \frac{\alpha}{m}.$$

The prices P_{CK} and P_{PK} of (respectively) call and put options are given by

$$P_{CK} = (P_1 - K)\mathcal{N}\big(\sqrt{G}(P_1 - K)\big) + \frac{1}{\sqrt{2\pi G}} \exp\left(- \frac{G(P_1 - K)^2}{2}\right) \quad \textit{for } K \geq 0, \tag{3.4}$$

$$P_{PK} = (K - P_1)\mathcal{N}\big(\sqrt{G}(K - P_1)\big) + \frac{1}{\sqrt{2\pi G}} \exp\left(- \frac{G(K - P_1)^2}{2}\right) \quad \textit{for } K < 0, \tag{3.5}$$

where $\mathcal{N}(\cdot)$ is the standard normal cumulative distribution function. The liquidity demander's demands for the risky asset and the corresponding options are

$$X_d = \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\alpha \operatorname{Var}[D|\mathcal{F}_d]} - \frac{G - G_d}{\alpha} P_1 - z, \tag{3.6}$$

$$X_{d,CK} = \frac{G - G_d}{\alpha}, \tag{3.7}$$

$$X_{d,PK} = \frac{G - G_d}{\alpha}. \tag{3.8}$$

The liquidity supplier's demands for the risky asset and the corresponding options are

$$X_s = \frac{\mathrm{E}[D|\mathcal{F}_s] - P_1}{\alpha \operatorname{Var}[D|\mathcal{F}_s]} - \frac{G - G_s}{\alpha} P_1, \tag{3.9}$$

$$X_{s,CK} = \frac{G - G_s}{\alpha}, \tag{3.10}$$

$$X_{s,PK} = \frac{G - G_s}{\alpha}. \tag{3.11}$$

In (3.6)–(3.11), $G = h + \pi m + (1-\pi)q$, $G_d = h + m$, $G_s = h + q$, and $\frac{1}{q} = \frac{1}{m} + C^2 \frac{1}{n}$. The terms G_d and G_s represent the conditional precision of D for liquidity demanders and suppliers, and G denotes the average precision for all investors.

There are several interesting features of the equilibrium at date 1. First, options are not redundant securities and are traded by investors. In equilibrium, liquidity demanders take short positions in the options whereas liquidity suppliers take long positions. Given these payoff structures, options are able to provide hedging benefits for the second moment of the risky asset's payoff D. In this sense, options can act as a substitute for information in reducing risk. Since liquidity demanders have more precise information, they always have less incentive than suppliers to hedge against the second moment. Because options are in zero net supply, liquidity demanders sell the options to earn profit while liquidity suppliers take long positions to hedge against the second moment. Second, introducing options into an economy with asymmetric information has no direct effect on the underlying asset's equilibrium price at date 1, which accords with the result reported in Brenna and Cao (1996). Third, option prices are a function of the risky asset's price but carry no additional information; hence the option prices are considered to be "informationally redundant" (Chabakauri, Yuan, and Zachariadis, 2014).

Lemma 3.1. *At interim date $t = \frac{1}{2}$, if options are available then the utilities of liquidity demanders are given by*

$$U_d = \frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{G/G_d}} \mathrm{E}\Big\{ - \exp\Big[-\alpha\Big(W_0 + X_0(P_1 - P_0) + z(P_1 - \bar{D}) + \frac{[\bar{D} + \beta_s(s - \bar{D}) - P_1]^2}{2\alpha \operatorname{Var}[D|\mathcal{F}_d]} \Big)\Big]\Big\} \tag{3.12}$$

and the utilities of liquidity suppliers are given by

$$U_s = \frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{G/G_s}} \mathrm{E}\Big\{ - \exp\Big[-\alpha\Big(W_0 + X_0(P_1 - P_0) + \frac{[\bar{D} + \beta_P(s - \bar{D} + \frac{\alpha}{m}z) - P_1]^2}{2\alpha \operatorname{Var}[D|\mathcal{F}_s]} \Big)\Big]\Big\}. \tag{3.13}$$

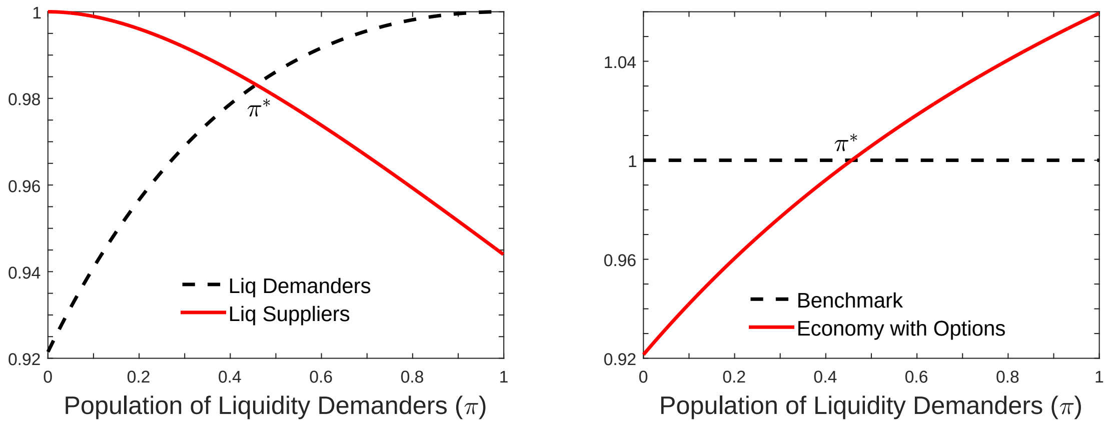
For $\pi \in (0,1)$ we have that both $\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{G/G_d}} < 1$ and $\frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{G/G_s}} < 1$, where E is the expectation over (s,z).

After an options market is introduced, the interim utilities of liquidity demanders and suppliers are the product of two terms: one reflecting the impact of introducing options and one capturing the interim utilities of agents *before* options are introduced (see Appendix A). The left panel of Figure 1 illustrates that, for all agents, the additional term induced by options is less than 1. Owing to the negative exponential utility, a less negative number indicates greater utility.[20] Hence introducing options improves the utilities of both groups of agents thanks to the improved risk sharing that options allow. When equipped with options, uninformed liquidity suppliers are more willing to accommodate the trades of informed demanders. More importantly, the effects of these financial derivatives on the interim utilities of the two groups of agents are not the same: the value of the extra term—which appears when options are incorporated into the model—depends on the conditional precision of the payoff D, which differs between liquidity demanders and liquidity suppliers. Lemma 3.2 formally characterizes this distinction.

Lemma 3.2. *Let $\pi^* \in (0,1)$ be as defined in the Appendix. Then the following statements hold:*

(1) *if $0 < \pi < \pi^*$, then the welfare gain induced by options is greater for liquidity demanders than for liquidity suppliers;*

[20]The parameters for all the figures are $h = 1$, $m = 1$, $n = 1$, and $\alpha = 0.7$.



(a) Welfare gains

(b) Relative welfare gains

Figure 1: Welfare gains from options. The left panel plots the welfare gains for liquidity demanders (the first term in (3.12)) and liquidity suppliers (the first term in (3.13)); the right panel plots the ratio—demanders/suppliers—of welfare gains. The solid (resp. dashed) line marks the ratio of welfare gains with (resp. without) options.

(2) *if* $\pi^* < \pi < 1$, *then the welfare gain induced by options is less for liquidity demanders than for liquidity suppliers;*

(3) *if* $\pi = \pi^*$, *then the welfare gain induced by options for liquidity demanders is the same as that for liquidity suppliers.*

According to this lemma, the welfare gain due to options is much greater for liquidity demanders than for liquidity suppliers when the market consists predominantly of the latter. It is intuitive that if the population of liquidity suppliers is large then the long side of options is higher than the short side; this, in turn, drives up option price and increases the profits of informed liquidity demanders who sell options. Hence the welfare gain for liquidity demanders is greater than that for liquidity suppliers. When the market is dominated by liquidity demanders, these results are reversed. That is, the relative benefit from more risk-sharing opportunities is determined by the competition within each group. The right panel of Figure 1 shows this pattern clearly. Options benefit both types of agents—but not to the same extent, which affects agents' trading incentives at date 0.

Proposition 3.2. *At date 0, the equilibrium price of the risky asset is given by*

$$P_0 = \bar{D} - \alpha \frac{1}{h} \bar{X} - \frac{\pi M^O}{1 - \pi + \pi M^O} \Delta_1 \bar{X}, \tag{3.14}$$

where

$$\Delta_0 = \frac{(B-\beta_P)^2\left(\frac{1}{h}+\frac{1}{q}\right)}{\pi^2 \operatorname{Var}[D|\mathcal{F}_s]}, \tag{3.15}$$

$$\Delta_1 = \frac{\alpha^3 B \frac{1}{nh}\left(\frac{1}{h}+\frac{1}{m}\right)}{1+\Delta_0(1-\pi)^2-\alpha^2\frac{1}{nh}}, \tag{3.16}$$

$$\Delta_2 = \frac{\alpha^3 \frac{1}{nh^2}\left[1+\frac{(B-\beta_s)^2\left(\frac{1}{h}+\frac{1}{m}\right)}{\operatorname{Var}[D|\mathcal{F}_d]}\right]}{1+\Delta_0(1-\pi)^2-\alpha^2\frac{1}{nh}}, \tag{3.17}$$

and

$$M^O = \exp\left(\frac{G_s-G_d}{2G}\right)\sqrt{\frac{G_d}{G_s}}\,\exp\left(\frac{1}{2}\alpha\Delta_2\bar{X}^2\right)\sqrt{\frac{1+\pi^2\Delta_0}{1+\Delta_0(1-\pi)^2-\alpha^2\frac{1}{nh}}}. \tag{3.18}$$

This proposition states that there are also three terms in the ex ante price P_0 when options are available. Introducing options affects only the *ratio* of marginal utilities (liquidity demanders to liquidity suppliers)—that is, M^O in the third term of P_0. Although introducing derivatives improves agents' welfare and the extent of risk sharing between them, it need not increase their willingness to hold the risky asset at date 0 and thereby raise the price P_0. Intuitively, agents are unsure at date 0 about whether they will become suppliers or demanders. As a result, agents are naturally reluctant to buy the risky asset at date 0 because with probability π they will receive a liquidity shock that could increase their risk exposure. The uncertainty about this liquidity shock is costly to risk-averse agents and hence reduces their willingness to hold the asset at date 0, which explains the illiquidity discount reflected in P_0. Asymmetric information hampers risk sharing and makes liquidity demanders less able to hedge their liquidity risk, which increases the illiquidity discount component of P_0.

Options can improve the welfare of all agents, but this does not ameliorate the aversion to holding the underlying asset at date 0 that results from the two agent groups enjoying different levels of welfare gains (see Figure 1). Next I shall explore in detail how introducing an options market affects the illiquidity discount in P_0 as well as the illiquidity measures defined by (2.15) and (2.16).

3.2 Options and Illiquidity

We have seen that introducing options affects liquidity demanders and suppliers in different ways. By Lemma 3.2, the population of liquidity demanders/suppliers determines the relative benefits for the two groups of agents and is a key component of the illiquidity discount component of P_0. Hence the population π plays an important role in the ex ante price P_0. Proposition 3.3 summarizes the results.

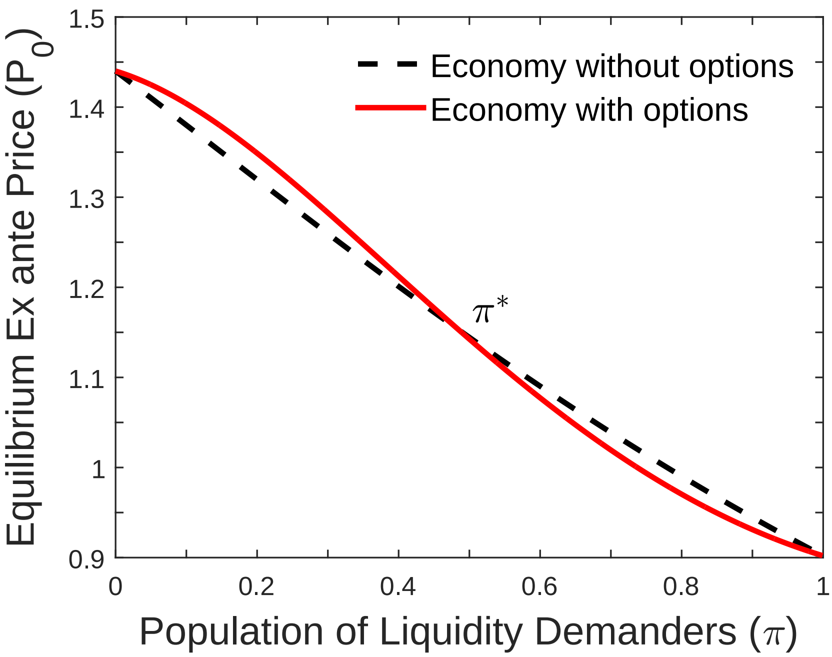


Figure 2: Effects of options on the ex ante price of the risky asset. The solid (resp. dashed) line plots the equilibrium ex ante price with (resp. without) options.

Proposition 3.3. *Let π^* be as defined in the Appendix. Then the following statements hold:*

(1) *if $0 < \pi < \pi^*$, then P_0 is higher in the presence of options than in the absence of options;*

(2) *if $\pi^* < \pi < 1$, then P_0 is lower in the presence of options than in the absence of options;*

(3) *if $\pi = \pi^*$, then P_0 is the same in the presence as in the absence of options.*

This proposition shows that introducing an options market has non-monotonic effects—on the ex ante price P_0—that vary with the likelihood of being a liquidity demander. Specifically: when the market consists mainly of liquidity suppliers (i.e., when liquidity provision is sufficient), the benefits of options are greater for liquidity demanders than for liquidity suppliers. At date 0, the identical investor can anticipate the effects of options and so is less worried about liquidity shock. As a result, they are now more willing to hold the risky asset at date 0; that change leads to a higher ex ante price P_0 and a lower expected return. This mechanism is clearly evident in Figure 2. In the case of a population dominated by liquidity suppliers, an options market benefits liquidity demanders more than it does suppliers. In this case, agents are induced to take larger positions in the risky asset, which raises the price at date 0. The same line of reasoning establishes that, when the population is dominated by liquidity demanders, introducing options results in a lower price P_0. Furthermore, P_0 is a decreasing function

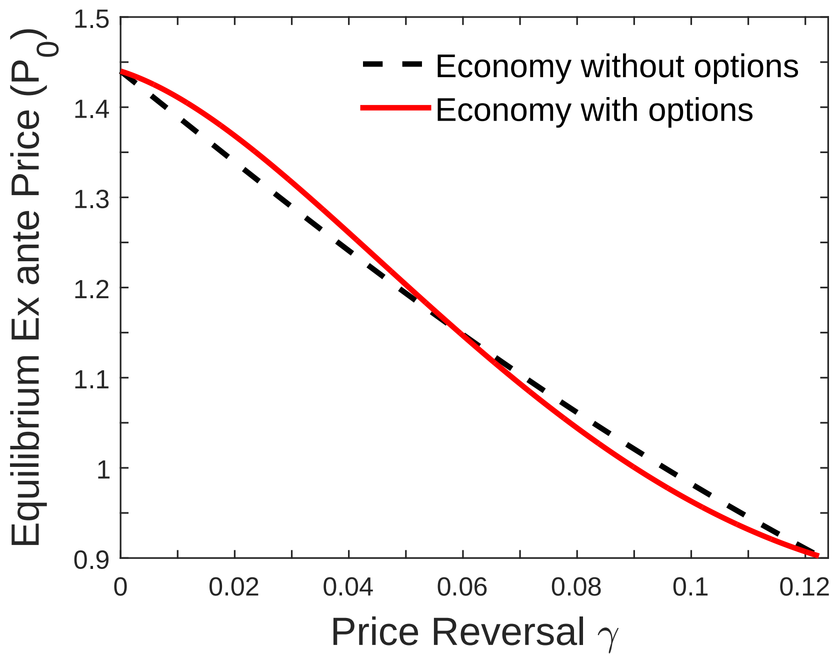


Figure 3: Price reversal γ versus equilibrium ex ante price as a function of whether options are (solid line) or are not (dashed line) available; here the population π drives the cross-sectional variation in γ, and P_0.

of π because a lower probability of receiving the liquidity shock translates, at date 0, into both a lower discount and a higher price.

Before options are introduced to trade, two illiquidity measures are affected by the population π of informed liquidity demanders.[21] The price reversal measure γ defined in (2.16) captures the price deviation from a fundamental value that liquidity suppliers require to absorb the liquidity shock. A large population of liquidity demanders leads to a large price deviation, which implies that γ is an increasing function of π. As illustrated in Proposition 3.3, the effect of an options market depends on π; hence I can use π to establish a link between illiquidity and asset price as a function of whether options are available. I find that the price of relatively more liquid (low-γ) stocks rises in response to the introduction of options; in contrast, the price of relatively less liquid (high-γ) stocks declines. This implication is clearly illustrated in Figure 3. If stocks are sorted in terms of price reversal γ then I find, after the corresponding stock options are listed, that more liquid stocks generally have a higher price and a lower expected return; conversely, less liquid stocks tend to have a lower price and a higher expected return.

The results could also be interpreted in other ways. For example, if liquidity provision in

[21]The price impact measure λ is not a monotonic function of π because it is affected by the extent of risk sharing between the groups of agents. For that reason, in this section the focus is on the price reversal measure γ.

the market is scarce (which it usually is during “bad” states) then introducing new assets, such as derivatives, increases the underlying asset’s expected return. In contrast, introducing derivatives lowers that asset’s expected return when there is sufficient liquidity in the market. As argued by Vayanos and Wang (2012a,b), the liquidity shock could be a consequence of institutional frictions. One can therefore view liquidity demanders as institutional investors who usually have more precise information yet are subject to some frictions (as might result from a change in fund flows). At the same time, liquidity suppliers can be interpreted as designated market makers who supply liquidity.[22] Introducing an options market can increase expected returns of the underlying stocks when agents include a high proportion of institutional investors (e.g., large firms and firms with extensive coverage by analysts). Conversely, introducing an options market can reduce expected returns of the underlying stocks when agents include a low proportion of institutional investors (e.g., small firms and firms with low analyst coverage). These dynamics yield new implications—which can be tested cross-sectionally—regarding how options affect the underlying assets.

These results illustrate the relationship between illiquidity measures and asset prices when cross-sectional variation in the illiquidity measures (λ and γ) and the illiquidity discount in P_0 is driven by the population π. One can likewise examine this relationship when the cross-sectional variation is driven instead by the precision of the private signal and/or of the liquidity shock (i.e., the parameters m and n). A higher m, the precision of ϵ’s distribution in (2.2), increases both the illiquidity discount and λ but decreases γ. The intuitive explanation is that, when the private signal is more precise, the consequent adverse selection becomes more severe and thus increases the illiquidity discount and the price impact λ. In contrast, the effects of the liquidity shock captured by γ are not that important when the private signal is precise. Hence an increase in m reduces the price reversal γ. Combining the effects of m on P_0, on λ, and on γ yields that the illiquidity measure λ is negatively correlated with the price P_0 even as the illiquidity measure γ is positively correlated with P_0 (this is clearly shown by the dashed lines in Figure 4).

After an options market is introduced, Proposition 3.3 establishes the existence of a threshold π^* that determines the sign of the effects generated by options. When $\pi = \pi^*$, the welfare gains for liquidity demanders and suppliers are the same. The threshold π^* increases with the parameter m in that liquidity demanders who observe a more accurate private signal can extract more profits thanks to this information advantage, which compensates for the cost of more intense competition due to more demanders. As the parameter m increases, a certain population of liquidity demanders close to π^* (say, $\pi = 0.49$) is first larger and then smaller

[22] It is worth remarking that the option positions taken by liquidity demanders and suppliers are in line with results reported in Garleanu, Pedersen, and Poteshman (2009): market makers take long positions in individual stock options while end users take short positions.

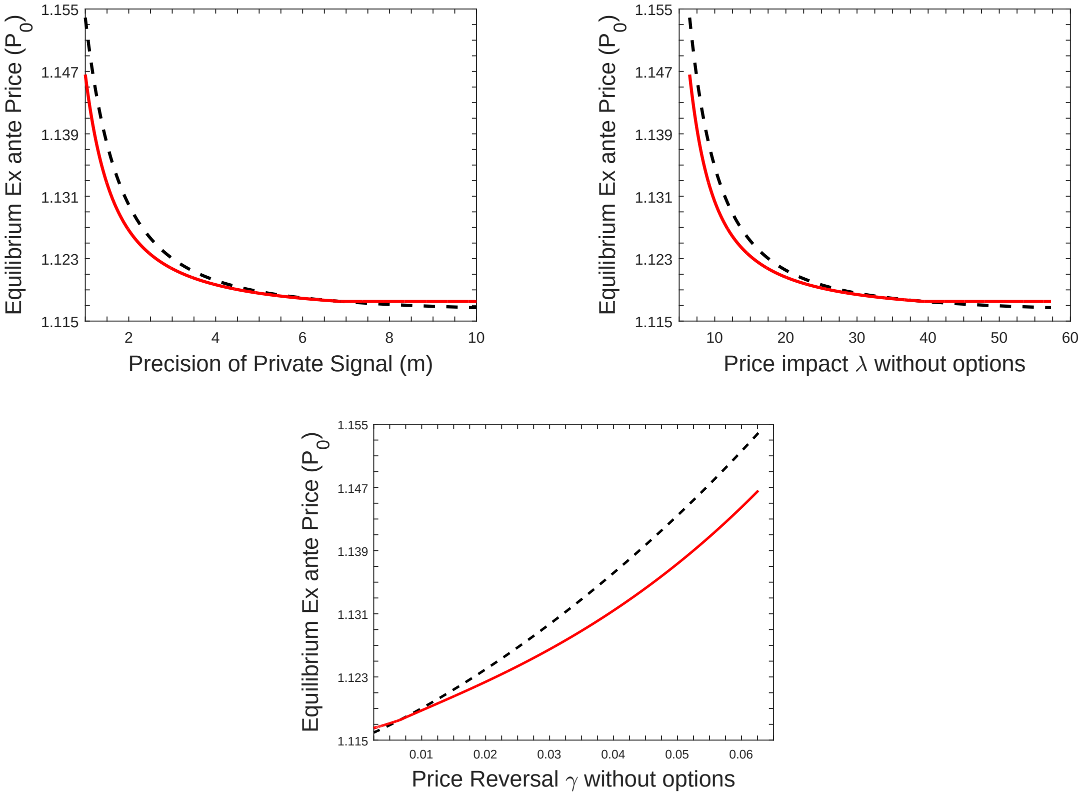


Figure 4: Parameters m, λ, and γ (the latter two are illiquidity measures) versus equilibrium ex ante price as a function of whether options are (solid line) or are not (dashed line) available; here the precision parameter m (for the private signal) drives the cross-sectional variation in λ, γ, and P_0 ($\pi = 0.49$).

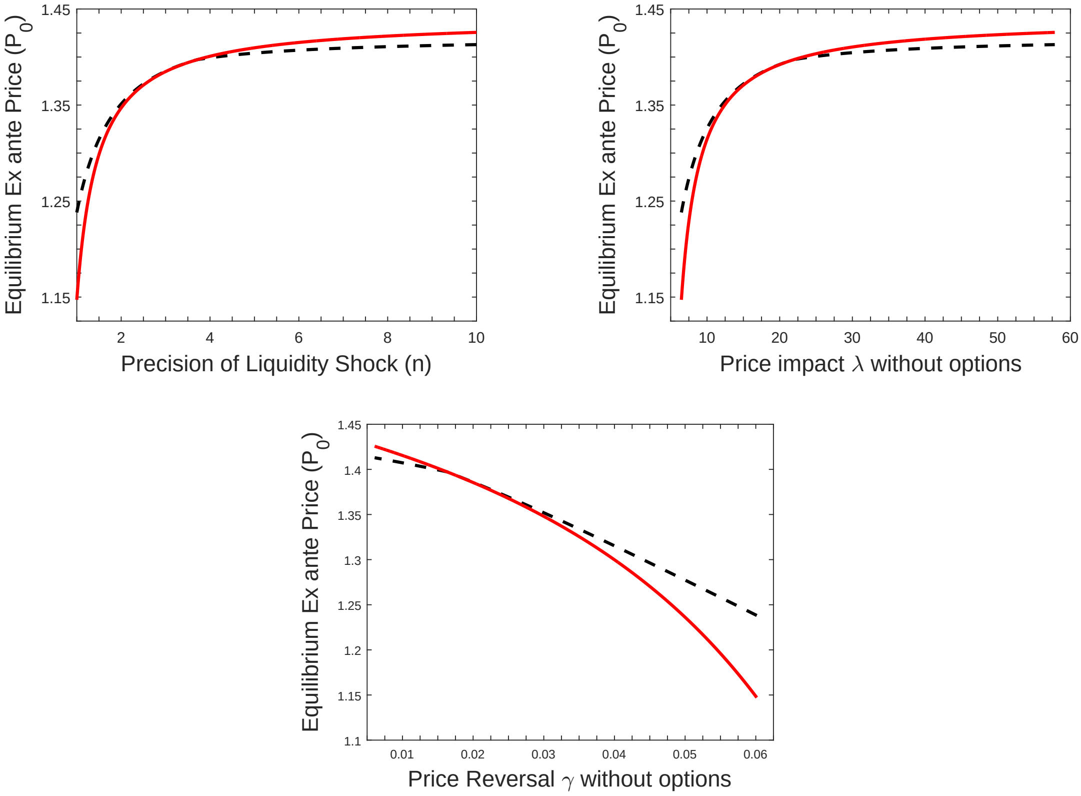


Figure 5: Parameters n, λ, and γ (the latter two are illiquidity measures) versus equilibrium ex ante price as a function of whether options are (solid line) or are not (dashed line) available; here the precision parameter n (for the liquidity shock) drives the cross-sectional variation in λ, γ, and P_0 ($\pi = 0.49$).

than this threshold π^*. Consequently, one can observe a negative (resp. positive) effect of options on the price of the underlying asset when the private signal is less (resp. more) accurate. This pattern can be seen clearly in the upper left panel of Figure 4. Because λ is an increasing function of m, we observe a similar pattern in the upper right panel of that figure. The results are reversed in the lower panel because γ decreases with m. The upper right panel of Figure 4 shows that, if illiquidity is measured by λ and if the cross-sectional variation in λ is driven by m, then introducing options lowers the prices of more liquid stocks but raises the price of less liquid stocks. Yet if illiquidity of stocks is measured by γ then the opposite result obtains, as seen in the lower panel of Figure 4.

I also examine how the underlying asset price is affected by derivatives if the cross-sectional variations in asset-specific characteristics all stem from the liquidity shock's precision n, which is simply the *reciprocal* of the liquidity shock's magnitude.[23] In contrast to the effects of m, an increase in n reduces both the illiquidity discount and γ because the shock's effect is attenuated as its magnitude becomes smaller. Consider the extreme case where n is infinite. At date 0, agents know that the liquidity shock's size is zero and so they do not expect the price P_0 to reflect an illiquidity discount. The price reversal γ, which captures the effect of a liquidity shock, is likewise attenuated when that shock is of relatively low magnitude. The price impact λ increases with n because a smaller liquidity shock renders price more informative and amplifies the learning effect embedded in λ. After combining the effects of n on P_0, on λ, and on γ, one obtains that the illiquidity measure λ (resp. γ) is positively (resp. negatively) correlated with the price P_0. This relationship between illiquidity and asset price, which is plotted in Figure 5, is totally opposite to the one in Figure 4.

Just as for the precision m, an increase in the precision parameter n increases π^* as well. One can therefore observe that options have a negative (resp. positive) effect on P_0 when the liquidity shock is large (resp. small); these effects are displayed in the upper left panel of Figure 5. The upper right and lower panels of this figure show that, after an options market is introduced, a liquid stock has a lower price if illiquidity is measured by λ but a higher one if illiquidity is measured by γ; these results are similar to those plotted in the corresponding two panels of Figure 4.

There is a large body of empirical work that investigates the link between illiquidity and expected asset returns. Among these studies, a basic premise is that a less liquid asset can offer a higher expected return; in other words, there is a positive relationship between illiquidity and expected returns. Figures 4 and 5 demonstrate that the empirical relationship between illiquidity and asset returns depends on the source of cross-sectional variation and also on how illiquidity is measured. If cross-sectional variation is driven by m, for instance, then the

[23]Because its mean is zero, the liquidity shock's variance (i.e., $1/n$) measures its magnitude.

relationship between illiquidity and expected returns is positive when illiquidity is measured by λ but negative when measured by γ. Hence it is critical to identify the factors that drive cross-sectional variation.

Proposition 3.4. *If options are available, then the price impact measure is*

$$\lambda = \frac{\alpha \operatorname{Var}[D|\mathcal{F}_s]}{(1-\pi)\big[\pi(m-q)\operatorname{Var}[D|\mathcal{F}_s] + 1 - \frac{\beta_P}{B}\big]} \tag{3.19}$$

and the price reversal measure is

$$\gamma = B(B-\beta_P)\left(\frac{1}{h}+\frac{1}{q}\right). \tag{3.20}$$

Introducing options reduces λ but has no effect on γ.

Compared with the symmetric information case, asymmetric information strengthens the price impact λ because the information is dispersed across agents (Vayanos and Wang, 2012a,b). In particular, uninformed liquidity suppliers learn the signal from the price; the consequent learning effect, which corresponds to the term β_p/B, reduces the magnitude of (3.19)'s denominator and so increases the price impact λ. Yet if options are available, then liquidity suppliers can hedge against the uncertainty of the risky asset's payoff despite not observing the signal. In other words, options act as a substitute for information in reducing risks. This effect is clearly reflected by an increase in the magnitude of (3.19)'s denominator and a decrease in λ. By Proposition 3.1, options carry no additional information beyond—and have no effect on—the price P_1 of the risky asset. The implication is that introducing an options market is unrelated to price reversals of the risky asset. Options reduce the price impact per trade (as captured by λ), yet because their availability increases trade size, the price impact of the *entire* trade (as measured by γ) is unaffected. Therefore, the introduction of derivatives reduces the price impact measure λ but has no effect on the price reversal measure γ.

I have shown that the effect of options on the underlying asset's price depends both on precision parameters and on the illiquidity measure chosen. A similar study could be conducted to examine how the beneficial effect of options on liquidity is distributed across stocks of various liquidity levels. As discussed previously, an increase in either m or n raises λ and lowers γ. When information becomes more asymmetric, the consequent adverse selection is more severe and so the price impact λ is greater. In other words, high-λ stocks are more subject to information asymmetry. In comparison to the setting without options, the additional term (viz., $\pi(m-q)\operatorname{Var}[D|\mathcal{F}_s]$) in the denominator of (3.19) implies that the more asymmetric is information, the greater is the hedging benefit provided by options. Taken together, the liquidity improvement of more liquid (low-λ) stocks is less than that of less liquid (high-λ)

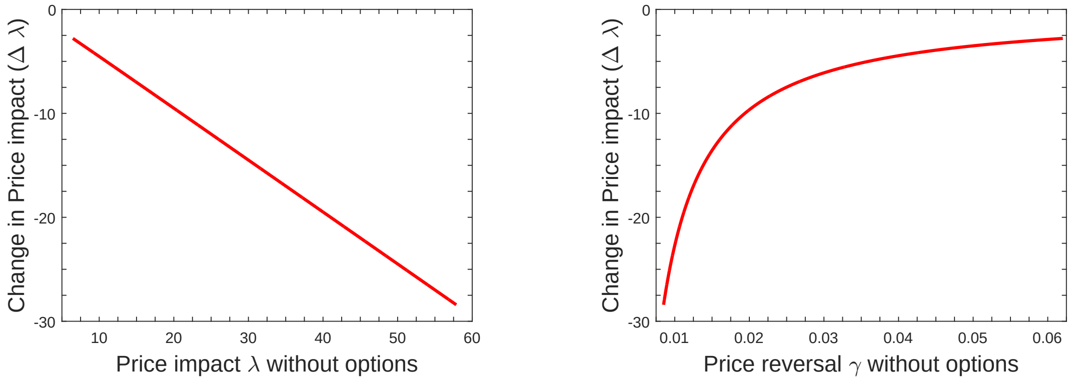


Figure 6: Relationship between illiquidity measures and liquidity improvement.

stocks. However, if illiquidity is measured by γ then the liquidity improvement of more liquid (low-γ) stocks is *greater* than that of less liquid (high-γ) stocks. This converse effect is observed because asymmetric information can reduce γ. Figure 6 plots these results.[24] The findings based on illiquidity as measured by λ are consistent with the empirical evidence reported by Kumar, Sabin, and Shastri (1998). In this sense, the measure λ fits the empirical evidence well and seems to reflect frictions more accurately (see also Vayanos and Wang, 2012a,b).

3.3 Welfare

Proposition 3.5. *As defined in* (2.12)*, an agent's ex ante utility at date 0 is higher in the presence than in the absence of options.*

The identical investor's expected utility at date 0 can be calculated as the weighted average of the liquidity suppliers' and demanders' interim utilities. Introducing options will, of course, increase expected utilities at date 0 because the options improve risk sharing among agents.

4 Analysis with Participation Costs

The results described so far all relate to an economy without participation costs. Recall that, in the absence of such costs, introducing derivatives increases the utility of all investors; yet the welfare gains derived by the two groups of agents are of different magnitudes, leading to distinct effects on the ex ante price at date 0. In this section, I endogenize the participation decisions of agents. More specifically, I assume that all investors must pay a fixed cost f in order to participate in the market. My analysis of participation decisions and equilibrium

[24]Because the precision parameters m and n have similar effects on these two illiquidity measures, this figure plots only the case where cross-sectional variation is driven by m.

at date 1 is closely related to the work of Huang and Wang (2009, 2010) and Vayanos and Wang (2012b), who examine a setting in which investors have hedging (non-informational) motives for trading. In contrast, I study the equilibrium when both non-informational and informational motives are present.

4.1 Equilibrium without an Options Market

Like Vayanos and Wang (2012b), I seek to establish the existence of an equilibrium in which, at date 1, all liquidity demanders participate in the market but only a (positive) proportion μ of liquidity suppliers participate. Such a *partial participation* equilibrium reflects that only liquidity demanders face the risk of liquidity shock and so can benefit more (than liquidity suppliers) from participation; moreover, of most importance for my model is the *relative* measure of participating suppliers and demanders. I assume that the decision to participate is made ex ante and therefore that investors decide whether or not to pay the cost at date $\frac{1}{2}$—in other words, after learning whether or not they will receive the random endowment but before observing the price at date 1. Once the date-1 price is observed, investors can make decisions contingent on that price. This setup implies that the cost f is a fixed transaction cost, not a participation cost.[25]

I denote by $U_{d,P}$ and $U_{d,NP}$ the interim utilities of participating and non-participating liquidity demanders at date $\frac{1}{2}$ and likewise use $U_{s,P}$ and $U_{s,NP}$ with respect to liquidity suppliers. Much as in the previous section without participation costs, I conjecture a linear price function of the risky asset in the following form:

$$P_1 = A + B(s - \bar{D} - Cz). \tag{4.1}$$

Conditional on the information set $\mathcal{F}_d$, liquidity demanders maximize their utilities over the wealth at date 2, $W_{d2} = W_1 + X_d(D - P_1) + z(D - \bar{D}) - f$, and submit a demand schedule for the risky asset:[26]

$$X_d = \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\alpha \operatorname{Var}[D|\mathcal{F}_d]} - z. \tag{4.2}$$

Similarly, the wealth of liquidity suppliers is $W_{s2} = W_1 + X_s(D - P_1) - f$ and their demand schedule is

$$X_s = \frac{\mathrm{E}[D|\mathcal{F}_s] - P_1}{\alpha \operatorname{Var}[D|\mathcal{F}_s]}. \tag{4.3}$$

[25] If the participation cost is introduced at date 0 then it can be viewed as an entry cost. An entry cost similarly reduces the participation rate of agents and thereby reduces the underlying asset's ex ante price. See Huang and Wang (2009) for more details.

[26] The information structure does not differ in the case of participation costs, so agents have the same information sets as before.

After both groups of investors submit their demand schedules, the equilibrium price clears the market and thus equates investors' total demands and the asset supply $\bar{X}$:

$$\pi X_d + (1-\pi)\mu X_s = [\pi + (1-\pi)\mu]\bar{X}. \tag{4.4}$$

The equilibrium price of the risky asset is obtained as follows.

Proposition 4.1. *At date 1, the price P_1 of the risky asset in the presence of participation costs is given by*

$$P_1 = A + B(s - \bar{D} - Cz), \tag{4.5}$$

here

$$A = \bar{D} - \frac{\alpha}{h + \tilde{\pi}m + (1-\tilde{\pi})q}\bar{X}, \quad B = \frac{\tilde{\pi}m + (1-\tilde{\pi})q}{h + \tilde{\pi}m + (1-\tilde{\pi})q}, \quad \text{and} \quad C = \frac{\alpha}{m} \tag{4.6}$$

for $\tilde{\pi} = \frac{\pi}{\pi + (1-\pi)\mu}$.

Comparing (2.9) and (4.6) reveals that the coefficients in the price function with participation costs take the same form as in the benchmark case without participation costs. Note that $\mathrm{E}[P_1]$, the expected equilibrium price at date 1, depends on the measure μ of participating liquidity suppliers. This finding is in contrast to results for the no-information case studied by Vayanos and Wang (2012b). Substituting the demands of liquidity demanders and liquidity suppliers into their utility functions yields their expected utilities. The agents' interim utilities can be computed as in Section 2.2.

To find the equilibrium where all liquidity demanders and a positive fraction of liquidity suppliers participate in the market, I first determine the participation decision by comparing $U_{d,P}$ to $U_{d,NP}$ and $U_{s,P}$ to $U_{s,NP}$. In particular, I discuss several scenarios defined by the participation fraction μ of liquidity suppliers. It is intuitive that all liquidity suppliers choose to enter the market when the participation cost f is low and that none of them will participate when the cost f is high. However, liquidity demanders in this study receive a private signal s in addition to the liquidity shock z. New findings will be compared with results from the no-information case examined in Vayanos and Wang (2012b). I derive the following results concerning the participation decision of liquidity suppliers.

Proposition 4.2. *Suppose that all liquidity demanders participate in the market. For c as defined in the Appendix, there are three cases of the liquidity suppliers' participation decisions as follows.*

Case 1: All liquidity suppliers participate in the market if $f \leq f^1 \equiv \frac{\log\left(1 + \frac{\pi^2}{[h+q+\pi(m-q)]^2}c\right)}{2\alpha}$.

Case 2: A positive fraction μ of liquidity suppliers participate in the market if $f^1 < f < f^2$, *where*

$$\mu = \frac{\pi}{1-\pi}\left\{\frac{m-q}{h+q}\left[\sqrt{\frac{h}{q(e^{2\alpha f}-1)}}-1\right]-1\right\} \quad \textit{and} \quad f^2 \equiv \frac{\log\left(1+\frac{1}{(h+m)^2}c\right)}{2\alpha}.$$

Case 3: No liquidity suppliers participate in the market if $f \geq f^2$.

Given the participation decision of liquidity suppliers, our next step is to find a sufficient condition for all liquidity demanders to enter the market. To ensure their participation it is enough that $U_{d,P}$ be larger than $U_{d,NP}$. The sufficient condition for full participation of liquidity demanders is formalized as follows.

Proposition 4.3. (1) *Suppose that a positive fraction of liquidity suppliers participate in the market. Then a sufficient condition for all liquidity demanders to participate is* $\pi \leq (1-\pi)\mu$.

(2) *An equilibrium where all liquidity demanders and a positive fraction of liquidity suppliers participate in the market exists under the sufficient conditions* $\pi \leq \frac{1}{2}$ *and* $f \leq \hat{f}$, *where*

$$\hat{f} \equiv \frac{\log\left[1+\left(4[h+q+\frac{1}{2}(m-q)]^2\right)^{-1}c\right]}{2\alpha}$$

and c *is as defined in the Appendix.*

This sufficient condition for the existence of a partial participation equilibrium is similar to that given in Vayanos and Wang (2012b). My results differ from theirs in the condition for the fixed participation cost f owing to the presence of asymmetric information in the economy.[27] Proposition 4.3 leads to the following statement as regards the ex ante price.

Proposition 4.4. *At date 0, the equilibrium price of the risky asset is given by*

$$P_0 = \bar{D} - \alpha\frac{1}{h}\bar{X} - \frac{\pi M^P}{1-\pi+\pi M^P}\Delta_1\bar{X}, \tag{4.7}$$

where

$$\Delta_0 = \frac{(B-\beta_P)^2\left(\frac{1}{h}+\frac{1}{q}\right)}{(\tilde{\pi})^2\,\mathrm{Var}[D|\mathcal{F}_s]}, \tag{4.8}$$

$$\Delta_1 = \frac{\alpha^3 B\frac{1}{nh}\left(\frac{1}{h}+\frac{1}{m}\right)}{1+\Delta_0(1-\tilde{\pi})^2-\alpha^2\frac{1}{nh}}, \tag{4.9}$$

$$\Delta_2 = \frac{\alpha^3\frac{1}{nh^2}\left[1+\frac{(B-\beta_s)^2\left(\frac{1}{h}+\frac{1}{m}\right)}{\mathrm{Var}[D|\mathcal{F}_d]}\right]}{1+\Delta_0(1-\tilde{\pi})^2-\alpha^2\frac{1}{nh}}, \tag{4.10}$$

[27] As argued in Vayanos and Wang (2012a,b), there are two equilibria: the one described in the proposition and the one where no investors participate. The latter can be excluded (for details, see Vayanos and Wang, 2012a,b).

and

$$M^P = \exp\left(\frac{1}{2}\alpha\Delta_2\bar{X}^2\right)\sqrt{\frac{1+(\tilde{\pi})^2\Delta_0}{1+\Delta_0(1-\tilde{\pi})^2-\alpha^2\frac{1}{nh}}} \tag{4.11}$$

for $\tilde{\pi} = \frac{\pi}{\pi+(1-\pi)\mu}$.

In contrast to the benchmark case studied in Section 2.2, the new price P_0 is obtained by replacing π with $\tilde{\pi}$ when evaluating $(\Delta_0, \Delta_1, \Delta_2, M)$. After the equilibrium analysis, I turn to investigate how the illiquidity measures and the illiquidity discount are affected by participation costs when the economy is characterized by information asymmetry. The price impact measure and price reversal measure are calculated as follows.

Proposition 4.5. *In the presence of participation costs, the price impact measure is*

$$\lambda = \frac{\alpha[\pi m + (1-\pi)\mu q]}{h(1-\pi)\pi\mu(m-q)} \tag{4.12}$$

and the price reversal measure is

$$\gamma = B(B-\beta_P)\left(\frac{1}{h}+\frac{1}{q}\right), \tag{4.13}$$

where $B = \frac{\tilde{\pi}m+(1-\tilde{\pi})q}{h+\tilde{\pi}m+(1-\tilde{\pi})q}$.

The new illiquidity measures are akin to those in the benchmark case except that now they are affected by the participation fraction μ. It is therefore natural to posit that introducing options affects illiquidity measures through the participation fraction because options enhance agents' utilities and so induce them to participate. Thus it is crucial to examine the effects of participation costs f and of the participation rate μ on illiquidity measures and on the illiquidity discount; these effects can then be compared to those for the case (discussed in Section 4.2) where options are available. The following proposition illustrates these effects.[28]

Proposition 4.6. *A decrease in the fixed cost f leads to an increase in the participation rate μ; hence a reduced f lowers both the price impact λ and the price reversal γ while raising the ex ante price P_0.*

4.2 Equilibrium with an Options Market

In this section I study how the introduction of an options market, in the presence of participation costs, affects the illiquidity discount and the two illiquidity measures. As shown in

[28]Without loss of generality, I consider only the region $f > f^1$—that is, the region in which only some liquidity suppliers participate. If $f \leq f^1$ then all liquidity demanders and suppliers participate, which reduces to the benchmark case examined in Section 2.2.

Section 3, introducing derivatives reduces the price impact measure λ but has no effect on the price reversal measure γ. In the presence of participation costs, however, the introduction of derivatives can affect both liquidity measures through the participation rate μ. The reason is that introducing options attracts more uninformed liquidity suppliers who would otherwise refuse to participate because of the cost f. I formally present the effects of derivatives in the following results.

As in Section 3, I introduce a set of call and put options into the economy. I follow similar procedures to solve for the equilibrium at date 0 and date 1, except that only a proportion $\mu \in (0, 1)$ of liquidity suppliers participate. Proposition 4.7 characterizes the solution.

Proposition 4.7. *At date 1, there exists one equilibrium. The price of the risky asset is given by*

$$P_1 = A + B(s - \bar{D} - Cz), \tag{4.14}$$

where

$$A = \bar{D} - \frac{\alpha}{h + \tilde{\pi}m + (1 - \tilde{\pi})q}\bar{X},$$
$$B = \frac{\tilde{\pi}m + (1 - \tilde{\pi})q}{h + \tilde{\pi}m + (1 - \tilde{\pi})q},$$
$$C = \frac{\alpha}{m}.$$

The prices P_{CK} and P_{PK} of call and put options are given by

$$P_{CK} = (P_1 - K)\mathcal{N}\big(\sqrt{G^P}(P_1 - K)\big) + \frac{1}{\sqrt{2\pi G^P}} \exp\left(- \frac{G^P(P_1 - K)^2}{2}\right) \quad \text{for } K \geq 0,$$
$$P_{PK} = (K - P_1)\mathcal{N}\big(\sqrt{G^P}(K - P_1)\big) + \frac{1}{\sqrt{2\pi G^P}} \exp\left(- \frac{G^P(K - P_1)^2}{2}\right) \quad \text{for } K < 0,$$

where $\mathcal{N}(\cdot)$ is the standard normal cumulative distribution function. The liquidity demander's demands for the risky asset and the corresponding options are

$$X_d = \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\alpha \operatorname{Var}[D|\mathcal{F}_d]} - \frac{G^P - G_d}{\alpha}P_1 - z, \tag{4.15}$$
$$X_{d,CK} = \frac{G^P - G_d}{\alpha}, \tag{4.16}$$
$$X_{d,PK} = \frac{G^P - G_d}{\alpha}; \tag{4.17}$$

the liquidity supplier's demands for the risky asset and the corresponding options are

$$X_s = \frac{\mathrm{E}[D|\mathcal{F}_s] - P_1}{\alpha\,\mathrm{Var}[D|\mathcal{F}_s]} - \frac{G^P - G_s}{\alpha} P_1, \tag{4.18}$$

$$X_{s,CK} = \frac{G^P - G_s}{\alpha}, \tag{4.19}$$

$$X_{s,PK} = \frac{G^P - G_s}{\alpha}. \tag{4.20}$$

Here $G^P = h + \tilde{\pi} m + (1-\tilde{\pi})q$, $G_d = h + m$, *and* $G_s = h + q$. *The term* G^P *represents the average precision of* D *for all investors in the presence of participation costs.*

The equilibrium prices of the underlying asset and the derivatives take a form similar to their presentation in Section 3 except that the proportion π is replaced by $\tilde{\pi}$. Once again, I limit the scope of analysis to the equilibrium where liquidity demanders fully participate and liquidity suppliers partially participate. The new average precision of D for all investors (G^P) is obtained by replacing the exogenous fraction π with the endogenous fraction $\tilde{\pi}$, where the latter is determined by the fixed participation cost f. Therefore, it is first necessary to identify a sufficient condition for the existence of this equilibrium.

Lemma 4.1. *At interim date* $t = \frac{1}{2}$, *the utilities of liquidity demanders in the presence of options are given by*

$$U_d = \frac{\exp\left(\frac{G^P - G_d}{2G^P}\right)}{\sqrt{G^P/G_d}} \mathrm{E}\Bigg\{ \exp\Bigg[-\alpha\Bigg(W_0 + X_0(P_1 - P_0) + z(P_1 - \bar{D}) + \frac{[\bar{D} + \beta_s(s - \bar{D}) - P_1]^2}{2\alpha\,\mathrm{Var}[D|\mathcal{F}_d]} - f \Bigg)\Bigg]\Bigg\} \tag{4.21}$$

and the utilities of liquidity suppliers are given by

$$U_s = \frac{\exp\left(\frac{G^P - G_s}{2G^P}\right)}{\sqrt{G^P/G_s}} \mathrm{E}\Bigg\{ -\exp\Bigg[-\alpha\Bigg(W_0 + X_0(P_1 - P_0) + \frac{[\bar{D} + \beta_P(s - \bar{D} + Cz) - P_1]^2}{2\alpha\,\mathrm{Var}[D|\mathcal{F}_s]} - f \Bigg)\Bigg]\Bigg\}. \tag{4.22}$$

For $\tilde{\pi} \in (0,1)$ *we have that both* $\frac{\exp\left(\frac{G^P - G_d}{2G^P}\right)}{\sqrt{G^P/G_d}} < 1$ *and* $\frac{\exp\left(\frac{G^P - G_s}{2G^P}\right)}{\sqrt{G^P/G_s}} < 1$.

Lemma 4.1 states that the interim utility of any agent is the product of two terms after options are introduced, which is just as in the case without participation costs. The first term reflects the effect due to the presence of options, which compensates for the participation cost f and so induces more agents to participate. Hence the previous sufficient condition—for the equilibrium in which options are not available and liquidity suppliers participate partially—still holds.

Proposition 4.8. *In the presence of an options market, an equilibrium where all liquidity demanders and a positive fraction of liquidity suppliers participate in the market exists under the sufficient conditions* $\pi \leq \frac{1}{2}$ *and* $f \leq \hat{f}^o$, *where*

$$\hat{f}^o \equiv \frac{\log\left[\left(1+\frac{1}{4[h+q+\frac{1}{2}(m-q)]^2}c\right)^{\frac{h+q+\frac{1}{2}(m-q)}{h+q}}\right]+\frac{h+q}{h+q+\frac{1}{2}(m-q)}-1}{2\alpha}$$

and c *is as defined in the Appendix.*

Given this sufficient condition for the equilibrium of partial participation, one can obtain the equilibrium ex ante price at date 0 as follows.

Proposition 4.9. *At date 0, the equilibrium price of the risky asset is given by*

$$P_0 = \bar{D} - \alpha\frac{1}{h}\bar{X} - \frac{\pi M^{PO}}{1-\pi+\pi M^{PO}}\Delta_1\bar{X}, \tag{4.23}$$

where

$$\Delta_0 = \frac{(B-\beta_P)^2\left(\frac{1}{h}+\frac{1}{q}\right)}{(\tilde{\pi})^2\text{Var}[D|\mathcal{F}_s]}, \tag{4.24}$$

$$\Delta_1 = \frac{\alpha^3 B\frac{1}{nh}\left(\frac{1}{h}+\frac{1}{m}\right)}{1+\Delta_0(1-\tilde{\pi})^2-\alpha^2\frac{1}{nh}}, \tag{4.25}$$

$$\Delta_2 = \frac{\alpha^3\frac{1}{nh^2}\left[1+\frac{(B-\beta_s)^2\left(\frac{1}{h}+\frac{1}{m}\right)}{\text{Var}[D|\mathcal{F}_d]}\right]}{1+\Delta_0(1-\tilde{\pi})^2-\alpha^2\frac{1}{nh}}, \tag{4.26}$$

and

$$M^{PO} = \exp\left(\frac{G_s-G_d}{2G^P}\right)\sqrt{\frac{G_d}{G_s}}\exp\left(\frac{1}{2}\alpha\Delta_2\bar{X}^2\right)\sqrt{\frac{1+(\tilde{\pi})^2\Delta_0}{1+\Delta_0(1-\tilde{\pi})^2-\alpha^2\frac{1}{nh}}}. \tag{4.27}$$

Proposition 4.10. *The introduction of options raises the fraction* μ *of participating liquidity suppliers.*

When an options market is introduced into the economy, liquidity suppliers are more inclined to provide liquidity. Thus their participation fraction μ increases as a consequence of the welfare gains created by options. Yet when liquidity suppliers participate only partially, their interim utilities are the same as if they do not participate at all. For a liquidity demander, in contrast, expected utility is enhanced by options owing to her full participation (i.e., even before the options were introduced). As a result of these two effects, the identical investors become more willing to hold the risky asset at date 0, which in turn lowers the illiquidity discount and raises the equilibrium ex ante price P_0.

Proposition 4.11. *The introduction of options increases the ex ante price P_0, but it reduces both the price impact λ and the price reversal γ.*

This proposition indicates that, if liquidity suppliers participate partially (rather than fully) in the market, then introducing derivatives always raises the ex ante price regardless of the liquidity provision $1-\pi$. This result is in line with the finding of Cao (1999), who studies how stock prices can be affected—through the information acquisition channel—by options listing. At this point, a comparative statics analysis can be used to generate predictions about the effect of participation costs on trading volume in the derivatives market. The trading volume of options is calculated as $V^O = \frac{2\pi(1-\pi)\mu}{(\pi+(1-\pi)\mu)^2}(m-q)$ and depends on the participation cost f.

Proposition 4.12. *The trading volume of options exhibits an inverse U shape as a function of the participation cost; thus an increase in the participation cost f raises* (*resp. lowers*) *V^O when f is less* (*resp. greater*) *than f^o, where f^o is as defined in the Appendix.*

When the participation cost f is high, the participation rate μ of liquidity suppliers is low. In this scenario, reducing the participation cost f leads more suppliers to participate. Then these uninformed liquidity suppliers can buy options (which can be thought of as insurance) in order to hedge the uncertainty of the risky asset's payoff D. Hence the trading volume of options increases, and that volume is a decreasing function of the cost f. If instead the participation cost f is low, then the participation rate μ of liquidity suppliers is high; this scenario implies that the competition among suppliers is intense. So even though a decrease in f attracts more suppliers, the total trading volume of options becomes much lower because each supplier wants to hold a reduced volume of options. Because the long side of options comes from uninformed suppliers, the intense competition under a high participation rate μ makes options expensive. Accordingly, the option trading volume for each liquidity supplier—and thus the total option trading volume—declines. If participation is costly, then this result means that reducing the participation cost could accelerate the growth of an options market (Cao and Ou-Yang, 2009). That implication is consistent with the recent empirical literature on options. In addition, my model offers new predictions about the relation between market participation cost and options trading volume.

The options volume is also an increasing function of the information dispersion, which corresponds to the term $m-q$—that is, the difference in information precision between a liquidity demander and a liquidity supplier. Since the uninformed liquidity suppliers cannot perfectly identify the trading motive of liquidity demanders, they face the risk of trading against the private information of informed demanders. As the adverse selection problem arising from this information asymmetry worsens (higher $m - q$), the uninformed liquidity suppliers are more likely to buy the options as insurance to hedge against the uncertainty; hence the trading volume of options increases. A small peak is evident in the option trading volumes during the financial crisis, a deviation that can be explained by my model.

5 Other General Derivative Securities

In previous sections I examined the effect of introducing a set of vanilla options. This section turns to the case of more general derivatives. Specifically, I first analyze what effect the introduction of a generalized straddle has on the ex ante price of the risky asset and on our two liquidity measures; then I check the robustness of this mechanism by examining some concrete examples.

5.1 General Derivatives

I consider some general derivatives as modeled by Cao (1999). Specifically: I assume that a general derivative, whose price is denoted by P_G, pays off $g(|D - P_1|)$; here $g(\cdot)$ is a monotonic function.[29] Following the pattern of previous notation, the demand of liquidity demanders and suppliers for this derivative are denoted by (respectively) $X_{d,G}$ and $X_{s,G}$. At date 2, the wealth of liquidity demanders who are equipped with a *generalized straddle* is given by

$$W_{d2} = W_1 + X_d(D - P_1) + X_{d,G}(g(|D - P_1|) - P_G) + z(D - \bar{D}), \tag{5.1}$$

and the wealth of liquidity suppliers is given by

$$W_{s2} = W_1 + X_s(D - P_1) + X_{s,G}(g(|D - P_1|) - P_G). \tag{5.2}$$

I obtain a partially revealing rational expectations equilibrium that characterizes the effect of introducing the generalized straddle at date 1 (cf. Cao, 1999). This result is stated formally as follows.

Proposition 5.1. *At date 1, there exists one equilibrium. The price of the risky asset is given by*

$$P_1 = \bar{D} - \frac{\alpha}{h + \pi m + (1-\pi)q}\bar{X} + \frac{\pi m + (1-\pi)q}{h + \pi m + (1-\pi)q}\left(s - \bar{D} - \frac{\alpha}{m}z\right). \tag{5.3}$$

[29] Given that $g(\cdot)$ resides in the positive domain, the quadratic derivative is a special case of this general derivative. A more detailed analysis is given in Section 5.2.

Demand for the risky asset is given by

$$X_d = \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\alpha \, \mathrm{Var}[D|\mathcal{F}_d]} - z \tag{5.4}$$

for the liquidity demander and by

$$X_s = \frac{\mathrm{E}[D|\mathcal{F}_s] - P_1}{\alpha \, \mathrm{Var}[D|\mathcal{F}_s]} \tag{5.5}$$

for the liquidity supplier. The liquidity demander's demand for the general derivative satisfies the equality

$$\int_0^{+\infty} (g(u) - P_G) \exp\left[-\frac{1}{2} G_d u^2 - \alpha X_{d,G} \, g(u) \right] du = 0, \tag{5.6}$$

and that of the liquidity supplier satisfies

$$\int_0^{+\infty} (g(u) - P_G) \exp\left[-\frac{1}{2} G_s u^2 - \alpha X_{s,G} \, g(u) \right] du = 0. \tag{5.7}$$

As for the general derivative, the market-clearing condition is

$$\pi X_{d,G} + (1 - \pi) X_{s,G} = 0, \tag{5.8}$$

where $G_d = h + m$ and $G_s = h + q$.

Proposition 5.1 establishes that the introduction of a generalized straddle has no direct influence on the underlying asset's price at date 1, which is the same result as when calls and puts were introduced in Section 3. Unlike those vanilla options, a straddle does not affect the demand of agents for the underlying asset. Hence the price impact measure λ and the price reversal measure γ remain unchanged. As before, the introduction of derivatives changes investors' utilities; the next lemma summarizes the results that concern their interim utilities.

Lemma 5.1. *At interim date $t = \frac{1}{2}$, if a general derivative is available then the utilities of liquidity demanders are given by*

$$U_d^G = U_d \sqrt{\frac{2G_d}{\pi}} \int_0^{+\infty} \exp\left(-\frac{1}{2} G_d u^2 - \alpha X_{d,G} (g(|u|) - P_G) \right) du \tag{5.9}$$

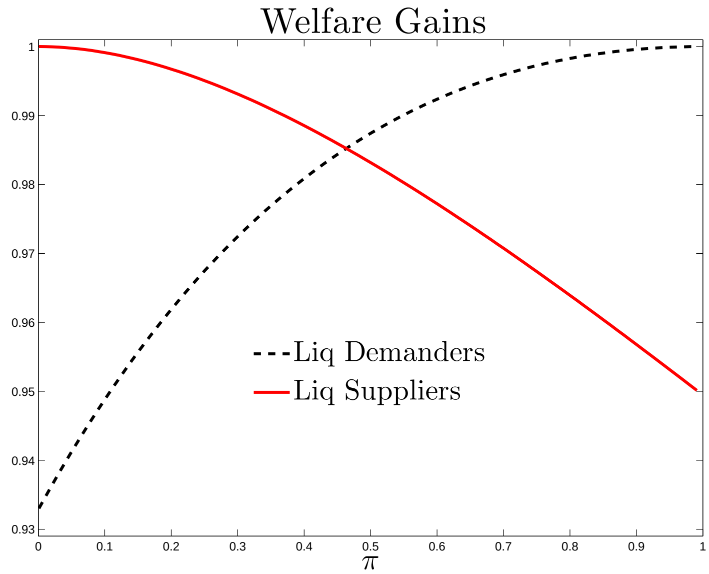


Figure 7: Welfare gains from derivatives; $g(y) = y$.

and those of liquidity suppliers are given by

$$U_s^G = U_s \sqrt{\frac{2G_s}{\pi}} \int_0^{+\infty} \exp\left(-\frac{1}{2} G_s u^2 - \alpha X_{s,G}(g(|u|) - P_G) \right) du; \tag{5.10}$$

here U_i^G and U_i $(i \in \{s,d\})$ are the interim utilities of agents with and without the general derivatives, respectively.

As suggested by Proposition 5.1 and Lemma 5.1, it is difficult to obtain analytical solutions for the asset price, demand for the derivative, and interim utilities. I then turn to numerical studies. Figure 7 illustrates the welfare gains to the two groups of agents when $g(y) = y$. The welfare gains from derivatives are clearly much greater for liquidity demanders than for liquidity suppliers when the latter dominate the market population (i.e., when π is small).[30] This asymmetric effect of the generalized straddle (as seen graphically in the figure) allows me to confirm the ex ante price results derived previously for the case when vanilla options are introduced to trade. The only difference here is that the two liquidity measures, λ and γ, are not changed by introducing the generalized straddle thanks to the unchanged demand for the underlying asset. To develop a clearer picture of the generalized straddle's effects, I shall next investigate a concrete example of generalized straddles that yields closed-form solutions.

[30]Because of the negative exponential utility, a smaller number indicates a larger welfare gain.

5.2 A Quadratic Derivative

Brenna and Cao (1996) analyze the effect of a new quadratic derivative, which can be seen as a special case of the aforementioned generalized straddle. In this section I study the effects of this particular derivative in the presence of a liquidity shock. In particular, I focus on the effects of the quadratic derivative on the illiquidity discount and our two liquidity measures.

In contrast to the case of vanilla options, here I introduce a derivative whose payoff is a quadratic function of the risky asset's payoff D. More specifically, this derivative pays off $(D - P_1)^2$ and its price, prior to expiration, is denoted by P_{G1}. Just as for the vanilla options, this derivative is in zero net supply and expires at date 2. I use $X_{d,G1}$ and $X_{s,G1}$ to denote the demand (of, respectively, liquidity demanders and liquidity suppliers) for this new derivative. If that quadratic derivative is available, then the wealth of liquidity demanders at date 2 is given by

$$W_{d2} = W_1 + X_d(D - P_1) + X_{d,G1}((D - P_1)^2 - P_{G1}) + z(D - \bar{D}) \tag{5.11}$$

and that of liquidity suppliers is given by

$$W_{s2} = W_1 + X_s(D - P_1) + X_{s,G1}((D - P_1)^2 - P_{G1}). \tag{5.12}$$

I have the following partially revealing rational expectations equilibrium regarding the prices P_1 and P_{G1} of different assets and the corresponding demands of the two groups of agents at date 1.

Proposition 5.2. *At date 1, there exists one equilibrium. The underlying risky asset's price is given by*

$$P_1 = \bar{D} - \frac{\alpha}{h + \pi m + (1-\pi)q}\bar{X} + \frac{\pi m + (1-\pi)q}{h + \pi m + (1-\pi)q}\left(s - \bar{D} - \frac{\alpha}{m}z\right), \tag{5.13}$$

and the price of the quadratic derivative is given by

$$P_{G1} = \frac{1}{h + \pi m + (1-\pi)q}. \tag{5.14}$$

The liquidity demander's demands for the risky asset and the corresponding derivative are

$$X_d = \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\alpha \operatorname{Var}[D|\mathcal{F}_d]} - z, \tag{5.15}$$

$$X_{d,G1} = \frac{1}{2\alpha}\left(\frac{1}{P_{G1}} - \frac{1}{\operatorname{Var}[D|\mathcal{F}_d]}\right); \tag{5.16}$$

the liquidity suppler's demands for the risky asset and the corresponding derivative are

$$X_s = \frac{\mathrm{E}[D|\mathcal{F}_s] - P_1}{\alpha \,\mathrm{Var}[D|\mathcal{F}_s]}, \tag{5.17}$$

$$X_{s,G1} = \frac{1}{2\alpha}\left(\frac{1}{P_{G1}} - \frac{1}{\mathrm{Var}[D|\mathcal{F}_s]}\right). \tag{5.18}$$

As in Proposition 3.1, liquidity demanders take short positions in derivatives while liquidity suppliers take long positions. Proposition 5.2 indicates that this result is robust to a derivative with quadratic payoff. However, such a derivative has no effect on demand for the risky asset. In contrast to the nonlinear price function of call and put options illustrated in Proposition 3.1, the quadratic derivative's price is the *reciprocal* of the average precision (over all investors) of the risky asset's payoff D (i.e., $1/G$). This difference simply reflects the different payoff structure of these derivatives and does not require any alterations in my mechanism. With regard to interim utilities, the welfare gains stemming from quadratic derivative are of a different magnitude for the two groups of investors; this finding, too, is in line with the results for vanilla options. The following statement formalizes that claim.

Lemma 5.2. *At interim date $t = \frac{1}{2}$, if a quadratic derivative is available then the interim utilities of liquidity demanders are given by*

$$U_d = \frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{G/G_d}}\mathrm{E}\Bigg\{ - \exp\Bigg[- \alpha\Bigg(W_0 + X_0(P_1 - P_0) + z(P_1 - \bar{D}) + \frac{[\bar{D} + \beta_s(s - \bar{D}) - P_1]^2}{2\alpha\,\mathrm{Var}[D|\mathcal{F}_d]}\Bigg)\Bigg]\Bigg\} \tag{5.19}$$

and the utilities of liquidity suppliers are given by

$$U_s = \frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{G/G_s}}\mathrm{E}\Bigg\{ - \exp\Bigg[- \alpha\Bigg(W_0 + X_0(P_1 - P_0) + \frac{[\bar{D} + \beta_P(s - \bar{D} + \frac{\alpha}{m}z) - P_1]^2}{2\alpha\,\mathrm{Var}[D|\mathcal{F}_s]}\Bigg)\Bigg]\Bigg\}. \tag{5.20}$$

For $\pi \in (0,1)$ we have that both $\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{G/G_d}} < 1$ and $\frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{G/G_s}} < 1$, where E is the expectation over (s, z).

For the two groups of agents, the welfare improvement resulting from a quadratic derivative is the same as that from a set of call and put options (the case analyzed previously). Hence the new quadratic derivative will benefit liquidity demanders and suppliers in the same way that options do and will also yield similar equilibrium prices at date 0.

Proposition 5.3. *At date 0, the equilibrium price of the risky asset is given by*

$$P_0 = \bar{D} - \alpha \frac{1}{h} \bar{X} - \frac{\pi M^G}{1 - \pi + \pi M^G} \Delta_1 \bar{X}, \tag{5.21}$$

where

$$\Delta_0 = \frac{(B - \beta_P)^2 \left(\frac{1}{h} + \frac{1}{q}\right)}{\pi^2 \operatorname{Var}[D|\mathcal{F}_s]}, \tag{5.22}$$

$$\Delta_1 = \frac{\alpha^3 B \frac{1}{nh} \left(\frac{1}{h} + \frac{1}{m}\right)}{1 + \Delta_0 (1-\pi)^2 - \alpha^2 \frac{1}{nh}}, \tag{5.23}$$

$$\Delta_2 = \frac{\alpha^3 \frac{1}{nh^2} \left[1 + \frac{(B-\beta_P)^2 \left(\frac{1}{h} + \frac{1}{m}\right)}{\operatorname{Var}[D|\mathcal{F}_d]}\right]}{1 + \Delta_0 (1-\pi)^2 - \alpha^2 \frac{1}{nh}}, \tag{5.24}$$

and

$$M^G = \exp\left(\frac{G_s - G_d}{2G}\right) \sqrt{\frac{G_d}{G_s}} \exp\left(\frac{1}{2} \alpha \Delta_2 \bar{X}^2\right) \sqrt{\frac{1 + \pi^2 \Delta_0}{1 + \Delta_0 (1-\pi)^2 - \alpha^2 \frac{1}{nh}}}. \tag{5.25}$$

Proposition 5.4. (1) *For π^* as defined in the Appendix: if $0 < \pi < \pi^*$, then P_0 is higher in the presence than in the absence of a quadratic derivative; if $\pi^* < \pi < 1$, then P_0 is lower in the presence than in the absence of a quadratic derivative; and if $\pi = \pi^*$, then P_0 is the same in the presence as in the absence of a quadratic derivative.*

(2) *The introduction of a quadratic derivative alters neither the price impact measure λ nor the price reversal measure γ.*

5.3 One Call Option with a Strike Price K

Suppose that only one call option, with strike price K, is introduced into the economy. Then the liquidity demanders' Euler equation for the risky asset can be written as

$$\int_{K-P_1}^{+\infty} u \exp\left\{ -[\alpha(X_d + z) - G_d(\mathrm{E}[D|\mathcal{F}_d] - P_1)]u - \frac{1}{2} G_d u^2 - \alpha X_{d,G}(u - (K - P_1)) \right\} du$$
$$+ \int_{-\infty}^{K-P_1} u \exp\left\{ -[\alpha(X_d + z) - G_d(\mathrm{E}[D|\mathcal{F}_d] - P_1)]u - \frac{1}{2} G_d u^2 \right\} du = 0,$$

where $u \equiv D - P_1$, $\mathrm{E}[u|\mathcal{F}_d] = \mathrm{E}[D|\mathcal{F}_d] - P_1$, and $\mathrm{Var}(u|\mathcal{F}_d) = \mathrm{Var}(D|\mathcal{F}_d) = G_d^{-1}$. When the call option is at the money (i.e., $K = P_1$), it collapses to the case of a generalized straddle:[31]

$$\int_{-\infty}^{+\infty} u \exp\left\{ - \left[\alpha(X_d + z) - \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\mathrm{Var}(D|\mathcal{F}_d)} + \frac{\alpha}{2} X_{d,G}\right] u - \frac{1}{2} G_d u^2 - \alpha X_{d,G}|u| \right\} du = 0.$$

Therefore, demand for the risky asset is given by

$$X_d = \frac{\mathrm{E}[D|\mathcal{F}_d] - P_1}{\alpha \,\mathrm{Var}(D|\mathcal{F}_d)} - \frac{1}{2} X_{d,G} - z, \tag{5.26}$$

$$X_s = \frac{\mathrm{E}[D|\mathcal{F}_s] - P_1}{\alpha \,\mathrm{Var}(D|\mathcal{F}_s)} - \frac{1}{2} X_{s,G}. \tag{5.27}$$

The results in this case are similar to those when a set of call and put options is introduced.[32]

6 Conclusion

This paper uses a rational expectations model to examine the effect—on asset returns and liquidity—of introducing an options market. The main results are as follows. First, liquidity demanders (who observe the private signal) take short positions in the introduced options whereas liquidity suppliers (who are informed only by asset prices) take long positions in those options. Second, options provide hedging benefits and increase risk sharing between the demanders and suppliers of liquidity; thus the welfare of market participants is improved. Yet this improvement in welfare is asymmetric across agent types, which affects their trading incentives at date 0. More importantly, I find that introducing derivatives has surprisingly non-monotonic effects on the underlying asset price and so could reconcile the mixed empirical evidence on the effects of options listing. I also find that introducing an options market reduces the price impact λ but leaves the price reversal γ unchanged. Finally, I show that the effects of derivatives on the price and liquidity of the underlying asset are sensitive to the measures of liquidity used and the factors driving asset-specific characteristics. These results constitute new and empirically testable implications concerning how the introduction of financial derivatives affects the underlying asset.

In addition, I endogenize the participation decisions of agents and examine the case where agents must pay a participation fee to enter the market. Such a participation cost naturally reduces the participation of liquidity suppliers and likewise their proportion in the overall agent

[31] More specifically, the at-the-money options are a special case of the generalized straddle in which $g(\cdot)$ is not strictly monotonic.

[32] Just as in the case of in-the-money and out-of-the-money options, for the risk asset a linear price function cannot be guaranteed. The scope of analysis is limited here to linear pricing functions of risky assets, so that topic is left for future research.

population. I find that introducing an options market always reduces the illiquidity discount component of the ex ante price P_0 as well as the expected return of the underlying assets—that is, irrespective of the supply of liquidity. Moreover, both illiquidity measures decline after derivatives are introduced. I also provide a new prediction that the trading volume of options exhibits an inverse U shape with respect to the participation cost. Finally, the mechanism proposed here is robust to derivatives with a general payoff structure.

There are several avenues for future research. First, extending the static setting to a dynamic model would be a worthwhile pursuit (see e.g. Cao, 1999). Moreover, because of the CARA-normal framework (adopted to make the model tractable), the options considered in this paper carry no additional information and so are informationally redundant. Relaxing these assumptions and examining the effects of derivatives under more general utility functions and asset payoff distributions may yield findings of considerable interest (see e.g. Chabakauri, Yuan, and Zachariadis, 2014; Malamud, 2015).

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Appendices

A Some useful results from Vayanos and Wang (2012a)

Following Vayanos and Wang (2012a), the interim utilities of liquidity suppliers and demanders at $t = \frac{1}{2}$, denoted by U_s and U_d, can be calculated as follows:

$$\begin{aligned} U_s &= \mathrm{E}\left\{-\exp\left\{-\alpha\left[W_0 + X_0(P_1 - P_0) + \frac{\left[\overline{D} + \beta_P(s - \overline{D} - \frac{\alpha}{m}z) - P_1\right]^2}{2\alpha \mathrm{Var}(D|P_1)}\right]\right\}\right\} \\ &= -\exp(-\alpha F_s)\frac{1}{\sqrt{1 + \frac{(B-\beta_P)^2}{\mathrm{Var}(D|\mathcal{F}_s)}\left(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n}\right)}}, \end{aligned} \tag{A.1}$$

where

$$F_s = W_0 + X_0(\overline{D} - P_0) - \frac{\alpha}{2h}X_0^2 + \frac{\alpha\frac{(1-B)^2\frac{1}{h^2}}{\mathrm{Var}(D|\mathcal{F}_s)}(X_0 - \overline{X})^2}{2\left[1 + \frac{(B-\beta_P)^2}{\mathrm{Var}(D|\mathcal{F}_s)}(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n})\right]}, \tag{A.2}$$

and

$$\begin{aligned} U_d &= \mathrm{E}\left\{-\exp\left\{-\alpha\left[W_0 + X_0(P_1 - P_0) + z(P_1 - \overline{D}) + \frac{[\overline{D} + \beta_s(s - \overline{D}) - P_1]^2}{2\alpha\mathrm{Var}(D|\mathcal{F}_d)}\right]\right\}\right\} \\ &= -\exp(-\alpha F_d)\frac{1}{\sqrt{1 + \frac{(B-\beta_s)^2}{\mathrm{Var}(D|\mathcal{F}_d)}(\frac{1}{h} + \frac{1}{m})(1 + \alpha^2\frac{1}{m}\frac{1}{n}) - \alpha^2\frac{1}{h}\frac{1}{n}}}, \end{aligned} \tag{A.3}$$

where

$$\begin{aligned} F_d =& W_0 + X_0(\overline{D} - P_0) - \frac{\alpha}{h}X_0\overline{X} + \frac{1}{2}\frac{\alpha}{h}\overline{X}^2 \\ &- \frac{\alpha\left\{B^2(\frac{1}{h} + \frac{1}{m})(1 + \alpha^2\frac{1}{m}\frac{1}{n})(X_0 - \overline{X})^2 + (\frac{1}{h} + \frac{1}{m})^2\frac{\alpha^2}{n}\left[\frac{2B\frac{1}{h}X_0\overline{X}}{\frac{1}{h} + \frac{1}{m}} + [\frac{(B-\beta_s)^2}{h\mathrm{Var}(D|s)} - B^2]\overline{X}^2\right]\right\}}{2\left[1 + \frac{(B-\beta_s)^2}{\mathrm{Var}(D|\mathcal{F}_d)}(\frac{1}{h} + \frac{1}{m})(1 + \alpha^2\frac{1}{m}\frac{1}{n}) - \alpha^2\frac{1}{h}\frac{1}{n}\right]}. \end{aligned} \tag{A.4}$$

Furthermore, it is easy to show that

$$\frac{(B - \beta_s)^2}{\mathrm{Var}(D|\mathcal{F}_d)}(\frac{1}{h} + \frac{1}{m})(1 + \alpha^2\frac{1}{m}\frac{1}{n}) = \Delta_0(1 - \pi)^2, \tag{A.5}$$

$$\frac{(B - \beta_P)^2}{\mathrm{Var}(D|\mathcal{F}_s)}(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n}) = \Delta_0\pi^2. \tag{A.6}$$

When $X_0 = \overline{X}$, we have four useful equations:

$$\frac{dF_s}{dX_0} = \overline{D} - P_0 - \frac{\alpha}{h}\overline{X}, \tag{A.7}$$

$$F_s = W_0 + \overline{X}(\overline{D} - P_0) - \frac{\alpha}{2h}\overline{X}^2, \tag{A.8}$$

$$\frac{dF_d}{dX_0} = \frac{dF_s}{dX_0} - \Delta_1\overline{X}, \tag{A.9}$$

$$F_d = F_s - \frac{1}{2}\Delta_2\overline{X}^2. \tag{A.10}$$

An agent chooses X_0 to maximize his/her utility $U \equiv \pi U_d + (1-\pi)U_s$ at $t = 0$. The first-order condition is given by

$$\pi\exp(-\alpha F_d)\frac{dF_d}{dX_0}\frac{1}{\sqrt{1+\Delta_0(1-\pi)^2-\alpha^2\frac{1}{nh}}} + (1-\pi)\exp(-\alpha F_s)\frac{dF_s}{dX_0}\frac{1}{\sqrt{1+\Delta_0\pi^2}} = 0 \tag{A.11}$$

$$\Leftrightarrow \pi(\overline{D} - P_0 - \frac{\alpha}{h}\overline{X} - \Delta_1\overline{X})M + (1-\pi)(\overline{D} - P_0 - \frac{\alpha}{h}\overline{X}) = 0. \tag{A.12}$$

Then we obtain the equilibrium price of the risky stock as shown in (2.13).

B Asymmetric Information with an Options Market

Proof of Proposition 3.1. To prove that the proposed prices and demands are obtained in equilibrium, we should verify that the market clears and the Euler condition holds for the risky stock and options. We first prove that the proposed investors' demands do clear the market at the equilibrium prices. Specifically, for the risky stock, it is easy to check that

$$\pi X_d + (1-\pi)X_s = \overline{X}. \tag{B.1}$$

Given the demands for the risky stock, checking the market clearing condition for options is equivalent to verifying that the following relation holds

$$\frac{\pi}{\alpha}(G - G_d) + \frac{1-\pi}{\alpha}(G - G_s) = 0. \tag{B.2}$$

For a liquidity demander, his wealth at $t = 2$ is given by:

$$\begin{aligned} W_{d2} =& W_1 + X_d(D - P_1) + z(D - \overline{D}) \\ &+ \int_0^{+\infty} X_{d,CK}\left[(D-K)^+ - P_{CK}\right]dK + \int_{-\infty}^{0} X_{d,PK}\left[(K-D)^+ - P_{PK}\right]dK. \end{aligned} \tag{B.3}$$

Given the proposed equilibrium prices, we then show that the equilibrium demands for the risky stock and call and put options satisfy the first-order conditions.

At $t = 1$, a liquidity demander holds X_d shares of the risky stock to maximize the expected utility conditional on signal s,

$$-\operatorname{E}\exp\left(-\alpha W_{d2}\right). \tag{B.4}$$

We compute the terms in (B.4) separately. Firstly, it can be easily verified

$$\int_0^{+\infty}(D-K)^+dK+\int_{-\infty}^0(K-D)^+dK=\frac{D^2}{2}.$$

Then we compute the integration of option prices as follows

$$\begin{aligned}
&\int_0^{+\infty}P_{CK}dK+\int_{-\infty}^0 P_{PK}dK\\
&=\int_0^{+\infty}(P_1-K)\mathcal{N}\left(\sqrt{G}\,(P_1-K)\right)+\int_{-\infty}^0(K-P_1)\mathcal{N}\left(\sqrt{G}\,(K-P_1)\right)+\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi G}}\exp\left(-\frac{G(P_1-K)^2}{2}\right)dK\\
&=\int_0^{+\infty}(P_1-K)\mathcal{N}\left(\sqrt{G}\,(P_1-K)\right)+\int_{-\infty}^0(K-P_1)\mathcal{N}\left(\sqrt{G}\,(K-P_1)\right)+\frac{1}{G}\\
&=\int_0^{+\infty}(P_1-K)\int_{-\infty}^{\sqrt{G}(P_1-K)}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}x^2\right)dxdK+\int_{-\infty}^0(K-P_1)\int_{-\infty}^{\sqrt{G}(K-P_1)}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}x^2\right)dxdK+\frac{1}{G}\\
&=\frac{1}{\sqrt{2\pi}}\left\{\int_{-\infty}^{\sqrt{G}(P_1-K)}\int_0^{P_1-\frac{x}{\sqrt{G}}}(P_1-K)dK\exp\left(-\frac{1}{2}x^2\right)dx\int_{-\infty}^{\sqrt{G}(K-P_1)}\int_{P_1+\frac{x}{\sqrt{G}}}^0(K-P_1)dK\exp\left(-\frac{1}{2}x^2\right)dx\right\}+\frac{1}{G}\\
&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\sqrt{G}(P_1-K)}\left(\frac{1}{2}P_1^2-\frac{1}{2G}x^2\right)\exp\left(-\frac{1}{2}x^2\right)dx+\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\sqrt{G}(K-P_1)}\left(\frac{1}{2}P_1^2-\frac{1}{2G}x^2\right)\exp\left(-\frac{1}{2}x^2\right)dx+\frac{1}{G}\\
&=\frac{1}{2}P_1^2-\frac{1}{2G}+\frac{1}{G}\\
&=\frac{1}{2}\left(P_1^2+\frac{1}{G}\right)
\end{aligned}$$

Substituting above terms into liquidity demander's wealth (B.3) yields

$$W_{d2}=W_1+X_d(D-P_1)+z(D-\overline{D})+\frac{G-G_d}{2\alpha}D^2-\frac{G-G_d}{2\alpha}\left(P_1^2+\frac{1}{G}\right), \tag{B.5}$$

and the optimization problem of a liquidity demander can be rewritten as[33]

$$
\begin{aligned}
& -\mathrm{E}\left[\exp\left(-\alpha W_{d2}\right)|\mathcal{F}_d\right] \\
&= -\mathrm{E}\left[\exp\left(-\alpha W_1 - \alpha X_d(D-P_1) - \alpha z(D-\overline{D}) - \frac{G-G_d}{2}D^2 + \frac{G-G_d}{2}\left(P_1^2+\frac{1}{G}\right)\right)|s\right] \\
&= -\frac{1}{\sqrt{1+\frac{1}{G_d}(G-G_d)}} * \\
& \exp\left\{ ... + \alpha X_d P_1 - \alpha(X_d+z)\mathrm{E}(D|\mathcal{F}_d) - \frac{G-G_d}{2}\mathrm{E}^2(D|\mathcal{F}_d) + \frac{1}{2}\frac{(\alpha(X_d+z)+(G-G_d)\mathrm{E}(D|\mathcal{F}_d))^2}{\left(1+\frac{1}{G_d}(G-G_d)\right)G_d} \right\}.
\end{aligned}
$$

The first order condition with respect to X_d is

$$\alpha P_1 - \alpha \mathrm{E}(D|\mathcal{F}_d) + \alpha \frac{\alpha(X_d+z)+(G-G_d)\mathrm{E}(D|\mathcal{F}_d)}{\left(1+\frac{1}{G_d}(G-G_d)\right)G_d} = 0, \tag{B.6}$$

which implies that

$$X_d = \frac{\mathrm{E}(D|\mathcal{F}_d)-P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_d)} - \frac{G-G_d}{\alpha}P_1 - z. \tag{B.7}$$

We have shown that the Euler equation holds for the risky stock at the proposed demand. Next we check whether the Euler condition holds for options. In other words, we need to prove

$$\mathrm{E}[((D-K)^+ - P_{CK})\exp(-\alpha W_{d2})|\mathcal{F}_d] = 0, \tag{B.8}$$

$$\mathrm{E}[((K-D)^+ - P_{PK})\exp(-\alpha W_{d2})|\mathcal{F}_d] = 0. \tag{B.9}$$

Based on the above results, we can write the liquidity demanders' wealth at $t=2$ as

$$W_{d2} = W_1 + \frac{1}{\alpha}(G_d\mathrm{E}(D|\mathcal{F}_d) - GP_1 - z)(D-P_1) + z(D-\overline{D}) + \frac{G-G_d}{2\alpha}\left(D^2 - P_1^2 - \frac{1}{G}\right), \tag{B.10}$$

Substituting (B.7) into (B.4) yields

$$
\begin{aligned}
& -\mathrm{E}\left[\exp\left(-\alpha W_{d2}\right)|\mathcal{F}_d\right] \\
&= -\frac{1}{\sqrt{\frac{G}{G_d}}}\exp\left\{-\alpha W_1 + \alpha z(\overline{D}-P_1) + \frac{G-G_d}{2G} - \frac{G_d}{2}\left(E(D|\mathcal{F}_d)-P_1\right)^2\right\}.
\end{aligned} \tag{B.11}
$$

[33]To make the first-order condition clear, the constant mean term is dropped in our calculation and is replaced by ....The dropped term is $-\alpha W_{d1} + \alpha z \overline{D} + \frac{G-G_d}{2}\left(P_1^2 + \frac{1}{G}\right)$.

Let $x = D - P_1$, $\mu = \mathrm{E}(D|\mathcal{F}_d) - P_1$, and $G_d^{-1} = Var(D - P_1|\mathcal{F}_d)$, we have

$$
\begin{aligned}
&\mathrm{E}\left[(D-K)^+ \exp(-\alpha W_{d2})|\mathcal{F}_d\right] \\
=&\frac{1}{\sqrt{\frac{G}{G_d}}} \exp\left(-\alpha W_1 + \alpha z(\overline{D} - P_1) + \frac{G - G_d}{2G}\right) \\
&\times \int_{K-P_1}^{+\infty} [x - (K - P_1)] \sqrt{\frac{G_d}{2\pi}} \exp\left(-x(\mu G_d - \frac{1}{2}x(G - G_d)) - \frac{G_d(x-\mu)^2}{2}\right) dx \\
=&\sqrt{\frac{G}{G_d}} \mathrm{E}\left[\exp\left(-\alpha W_{d2}\right)|\mathcal{F}_d\right] \int_{K-P_1}^{+\infty} [x - (K - P_1)] \sqrt{\frac{G_d}{2\pi}} \exp\left(-\frac{G}{2}x^2\right) dx \\
=&\sqrt{\frac{G}{G_d}} \mathrm{E}\left[\exp\left(-\alpha W_{d2}\right)|\mathcal{F}_d\right] \left\{\frac{\sqrt{G_d}}{G} \frac{1}{\sqrt{2\pi}} \exp(-\frac{G(P_1 - K)^2}{2}) + (P_1 - K)\sqrt{\frac{G_d}{G}} \mathcal{N}\left(\sqrt{G}(P_1 - K)\right)\right\} \\
=&\mathrm{E}\left[\exp\left(-\alpha W_{d2}\right)|\mathcal{F}_d\right] \left\{\frac{1}{\sqrt{2\pi G}} \exp\left(-\frac{G\left(P_1 - K\right)^2}{2}\right) + \left(P_1 - K\right) \mathcal{N}\left(\sqrt{G}\left(P_1 - K\right)\right)\right\}.
\end{aligned} \tag{B.12}
$$

Combining (B.8) and (B.12) results in the equilibrium price of call option in (3.4). This verifies the proposed prices in the proposition. Following similar procedures, it is easy to demonstrate that for liquidity suppliers, demands of risky asset and options do take the forms in the proposition.

□

Proof of Lemma 3.1. As shown in the above proof of Proposition 3.1, the expected utility of a liquidity demander at $t = 1$ can be written as

$$
\begin{aligned}
&-\mathrm{E}\left[\exp\left(-\alpha W_{d2}\right)|\mathcal{F}_d\right] \\
=&-\frac{1}{\sqrt{\frac{G}{G_d}}} \exp\left\{-\alpha W_1 + \alpha z(\overline{D} - P_1) + \frac{G - G_d}{2G} - \frac{G_d}{2}\left(E(D|\mathcal{F}_d) - P_1\right)^2\right\} \\
=&-\frac{\exp\left(\frac{G - G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}} \left\{\exp\left[-\alpha\left(W_0 + X_0(P_1 - P_0) + z(P_1 - \overline{D}) + \frac{\left[\,\overline{D} + \beta_s(s - \overline{D})\ - P_1\right]^2}{2\alpha \mathrm{Var}(D|\mathcal{F}_d)}\right)\right]\right\}.
\end{aligned} \tag{B.13}
$$

Taking the expectation of (B.13) over (s, z) yields the interim utility U_d of the liquidity demander at $t = \frac{1}{2}$, i.e. (3.12). Similarly, we can obtain the interim utilities of liquidity supplies taking the form of (B.15).

As $0 < \frac{G}{G_d} < 1$, $\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}}$ is a decreasing function of $\frac{G_d}{G}$. Therefore, $\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}} < 1$. Likewise, $\frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{\frac{G}{G_s}}}$ is an increasing function of $\frac{G_s}{G}$ due to the fact that $\frac{G}{G_s} > 1$. Then we get $\frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{\frac{G}{G_s}}} < 1$.

□

Proof of Lemma 3.2. The ratio of the additional factor in the interim utilities resulting from options is given by

$$\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}} \Big/ \frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{\frac{G}{G_s}}} = \exp\left(\frac{G_s - G_d}{2G}\right)\sqrt{\frac{G_d}{G_s}}. \tag{B.14}$$

Define $\pi^* \equiv \frac{1}{\log\left(1+\frac{m-q}{h+q}\right)} - \frac{h+q}{m-q}$, we have

(1) $\exp\left(\frac{G_s-G_d}{2G}\right)\sqrt{\frac{G_d}{G_s}} > 1$ only if $\frac{m-q}{h+\pi m+(1-\pi)q} + \log\left(\frac{h+q}{h+m}\right) < 0$, which is equivalent to $\pi > \pi^*$.

(2) $\exp\left(\frac{G_s-G_d}{2G}\right)\sqrt{\frac{G_d}{G_s}} < 1$ only if $\frac{m-q}{h+\pi m+(1-\pi)q} + \log\left(\frac{h+q}{h+m}\right) > 0$, which is equivalent to $\pi < \pi^*$.

(3) $\exp\left(\frac{G_s-G_d}{2G}\right)\sqrt{\frac{G_d}{G_s}} = 1$ only if $\frac{m-q}{h+\pi m+(1-\pi)q} + \log\left(\frac{h+q}{h+m}\right) = 0$, which is equivalent to $\pi = \pi^*$.

As $\log\left(1+\frac{m-q}{h+q}\right) < \frac{m-q}{h+q}$, we have $\frac{1}{\log\left(1+\frac{m-q}{h+q}\right)} - \frac{h+q}{m-q} > 0$.

□

Proof of Proposition 3.2. According to the calculation from Vayanos and Wang (2012a), we compute the expected utilities of liquidity demanders and liquidity suppliers at $t = \frac{1}{2}$ as following. For liquidity suppliers, as shown in Lemma 3.1

$$U_s = \frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{\frac{G}{G_s}}}\mathrm{E}\left\{-\exp\left[-\alpha\left(W_0 + X_0(P_1 - P_0) + \frac{[\,\overline{D} + \beta_P(s - \overline{D} + Cz)\ \ - P_1]^2}{2\alpha\mathrm{Var}(D|\mathcal{F}_s)}\right)\right]\right\}, \tag{B.15}$$

then we have

$$U_s = -\frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{\frac{G}{G_s}}}\exp(-\alpha F_s)\frac{1}{\sqrt{1 + \frac{(B-\beta_{P_1})^2}{\mathrm{Var}(D|\mathcal{F}_s)}(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n})}}, \tag{B.16}$$

where F_s is the same as the case of Vayanos and Wang (2012a).

For liquidity suppliers, the interim utilities are given as

$$U_d = \frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}} \mathrm{E}\left\{-\exp\left[-\alpha\left(W_0 + X_0(P_1 - P_0) + z(P_1 - \overline{D}) + \frac{[\,\overline{D} + \beta_s(s - \overline{D}) \;\; - P_1]^2}{2\alpha \mathrm{Var}(D|\mathcal{F}_d)}\right)\right]\right\}, \tag{B.17}$$

then we have:

$$U_d = -\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}} \exp(-\alpha F_d) \frac{1}{\sqrt{1 + \frac{(B-\beta_s)^2}{\mathrm{Var}(D|\mathcal{F}_d)}(\frac{1}{h} + \frac{1}{m})(1 + \alpha^2 \frac{1}{h}\frac{1}{n}) - \alpha^2 \frac{1}{m}\frac{1}{n}}}, \tag{B.18}$$

where F_d is the same as the case of Vayanos and Wang (2012a). Based on the above expected utilities, the identical investor's expected utility at $t = 0$ is given by

$$U \equiv \pi U_d + (1 - \pi) U_s. \tag{B.19}$$

The first-order condition of the optimization problem is given by

$$\pi \exp(-\alpha F_d) \frac{dF_d}{dX_0} \frac{\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}}}{\sqrt{1 + \Delta_0(1 - \pi)^2 - \alpha^2 \frac{1}{nh}}} + (1 - \pi) \exp(-\alpha F_s) \frac{dF_s}{dX_0} \frac{\frac{\exp\left(\frac{G-G_s}{2G}\right)}{\sqrt{\frac{G}{G_s}}}}{\sqrt{1 + \Delta_0 \pi^2}} = 0 \tag{B.20}$$

$$\Leftrightarrow \pi(\overline{D} - P_0 - \frac{\alpha}{h}\overline{X} - \Delta_1 \overline{X}) M^O + (1 - \pi)(\overline{D} - P_0 - \frac{\alpha}{h}\overline{X}) = 0. \tag{B.21}$$

Then we obtain the ex ante price of the risky asset

$$P_0 = \overline{D} - \frac{\alpha}{h}\overline{X} - \frac{\pi M^O}{1 - \pi + \pi M^O} \Delta_1 \overline{X}, \tag{B.22}$$

where

$$M^O = \exp\left(\frac{G_s - G_d}{2G}\right) \sqrt{\frac{G_d}{G_s}} \exp\left(\frac{1}{2}\alpha \Delta_2 \overline{X}^2\right) \sqrt{\frac{1 + \pi^2 \Delta_0}{1 + \Delta_0(1 - \pi)^2 - \alpha^2 \frac{1}{nh}}}. \tag{B.23}$$

□

Proof of Proposition 3.3. See the proofs of Lemma 3.2 and Proposition 3.2. □

Proof of Proposition 3.4. The signed volume of liquidity suppliers is

$$(1-\pi)(X_s-\overline{X}) =(1-\pi)\left(\frac{\mathrm{E}(D|\mathcal{F}_s)-P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_s)}-\frac{G-G_s}{\alpha}P_1-\overline{X}\right)$$
$$=(1-\pi)\left(\frac{\overline{D}+\beta_P\frac{P_1-A}{B}-P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_s)}-\frac{\pi\,(m-q)}{\alpha}P_1-\overline{X}\right). \tag{B.24}$$

According to the definition of the price impact shown in (2.15), we have

$$\lambda =\frac{\mathrm{Cov}\left(P_1-P_0,\pi(X_d-\overline{X})\right)}{\mathrm{Var}\left[\pi(X_d-\overline{X})\right]}$$
$$=\frac{-\mathrm{Cov}\left(P_1-P_0,(1-\pi)\left(X_s-\overline{X}\right)\right)}{\mathrm{Var}\left[(1-\pi)\left(X_s-\overline{X}\right)\right]}$$
$$=\frac{\alpha \mathrm{Var}(D|\mathcal{F}_s)}{(1-\pi)\left[\pi\,(m-q)\,\mathrm{Var}(D|\mathcal{F}_s)+1-\frac{\beta_P}{B}\right]}. \tag{B.25}$$

As $m>q$, we have $\lambda<\frac{\alpha \mathrm{Var}(D|\mathcal{F}_s)}{(1-\pi)(1-\frac{\beta_P}{B})}$, which completes the proof of the first part. From Proposition 3.1, P_1 is unchanged after the introduction of options. As a result, the price reversal γ is not affected according to the definition of γ.

□

Proof of Proposition 3.5. The interim utilities of liquidity suppliers and demanders in the presence of options are provided in Lemma 3.1, whereas the interim utilities in the absence of options are given in (A.1) and (C.3). Because the ex ante utility is the expectation of the interim utilities defined in (2.12), we can calculate the ratio of utilities of the case with options to the case without options as follows:

$$\frac{\exp\left(\frac{G-G_d}{2G}\right)}{\sqrt{\frac{G}{G_d}}}\exp\left(-\alpha\Delta_1\overline{X}\left(\frac{\pi M^O}{1-\pi+\pi M^O}-\frac{\pi M}{1-\pi+\pi M}\right)\right)\frac{\pi M^O+1-\pi}{\pi M^O+(1-\pi)\frac{M^O}{M}}. \tag{B.26}$$

The ratio of ex ante utilities increases in π when the precision of the risky asset payoff is small (i.e. $h<\overline{h}$), whereas it first decreases and then increases in π when the precision of the risky asset payoff is large (i.e. $h>\overline{h}$). The maximum ratio is smaller than or equal to one. As we are using the exponential utility which are negative, the ex ante utility in the presence of options is higher than that in the absence of options. □

C Participation Costs

Proof of Proposition 4.1. Substituting the demand schedules of liquidity demanders (4.2) and liquidity suppliers (4.3) into the market clear condition (4.4) yields the equilibrium price of the risky asset. □

Proof of Proposition 4.2. Following Vayanos and Wang (2012a), the interim utilities of participating liquidity suppliers and demanders, $U_{s,P}$ and $U_{d,P}$, can be calculated as follows:

$$U_{s,P} = -\frac{\exp(-\alpha F_s)}{\sqrt{1 + \frac{(B-\beta_P)^2}{\mathrm{Var}(D|\mathcal{F}_s)}\left(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n}\right)}}, \tag{C.1}$$

where

$$F_s = W_0 + X_0(\overline{D} - P_0) - \frac{\alpha}{2h}X_0^2 + \frac{\alpha\frac{(1-B)^2\frac{1}{h^2}}{\mathrm{Var}(D|\mathcal{F}_s)}(X_0 - \overline{X})^2}{2\left[1 + \frac{(B-\beta_P)^2}{\mathrm{Var}(D|\mathcal{F}_s)}(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n})\right]} - f, \tag{C.2}$$

and

$$U_{d,P} = -\frac{\exp(-\alpha F_d)}{\sqrt{1 + \frac{(B-\beta_s)^2}{\mathrm{Var}(D|\mathcal{F}_d)}(\frac{1}{h} + \frac{1}{m})(1 + \alpha^2\frac{1}{m}\frac{1}{n}) - \alpha^2\frac{1}{h}\frac{1}{n}}}, \tag{C.3}$$

where

$$\begin{aligned} F_d =& W_0 + X_0(\overline{D} - P_0) - \frac{\alpha}{h}X_0\overline{X} + \frac{1}{2}\frac{\alpha}{h}\overline{X}^2 \\ &- \frac{\alpha\left\{B^2(\frac{1}{h} + \frac{1}{m})(\alpha^2\frac{1}{m}\frac{1}{n})(X_0 - \overline{X})^2 + (\frac{1}{h} + \frac{1}{m})^2\frac{\alpha^2}{n}\left[\frac{2B\frac{1}{h}X_0\overline{X}}{\frac{1}{h}+\frac{1}{m}} + [\frac{(B-\beta_s)^2}{h\mathrm{Var}(D|s)} - B^2]\overline{X}^2\right]\right\}}{2\left[1 + \frac{(B-\beta_s)^2}{\mathrm{Var}(D|\mathcal{F}_d)}(\frac{1}{h} + \frac{1}{m})(1 + \alpha^2\frac{1}{m}\frac{1}{n}) - \alpha^2\frac{1}{h}\frac{1}{n}\right]} - f. \end{aligned} \tag{C.4}$$

In the case of not participating, the interim utilities of liquidity suppliers are:

$$U_{s,NP} = -\exp\left\{-\alpha\left[W_0 + X_0(\overline{D} - P_0) - \frac{1}{2}\alpha X_0^2\frac{1}{h}\right]\right\}. \tag{C.5}$$

Liquidity suppliers are willing to enter the market if $U_{s,P} \geq U_{s,NP}$, that is

$$-\frac{\exp(-\alpha F_s)}{\sqrt{1 + \frac{(B-\beta_P)^2}{\mathrm{Var}(D|\mathcal{F}_s)}\left(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n}\right)}} \geq -\exp\left\{-\alpha\left[W_0 + X_0(\overline{D} - P_0) - \frac{1}{2}\alpha X_0^2\frac{1}{h}\right]\right\} \tag{C.6}$$

$$\Leftrightarrow \exp(2\alpha f) \leq 1 + \frac{(\pi^*)^2}{[h + q + \pi^*(m-q)]^2}\frac{h^2(m-q)^2}{h+q}\left(\frac{1}{h} + \frac{1}{m} + \frac{\alpha^2}{m^2}\frac{1}{n}\right). \tag{C.7}$$

Let $c = \frac{h^2(m-q)^2}{h+q}\left(\frac{1}{h}+\frac{1}{m}+\frac{\alpha^2}{m^2}\frac{1}{n}\right) = \frac{h}{q}(m-q)^2$. Based on the fact that $\frac{(\pi^*)^2}{[h+q+\pi^*(m-q)]^2}$ increases in π^* and thereby decreases in μ, we have the following three scenarios:

Case 1: if $f \leq f^1 \equiv \frac{\log\left(1+\frac{(\pi)^2}{[h+q+\pi(m-q)]^2}c\right)}{2\alpha}$, then (C.7) holds for $\mu = 1$, indicating that all liquidity suppliers participate in the market.

Case 2: if $f^1 < f < f^2$, where $f^2 \equiv \frac{\log\left(1+\frac{1}{(h+m)^2}c\right)}{2\alpha}$, then (C.7) holds as an equality for $\mu \in (0,1)$ given by $\mu = \frac{\pi}{1-\pi}\left\{\frac{m-q}{h+q}\left[\sqrt{\frac{h}{q(e^{2\alpha f}-1)}}-1\right]-1\right\}$, indicating that a positive fraction of liquidity suppliers participate in the market.

Case 3: if $f \geq f^2$, then (C.7) does not hold for any $\mu \in (0,1]$, indicating no liquidity suppliers participate. $\square$

Proof of Proposition 4.3. In the presence of participation costs, (A.5) and (A.6) can be rewritten as

$$\frac{(B-\beta_s)^2}{\text{Var}(D|\mathcal{F}_d)}(\frac{1}{h}+\frac{1}{m})(1+\alpha^2\frac{1}{m}\frac{1}{n}) = \Delta_0(1-\pi^*)^2, \tag{C.8}$$

$$\frac{(B-\beta_P)^2}{\text{Var}(D|\mathcal{F}_s)}(\frac{1}{h}+\frac{1}{m}+\frac{\alpha^2}{m^2}\frac{1}{n}) = \Delta_0(\pi^*)^2. \tag{C.9}$$

Similarly, when $X_0 = \overline{X}$, (A.7)-(A.10) are rewritten as

$$\frac{dF_s}{dX_0} = \overline{D} - P_0 - \frac{\alpha}{h}\overline{X}, \tag{C.10}$$

$$F_s = W_0 + \overline{X}(\overline{D}-P_0) - \frac{\alpha}{2h}\overline{X}^2 - f, \tag{C.11}$$

$$\frac{dF_d}{dX_0} = \frac{dF_s}{dX_0} - \Delta_1\overline{X}, \tag{C.12}$$

$$F_d = F_s - \frac{1}{2}\Delta_2\overline{X}^2. \tag{C.13}$$

In the case of not participating in the market, the interim utilities of liquidity demanders are

$$U_{d,NP} = -\frac{\exp\left\{-\alpha\left[W_0 + X_0(\overline{D}-P_0) - \frac{\alpha\frac{1}{h}}{2(1-\alpha^2\frac{1}{h}\frac{1}{n})}\overline{X}^2\right]\right\}}{\sqrt{1-\alpha^2\frac{1}{h}\frac{1}{n}}} \tag{C.14}$$

When liquidity demanders fully participate, we have $U_{d,P} > U_{d,NP}$, that is

$$\exp\left\{2\alpha\left[-\frac{\alpha^3\frac{1}{h^2}\frac{1}{n}}{2(1-\alpha^2\frac{1}{h}\frac{1}{n})}\overline{X}^2 + \frac{1}{2}\Delta_2\overline{X}^2 + f\right]\right\} < \frac{1+\Delta_0(1-\pi^*)^2 - \alpha^2\frac{1}{h}\frac{1}{n}}{1-\alpha^2\frac{1}{h}\frac{1}{n}} \tag{C.15}$$

The sufficient condition for (C.15) to hold is

$$\frac{\exp\left\{2\alpha\left[-\frac{\alpha^3\frac{1}{h^2}\frac{1}{n}}{2(1-\alpha^2\frac{1}{h}\frac{1}{n})}\overline{X}^2+\frac{1}{2}\Delta_2\overline{X}^2+f\right]\right\}}{\exp(2\alpha f)} < \frac{\frac{1+\Delta_0(1-\pi^*)^2-\alpha^2\frac{1}{h}\frac{1}{n}}{1-\alpha^2\frac{1}{h}\frac{1}{n}}}{1+\frac{(B-\beta_P)^2}{\text{Var}(D|\mathcal{F}_s)}(\frac{1}{h}+\frac{1}{m}+\frac{\alpha^2}{m^2}\frac{1}{n})} \tag{C.16}$$

$$\Leftrightarrow \exp\left\{2\alpha\left[-\frac{\alpha^3\frac{1}{h^2}\frac{1}{n}}{2(1-\alpha^2\frac{1}{h}\frac{1}{n})}\overline{X}^2+\frac{1}{2}\Delta_2\overline{X}^2\right]\right\} < \frac{\frac{1+\Delta_0(1-\pi^*)^2-\alpha^2\frac{1}{h}\frac{1}{n}}{1-\alpha^2\frac{1}{h}\frac{1}{n}}}{1+\Delta_0(\pi^*)^2} \tag{C.17}$$

It is easy to show that $-\frac{\alpha^3\frac{1}{h^2}\frac{1}{n}}{2(1-\alpha^2\frac{1}{h}\frac{1}{n})}\overline{X}^2+\frac{1}{2}\Delta_2\overline{X}^2 < 0$, then we transform the sufficient condition as

$$\frac{\frac{1+\Delta_0(1-\pi^*)^2-\alpha^2\frac{1}{h}\frac{1}{n}}{1-\alpha^2\frac{1}{h}\frac{1}{n}}}{1+\Delta_0(\pi^*)^2} > 1, \tag{C.18}$$

which is equivalent to

$$\Delta_0(1-\pi^*)^2 > (1-\alpha^2\frac{1}{h}\frac{1}{n})\Delta_0(\pi^*)^2 \tag{C.19}$$

Therefore, the sufficient condition is $1-\pi^* \geq \pi^* \Leftrightarrow \pi^* \leq \frac{1}{2} \Leftrightarrow \frac{\pi}{\pi+(1-\pi)\mu} \leq \frac{1}{2} \Leftrightarrow \pi \leq (1-\pi)\mu$. Based on (C.7) and $\pi^* \leq \frac{1}{2}$, we have the sufficient condition with respect to f as follows

$$f \leq \widehat{f} = \frac{\log\left(1+\frac{1}{4[h+q+\frac{1}{2}(m-q)]^2}\frac{h^2(m-q)^2}{h+q}(\frac{1}{h}+\frac{1}{m}+\frac{\alpha^2}{m^2}\frac{1}{n})\right)}{2\alpha}. \tag{C.20}$$

Then we check the sufficient condition for the existence of the equilibrium where all liquidity demanders and a positive fraction of liquidity suppliers participate in the market.

Since $\pi \leq \frac{1}{2}$, $f^1 \leq \widehat{f} < f^2$. When $f^1 < f \leq \widehat{f}$ and all liquidity demanders participate, then μ is in $(0,1)$ and determined by (C.7) as an equality. According to (C.20), $f \leq \widehat{f}$ results in $\pi^* \leq \frac{1}{2}$, satisfying the sufficient condition for the full participation of liquidity demanders. When $f \leq f^1$ and all liquidity demanders enter the market, then all liquidity suppliers enter the market as well. On the other hand, $\pi \leq \frac{1}{2}$ implies that the sufficient condition for all demanders to participate, $\pi \leq (1-\pi)\mu$, is satisfied when $\mu = 1$, hence all liquidity demanders participate in the market. □

Proof of Proposition 4.4. The investors at $t = 0$ choose X_0 to maximize their utilities $U = \pi\max\{U_d, U_{d,NP}\} + (1-\pi)\max\{U_s, U_{s,NP}\}$. The above Proposition implies that $U_d \geq U_{d,NP}$ and $U_s \geq U_{s,NP}$. Then the optimization problem can be rewritten $U = \pi U_d + (1-\pi)U_s$, and the consequent first-order condition is given by

$$\pi \exp(-\alpha F_d) \frac{dF_d}{dX_0} \frac{1}{\sqrt{1+\Delta_0(1-\pi^*)^2 - \alpha^2 \frac{1}{nh}}} + (1-\pi)\exp(-\alpha F_s)\frac{dF_s}{dX_0}\frac{1}{\sqrt{1+\Delta_0(\pi^*)^2}} = 0$$
$$\Leftrightarrow \pi(\overline{D} - P_0 - \frac{\alpha}{h}\overline{X} - \Delta_1 \overline{X}) M^P + (1-\pi)(\overline{D} - P_0 - \frac{\alpha}{h}\overline{X}) = 0.$$

Then we obtain the ex ante price of the risky asset in the presence of participation costs

$$P_0 = \overline{D} - \frac{\alpha}{h}\overline{X} - \frac{\pi M^P}{1-\pi+\pi M^P}\Delta_1 \overline{X}. \tag{C.21}$$

□

Proof of Proposition 4.5. The signed volume of liquidity demanders is

$$\pi(X_d - \overline{X}) = -(1-\pi)\mu(X_s - \overline{X}) = -(1-\pi)\mu\left(\frac{\mathrm{E}(D|\mathcal{F}_s) - P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_s)} - \overline{X}\right) \tag{C.22}$$
$$= -(1-\pi)\mu\left(\frac{\overline{D} + \frac{\beta_P}{B}(P_1 - A) - P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_s)} - \overline{X}\right). \tag{C.23}$$

The price impact measure is calculated as

$$\lambda = \frac{\mathrm{Cov}\left(P_1 - P_0, \pi(X_d - \overline{X})\right)}{\mathrm{Var}\left[\pi(X_d - \overline{X})\right]}$$
$$= \frac{-\mathrm{Cov}\left(P_1 - P_0, (1-\pi)\left(X_s - \overline{X}\right)\right)}{\mathrm{Var}\left[(1-\pi)\left(X_s - \overline{X}\right)\right]}$$
$$= \frac{\alpha \mathrm{Var}(D|\mathcal{F}_s)}{(1-\pi)\,\mu\left(1 - \frac{\beta_P}{B}\right)} \tag{C.24}$$
$$= \frac{\alpha[\pi m + (1-\pi)\mu q]}{h(1-\pi)\pi\mu(m-q)}. \tag{C.25}$$

Similarly, the price reversal measure is computed as

$$\gamma = -Cov(D - P_1, P_1 - P_0) \tag{C.26}$$
$$= B(B - \beta_P)(\frac{1}{h} + \frac{1}{q}), \tag{C.27}$$

where $B = \frac{\pi m + (1-\pi)\mu q}{(\pi + (1-\pi)\mu)h + \pi m + (1-\pi)\mu q}$. □

Proof of Proposition 4.6. Firstly, Proposition 4.2 indicates that a decrease in f leads to an increase in μ. Secondly, it is easy to show that $\frac{\partial \lambda}{\partial \mu} < 0$ and B is a decreasing function of μ. Then, (4.13) implies that $\frac{\partial \gamma}{\partial \mu} < 0$. To prove that the ex ante price is an increasing function of

μ, we check each terms of P_0 separately. According to its definition, Δ_0 can be rewritten as $\frac{h(m-q)^2}{q[h+q+\pi^*(m-q)]^2}$, indicating that Δ_0 decreases in π^* and thereby increases in μ. Additionally, $\Delta_0(\pi^*)^2$ increases in π^* and $\Delta_0(1-\pi^*)^2$ decreases in π^*. Further, Eqs. (4.9) and (4.10) imply that Δ_1 and Δ_2 are both decreasing functions of μ. As a result, M decreases in μ, and we conclude that an increase in μ raises the ex ante price P_0. □

Proof of Proposition 4.7, Lemma 4.1, Proposition 4.9. These proofs are very similar to the proofs in Section 3 where agents fully participate. The only difference is that we replace π with π^*. □

Proof of Proposition 4.8. Based on the results in Lemma 4.1, the condition that liquidity suppliers are willing to participate in the market, i.e. (C.7), can be rewritten as

$$\exp(2\alpha f) \leq \left(1 + \frac{(\pi^*)^2}{[h+q+\pi^*(m-q)]^2}c\right) \frac{\frac{G^P}{G_s}}{\exp\left(\frac{G^P - G_s}{G^P}\right)}. \tag{C.28}$$

Denote by RHS the right hand side of the above inequality, and we can get that $\frac{\partial RHS}{\partial \pi^*} > 0$ and $\frac{\partial RHS}{\partial \mu} < 0$. Then we have three scenarios similar to those in Proposition 4.2:

Case 1: if $f \leq \tilde{f}^1 \equiv \frac{\log\left[\left(1+\frac{(\pi)^2}{[h+q+\pi(m-q)]^2}c\right)\frac{h+q+\pi(m-q)}{h+q}\right] + \frac{h+q}{h+q+\pi(m-q)} - 1}{2\alpha}$, then (C.28) holds for $\mu = 1$, indicating that all liquidity suppliers participate in the market.

Case 2: if $\tilde{f}^1 < f < \tilde{f}^2$, where $\tilde{f}^2 \equiv \frac{\log\left[\left(1+\frac{1}{(h+m)^2}c\right)\frac{h+m}{h+q}\right] + \frac{h+q}{h+m} - 1}{2\alpha}$, then (C.28) holds as an equality for $\mu \in (0,1)$, indicating that a positive fraction of liquidity suppliers participate in the market.

Case 3: if $f \geq \tilde{f}^2$, then (C.28) does not hold for any $\mu \in (0,1]$, indicating no liquidity suppliers participate.

Here, $\tilde{f}^1 \geq f^1$, and $\tilde{f}^2 \geq f^2$.

As for the participation decision of liquidity demanders, the additional term in the interim utilities reflecting the introduction of options ensures that (C.15) holds. Following the proof in Proposition 4.3, we are able to achieve a similar sufficient condition for the equilibrium where all liquidity demanders and a positive fraction μ of liquidity suppliers participate. □

Proof of Proposition 4.10. Under the sufficient condition for the equilibrium where liquidity suppliers partially participate, (C.28) holds as an equality for $\mu \in (0,1)$. the introduction of derivative enhances the expected utilities of liquidity suppliers and thus increases the participation rate μ. □

Proof of Proposition 4.11. This proof is similar to the proof of Proposition 3.3 and 4.6. □

Proof of Proposition 4.12. The trading volume is calculated as follows

$$V^O = \frac{2\pi(1-\pi)\mu}{(\pi+(1-\pi)\mu)^2}(m-q). \tag{C.29}$$

When the participation cost f decreases, μ will increase. It is easy to show that V^O exhibits a hump shape as a function of μ. Then we can complete the proof. $\square$

D Asymmetric Information with General Derivatives

D.1 Generalized Straddles

Proof of Proposition 5.1 and Lemma 5.1. For a liquidity demander, his wealth at $t=2$, i.e. (5.1), can be rewritten as:

$$W_{d2} = W_1 + z(P_1 - \overline{D}) + (X_d + z)(D - P_1) + X_{d,G}\left(g(|D - P_1|) - P_G\right), \tag{D.1}$$

The expected utilities of liquidity demanders at $t=1$ can be expressed as

$$\begin{aligned}
&-\mathrm{E}(e^{-\alpha W_{d2}}|\mathcal{F}_d)\\
=&-e^{-\alpha\left(W_1+z(P_1-\overline{D})-X_{d,G}P_G\right)}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi\mathrm{Var}(u|\mathcal{F}_d)}}e^{-\frac{1}{2}\frac{(u-\mathrm{E}(u|\mathcal{F}_d))^2}{\mathrm{Var}(u|\mathcal{F}_d)}}e^{-\alpha\left[(X_d+z)u+X_{d,G}g(|u|)\right]}du\\
=&-e^{-\alpha\left(W_1+z(P_1-\overline{D})-X_{d,G}P_G\right)}\sqrt{\frac{G_d}{2\pi}}\\
&\times\int_{-\infty}^{+\infty}\exp\left(-\left[\alpha(X_d+z)-G_d(\mathrm{E}(D|\mathcal{F}_d)-P_1)\right]u-\frac{1}{2}G_du^2-\frac{1}{2}G_d(\mathrm{E}(D|\mathcal{F}_d)-P_1)^2-\alpha X_{d,G}g(|u|)\right)du,
\end{aligned} \tag{D.2}$$

where $u \equiv D - P_1$, $\mathrm{E}(u|\mathcal{F}_d) = \mathrm{E}(D|\mathcal{F}_d) - P_1$ and $\mathrm{Var}(u|\mathcal{F}_d) = \mathrm{Var}(D|\mathcal{F}_d) = G_d^{-1}$. Dropping irrelevant terms, we obtain two Euler equations for liquidity demanders:

$$\int_{-\infty}^{+\infty} u\exp\left\{-[\alpha(X_d+z)-G_d(\mathrm{E}(D|\mathcal{F}_d)-P_1)]u-\frac{1}{2}G_du^2-\alpha X_{d,G}g(|u|)\right\}du = 0,$$

$$\int_{-\infty}^{+\infty}(g(|u|)-P_G)\exp\left\{-[\alpha(X_d+z)-G_d(\mathrm{E}(D|\mathcal{F}_d)-P_1)]u-\frac{1}{2}G_du^2-\alpha X_{d,G}g(|u|)\right\}du = 0.$$

When $X_d = \frac{E(D|\mathcal{F}_d)-P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_d)} - z$, the linear term in the first Euler equation vanishes, and we obtain an odd function of u as the integrand. Then the integral on the left-hand side of the Euler equation with regard to the underlying asset is equal to zero. Along the same line of reasoning, we employ the Euler equation for liquidity suppliers and the market clearing condition to obtain the demands of liquidity suppliers and the equilibrium price for the risky asset. Substituting the demands for the risky asset into the Euler equations for the general derivative yields (5.6) and (5.7). Finally, we get the price of the general derivatives through the market clearing condition.

Based on the above results, we can write the liquidity demanders' expected utilities at $t = 2$ as

$$\begin{aligned}
&- \mathrm{E}(e^{-\alpha W_{d2}}|\mathcal{F}_d) \\
=& - e^{-\alpha\left(W_1+z(P_1-\overline{D})\right)}\sqrt{\frac{G_d}{2\pi}} \times 2 \int_0^{+\infty} \exp\left(-\frac{1}{2}G_d u^2 - \frac{1}{2}G_d(\mathrm{E}(D|\mathcal{F}_d)-P_1)^2 - \alpha X_{d,G}\left(g(|u|)-P_G\right)\right) du \\
=& - e^{-\alpha\left(W_0+X_0(P_1-P_0)+z(P_1-\overline{D})+\frac{1}{2}\frac{(\mathrm{E}(D|\mathcal{F}_d)-P_1)^2}{\alpha \mathrm{Var}(D|\mathcal{F}_d)}\right)}\sqrt{\frac{2G_d}{\pi}} \int_0^{+\infty} \exp\left(-\frac{1}{2}G_d u^2 - \alpha X_{d,G}\left(g(|u|)-P_G\right)\right) du.
\end{aligned} \tag{D.3}$$

Then we get (5.9) and (5.10). □

D.2 Quadratic Derivatives

Proof of Proposition 5.2. The liquidity demanders' wealth at $t = 2$ is given by

$$W_{d2} = W_1 + X_d(D-P_1) + X_{d,G1}((D-P_1)^2 - P_G) + z(D-\overline{D}), \tag{D.4}$$

The optimization problem of a liquidity demander can be written as

$$- \mathrm{E}\left[\exp\left(-\alpha W_{d2}\right)|\mathcal{F}_d\right] \tag{D.5}$$

$$= - \frac{1}{\sqrt{1+2\alpha X_{d,G1}\mathrm{Var}(D|\mathcal{F}_d)}} \exp\left[\begin{array}{c} \frac{\mathrm{Var}(D|\mathcal{F}_d)\left[\alpha(X_d+z)-\frac{E(D|\mathcal{F}_d)-P_1}{\mathrm{Var}(D|\mathcal{F}_d)}\right]^2}{2(1+2\alpha X_{d,G1}\mathrm{Var}(D|\mathcal{F}_d))} - \frac{1}{2}\frac{(\mathrm{E}(D|\mathcal{F}_d)-P_1)^2}{\mathrm{Var}(D|\mathcal{F}_d)} \\ -\alpha(W_1 - X_{d,G1}P_G + z(P_1-\overline{D})) \end{array}\right] \tag{D.6}$$

Then we have the demands for the stock and the derivative in the following forms

$$X_d = \frac{\mathrm{E}(D|\mathcal{F}_d) - P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_d)} - z, \tag{D.7}$$

$$X_{d,G1} = \frac{1}{2\alpha}\left(\frac{1}{P_{G1}} - \frac{1}{\mathrm{Var}(D|\mathcal{F}_d)}\right). \tag{D.8}$$

Along the same line of reasoning, we have

$$X_s = \frac{\mathrm{E}(D|\mathcal{F}_s) - P_1}{\alpha \mathrm{Var}(D|\mathcal{F}_s)}, \tag{D.9}$$

$$X_{s,G1} = \frac{1}{2\alpha}\left(\frac{1}{P_{G1}} - \frac{1}{\mathrm{Var}(D|\mathcal{F}_s)}\right). \tag{D.10}$$

Employing the market clearing conditions for both stocks and derivatives markets, we can complete the proof. □

Proof of Lemma 5.2. See the proof of Lemma 3.1 regarding the vanilla options. □

Proof of Proposition 5.3. See the proof of Proposition 3.2 regarding the vanilla options. □

Proof of Proposition 5.4. See the proof of Proposition 3.3 regarding the vanilla options. □