Aggregate Volatility Expectations and Threshold CAPM

July 2013

Abstract

We propose a volatility-based threshold capital asset pricing model (V-CAPM) in which asset betas change discretely with respect to investorsô expectations regarding innovations in near-term aggregate volatility. Using a novel measure to proxy for changes in aggregate volatility, i.e. monthly range of the VIX index (RVIX), we find that portfolio betas change significantly when aggregate volatility expectations is beyond a certain threshold level. Due to changes in their market betas, small and value stocks are perceived as riskier than their big and growth counterparts in bad times, when aggregate volatility is high. The findings support a rational volatility-based time-varying risk explanation.

Keywords: Aggregate Volatility; Threshold Regression; Conditional CAPM; Range; VIX
JEL Codes: C13; G12

1. Introduction

Capital asset pricing model (CAPM) assumes that a firm's riskiness, which is captured by its beta, is constant through time. However, changes in business conditions, technology, and taste might induce shifts in investment opportunity set and investors' associated risk-return tradeoffs. For example, according to Jagannathan and Wang (1996), betas and expected returns vary over time because of changes in the set of firm-specific information available to investors and overall economic conditions. Yet, despite a strong theory and considerable evidence on time variation in betas, there is no consensus on how this variation should be modelled. Many studies model the variation in betas using continuous approximation and the theoretical framework of the conditional CAPM.[1] However, Ghysels (1998) shows that this approximation fails to capture the true dynamics of betas because of significant structural breaks in parameter estimates. He argues that the actual time variation in betas is slower than assumed by linear factor models such as the conditional CAPM, and advocates the use of the static CAPM until researchers come up with a model that captures time variation in betas correctly.

In this paper, we model an asset's beta neither as static nor as a continuous approximation implied by conditional models, rather we assume that asset betas change slowly and discretely in time.[2] More particularly, we assume that investors update betas with respect to changes in aggregate risk conditions, which is represented by their expectations regarding innovations in aggregate volatility. There are several reasons why we assume asset betas should change with

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[1] See Harvey (1989), Ferson and Harvey (1991, 1993, 1999), Ferson and Korajczyk (1995), and Jagannathan and Wang (1996).

[2] The intuition similar to downside-upside beta approach in Ang, Cheng, and Xing (2006) who show that asset betas change during downside and upside markets and downside risk is priced. Methodologically, our approach is also related to Markov chain regime switching models as in Guidolin and Timmermann (2008), and Chen, Gerlach, and Lin (2011), and optimal changepoint approach as in Bollen and Whaley (2009), and Patten and Ramadorai (2013).

respect to changes in aggregate volatility. First, it is well documented that both equity and aggregate volatility is time-varying.[3] Therefore, an asset pricing model, which incorporates time-variation in aggregate volatility would naturally imply that asset betas also change accordingly. Second, time-varying risk literature suggests that stocks have different exposures to market risk during recessions and expansions. For example, under conditional CCAPM and CAPM settings, respectively, Lettau and Ludvigson (2001) and Petkova and Zhang (2005) find that value and small stocks correlate with consumption growth and market returns more during bad times relative to big and growth stocks, while the opposite holds during good times. Driven by the fact that aggregate volatility coincides with business conditions (see Hsu and Li (2009)), rather than using macro variables as in previous studies, we condition time variation in aggregate risk and expected returns as a factor of innovations in aggregate volatility. Our approach contributes to the literature by proposing a novel conditioning variable, which has a forward-looking feature by construction and which models time variation in an assetôs riskiness in a parsimonious way.

More particularly, we propose a volatility based threshold CAPM (V-CAPM) where asset betas change with respect to investors' assessment of aggregate risk conditions, which is proxied by marketôs expectations of changes in near-term aggregate volatility. The contribution of the proposed V-CAPM is fourfold. First, we propose a novel volatility measure to proxy changes in aggregate risk conditions, i.e. range of the VIX index (RVIX).[4] VIX is inherently a forward-

[3] For theoretical background and empirical evidence on stochastic volatility of equity and stock market returns, see Engle and Bollerslev (1986), French, Schwert, and Stambaugh (1987), Schwert (1989), Engle and Ng (1993), Canina and Figlewski (1993), Duffee (1995), Braun, Nelson, and Sunier (1995), Andersen (1996), Bollerslev and Mikkelsen (1999), and Bekaert and Wu (2000).

[4] We tested a battery of volatility measures ranging from statistical and historical measures of volatility such as standard deviation of returns, squared returns, GARCH based volatility estimates to forward-looking measures of volatility such as change in the VIX index and S&P 500 straddle returns. The proposed RVIX together with S&P 500 straddle returns were the most successful in capturing time-variation in betas. The reader is referred to Section 4.5 for a detailed discussion of results.

looking volatility measure and it reveals important information about investorsô expectations of near term volatility in the market.[5] Using the difference between the maximum and minimum level of the VIX index, our proposed measure is nicely able to capture investorôs expectations about changes in near-term aggregate volatility and aggregate risk. Second contribution is our approach to model time variation in betas. In standard conditional CAPM models, betas practically change at each point in time, however this approach might have a tendency to overstate the time variation in betas and result in estimates that are highly volatile. The proposed V-CAPM differs from standard conditional CAPM models by allowing investors to update betas and re-assess an asset's riskiness only when aggregate volatility moves beyond a certain threshold level, admitting a slower and discrete variation in betas as suggested by Ghysels (1998). Third, the model inherently allows for time variation in aggregate volatility which is not possible in a static CAPM. By endogenously incorporating aggregate volatility to CAPM, our model allows betas to change contemporaneously with respect to innovations in aggregate volatility, thus helps better represent changes in investment opportunity set with respect to aggregate risk conditions. Our fourth contribution is econometric. We formally test volatility related regime changes in beta risk, and estimate corresponding betas using Hansen's (2000) threshold regression methodology, which is intuitive and fully supported by econometric theory.[6] The model is rich in its predictions and offers alternative explanations to some of the empirical failures of CAPM.

[5] Often referred to as the ñfearò or ñmarket sentimentò index, VIX estimates near-term (roughly next 30-day) expected volatility by weighted-averaging the prices of puts and calls written on the S&P 500 index over a range of strike prices.

[6] See Hansen (2000) and Akdeniz, Altay-Salih and Caner (2003) for a detailed explanation of threshold estimation methodology.

Using RVIX as proxy for changes in aggregate volatility risk, and portfolios sorted with respect to market capitalizations and book-to-market ratios as test assets, our results can be summarized as follows. First, using the modified sup LM test suggested by Hansen (1996), we document significant time variation in betas. 15 out of 22 test portfolios have significant bootstrap p-values at 5% level.[7] Changes in betas are especially significant for the extreme size and book-to-market portfolios as well as SMB and HML portfolios. The initial results confirm the hypothesis that asset betas change slowly and discretely in time and changes in aggregate volatility is a key determinant in investors' assessment of market risk. Next, we test whether different size and book-to-market portfolios have different beta sensitivities with respect to investorsô expectations about changes in aggregate volatility and risk conditions. Threshold aggregate volatility estimates imply that investors update their beta risk assessments when range of the VIX index in a given month is beyond 9.33 points. The threshold estimates are quite stable for portfolios that exhibit significant beta changes (ranging from 6.07 to 11.10) confirming the robustness of the chosen threshold variable, RVIX. Note that, the range of VIX index measures marketôs expectation of changes in aggregate volatility over the next 30-day period. When RVIX is high in a given month, investors expect the market to be highly volatile over the next month. The estimated threshold level for RVIX, therefore, implies that investors update betas depending on whether marketôs expectations about the change in near-term aggregate volatility is high or low, i.e. when RVIX is roughly above or below 9.33. What makes the results further interesting is the direction of this update. Looking at changes in portfolio betas, one can see that stocks in small (and value) portfolios have consistently higher betas at times of high volatility and it is only largest market capitalization (and growth) portfolio that exhibits lower betas during volatile

[7] 9 portfolios exhibit significant change in betas at 1% level, 17 portfolios at 10% level, and 20 portfolios at 15% level.

times. The increase in betas is most pronounced for the smallest decile, highest book-to-market decile, SMB and HML portfolios.

French, Schwert, and Stambaugh (1987), and Campbell and Hentschel (1992) document that periods of high volatility usually coincide with downward market moves. Furthermore, risk-averse investors are reluctant to lose wealth in periods of high volatility because it represents a deterioration in investment opportunities, which usually coincides with periods of low consumption (recessions).[8] The increase in betas of small and value portfolios implies that stocks with these characteristics are perceived to be riskier at times of high volatility. This also holds for SMB and HML portfolios whose sensitivity to market returns becomes higher at volatile times. Investors view small and value firms riskier because their returns correlate strongly with market returns in volatile times. On the other hand, returns on big and growth stocks correlate less with market returns at those times. Our results are consistent with those of Lettau and Ludvigson (2001) and Petkova and Zhang (2005). Under conditional CCAPM and CAPM settings respectively, the authors find that value and small stocks correlate with consumption growth (market returns) more during bad times relative to big and growth stocks, while the opposite holds during good times. We argue that investors view small and value stocks riskier than their big and growth counterparts because their returns are much more sensitive to market risk at times of high volatility.

To test the robustness of the above results and to further examine the effect of time variation in investorsô expectations of near term volatility on asset risk-return dynamics, we next compare Jensenôs alphas and Sharpe ratios of test assets within full sample and in high and low volatility regimes implied by the threshold level of RVIX estimated via the V-CAPM. It is well-documented that small and value stocks (and the associated SMB and HML strategies) on

[8] Hsu and Li (2009) document that equity market volatility is higher in bear markets and recessions.

average produce significantly higher returns than their large and growth counterparts. However, looking at Jensenôs alphas and betas of different size and book-to-market portfolios, we confirm previous studies that static CAPM is unable to offer a risk-based explanation to these abnormal returns. On the other hand, alphas implied by the proposed V-CAPM help us uncover an important aspect of size and value vs. growth puzzles, offering a volatility-based time-varying risk explanation. We document that size and value strategies pay off at times of low volatility yielding significant and positive risk-adjusted returns. However the trade-off for these strategies is that they have extremely bad (significant and negative) risk-adjusted returns at times of high volatility.

Portfolio Sharpe ratios also offer a similar volatility-based time-varying risk explanation to size and value vs. growth anomalies. In calm periods, the strategy in the smallest (value) decile portfolio commands a higher reward-to-variability ratio compared to the biggest (growth) decile portfolio, however in volatile periods, the situation is the opposite where investors experience a much worse reward-to-variability ratios for the smallest (value) decile portfolio against the biggest (growth) decile portfolio. The results confirm our hypothesis that marketôs expectation of changes in aggregate volatility is an important determinant of investorsô assessment of risk and expected returns. Changes in betas and risk-adjusted returns explain why small and value stocks on average earn higher returns than their big and growth counterparts, they are risky strategies when volatility in the market is high and investors get compensated for the risk that they are taking. We further document that root-mean squared pricing errors of V-CAPM are always smaller than those of CAPM, indicating that the proposed model not only offers a potential explanation to the well-documented size and value anomalies but also does a marginally better job in pricing than the static CAPM.

We finally test the pricing implications of the proposed model by dividing the sample into high and low volatility periods and estimating the corresponding risk premia in the cross-section. To avoid the problem of factor structure related biases in the estimation procedure, we extend the cross-section of test assets to 55 portfolios and estimate the risk premia using returns on 25 (5x5) portfolios sorted with respect to market capitalisation and book-to-market ratios and 30 industry portfolios.[9] Once again, with its significant alpha and insignificant and negative risk premium, static CAPM is insufficient to explain the cross-section of expected returns throughout the sample period. Looking at the risk premia of V-CAPM in volatile and calm periods, we find that although the market risk premium is insignificant in low volatility regime, it commands a significant and negative risk premium in the high volatility regime, implying that investors require a compensation for holding stocks that correlate highly with the market when volatility is high. Although a negative market risk premium might at first sight seem at odds with rational asset pricing theories, following the works of Campbell (2007), Damodaran (2012) and data compiled by Shiller (2013), the result is not surprising when the sample period of 1986-2012 is considered. The findings are robust to inclusion of a separate volatility risk factor as well as Fama-French factors, SMB and HML.

The remainder of the paper is organized as follows. Section 2 introduces the threshold V-CAPM and the related econometric framework. Section 3 presents data and some stylized facts. Section 4 documents the empirical findings associated with the proposed V-CAPM. The final section offers concluding remarks.

[9] This is also suggested by Lewellen, Nagel, and Shanken (2010) who point out that size and book-to-market portfolios have a strong factor structure, thus advocating to extend cross-sectional asset pricing tests using alternative portfolio sets, such as industry portfolios, momentum portfolios, etc.

2. Threshold Volatility CAPM

To capture the effect of changes in aggregate volatility on market beta, we start with the following conditional CAPM:

$$E\left[r_{i,t+1} \middle| Z_t\right] = \alpha_i + \beta_t E\left[r_{m,t+1} \mid Z_t\right] + \varepsilon_{it+1} \ , \qquad (1)$$

where $r_{i,t+1}$ is the excess return on asset *i*, $r_{m,t+1}$ is the excess return on the market portfolio and *E* is the expectation operator. β_t captures time-variation in market betas, and Z_t is the conditioning information on investorsô assessment of near-term aggregate volatility risk. Using monthly range of the VIX index as a proxy for investorsô information set for changes in aggregate volatility, we model time-variation in betas as in Ferson and Harvey (1999):[10]

$$\beta_t = \beta_1 1_{\{Z_t \le \lambda\}} + \beta_2 1_{\{Z_t > \lambda\}} \ , \qquad (2)$$

where $1_{\{\}}$ is the indicator function and λ is the threshold parameter for aggregate volatility. Combining equations (1) and (2), we have the following threshold volatility CAPM:

$$r_{i,t+1} = \left(\alpha_1 1_{\{Z_t \le \lambda\}} + \alpha_2 1_{\{Z_t > \lambda\}}\right) + \left(\beta_1 1_{\{Z_t \le \lambda\}} + \beta_2 1_{\{Z_t > \lambda\}}\right) r_{m,t+1} + \varepsilon_{i,t+1} \ , \qquad (3)$$

where Z_t is the range of the VIX index that summarizes investorsô information set on the evolution of near-term aggregate volatility.

[10] See Section 3 for details on the construction of the conditioning variable RVIX.

2.1. Econometric Model

The observed sample is $\{r_{t+1}, r_{m,t+1}, Z_t\}$, t = 1, é , T-1. The random variables r_t, $r_{m,t}$, and Z_t are real-valued. The threshold variable Z_t is assumed to have a continuous distribution. Threshold regression has the same format as in equation (3).

We can rewrite equation (3) in the following form:

$$r_{t+1} = \theta' x_{t+1} + \delta' x_{t+1}(\lambda) + e_{t+1} \tag{4}$$

where $x_{t+1} = r_{m,t+1}$, $x_{t+1}(\lambda) = x_{t+1} 1_{\{z_t \leq \lambda\}}$, $\theta = \beta_2$, and $\delta = \beta_1 - \beta_2$.

The results can further be generalized to the case where only a subset of parameters switches between the regimes and to the case where some regressors only enter in one of the two regimes. Also, λ takes values in a bounded subset of the real line, Γ. This applies to the case of our conditioning variable RVIX, which is bounded below by zero by definition. We assume r_{mt}, Z_t, and e_t are strictly stationary ergodic and ρ-mixing, with ρ-mixing coefficients satisfying $\sum \rho_m^{1/2} < \infty$. The ρ-mixing assumption controls the degree of time series dependence and allows the processes to be autocorrelated and heteroskedastic, and is sufficiently flexible to embrace many non-linear time series processes, including threshold autoregressions.[11]

2.1.1 Testing for a Threshold

We use the heteroskedasticity-consistent Lagrange Multiplier (LM) test for a threshold, as in Hansen (1996). We test for the null of H_0: $\delta = 0$ against H_1: $\delta \neq 0$. If the null is rejected, this implies a significant change in betas with respect to levels above or below threshold RVIX.

[11] See Hansen (2000) for detailed explanations related to the assumptions.

For all λ ∈ Γ we have the following LM statistics for the null of no threshold:

$$LM_T(\lambda) = T\left[R\breve{\gamma}(\lambda)\right]'\left[R\breve{V}_T^{*}(\lambda)R'\right]^{-1}\left[R\breve{\gamma}(\lambda)\right],$$

where,

$$R = [0, I],$$
$$\breve{\gamma}(\lambda) = \left[\breve{\theta}(\lambda)', \breve{\delta}(\lambda)'\right]' = \left[\sum_{t=1}^{T} x_{t+1}^{*}(\lambda)x_{t+1}^{*}(\lambda)'\right]^{-1}\left[\sum_{t=1}^{T} x_{t+1}^{*}(\lambda)r_{t+1}\right],$$
$$\mathrm{x}_{t+1}^{*}(\lambda) = \left[x_{t+1}, x_{t+1}(\lambda)\right],$$
$$\breve{\mathrm{V}}_{\mathrm{T}}^{*}(\lambda) = M_T(\lambda)^{-1}\tilde{V}_T(\lambda)M_T(\lambda)^{-1},$$
$$\mathrm{M}_{\mathrm{T}}(\lambda) = \frac{1}{T}\sum_{t=1}^{T} x_{t+1}^{*}(\lambda)x_{t+1}^{*}(\lambda)',$$
$$\tilde{\mathrm{V}}_{\mathrm{T}}(\lambda) = \frac{1}{T}\sum_{t=1}^{T} x_{t+1}^{*}(\lambda)x_{t+1}^{*}(\lambda)'\tilde{e}_{t+1}^{\,2},$$

and where $\tilde{e}_t$ is obtained from the restricted least squares. One limitation of the above Lagrange Multiplier test is the large sample limit for the sup-LM, which is not nuisance free because the threshold is not identified under the null of no-threshold effect. Because of this issue, Hansen (1996) suggests a bootstrap analog of the sup-LM test and shows that this bootstrap method yields asymptotically correct p-values. In this paper, we use the bootstrap analog following the steps outlined in Hansen (1996).

2.1.2 Estimation

This section presents the estimation of the unknown threshold parameter, λ. We slightly modify the model in Equation (4) to have:

$$r_{t+1} = \theta' x_{t+1} + \delta_T' x_{t+1}(\lambda) + e_{t+1}, \quad \mathrm{t} = 1, \ldots, \mathrm{T} - 1, \qquad (5)$$

where δ_T is the “threshold effect”. The only modification is we let $\delta_T \to 0$ as $T \to \infty$ in order to have a nuisance free parameter asymptotic distribution. However, confidence intervals for λ can be built even when the threshold effect does not decrease with the sample size. We can rewrite Equation (5) in a matrix form where X and X_λ are $T \times 2$ matrices and R is a Tx1 vector:

$$R = X\theta + X_\lambda \delta_T + e \tag{6}$$

We use least squares (LS) estimation:

$$S_T(\theta, \delta, \lambda) = (R - X\theta - X_\lambda \delta)'(R - X\theta - X_\lambda \delta) ,$$

where S_T is the sum of squared errors. To estimate the slope parameters and the threshold parameter, we observe that first, given λ, equation (5) is linear in θ and δ_T. We can have the conditional LS estimates $\hat{\theta}(\lambda)$ and $\hat{\delta}_T(\lambda)$ by regressing Y on [X X_λ]. Then, setting

$$S_T(\lambda) = S_T(\hat{\theta}(\lambda), \hat{\delta}(\lambda), \lambda) ,$$

the estimate of threshold parameter $\hat{\lambda}$ can be uniquely defined as

$$\hat{\lambda} = \arg\min S_T(\lambda). \tag{7}$$

where λ is minimized over the set $\Gamma_T = \Gamma \cap \{Z_1, \ldots, Z_T\}$. Therefore, λ can be derived by less than T function evaluations. The asymptotic distribution for the threshold estimate follows from Hansen’s (2000) Theorem 1.

3. Data

Market return data is from Center for Research in Security Prices (CRSP) value-weighted market index for all NYSE, AMEX, and NASDAQ stocks. The risk-free rate is the one-month T-Bill rate obtained from Ibbotson Associates. Data on VIX and VXO is obtained from Chicago Board Options Exchangeǒs (CBOE). The sample covers the period from January 1986 to December 2012, with a total of 324 months.[12]

The test portfolios consist of stocks sorted according to their market capitalizations, and book-to-market ratios. More precisely, we use 10 portfolios sorted according to their market capitalizations, 10 portfolios sorted according to their book-to-market ratios, and 2 factor portfolios SMB and HML.[13] For cross-sectional tests performed in Section 4.3, we use 25 portfolios (5x5) sorted according to size and book-to-market ratios and 30 industry portfolios, all of which are downloaded from Kenneth Frenchǒs data library.[14]

In order to proxy investorsô expectations about the evolution of near-term aggregate volatility, we use monthly range of the VIX index (RVIX).[15] Similar to Chou (2005), we define RVIX in a given month as:

[12] VIX data is available from January 1990 onwards. In order to have as much data as possible, we use the VXO index (which is based on S&P 100 index options) from January 1986 to December 1989, and the VIX index from its introduction in January 1990 onwards. The results remain unaffected when we limit the sample period to 1990-2012 using the VIX index only, or omitting VIX and using VXO through the 1986-2012 period.

[13] SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios, and HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios.

[14] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[15] We have also tested a battery of volatility measures proposed in the literature ranging from statistical and GARCH based measures of volatility to forward-looking measures such as monthly change in the VIX index and S&P 500 at-the-money straddle returns. RVIX and straddle returns had the most significant results among all competing volatility measures implying that forward looking measures of volatility are much better in capturing investorsô time-varying risk-return tradeoffs.

$$RVIX_t = Max\{VIX_\tau\} - Min\{VIX_\tau\}, \tau = 1,2, \ldots, T \qquad (8)$$

where τ denotes trading days in a given month, and t denotes months. Taking the difference between the maximum and minimum level of VIX index in a given month, RVIX summarizes investors' expectations regarding changes in near-term aggregate volatility.

Range based volatility measures have gained recent interest, and they fare quite well in predicting future volatility.[16] To the best of our knowledge, this is the first study to propose a range based measure of the VIX index. Combining the explanatory power of VIX and its information content about future market conditions with the predictive power of range based volatility measures, the proposed RVIX measure is expected to have implications regarding asset pricing, portfolio allocation and stock return predictability.

Table 1 reports the summary statistics of RVIX and the market portfolio. Looking at the mean (6.66), median (4.98), the minimum (0.92) and the maximum (129.04) of the RVIX, one can say that VIX index (thus expectations of near-term market volatility) is quite stable and does not move significantly in most of the months during our sample period. Without much surprise, the maximum level of RVIX was recorded in October 1987, where the VXO index skyrocketed from its minimum value on 3rd October 1987 of 21.15 points to its historical maximum of 150.19 points on black Monday. Finally, similar to the negative correlation documented in previous studies between the VIX index and market returns, the correlation between RVIX and the market is -0.41.

<< Insert Table 1 about here >>

[16] See Alizadeh et al. (2002), Chou (2005), Brandt and Jones (2006), Chou and Liu (2010), and Harris et al. (2011) for articles that motivate the use of range based volatility measures in different settings.

3.1 Some Stylized Facts

Before moving on to tests of the proposed V-CAPM, this section documents some stylized facts about the chosen threshold parameter, market returns, and the empirically documented size and value vs. growth anomalies.

First, looking at Figure 1, one can see that our proposed conditioning variable RVIX indeed tracks significant negative market moves. Given the empirical evidence that negative market moves are most associated with increases in aggregate volatility, our novel measure RVIX is capable of providing a relationship between the evolution of near-term market volatility and downward market moves.

<< Insert Figure 1 about here >>

<< Insert Table 2 about here >>

Next, we conduct a simple exercise to examine returns on different size and book-to-market portfolios in different volatility regimes in more detail. Using threshold estimates of the RVIX index, we divide the sample into two regimes, i.e. regime 1 (2) represents calm (volatile) months where RVIX is below (above) the estimated threshold level for the associated portfolio. This way of decomposing returns into calm and volatile month gives us interesting insights regarding aggregate volatility and portfolio return dynamics. For example, looking at columns 5 and 10 of Table 2, one can see that asset classes, regardless of their portfolio characteristics, lose much more when market is highly volatile. This is in line with Hsu and Liu (2009) who document that volatile periods coincide with bear markets. On the other hand, columns 2, 3, 7 and 8 document the typical size and value vs. growth anomalies. More particularly, static CAPM fails to offer a clear and linear relationship between betas and portfolio returns, i.e. high (low) returns are not always justified by high (low) CAPM betas. However, looking at columns 4, 5, 9, and 10 of Table 2, one can gain interesting insights. For example, in calm months (when RVIX is below the estimated threshold), one can see almost a monotonous decrease in returns going from small and value portfolios through big and growth portfolios. The opposite is true for volatile episodes, where small and value portfolios become the worst performers. Despite their higher average returns relative to big and growth portfolios, small and value stock portfolios become the worst performers at times of high volatility and when the market is doing badly.[17] On the other hand, by losing less than the market portfolio, big and growth portfolios can be seen as relatively safer asset classes during volatile market conditions.

The preliminary findings informally confirm our hypotheses that size and book-to-market portfolios have different sensitivities to market risk during periods of high and low volatility. A static CAPM is unable to capture this time variation in risk and expected returns, thus, an asset pricing model that correctly takes into account this volatility-based time variation in risk and returns is expected to do better in pricing and in explaining size and value vs. growth anomalies.

4. Tests of V-CAPM

We begin by examining whether there are statistically significant regime shifts in betas due to changes in investorsô expectations of aggregate volatility risk and market conditions. Our conditioning variable is range of the VIX index. Table 3 reports the associated bootstrap p-values

[17] This also holds for the zero-cost SMB and HML portfolios, which earn on average 58 and 71 basis points per month during calm market conditions, but which become extremely risky strategies and lose 269 and 231 basis points, respectively, during volatile markets.

for the sup-LM test. The null hypothesis is that there is no significant regime shift in portfolio betas. According to bootstrap p-values presented in Table 3, there are significant regime changes in betas of most portfolios. For example, for portfolios sorted with respect to market capitalizations, eight out of ten experience significant changes in their betas between high and low volatility periods. For portfolios sorted with respect to book-to-market ratios, the evidence indicates a regime shift in betas of seven out of ten portfolios. SMB and HML portfolios also exhibit significant regime shifts in betas. Taking into account that the threshold parameter RVIX captures investorsô expectations about changes in near-term volatility, investorsô expectations about future volatility seems to be an important determinant of their assessment of aggregate risk conditions and an assetôs sensitivity to overall market risk. The results, if persistent, offer new evidence and an alternative explanation to the empirically observed size and value vs. growth anomalies.

<< Insert Table 3 about here >>

The conditional CAPM models may have a tendency to overstate the time variation, and as a result, continuous approximations of CAPM produce highly volatile beta estimates. This is further confirmed with the evidence reported in Braun, Nelson, and Sunier (1995), who use a bivariate EGARCH model to estimate conditional betas and document weak evidence of time variation. On the other hand, our methodology with RVIX as a conditioning variable suggests that portfolio betas are stable during different volatility regimes, however investors update their beta estimates when their expectations regarding the evolution of near-term volatility change considerably. Our findings are also in line with Ghysels (1998), who argues that betas change through time discretely and slowly.

4.1 Relationship Between Expected Aggregate Volatility and Beta

Having detected significant regime shifts in betas for most of the portfolios, we proceed to test the magnitude of this change, and estimate market betas and their associated threshold parameters under high and low aggregate volatility regimes. Table 4 reports the static CAPM betas, betas estimated via the V-CAPM in low (regime 1) and high (regime 2) volatility regimes, together with the threshold estimate of RVIX, which determines the change in the level of aggregate volatility, above (or below) which investors re-assess a stockôs riskiness.

Before going into detailed analysis of portfolio betas Table 4, looking at the last column of Panel A, one can see that the estimated threshold level of RVIX is very stable across size portfolios, which is estimated at 9.33 in 7 of the 11 cases.[18] Given that RVIX can take on any positive real number, this consistent level of the threshold estimate affirms the robustness of the threshold estimation procedure, the proposed model, and the chosen threshold parameter, RVIX. Next, a detailed analysis of columns 3 and 4 of Panel A reveals important insights about how the riskiness of different size sorted portfolios changes from one volatility regime to the other. We note significant changes in beta risk of size sorted portfolios. In particular, betas of small stock portfolios increase considerably at times of high volatility. Furthermore, it is only the biggest decile portfolio, which exhibits a decrease in its beta during volatile market episodes.

<< Insert Table 4 about here >>

[18] The threshold levels of 17.69 and 6.07 for deciles 6 and 8 might seem as big deviations from 9.33 at first sight, however, note that these are the two portfolios where the sup-LM test was unable to detect significant regime changes. This is also the case for decile 8 of book-to-market sorted portfolios, which is detected as a portfolio with insignificant regime shift and has a relatively high threshold estimate of 15.00.

The above findings imply that investors re-assess the riskiness of size sorted portfolios when range of the VIX index is above (or below) the threshold level of 9.33. For example, when aggregate volatility is expected to increase significantly (i.e. when RVIX is above the threshold), investors re-estimate the beta for the smallest decile portfolio, and update it from 0.91 in low volatility periods to 1.24 in high volatility periods. Similarly, the riskiness of the biggest decile portfolio changes when the RVIX is above (or below) the threshold level of 9.33. More specifically, the beta for the biggest portfolio drops from 0.98 in low volatility periods to 0.92 in high volatility periods. Furthermore, the beta differential between the smallest and biggest portfolios (SMB) increases from -0.06 in the low volatility regime to 0.30 in the high volatility regime. Our findings imply that the sensitivity of an assetôs return with respect to the level of aggregate volatility is an important determinant of an assetôs riskiness. This has clear implications on pricing and portfolio allocation. For example, by having a lower covariance with the market at times of high volatility, biggest decile portfolio tends to lose less than any other size-based strategy during volatile periods. Also given that volatile episodes usually coincide with downward market moves and recessions, a strategy invested in the biggest decile portfolio appears to be relatively less risky for risk-averse investors, who are reluctant to lose wealth during those times. This implies a demand for big stocks, thus pushing their prices up and resulting in lower average returns. Similarly, investors require a premium for holding stocks in small portfolios because their risk goes up when near-term aggregate volatility is expected to increase. Because increases in aggregate volatility are mostly associated with bad market conditions and deteriorations in investor wealth, by correlating highly with the market at times of high aggregate volatility, small stocks are viewed as riskier at times when extra dollar of loss is much more important.

Panel B of Table 4 offers similar results for potfolio sorted with respect to book-to-market ratios. Value portfolios have consistently higher betas at times of high volatility, whereas it is only the growth portfolio whose beta decreases during volatile times. The results indicate significant time variation in the risk assessments of value and growth portfolios with respect investorsô expectations about changes in near-term aggregate volatility. Investors view value stocks much riskier because they have a higher correlation with the market at times of high volatility. Similarly, growth portfolio strategies tend to be less risky at those times. The results are in line with Lettau and Ludvigson (2001) and Petkova and Zhang (2005), who also document time variation in riskiness and expected returns of value and growth stocks, in conditional CCAPM and conditional CAPM settings, respectively.

4.2. Relationship Between Expected Aggregate Volatility and Risk-Adjusted Returns

The documented evidence so far indicates that asset betas change significantly between volatility regimes. Furthermore, the proposed V-CAPM reveals a distinctive pattern regarding change of beta risk among different asset classes. More particularly, small market capitalization and high book-to-market (value) portfolios become riskier at times of high aggregate volatility. On the other hand, big market capitalization and low book-to-market (growth) portfolios become less risky at those times. The findings of our model offer a potential remedy to the static CAPM and its failure in explaining the well documented size and value vs. growth anomalies. In order to examine the robustness of the proposed volatility-based time-varying beta risk explanation, and to see whether investorsô expectations about changes in aggregate volatility has a similar time-varying effect on risk-adjusted returns, we next compare Jensenôs alphas and Sharpe ratios withinin the full sample, and in low and high volatility regimes determined by the threshold level of RVIX implied by the proposed V-CAPM.

4.2.1 Comparison of Jensenǿs alphas

Table 5 presents Jensenǿs alphas for different test assets. It is well-documented that small and value stocks (and the associated SMB and HML strategies) on average produce significantly higher returns than their large and growth counterparts.[19] However, looking at Jensenǿs alphas and betas of different size and book-to-market portfolios, we confirm previous studies that static CAPM is unable to offer a risk-based explanation to these abnormal returns. On the other hand, alphas implied by different volatility regimes help us uncover an important aspect of size and value vs. growth puzzles, offering a volatility-based time-varying risk explanation. We document that size and value strategies pay off at times of low volatility yielding significant and positive risk-adjusted returns. However, the trade-off for these strategies is that they have extremely bad (significant and negative) risk-adjusted returns at times of high volatility. For example, a strategy invested in the smallest decile portfolio earns an average risk-adjusted return of 56 basis points during calm months, whereas the same strategy yields a risk-adjusted return of -211 basis points in volatile months. Similarly, a strategy invested in the highest book-to-market (value) portfolio earns an average risk-adjusted return of 59 basis points during calm months, but yields an average risk-adjusted return of -142 basis points in volatile months. SMB and HML strategies also yield similar and significant risk-adjusted returns over calm and volatile periods. On the contrary, although strategies in biggest and lowest book-to-market (growth) portfolios disappoint

[19] Although excess returns on small stocks over big stocks have been disappearing during the last two decades, excess returns on value stocks over growth stocks have been significantly persistent over years.

their investors in calm months with average risk-adjusted returns of 8 and 24 basis points, respectively, these strategies yield positive and significant risk-adjusted returns in volatile months (24 and 75 basis points, respectively).

<< Insert Table 5 about here >>

4.2.2 Comparison of Sharpe ratios

Next, we look at another popular measure of risk-adjusted return proposed by Sharpe (1966, 1975). Sharpe ratio is a commonly used measure to track the performance of mutual funds and it can be easily applied to measure the reward-to-variability of any investment asset or portfolio. By scaling an assetôs excess return to the standard deviation of excess returns on the asset, it is an ideal way of measuring of reward-to-variability of a managed fund and the sensitivity of returns on an investment asset or a trading strategy per unit of risk taken.[20] The measure is model free, hence it provides an indirect test for the robustness of our chosen volatility parameter RVIX, as we will compare the Sharpe ratios of test assets within the whole sample with those obtained in two different volatility regimes determined by RVIX. Analyzing reward-to-variability ratios in different volatility regimes will give us further insight about the risk-return dynamics of the test assets with respect investorsô expectations regarding the evolution of near-term volatility.

<< Insert Table 6 about here >>

[20] The excess return on the asset can be on any benchmark such as the S&P 500 returns or the risk-free rate. As in most studies, we choose returns in excess of the risk-free rate to measure an assetôs excess return.

Looking at Panel A of Table 6, within the full sample there is no clear pattern in Sharpe ratios of portfolios sorted with respect to market capitalizations. One can even say that an investment strategy based on stocks in the smallest size decile commands a lower reward per unit of risk taken as opposed to a strategy based on stocks in the highest decile. This is not consistent with a rational risk-based explanation, where one would expect from stocks with higher risk to compensate their investors for the risks that they are taking. On the other hand, when we decompose the sample into two volatility regimes determined by the RVIX, we see different risk-return dynamics across size sorted portfolios in different volatility regimes. In calm periods, the strategy in the smallest decile portfolio commands a higher reward-to-variability ratio compared to the biggest decile portfolio (0.2727 vs. 0.2250), however in volatile periods, the situation is reversed, investors experience a much worse reward-to-variability ratio for the smallest decile portfolio against the biggest decile (-0.4513 vs. -0.1727). This different pattern in Sharpe ratios is also consistent with our previous results documenting significant differences in betas and Jensenôs alphas of those strategies and explains why investors would want to be compensated for the extra risk that they are taking by investing in small stocks.

We observe a similar pattern for the Sharpe ratios of portfolios sorted with respect to book-to market ratios. Although there is not a significant in Sharpe ratios in the full sample, we document that value portfolios command higher (lower) reward-to-variability ratios compared to growth portfolios in calm (volatile) periods, offering a coherent volatility-based risk-return explanation to the empirically documented value vs. growth anomaly.

The results confirm our hypothesis that marketôs expectation of changes in aggregate volatility is an important determinant of investorsô assessment of risk and expected returns. Changes in betas and risk-adjusted returns explain why small and value stocks on average earn higher returns than their big and growth counterparts - they are risky strategies when volatility in the market is high and investors get compensated for the risk that they are taking. By conditioning asset returns using a novel forward-looking volatility measure (RVIX), which summarizes investorsô expectations about the evolution of near-term aggregate volatility, the proposed V-CAPM offers a volatility-based time-varying risk explanation to the size and value vs. growth anomalies.

4.3. Comparison of Root Mean Square Errors

In order to check the robustness of the above results, we further calculate the pricing errors of the proposed threshold V-CAPM as described by equation (4), and compare them with the pricing errors of the unconditional CAPM, and with the Fama-French (FF) three-factor model (1992). The following Root Mean Square Error (RMSE) formula is used to calculate the pricing errors of each model:

$$RMSE = \left[T^{-1} \sum_{t}^{T} \left(r_{i,t} - \hat{r}_{i,t} \right)^2 \right]^{1/2} . \tag{9}$$

Table 7 reports in-sample root mean square pricing errors for the three models. One can see that pricing errors of the V-CAPM are always smaller than those of unconditional CAPM regardless of portfolio characteristics, indicating that the proposed model not only offers a potential explanation to the well-documented size and value anomalies but also does a better job in pricing than the static CAPM. However, one should also note that the benchmark FF 3-factor model is always superior in yielding lowest pricing errors among all asset classes.

<< Insert Table 7 about here >>

4.4. Fama-MacBeth (1973) Tests for Risk Premia

There is now a consensus on time variation in market risk. The conditional CAPM is an attempt to capture this variation. However, Ghysels (1998) shows that the conditional CAPM is unable to specify time variation accurately, leading to higher pricing errors compared to the unconditional CAPM. In view of these findings, we believe that it is crucial to understand the true dynamics of time variation in beta risk and incorporate this dynamics in the pricing model. Our previous findings establish that beta risk exhibits significant shifts triggered by changes in investorsô expectations regarding the evolution of near-term aggregate volatility. Hence, the next natural step would be to analyze whether this risk is priced in the cross-section.

We employ standard Fama-MacBeth (1973) two-pass regressions. The full model to be tested is,

$$r_t^i = \alpha^i + \beta^i_{MKT,j}\lambda_{MKT,j} + \beta^i_{RVIX,j}\lambda_{RVIX,j} + \beta^i_{SMB,j}\lambda_{SMB,j} + \beta^i_{HML,j}\lambda_{HML,j} + \varepsilon^i_t, \quad j = 0,1,2 \qquad (10)$$

where əĉs represent unconditional prices of risk for various factors, and j = 0, 1, and 2 represent full sample, regime 1 (low volatility) and regime 2 (high volatiltity), respectively.

In the first pass, portfolio betas are estimated from a single time-series using the full sample.[21] In the second pass, a cross-sectional regression is run at each time period, with full-sample betas obtained from the first pass regressions. The associated estimates for the intercept term, Ǔ, and the risk premia, əĉs, are given by the average of those cross-sectional regression

[21] More specifically, we use 55 test portfolios, i.e. 25 (5x5) portfolios sorted with respect to market capitalization and book-to-market ratios and 30 industry portfolios, all of which are downloaded from Ken Frenchĉs data library.

estimates. Table 8 summarizes the risk premium estimates of associated models implied by Equation (10).

<< Insert Table 8 about here >>

We test 11 different specifications of Equation (10). The first row represents the market model. Consistent with earlier findings, CAPM is not a true representation of the pricing kernel of the economy. The market risk premium is negative and insignificant, and a single factor market model poorly explains the cross-section of returns with an adjusted R^2 of 12%. The second row estimates the price of market risk in low volatility regime. Consistent with previous findings, the proposed V-CAPM is not successful in establishing a significant relationship for the price of market risk when market volatility is low. Row 3 estimates the price of market risk in high volatility regime. We document a negative and significant market risk premium when the market exhibits high volatility. Previous studies argue that market risk factor is not sufficient to capture sensitivity of stock returns to innovations in volatility. By including volatility risk as a separate risk factor, Ang et al. (2006) and Moise (2007) find a negative price for volatility risk. We show that when markets are divided into low and high volatility regimes using our threshold parameter, market risk captures this negative price of volatility risk. Furthermore, an adjusted R^2 of 0.40 indicates that the proposed V-CAPM performs much better than the static CAPM in explaining the cross section of returns in volatile markets.

To see whether the above result is robust and to make sure the cross-sectional tests are not suffering from an omitted variables bias, we test the proposed V-CAPM in the presence of a separate volatility risk factor based on RVIX, SMB and HML factors. Rows 5 and 7 reveal that the market risk premium implied by the proposed V-CAPM is still insignificant in calm markets neither in the presence of a separate volatility risk factor nor SMB and HML factors. However, Rows 6 and 8 indicate a consistent negative price for market risk in volatile markets, which is robust to addition RVIX, SMB and HML factors. Furthermore, Shanken (1992) corrected t-statistics for the intercept term becomes insignificant for specification 8, which confirms our previous findings that the proposed model does a good job in pricing especially in volatile episodes of the market. Rows 9, 10, 11 present the results for the most comprehensive models, including market risk, volatility risk, SMB and HML factors, and the results are similar.

A negative and significant price for market risk at volatile times implies that the market is expected to have returns lower than the risk-free rate at those times. This finding might seem counter-intuitive at first sight and at odds with theoretical predictions of CAPM and relevant risk-based asset pricing theories where investors should be compensated with a positive risk premium for holding risky assets such as the market portfolio. However, the history of U.S. stock returns shows us that although the equity risk premium is positive on average in the long-run, investors also witnessed episodes of negative market risk-premium. For example, according to data compiled by Shiller (2013), equity risk premium has been negative around 75% of the time during the period starting with Nifty Fifty of the 1970ŝ to the end of 2007 just before the crisis.[22] One explanation for the negative market risk premium is that investors expect rapid earnings growth for the stock market to compensate them for the additional risk of holding equities. This would result in the bidding up of share prices and a consequent decline in the equity risk premium. Furthermore, Campbell (2007) shows that valuation ratios and the cross-section of stock prices fell considerably implying negative levels of equity premium in the late

[22] http://www.econ.yale.edu/~shiller/data.htm

20th century.[23] Damodaran (2012) further highlights the importance of the sampling period, documenting a negative market risk premium over the 2002-2011 period. Overall, regarding the sample period of our study, the significant negative market risk premium that we document during volatile episodes of 1986-2012 is consistent with the above studies.

A significant and negative price for market risk during volatile episodes of the market implies that equity investors are expected to have returns lower than the risk-free rate at those times. In addition, stocks with higher betas are expected to lose even more at those times. Thus, risk-averse agents will demand additional compensation for holding stocks that have high sensitivities to market risk at volatile periods when the price of market risk is significantly negative. In other words, a stock whose return correlates highly with market returns at times of high aggregate volatility will be deemed as riskier. To give an example, because of its high beta at volatile times, stocks in the smallest decile portfolio are expected to lose on average 3.10% [4.66 + 1.235*(-6.28)] per month when the market is expected to experience high volatility, i.e. RVIX is above 9.33. Again, due to differences in their sensitivities to aggregate volatility, a portfolio which longs the smallest decile firms and shorts the biggest decile firms will be expected to lose additional return of 1.96% per month [{4.66 + 1.24*(-6.28)}-{4.66 + 0.92*(-6.28)}]. Overall, the results imply that the market risk-premium is time-varying and the cross-section of stock returns exhibits a negative market risk premium when the market is highly volatile.

[23] Campbell and Shiller (1998) also find extraordinarily low valuation ratios during late 1990ǒs resulting in negative regression forecasts of the equity premium. Furthermore, using expected real stock returns and expected real bond returns, Arnott and Bernstein (2002) document similar negative levels of market risk premium during most of the 1983-2002 period.

4.5. Alternative Measures of Volatility and Further Robustness Tests

To test whether the sensitivity of results to alternative measures of volatility, we repeat tests presented in previous sections using several historical, statistical and option-based measures as a proxy for aggregate volatility. These include the standard deviation of market returns, squared returns, monthly range of the market index (given by the maximum and minimum level of the market index in a given month), GARCH (1, 1) volatility, level of the VIX index, change in VIX index (φVIX), and monthly returns on at-the-money straddles written on the S&P 500 index. Neither of the four backward-looking statistical measures, nor two of the forward-looking option-based measures (VIX and φVIX) proved to yield significant results to model time variation in betas.[24] However, monthly ATM S&P 500 straddle returns produce very similar results to those obtained by range of the VIX index.[25]

The results using monthly returns on ATM S&P 500 straddles as the conditioning volatility proxy can be summarized as follows.[26] Similar to previous findings, most portfolios exhibit significant bootstrap p-values indicating a significant change in beta risk due to changes in returns on ATM straddles. Second, the direction of change in betas is also striking. Similar to results obtained using RVIX, we observe an increase in beta risk for small and value portfolios and a decrease in the risk of big and growth portfolios at times of high volatility, confirming our

[24] This confirms our argument that forward-looking market-based volatility measures do a better job in capturing investorsô expectations on aggregate market risk compared to statistical measures. The results also lend indirect support to Ang et al. (2006) who find that statistical measures of aggregate volatility, such as sample volatility, extreme value volatility estimates, and realized volatility estimates, do not produce enough spread in the cross-section.

[25] The significance of ATM straddle returns over φVIX supports Cremers et al. (2012) who document that φVIX loses its significance in capturing volatility risk premium in the presence of measures constructed using ATM straddle returns.

[26] The results in detail related to ATM straddle returns are available upon request.

explanation that investors see small and value stocks much riskier in volatile market episodes, which usually coincides with deteriorations in investment opportunities and reductions in wealth.

We further examine Jensenôs alphas, Sharpe ratios and the pricing performance of the proposed V-CAPM using ATM straddle returns as the conditioning variable and confirm that our volatility-based time-varying risk explanation to the observed size and value vs. growth anomalies is robust to the use of an alternative forward-looking market based volatility measure. Finally, looking at the cross-sectional price of risk, we find that market risk is priced in the cross-section at times of high volatility and is negative.

5. Conclusion

We propose an asset pricing model where the covariance of an assetôs return with market return changes discretely at different points in time. This change is due to investorsô assessment of expected changes in near-term aggregate volatility. We argue that risk-averse investors use information embedded in the VIX index (and S&P 500 straddle returns) to assess changes in an assetôs riskiness and expected returns with respect to time-varying risk conditions. This forward-looking information becomes crucial in pricing and assessing beta risk. Proxying investorsô expectations about the evolution of near-term volatility with the range of the VIX index, RVIX, (and S&P 500 straddle returns) we document the following.

There exists significant time variation in market betas with respect to changes in aggregate market volatility. In particular, small market capitalization and value portfolios have consistently higher betas at times of high volatility compared to big market capitalization and growth portfolios. Moreover, the beta dispersions between small-big and value-growth portfolios are negative during low volatility periods and positive during high volatility periods. Because they correlate more with the market especially at times of high aggregate volatility, small and value portfolios are viewed as riskier than big and growth portfolios in bad times when aggregate market volatility is high. This volatility-based risk explanation is further confirmed by the risk-adjusted returns implied by the V-CAPM. During calm periods, small and value portfolios earn on average higher risk-adjusted returns than big and growth portfolios, however they become extremely risky strategies at volatile times with negative and significant risk-adjusted returns. Furthermore, the pricing errors for the proposed threshold V-CAPM are lower than the static CAPM implying that it does a better job in pricing. Finally, the proposed V-CAPM cannot establish a significant market risk premium and still suffers from the problems of static CAPM at times of low volatility. However, we are able to identify a negative market price of risk at times of high volatility.

High volatility periods are usually associated with downward market moves and recessions, i.e. periods when investors are more averse to losing an extra dollar from their wealth. Due to their significantly higher betas and negative risk-adjusted returns at times of high aggregate volatility, small and value portfolios are expected to experience far worse returns than big and growth portfolios when volatility is high. This volatility-based time-varying risk phenomenon can explain why small and value stocks generally have higher returns, because they are seen as riskier compared to their big and growth counterparts. The results support the view of a risk-based rational asset pricing theory and offers a volatility-based threshold CAPM where asset return sensitivities to market risk change discretely in time with respect to innovations in investorsô expectations regarding near-term aggregate volatility.

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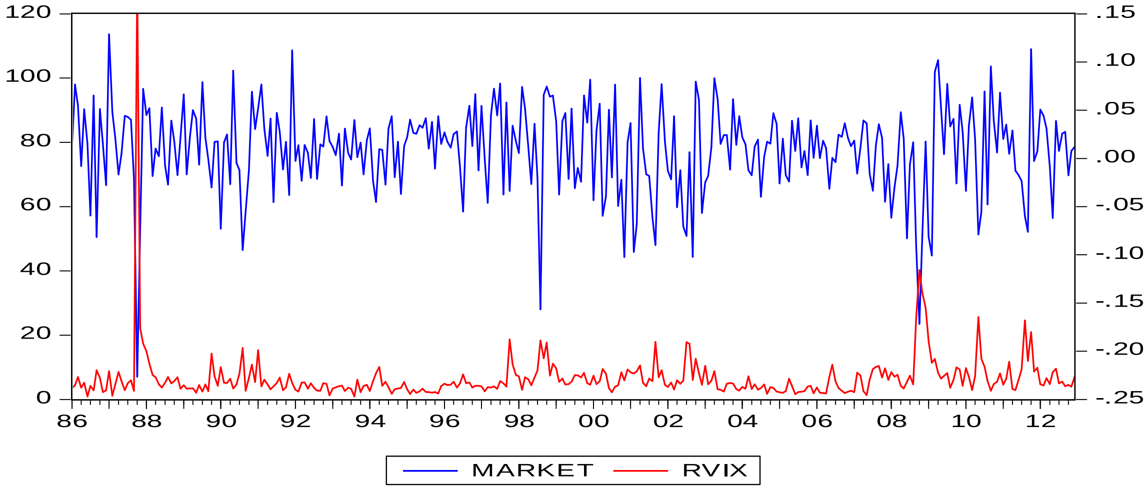
Table 1
Descriptive Statistics

	MKT	RVIX
Mean	0.89	6.66
Median	1.49	4.98
Maximum	12.88	129.04
Minimum	-22.64	0.92
Std.Dev.	4.61	8.36
Skewness	-0.89	10.25
Kurtosis	5.48	144.02
MKT	1	
RVIX	-0.41	1

Note: This table reports the descriptive statistics for monthly returns on the market portfolio (MKT), and monthly range of VIX index (RVIX). The market portfolio is the CRSP value-weighted index for all NYSE, AMEX, and NASDAQ stocks. RVIX is the difference between maximum and minimum level of VIX in a given month. The sample covers the period from January 1986 to December 2012 (324 months). VXO data based on S&P 100 index options is used for the period covering January 1986 to December 1989. All return figures are given in percentages.

Figure 1. Time-series of RVIX and market returns



Note: This figure plots the monthly time-series of RVIX (red line, left axis) and market returns (blue line, right axis) from January 1986 through December 2012 (324 months). The market portfolio is the CRSP value-weighted index for all NYSE, AMEX, and NASDAQ stocks, and RVIX is the monthly range of VIX index, which is defined as the difference between the maximum and minimum level of the VIX index within a calendar month.

Table 2. Stylized facts about portfolio returns

Size	Beta	Full sample	Regime1 (Calm)	Regime2 (Volatile)	B/M	Beta	Full sample	Regime1 (Calm)	Regime2 (Volatile)
Small	1.0163	0.0097	0.0176	-0.0365	High	1.0557	0.0115	0.0187	-0.0298
Decile2	1.1604	0.0096	0.0170	-0.0330	Decile2	0.9864	0.0107	0.0160	-0.0203
Decile3	1.1434	0.0104	0.0173	-0.0297	Decile3	0.9716	0.0091	0.0149	-0.0239
Decile4	1.1262	0.0094	0.0158	-0.0271	Decile4	0.9512	0.0102	0.0147	-0.0156
Decile5	1.1380	0.0104	0.0164	-0.0240	Decile5	0.9467	0.0090	0.0135	-0.0167
Decile6	1.0713	0.0104	0.0157	-0.0213	Decile6	0.9029	0.0096	0.0143	-0.0179
Decile7	1.0673	0.0108	0.0164	-0.0205	Decile7	0.8487	0.0098	0.0153	-0.0221
Decile8	1.0795	0.0103	0.0156	-0.0203	Decile8	0.8445	0.0102	0.0141	-0.0124
Decile9	1.0101	0.0101	0.0148	-0.0170	Decile9	0.9228	0.0093	0.0129	-0.0109
Big	0.9449	0.0087	0.0118	-0.0096	Low	1.0501	0.0090	0.0116	-0.0066
SMB	0.0716	0.0010	0.0058	-0.0269	HML	0.0059	0.0026	0.0071	-0.0231
Market		0.0089	0.0129	-0.0143	Market		0.0089	0.0128	-0.0143

Note: This table presents the returns on several portfolios that have been used as test assets in this study and the market portfolio during the full sample period from January 1986 through December 2012 (324 months) and two different volatility regimes. Size represents portfolios which contain stocks sorted with respect to their market capitalizations. B/M represents portfolios which contain stocks sorted with respect to their book-to-market ratios. SMB is a portfolio that is long in stocks in the smallest decile and short in stocks in the biggest decile. HML is a portfolio that is long in stocks which are in the highest B/M decile and short in stocks which are in the lowest B/M decile.

Table 3
Bootstrap p-values for 10 size and B/M portfolios

Size	RVIX	B/M	RVIX
Small	0.000***	High	0.012**
Decile2	0.000***	Decile2	0.009***
Decile3	0.000***	Decile3	0.013**
Decile4	0.015**	Decile4	0.144
Decile5	0.025***	Decile5	0.029**
Decile6	0.140	Decile6	0.067*
Decile7	0.016**	Decile7	0.008***
Decile8	0.144	Decile8	0.435
Decile9	0.056*	Decile9	0.219
Big	0.008***	Low	0.000***
SMB	0.009***	HML	0.001***

Note: This table reports the bootstrap p-values of the modified sup-LM test suggested by Hansen (1996). We test the null hypothesis of no significant regime shifts in portfolio betas due to changes in the level of aggregate volatility. RVIX is the threshold parameter. Size represents portfolios which contain stocks sorted with respect to their market capitalizations. B/M represent portfolios which contain stocks sorted with respect to their book-to-market ratios. SMB is a portfolio that is long in stocks in the smallest decile and short in stocks in the biggest decile. HML is a portfolio that is long in stocks which are in the highest B/M decile and short in stocks which are in the lowest B/M decile. Each test is estimated with monthly data from January 1986 to December 2012 (324 months) for RVIX threshold parameter. *, **, *** denote significance levels at 10%, 5%, and 1%, respectively.

Table 4
Threshold estimates for 10 size and 10 book-to-market portfolios

Panel A: 10 size portfolios				
	CAPM Beta	*Beta for Regime 1*	*Beta for Regime 2*	*Threshold Estimate*
Small	1.0163	0.9093	1.2350	8.80
Decile2	1.1604	1.0772	1.1662	8.42
Decile3	1.1434	1.0453	1.2060	9.33
Decile4	1.1262	1.0401	1.1874	9.33
Decile5	1.1380	1.0684	1.1884	9.33
Decile6	1.0713	1.0330	1.2516	17.69
Decile7	1.0673	1.0109	1.1067	9.33
Decile8	1.0795	1.0478	1.0919	6.07
Decile9	1.0101	0.9723	1.0190	9.33
Big	0.9449	0.9762	0.9223	9.33
SMB	0.0716	-0.0616	0.3026	9.33
Panel B: 10 B/M portfolios				
	CAPM Beta	*Beta for Regime 1*	*Beta for Regime 2*	*Threshold Estimate*
High	1.0557	0.9776	1.0831	9.58
Decile2	0.9864	0.9357	1.0513	9.58
Decile3	0.9716	0.9247	1.0606	9.58
Decile4	0.9512	0.9038	1.0399	9.58
Decile5	0.9467	0.8687	1.0648	11.10
Decile6	0.9029	0.8308	1.0127	11.10
Decile7	0.8487	0.7983	0.8906	10.92
Decile8	0.8445	0.7425	0.9266	15.00
Decile9	0.9228	0.8373	0.9944	10.78
Low	1.0501	1.1249	0.9892	9.33
HML	0.0059	-0.1514	0.0930	9.33

Note: This table reports the unconditional CAPM betas, the threshold beta estimates with respect to low and high volatility regimes, and their associated threshold volatility estimates, proxied by S&P 500 at-the-money straddle returns. Panels A and B present results for portfolios sorted with respect to market capitalizations, and book-to-market ratios, respectively. SMB is a portfolio that is long in stocks in the smallest decile and short in stocks in the biggest decile. HML is a portfolio that is long in stocks which are in the highest B/M decile and short in stocks which are in the lowest B/M decile. The sample covers the period from January 1986 to December 2012 (324 months). Regime 1 (2) corresponds to low (high) volatility regimes where monthly RVIX is lower (higher) than the estimated threshold level.

Table 5

Comparison of Jensenǒs alphas

Panel A: 10 Size portfolios			
	Ŭ$_{\text{CAPM}}$	Ŭ$_{\text{V-CAPM, Regime1}}$	Ŭ$_{\text{V-CAPM, Regime2}}$
Small	0.0636	0.5601	-2.1113
	(0.25)	(1.98**)	(-4.47***)
Decile2	-0.0221	0.3322	-1.5210
	(-0.11)	(1.37)	(-3.50***)
Decile3	0.0680	0.3908	-1.1887
	(0.41)	(1.96*)	(-2.85***)
Decile4	-0.0177	0.2389	-0.9651
	(-0.12)	(1.32)	(-2.48**)
Decile5	0.1753	0.2750	-0.6550
	(0.58)	(1.80*)	(-2.22**)
Decile6	0.1089	0.2543	-0.4421
	(0.94)	(2.00)	(-1.50)
Decile7	0.1521	0.3319	-0.5203
	(1.43)	(3.02***)	(-1.77*)
Decile8	0.0938	0.2158	-0.4439
	(0.92)	(1.94*)	(-1.86*)
Decile9	0.1147	0.1997	-0.2266
	(1.49)	(2.35**)	(-1.25)
Big	0.0111	-0.0792	0.3373
	(0.20)	(-1.27)	(2.37**)
SMB	-0.3088	0.2543	-2.6881
	(-0.85)	(1.33)	(-4.48***)

Panel B: 10 B/M portfolios			
	Ŭ$_{\text{CAPM}}$	Ŭ$_{\text{V-CAPM, Regime1}}$	Ŭ$_{\text{V-CAPM, Regime2}}$
High	0.2306	0.5930	-1.4147
	(0.94)	(2.26**)	(-3.40***)
Decile2	0.2241	0.4556	-0.6294
	(1.40)	(2.34**)	(-2.27**)
Decile3	0.1143	0.4352	-1.1103
	(0.61)	(2.29**)	(-2.54**)
Decile4	0.2190	0.3595	-0.3378
	(1.33)	(1.75*)	(-1.14)
Decile5	0.0443	0.1756	-0.1842
	(0.35)	(1.16)	(-0.63)
Decile6	0.1256	0.3015	-0.4080
	(0.89)	(1.77*)	(-1.21)
Decile7	0.1060	0.3309	-0.7462
	(0.73)	(1.95*)	(-2.50**)
Decile8	0.1566	0.1791	0.1433
	(1.58)	(1.75*)	(0.49)
Decile9	0.0547	0.0167	0.3516
	(0.62)	(0.15)	(1.61)
Low	-0.0222	-0.2426	0.7474
	(-0.18)	(-1.88*)	(2.66**)
HML	0.0600	0.4331	-2.4016
	(0.17)	(2.53**)	(-4.10***)

Note: This table reports Jensenǿs alphas for the unconditional CAPM and for the threshold volatility model (V-CAPM) with respect to low and high volatility regimes. Panels A and B presents results for portfolios sorted with respect to market capitalizations, and book-to-market ratios, respectively. SMB is a portfolio that is long in stocks in the smallest decile and short in stocks in the biggest decile. HML is a portfolio that is long in stocks which are in the highest B/M decile and short in stocks which are in the lowest B/M decile. The sample covers the period from January 1986 to December 2012 (324 months). Regime 1 (2) corresponds to low (high) volatility regimes where monthly RVIX is lower (higher) than the estimated threshold level. The numbers in parantheses denote the associated t-statistics with Newey-West corrected standard errors.

Table 6

Comparison of Sharpe ratios and volatilities

	S_{full}	$\mathring{u}_{full}$	$S_{Regime1}$	$\mathring{u}_{Regime1}$	$S_{Regime2}$	$\mathring{u}_{Regime2}$
Panel A: 10 Size portfolios						
Small	0.1047	6.1800	0.2727	5.2697	-0.4513	8.6296
Decile2	0.0995	6.4755	0.2486	5.5158	-0.3761	9.4052
Decile3	0.1197	6.0513	0.2833	4.9713	-0.3423	9.3709
Decile4	0.1073	5.8601	0.2605	4.8034	-0.3200	9.2201
Decile5	0.1264	5.7654	0.2774	4.7388	-0.2899	9.1022
Decile6	0.1369	5.2880	0.2894	4.3103	-0.2692	8.4899
Decile7	0.1475	5.1879	0.3159	4.1494	-0.2799	8.4778
Decile8	0.1374	5.1926	0.2912	4.2296	-0.2718	8.3341
Decile9	0.1451	4.7889	0.2965	3.8871	-0.2493	7.7732
Big	0.1238	4.4706	0.2250	3.8149	-0.1727	6.9428
SMB	-0.0664	3.2503	0.0570	3.2547	-0.5456	2.8644
Panel B: 10 B/M portfolios						
	S_{full}	$\mathring{u}_{full}$	$S_{Regime1}$	$\mathring{u}_{Regime1}$	$S_{Regime2}$	$\mathring{u}_{Regime2}$
High	0.1376	6.0808	0.3009	5.1259	-0.3601	8.9403
Decile2	0.1548	4.8706	0.3275	3.9062	-0.2871	7.8923
Decile3	0.1305	4.5919	0.3300	3.5221	-0.3430	7.6930
Decile4	0.1564	4.5171	0.3034	3.7634	-0.2560	7.0277
Decile5	0.1240	4.7423	0.2702	3.7813	-0.2403	7.9366
Decile6	0.1389	4.6370	0.2981	3.7196	-0.2668	7.6018
Decile7	0.1360	4.8830	0.3072	3.9252	-0.3128	7.8432
Decile8	0.1503	4.6758	0.2802	3.8612	-0.1958	7.5451
Decile9	0.1306	4.7558	0.2433	3.9492	-0.1732	7.6979
Low	0.1125	5.1623	0.1829	4.5820	-0.1185	7.5777
HML	0.0180	3.0837	0.0943	2.9435	-0.3368	3.5854

Note: This table reports portfolio ex-post Sharpe ratios and standard deviations for the full sample and two subsamples representing two different volatility regimes. Regime 1 (2) corresponds to low (high) volatility regimes where monthly RVIX is lower (higher) than the estimated threshold level. Panels A and B presents results for portfolios sorted with respect to market capitalizations, and book-to-market ratios, respectively. SMB is a portfolio that is long in stocks in the smallest decile and short in stocks in the biggest decile. HML is a portfolio that is long in stocks which are in the highest B/M decile and short in stocks which are in the lowest B/M decile. The sample covers the period from January 1986 to December 2012 (324 months).

Table 7
Root mean squared pricing errors for unconditional CAPM, FF three-factor model and threshold V-CAPM

	Panel A: 10 size portfolios		
	Unconditional CAPM	*FF 3-Factor Model*	*Threshold V-CAPM*
Small	4.0525	2.0303	3.9616
Decile2	3.6636	1.2056	3.5923
Decile3	2.9890	0.9409	2.9112
Decile4	2.7373	1.0262	2.6821
Decile5	2.4101	1.1109	2.3710
Decile6	1.9143	1.2770	1.9062
Decile7	1.6588	1.1893	1.6165
Decile8	1.4964	1.1474	1.4742
Decile9	1.1152	1.0228	1.0936
Big	0.9618	0.4535	0.9425
	Panel B: 10 B/M portfolios		
	Unconditional CAPM	*FF 3-Factor Model*	*Threshold V-CAPM*
High	3.6498	2.3739	3.5828
Decile2	2.3659	1.4966	2.2822
Decile3	2.4245	1.3243	2.3552
Decile4	2.2405	1.6734	2.1926
Decile5	1.8489	1.4782	1.8039
Decile6	2.0365	1.6291	2.0001
Decile7	1.9348	1.6289	1.8722
Decile8	1.6152	1.5593	1.6007
Decile9	1.3697	1.3479	1.3476
Low	1.7723	1.1737	1.7037

Note: This table reports the root mean squared pricing errors (RMSE) for unconditional CAPM, the Fama-French (1992) three-factor model and the proposed threshold V-CAPM for the period from January 1986 to December 2012 (324 months). The pricing errors are computed according to Equation (7) in Section 4. Panels A and B presents results for ten portfolios sorted with respect to market capitalizations and book-to-market ratios, respectively.

Table 8
Fama-MacBeth Risk Premium Estimates

	α_i	λ_{MKT}	$\lambda_{REGIME\,1}$	$\lambda_{REGIME\,2}$	λ_{RVIX}	λ_{SMB}	λ_{HML}	**Adj. R²**
Row 1	1.26	-0.25						0.12
	(3.88***)	(-0.58)						
Row 2	1.51		-0.30					0.20
	(3.64***)		(-0.63)					
Row 3	4.66			-6.28				0.40
	(2.30**)			(-2.45**)				
Row 4	1.16	-0.20			-1.33			0.20
	(3.40***)	(-0.48)			(-0.83)			
Row 5	1.76		-0.22		-1.49			0.14
	(5.01***)		(-0.52)		(-1.04)			
Row 6	1.47			-3.35	0.06			0.26
	(1.81*)			(-2.50**)	(0.01)			
Row 7	1.47		-0.10			0.28	0.34	0.26
	(4.94***)		(-0.27)			(1.37)	(1.79*)	
Row 8	1.00			-2.53		-1.39	-0.50	0.48
	(1.28)			(-1.85*)		(-3.24***)	(-0.88)	
Row 9	1.07	-0.11			-1.30	0.01	0.12	0.33
	(3.34***)	(-0.28)			(-0.88)	(0.04)	(0.64)	
Row 10	1.60		-0.24		-1.43	0.28	0.33	0.29
	(5.11***)		(-0.62)		(-1.10)	(1.36)	(1.74*)	
Row 11	0.80			-2.36	0.53	-1.36	-0.52	0.42
	(1.26)			(-1.77*)	(0.11)	(-3.20***)	(-0.92)	

Note: This table reports the estimates for the cross-sectional Fama-MacBeth (1973) regressions specified by equation (8), and/or subsets of it, using the excess returns on 55 test portfolios, i.e. 25 portfolios sorted with respect to market capitalization and book-to-market ratios and 30 industry portfolios. The sample period is from January 1986 to December 2012 (324 months). Regime 1 (Regime 2) corresponds to months where aggregate volatility is below (above) the threshold parameter. The numbers in parentheses are the Shanken (1992) corrected t-statistics for each coefficient estimate. The term adjusted R^2 denotes the cross-sectional R^2 statistic adjusted for the degrees of freedom.