Factors of the Term Structure of Realized Risk Premiums in Currency Forward Markets

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Abstract

We examine the term–structure of realized risk premiums in the Australian Dollar–US Dollar forward market. We find positive and time–varying risk premiums, while the magnitude and significance of the premiums increases with maturity of the contracts. We then examine the relationship between realized risk premiums and explanatory variables such as spot currency returns, the forward premium, realized variance, skewness and kurtosis of spot currency returns. Our results illustrate the significance of the considered variables for explaining the observed risk premiums. We apply principal component analysis to examine the term structure of risk premiums at different maturity levels of the forward contracts and explain the dynamics of the observed premiums through a smaller number of common factors. We obtain a four–factor model that explains over 90\% of the total variation in the term structure of risk premiums. Interestingly, the factors can be suitably labelled as ‘level’, ‘slope’ and ‘curvature’. Further, the first factor is found to be related to the realized variance of currency spot returns, the second factor can be related to the realized kurtosis, while the third factor is related to the forward premium and realized skewness of spot currency returns. Our results provide new and important insights on the behavior of currency forward risk premiums at different maturity levels.

Key Words: Currency Forward Markets, Term Structure, Risk Premiums, Principal Components, Factor Analysis.

JEL Classification: F31, G15, G13
1. Introduction

The forward premium anomaly is one of the most important and wide-spread puzzles in the area of international finance and relates to the crucial assumption that the forward exchange rate equals the expected future spot rate. In other words, a regression of the future change in spot rate at maturity on the current forward premium should yield a slope coefficient of one under the assumption of risk neutrality and rational expectations. However, the unbiasedness of the forward rate has been overwhelmingly rejected in a number of studies\(^1\) and the slope coefficient has been found to be significantly different from one, often even negative in a number of cases. In fact, as suggested by Jongen et al. (2008), it has become a well-established regularity in the international finance literature that the forward premium (discount) is a biased estimator of future changes in the exchange rates. Fama (1984) was among the first to suggest that a time-varying risk premium can be seen as the reason for the failure of unbiasedness hypothesis.

Many other studies\(^2\) have also established the presence of time-varying risk premiums in the currency futures market. However, typically these studies do focus on the risk premium for a single maturity contract and do not pay attention to the term structure of the risk premium. We provide a pioneer study to analyze the term structure of currency forward risk premiums in the AUD–USD currency market with maturities ranging from one month up to 24 months. For this purpose, we explore the time-series of the realized risk premiums of 14 different maturities and provide an analysis of its term structure through time using factor models. To the best of our knowledge, this is the first study ever to have undertaken the analysis of the term structure of realized risk premiums in currency forward markets. So far, term structures have been studied largely for interest rates, see, e.g., Knez et al. (1994), Diebold and Li (2006), Christensen et al. (2011) or Jungbacker and Koopman (2014), just to name a few.

In a first step we investigate AUD–USD risk premiums for different maturity levels of the forward contracts and examine the significance of the observed premiums. We report that the realized risk premiums for the one month and two month forward contracts are not significantly different from zero, while the risk premiums for all other maturities are found to be positive and significant. Moreover, similar to Inci and Lu (2007) and Kumar and Trück (2014), we find that the mean of the realized risk premiums increases with the maturity of the contract and is the highest for the 24–month contract. We believe that risk premiums becoming larger and more significant for contracts with longer maturities, can be attributed to hedging activities in the market which typically play a greater role for contracts with longer maturities. For example, Breuer (2000) states that longer maturity derivatives contracts have a great risk of valuation changes; risk premium increases as the time horizon is extended. Given the economic rationale for the existence of futures markets, i.e., the possibility to transfer the exchange rate risk from


\(^2\)Other studies that have confirmed the presence of a time-varying risk premium in currency futures markets are Wolff (1987), Canova and Ito (1991), Bessembinder and Chan (1992), Baum and Barkoulas (1996), Peresetsky and de Roon (1997), Jiang and Chiang (2000), Panigirtzoglou (2004), Kiani (2009), Frankel and Poonawala (2010) just to mention a few.
risk–averse investors to those most willing or able to take it, see e.g. Bessembinder (1993), the increasing risk premiums could be interpreted as a compensation for taking on these risks.

The inter–temporal risk return relationship is very much unexplored in the currency markets in contrast to stock markets (Christiansen, 2011). Therefore, we also examine the relationship between realized risk premiums and explanatory variables such as returns form the currency spot market, the forward premium, and additional measures based on currency spot returns such as realized variance, skewness and kurtosis. Jiang and Chiang (2000) suggest that volatility of equity or currency markets can explain excess results in these markets, since reward for the risk borne by investors is measured by volatility. Further, Guo and Whitelaw (2006) also report a positive relation between stock market risk and return. Therefore, we also expect risk premiums in the Australian currency market to be positively and significantly related to volatility of the spot currency returns. We also study the relationship of realized risk premiums with the skewness and kurtosis of spot currency returns. There are studies that suggest that the mean and variance may not be sufficient to explain the risk return relationship and, therefore, other moments of currency returns must be taken into consideration. For example, Harvey and Siddique (2000) state that if asset returns have significant skewness, returns should include the reward for accepting such a risk. They further state that investors would prefer those portfolios that are right–skewed over those that are left–skewed. Christiansen (2011) establish a positive relationship between excess returns in forex market and realized skewness of returns. Therefore, we examine whether the risk premiums in the Australian currency market can be explained by higher moments of currency returns.

Next, we model the term structure of time–varying risk premium using principal component analysis (PCA) framework and extract common factors that affect the movement of risk premiums over time. We attempt to measure and interpret the common factors that determine the term structure of risk premiums in the Australian currency market. The term structure of risk premiums on a currency measures the relationship among the high–dimensional risk premium series that differs only in their term to maturity. More than that, observations on the risk premium curve at different points in time suggest the presence of links among short–, medium– and long–term currency forward quotes. We model the term–structure of risk premiums of the AUD–USD exchange rate across different maturities and extract small number of latent factors from a large panel of multidimensional risk premium time–series. We analyze 14 different maturities for the risk premium ranging from one month up to 24 months for the AUD–USD currency pair. For this purpose, we assume a linear multi–factor model and investigate the decomposition of the unconditional covariance matrix of realized risk premiums into common and idiosyncratic factors and, thus, explain the relationship through a smaller number of common factors. The main motivation of this technique arises from the necessity of dimension reduction. Further, if this kind of simplification is subject to a negligible loss of information, we can obtain valuable insight into the core of the problem.

Term–structure modelling is quite prevalent in the area of interest rates. For example, Knez et al. (1994) study the term structure of money market returns and conclude that three factors, on average explain 86% of the total variation in money market returns. Diebold and Li (2006) and Diebold et al. (2006) extract three common factors from the term structure of bond yields and
label them as level, slope and curvature factors. However, to the best of our knowledge, no other study has analyzed the term structure of risk premiums in currency forward markets. In this paper, we provide a pioneer study with respect to analyzing term structure effects on time-varying risk premiums. Similar to the modelling of interest rates, we assume that the term structure of different maturities of risk premiums is driven by a small set of unobservable stochastic factors. Modelling the term structure of risk premiums in foreign exchange markets might provide new and important insights, given that it has not been studied extensively in the literature so far. Risk premium–curve models (similar to yield–curve models) are almost invariably expected to employ a structure that consists of a small set of factors and the associated factor loadings that relate risk premiums of different maturities to these factors. Besides providing a useful compression of information, a factor structure is also consistent with the celebrated ‘parsimony principle’.

We obtain a four–factor model which explains almost 93% of the total variation in the risk premiums for the AUD–USD forward market. Similar to Knez et al. (1994), our study is also restricted to an unconditional analysis of the realized risk premiums using a fixed factor model. The first factor explains almost 73% of the total explained variance by the first four factors. The second factor explains the second largest (16%) proportion of the total explained variance not accounted for by the first factor. Finally, the third and fourth factor explain almost seven and four percent respectively, of the total variation in the observed risk premiums. The first factor can be called as a ‘level’ factor (similar to Knez et al, 1994; Diebold et al. 2006) because risk premiums of all maturities load heavily and equally on it for most maturities and exhibit high correlations with a level measure of the risk premium. We further find that the first factor of the realized risk premiums is related to a measure of realized variance in the AUD–USD spot market. The second factor explains much of the variation in the risk premium curve at the very short end which leads us to interpret the second factor as a ‘slope’ factor. Interestingly, the factor is also related to the realized kurtosis of currency spot returns. The third and fourth factor can be labelled as a ‘curvature’ factors as they affect the very short and very long end of the risk premium curve to a larger extent and have very little effect on the medium–term risk premium, thereby changing the risk premium curvature.

The remainder of the paper is organized as follows. In Section 2, we provide a review of the literature review on risk premiums in currency futures and forward markets. We also briefly review important results on the application of factor models to the term structure of interest rates and other markets. Section 3 describes the applied methodological framework, while Section 4 provides empirical results on the significance of the observed risk premiums at different maturity levels and modelling the term structure and dynamics of these premiums. Finally, Section 5 concludes and provides suggestions for future research.

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Diebold at al. (2006) define the ‘level’ factor as the average of the 3-month, 24-month and 120-month yields; the ‘slope’ factor as the 120-month yield minus the 3-month yield and the ‘curvature’ factor as twice the two year yield minus the sum of the ten-year and three-month yields.
2. Literature

One of the most influential studies in the literature of international finance is provided by Fama (1984), who attributes the behavior of forward and spot exchange rates to a time-varying risk premium. In his analysis, Fama shows that the implied risk premium on a currency is negatively correlated with its expected rate of depreciation and that the premium must have greater variance. Hodrick and Srivastava (1984) examine the risk premium in foreign exchange markets using a statistical model based on theoretical models of asset pricing. They find evidence of heteroskedasticity in the premium and suggest that the conditional expectation of the risk premium is a non-linear function of the forward premium. Kaminsky and Peruga (1990) investigate the presence of time-varying risk premiums in currency futures markets using an inter-temporal asset pricing model. They conclude that although the time-varying risk premium is an important determinant of expected returns, still more flexible specifications are required for the model.

McCurdy and Morgan (1992) analyze weekly data for currency futures prices to investigate the nature of risk premiums. They measure the covariance risk and state that no significant risk premium is found when the parameters of the prices of the covariance risk are kept constant. However, they observed considerable premiums when the prices were allowed to vary along with the variances and conditional expected returns of the benchmark portfolio. Bessembinder (1993) tests the presence of risk premiums in a cross section of 22 markets, including financial, currency, metal and agricultural futures contracts. The author rejects the hypothesis of mean returns being equal to zero in only three out of a total twenty-two considered markets. Liu and Maddala (1992) test the market efficiency hypothesis (MEH) in foreign exchange markets using cointegration. Using survey data on expectations for one-week and one-month ahead exchange rates, they conclude that for the weekly data it is the risk premia and for the monthly data, it is both risk premia and expectational errors that lead to the violation of the MEH.

Baum and Barkoulas (1996) have also confirmed the presence of significant risk premiums in the currency future basis which in turn may be explained by variables stemming from stock and bond markets. Peresetsky and de Roon (1997) have used the current spot price, the current forward rate and the basis as a proxy for the risk premium. They confirm the presence of time varying risk premiums which are both statistically and economically significantly different from zero. Similarly, Roon, Nijman and Veld (1998) find that the difference in expected one period returns on futures contracts with different maturities could be attributed to the risk premiums present in the spread between futures and spot rates. Further, Wolff (2000) studies the nature of the risk premium for fifteen currencies relative to the US dollar. For almost all currencies, the presence of a time-varying risk premium is confirmed in the analysis.

Lustig and Verdelhan (2007) study currencies of countries where interest rates are either higher or lower than in the US market. Using aggregate consumption growth risk, they explain that low interest rate currencies do not appreciate as much as suggested by interest rate differentials, while high interest rate currencies do not depreciate as much as suggested by UIP and observed interest rate differentials. Thus, domestic investors would earn negative excess returns on currency portfolios with low interest rates and positive excess returns on currency portfolios.
having high interest rates. Kumar and Trück (2014) examine the relationship between currency futures and realized spot rates for the Indian rupee US dollar exchange rate. Using futures contracts with maturities of one, two and three months, they explore the unbiasedness of futures quotes as a predictor of the future spot exchange rate as well as the nature of time–varying risk premiums in this emerging market. Empirical estimates, obtained using monthly data, suggest the presence of significant time–varying risk premiums in the considered futures market, which increases in magnitude and become more significant with increasing maturity of the contracts.

All the above mentioned studies focus on the relationship between the spot and futures rate for a particular maturity usually a one–month or three–months forward exchange rate. None of them dig deeper into the term structure by observing the forward rates of all the possible maturities at a time. However, it is a common phenomenon that forward contracts of many maturities are traded simultaneously in the currency futures market. The hypothesis that the participants in the foreign exchange market use all the available information fully has implications for the joint behavior of forward contracts of all the maturities (Hakkio, 1981). Elaborating the same argument, McDonald and Taylor (1990) put forward that the efficient market hypothesis is related not only to the relationship between the contemporaneous spot and forward exchange rates, but also to the spot rate and forward exchange rates for different maturities.

Today, the increasingly availability of multi–dimensional time series data has intensified the need for computationally efficient estimation techniques. For example, Stock and Watson (2002) and Bai (2003) have gone deeply into the study of dynamic factor models. Stock and Watson (2002) have tried to forecast a macroeconomic time–series using a large number of explanatory variables which they have summarized through a small number of indexes using the principal component analysis. The interest rates of many maturities form a multidimensional time series which can be analyzed through a smaller number of hidden common factors. For example, Penna and Box (1987) suggest a model for identifying hidden factors in a multivariate time–series process. They state that when a time–series is driven by a set of common factors, for the adequate representation of the system, a large number of parameters may be needed which may be highly correlated. Hence, a relationship may appear very complex and badly defined when, in reality, a simpler and parsimonious model in terms of few common factors may be present.

Factor models have been extensively used in the financial literature, in particular for the estimation of the term structure of interest rates and yield curve modelling. The search for the parsimonious modelling of yield curve through factor models goes back to the seminal paper of Nelson and Siegel (1987), who introduce a parsimonious model for yield curves, generally known as the Nelson–Siegel model. Their suggested parametric model can explain the variety of yield curve shapes like monotonic, humped or S–shaped behavior. Their model extracts three factors namely short–, mid– and long–term components which explain a high percentage of variations in the dynamics of the yield curves of the US treasury bills of various maturities.

Two very popular approaches for term structure modelling are equilibrium models and no–arbitrage models. The first approach is based upon identifying a state variable that determines
the term structure and then both, yield curve and dynamic nature of interest rates are determined endogenously. Prominent contributions to the literature of affine equilibrium models include Cox et al. (1985), Dai and Singleton (2000) and Duffie (2002), just to name a few. The no–arbitrage model focuses upon fitting the term structure perfectly at a single point in time so that all the possibilities for arbitrage have been vanished. In these models, the currently observed yield curve is used as an input to model the dynamics of the term structure. The work of Hull and White (1990) and Heath at al. (1992) are significant for the models of no–arbitrage for the term–structure. More recently, Christensen et al. (2011) have suggested a class of arbitrage–free affine dynamic term–structure models.

Litterman and Scheinkman (1991) suggest that most of the variations in the returns of fixed income securities can be explained by three factors which they call level, steepness and curvature. They further explain that by considering the effect of each of the three factors on a portfolio, investors will have better hedged position than they would have achieved by simply holding a zero–duration portfolio. Knez et al. (1994) extract even a fourth factor that they describe as a private issue credit factor. Duffie and Singleton (1997) develop a multi–factor model for the term structure of interest rate swap yields and support a two factor model for this specification. Liu et al. (2006) use the same framework to estimate a five factor affine term structure model for interest rate swap spreads.

Mishkin (1990) examine the relationship between the term–structure of interest rates and future inflation and find that the shortest end of term structure provides no information about the future path of inflation. However, the term structure is found to have a great deal of information about the future path of inflation at the end of the term structure. Estrella and Mishkin (1997) examine the relationship between the term structure of interest rates and monetary instruments, subsequent real activity and inflation in Europe and the United States. The monetary policy emerges out to be an important (but not the only one) determinant of the term structure spread. There is also a significant predictive power for both real activity and inflation. Ang and Piazzesi (2003), estimate a three factor model for the term structure in which they include the unobservable factors as well as macroeconomic variables specifically inflation rate and real activity as additional variables. They state that in the long–end of the yield curve, and for the longer horizons predictions, the unobservable factors emerge out to be the most important elements of the exchange rate variability while for the short–end and middle of the yield curve, the macro factors have more explanatory power. Further, the models with macro factors forecast better than the models with only unobservable factors. Diebold et al. (2006) estimate the yield curve using latent factor models, providing for the inclusion of macroeconomic variables in the model and a bidirectional relationship between the economy and the yield curve. Diebold and Li (2006) forecast the yield curve of government bonds in the US. In line with Nelson–Siegel (1987), they obtain three time–varying parameters – level, slope and curvature – which may be interpreted as factors as also done by Litterman and Scheinkman (1991). Similarly, Koopman and Van der Wel (2011) also focus on forecasting the term structure of the US interest rates between 1970 and 2009.They have used a dynamic factor yield curve model which allows for the inclusion of macroeconomic factors. Carrying out likelihood based analysis; they jointly analyze a factor model for the yield curve and for the
macroeconomic series and their dynamic interactions with the latent dynamic factors. Their results indicate interdependence between the yield and macroeconomic factors and that the macroeconomic variables can lead to more accurate yield curve forecasts.

Lustig et al. (2011) incorporate principal component analysis for analysis of exchange rates. They identify factors by building monthly portfolios of the currencies sorted by their forward discounts. The first portfolio contains the lowest interest rate currencies and the last consists of the highest interest rate currencies. They find that the first two principal components of the currency portfolio returns account for most of the time-series variation in the currency returns. The first principal component they identify as a “level” factor which they define as the average excess return on all foreign currency portfolios and call it the dollar risk factor. The second principal component is labelled as a “slope” factor whose weights decreases monotonically from positive for high interest rate currency portfolios to negative for low interest rate currency portfolios. They further describe that the slope factor is very similar to a zero-cost strategy that that goes long in the last portfolio and short in the first. They call this factor the carry trade risk factor.

As mentioned earlier, to the best of our knowledge, so far no study has explicitly analyzed the term structure of risk premiums in currency forward or futures markets. We believe that such an analysis will provide important insights on the nature and dynamics of risk premiums for participants in these markets.

3. Methodology and Applied Models

3.1. Determinants of Risk Premiums

The unbiasedness of the forward rate has been overwhelmingly rejected in a number of studies as mentioned above and the slope coefficient has been found to be significantly different from one, often even negative in a number of cases. A review of the literature is available, for example, in Engel (1996) and more recently in Jongen et al. (2008). The puzzle of the biasedness of the forward rate has been coined as the forward premium puzzle by Fama (1984) who states that the reason for the failure of UIP is a time-varying risk premium. Tai (2003) states that the speculative return from holding a forward contract results from a risk premium that has to be paid to risk averse speculators who takes the risk of future changes in the exchange rates.

Given the presence of time-varying risk premium, we focus on explaining risk premiums in the AUD–USD currency forward market. We do so by examining the factors affecting the realized time-varying risk premium for contracts with different levels of maturity. The risk premium $Y_{t,j}$ at time $t$ for maturity $j$ can be defined as the difference between the future spot rate at maturity and the currency forward rate today $(E(s_{t+j}) - f_t^j)^4$. Hereby, $s_{t+j}$ denotes the

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4 This definition of risk premiums has been used in a number of studies like Hodrick and Srivastava (1984), Canova and Ito (1991), Peresetsky and de Roon (1997), Breuer (2000), Jiang and Chiang (2000), Frankel and Poonawala (2010) to name a few.
natural logarithm of the spot rate $S_{t+j}$ at time $t+j$, and $f^j_t$ denotes the natural logarithm of the current forward quote $F^j_t$ at time $t$ for a contract with maturity $j$ ($j = 1-, 2-, 3-, ..., 24-$months). However, in practice, it is difficult to get the quotes for the expected future spot rate $E(s_{t+j})$ and we have taken into consideration the realized spot rate, $s_{t+j}$ and calculated therealized risk premium as $(s_{t+j} - f^j_t)$. Under the assumption of rational expectations, the realized spot rate can be taken as a proxy for the expected futures spot rate (Pippenger, 2011). Therefore, instead of considering the expected spot exchange rate $E(s_{t+j})$ at time $t+j$, we consider the actual realized spot rate $s_{t+j}$ and the difference between the realized spot rate and the futures quote, $s_{t+j} - f^j_t$. The relationship can then be tested using the following model:

$$s_{t+j} - f^j_t = \alpha + \beta_1 r_t + \beta_2 (f^j_t - s_t) + \beta_3 Var_t + \beta_4 Skew_t + \beta_5 Kurt_t + \varepsilon_{t+j}$$  (1)

In equation (1), $r_t$ is the spot currency return ($s_t - s_{t-1}$); $(f^j_t - s_t)$ is the forward premium or the so-called basis, $Var_t$, $Skew_t$, and the $Kurt_t$ denote the realized variance, skewness and kurtosis of the daily spot currency returns respectively. $\varepsilon_{t+j}$ is the $j$th period prediction error. Therefore, in equation (1), we test for the joint explanatory power of the considered variables for the observed risk premiums.

The explanatory variables realized variance, skewness and kurtosis have been calculated on a monthly basis using daily observations of spot currency log returns. We have matched the time horizon that is used to calculate the considered realized measures with the maturity of the forward contract. For example, while analysing realized risk premiums for one–month forward contracts, we calculate the realized variance, skewness and kurtosis at time $t$ based on the previous months’ realizations of spot currency returns. In a similar manner, when examining the realized risk premiums of forward contracts with maturity of 18–months (24–months), we calculate these measures based on daily log return observations of the previous 18 (24) months and likewise for other contracts as well.

### 3.2. Term Structure of the Time–Varying Realized Risk Premiums

Next, similar to modelling of term–structure of interest rates, we apply principal component analysis to the term structure of risk premiums and estimate common factors that explain its variation over time. Modelling high–dimensional data is a challenging task which frequently requires a combination of a flexible approach and dimension reduction techniques. By providing a complete schedule of realized risk premiums across time, the term structure represents the markets’ anticipation of future events. Explaining the term structure leads us to extract this information. Standard models for the term structure of interest rates model the yield curve successfully by a limited number of underlying common factors. The models of the term structure attempts to replicate an observed yield curve. We rely on the estimation of the term structure of realized risk premiums which makes use of latent or unobservable factors. The

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5 For the entire methodology to calculate realized variance, skewness and kurtosis of spot currency returns, we refer to Kumar and Trück (2014).
factor approach assumes that the covariance matrix of a set of random variables (risk premium series in our case) can be decomposed into a set of common and idiosyncratic components. This decomposition into the common and specific components is further based on the assumption of a linear relationship between the risk premiums at different maturities and a set of common factors.

The first problem one faces in term–structure modelling is how to summarize the information set at any point in time for the large number of currency forward contracts that are traded. Such a model should be able both to reproduce the historical stylized facts of the average shape of the risk premium curve and to forecast future risk premiums. In this regard, since only a small number of sources of systematic risk appear to underlie the pricing of a number of tradable financial assets, nearly all currency price information can be summarized with just a few constructed variables or factors. For example, in the yield curve literature, most models are built on the assumption of the existence of only a few unobservable, or latent, factors underlying the pricing of tradable bonds (Litterman and Scheinkman, 1991; Balduzzi et al., 1996; and Dai and Singleton, 2000).

The objective of factor analysis is to explain the dynamics through time with a small set of factors. In factor analysis, the observed variables are supposed to be a linear combination of unobserved factors which are assumed to be orthogonal to each other. For example, in an orthogonal L–factor model, an observable J–dimensional random vector \( Y_t = (Y_{t,1}, \ldots, Y_{t,J}) \) can be represented as:

\[
Y_{t,j} = m_{0,j} + Z_{t,1}m_{1,j} + \ldots + Z_{t,L}m_{L,j} + \pi_{t,j}
\]  

(2)

Where \( Z_{t,l} \) is a common L–dimensional vector of factors, the coefficients \( m_{i,j} \) are factor loadings and \( \pi_{t,j} \) are specific factors or residual errors which explain the residual variance. The loadings \( m_{i,j} \) represent the sensitivity of the \( j \)th maturity to the \( i \)th factor. The index \( t = 1, \ldots, T \) reflects the time evolution of the whole system and \( Y_{t,j} \) is the multidimensional time–series (risk premium) under observation with \( j \) \( (j=1, 2, \ldots, J) \) maturities. In our case, \( Y_{t,j} \) is a 14–dimensional vector of the risk premium over 14 maturities. Hence, the high–dimensional time series \( Y_t \) is now simplified to the modelling of \( Z_t \) which would be highly feasible (when \( L \ll J \)) under the assumption that \( \pi_{t,j} \) is a zero mean random error.

As far as the yield curve is concerned, the factor models gained much of the popularity during the 1990s as explained in the work of Steeley (1990) and Litterman and Scheinkman (1991). These authors have incorporated factor analysis, or PCA to be more precise, to extract the common latent factors that drive the yield curve of different maturities for different countries and time periods. As has been explained in the literature above, that three principal components namely level, slope and curvature account for almost more than 95% of total variation in the term structure of interest rates. We plan to analyze the same model for the term structure of risk premiums of different maturities over a period of time. To the best of our knowledge, no other study has ever undertaken the examination of modelling the risk premium series through factor models.
Note that equation (2) can be written in matrix notation as:

\[ Y_t = MZ_t + \varepsilon_t \]  \hspace{1cm} (3)

Where, \( M \) is a \((J \times L)\) matrix of loadings \((m_{t,j})\), \( Z_t = (Z_{t,1}, ..., Z_{t,L}) \) is a \((L \times 1)\) vector of factors, and \( \pi_t = (\pi_{t,1}, ..., \pi_{t,J}) \) is a \((J \times 1)\) vector of specific or unique factors. \( Y_t = (Y_{t,1}, ..., Y_{t,J}) \) is again a \((J \times 1)\) vector of observations. In the context of currency markets, \( Y_{t,j} \) represents the risk premium on day \( t = 1, ..., T \) for contract of maturity \( j = 1, ..., J \).

In this study, we employ a four–factor model for the analysis of the term structure of risk premiums and obtain the factors in such a way that they are consistent with factors or parameters that typically describe the movement of the risk premiums.

4. Empirical Estimation and Results

4.1. Data Description

Data for all the currencies has been sourced from Bloomberg. We consider the series of forward contracts for the AUD–USD exchange rate. Note that in the following we will always quote the exchange rate as USD per unit of AUD. We consider data for the spot exchange rate as well as forward exchange rates for 14 different maturities viz. 1–M, 2–M, 3–M, 4–M, 5–M, 6–M, 7–M, 8–M, 9–M, 10–M, 11–M, 12–M, 18–M and 24–M on a monthly basis. The data period for the AUD–USD currency pair starts from October 1999 to December 2013 which provides us with 180 monthly observations at each maturity level.

To calculate the risk premium series for a given currency, we first arrange the exchange rates as spot rates followed by the forward rates of increasing maturities. Then the risk premiums can be calculated as the difference between the realized spot rate at time \( t+j \), and the currency forward rate at time \( t \), for delivery at time \( j \), as \((s_{t+j} - f^j_t)\). For example, when we calculate the realized risk premium at time \( t \) for a 24–month forward contract, we subtract the current 24–month forward rate from the spot rate realized 24–months ahead. Therefore, calculating the realized risk premium series this way, we have lost 24 observations in constructing the risk premium series for 24–months contract and we are now left with 156 observations of the risk premium running from October, 1999 to December, 2011 for the 24–month forward contracts.

4.2. Preliminary Empirical Results

Table 1 provides descriptive statistics for the AUD–USD monthly exchange rates for different maturities from January 1999 to December 2013. We find that during the considered time period, for all maturities, the mean of the forward rates is lower than the mean of the current spot rate. Further, there is generally a downward sloping term structure with maturity such that the mean is decreasing with maturity of the contracts. This trend is in line with interest rate parity (IRP) condition which suggests higher interest rates in the Australian market as compared to the US market. Further, for all maturities, the standard deviation of forward rates is lower than the standard deviation of the spot rate, with the standard deviation of the 24–
month forward rates being the lowest. The Jarque–Bera statistic suggests that for all of the considered series, normality can be rejected at the 5% significance level. Further, Table 1 also shows results for augmented Dickey–Fuller (ADF) tests for the spot and forward rates in their level and first difference. The ADF test shows that that the considered spot and forward rates time series are non–stationary, however, they are stationary at 1% level of significance if we take the first difference.

### Table 1

<table>
<thead>
<tr>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>SD</th>
<th>Skew</th>
<th>Kurt</th>
<th>J–B</th>
<th>ADF (Level)</th>
<th>ADF (Diff)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot</td>
<td>0.777</td>
<td>1.099</td>
<td>0.486</td>
<td>0.171</td>
<td>0.116</td>
<td>1.913</td>
<td>9.26***</td>
<td>-1.260</td>
</tr>
<tr>
<td>1-M</td>
<td>0.775</td>
<td>1.095</td>
<td>0.486</td>
<td>0.170</td>
<td>0.112</td>
<td>1.913</td>
<td>9.23***</td>
<td>-1.258</td>
</tr>
<tr>
<td>2-M</td>
<td>0.773</td>
<td>1.091</td>
<td>0.486</td>
<td>0.169</td>
<td>0.108</td>
<td>1.917</td>
<td>9.144**</td>
<td>-1.259</td>
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<tr>
<td>3-M</td>
<td>0.772</td>
<td>1.087</td>
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<td>0.106</td>
<td>1.917</td>
<td>9.139**</td>
<td>-1.254</td>
</tr>
<tr>
<td>4-M</td>
<td>0.770</td>
<td>1.082</td>
<td>0.486</td>
<td>0.167</td>
<td>0.103</td>
<td>1.919</td>
<td>9.082**</td>
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</tr>
<tr>
<td>5-M</td>
<td>0.768</td>
<td>1.079</td>
<td>0.485</td>
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<td>0.100</td>
<td>1.922</td>
<td>9.015**</td>
<td>-1.248</td>
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<tr>
<td>6-M</td>
<td>0.767</td>
<td>1.075</td>
<td>0.485</td>
<td>0.165</td>
<td>0.098</td>
<td>1.925</td>
<td>8.947**</td>
<td>-1.244</td>
</tr>
<tr>
<td>7-M</td>
<td>0.765</td>
<td>1.071</td>
<td>0.485</td>
<td>0.165</td>
<td>0.095</td>
<td>1.929</td>
<td>8.876**</td>
<td>-1.241</td>
</tr>
<tr>
<td>8-M</td>
<td>0.763</td>
<td>1.067</td>
<td>0.485</td>
<td>0.164</td>
<td>0.093</td>
<td>1.933</td>
<td>8.796**</td>
<td>-1.237</td>
</tr>
<tr>
<td>9-M</td>
<td>0.762</td>
<td>1.063</td>
<td>0.485</td>
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<td>0.091</td>
<td>1.937</td>
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<td>10-M</td>
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<td>0.485</td>
<td>0.162</td>
<td>0.088</td>
<td>1.941</td>
<td>8.639**</td>
<td>-1.230</td>
</tr>
<tr>
<td>11-M</td>
<td>0.759</td>
<td>1.056</td>
<td>0.484</td>
<td>0.161</td>
<td>0.086</td>
<td>1.946</td>
<td>8.553**</td>
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</tr>
<tr>
<td>12-M</td>
<td>0.757</td>
<td>1.052</td>
<td>0.484</td>
<td>0.160</td>
<td>0.084</td>
<td>1.950</td>
<td>8.474**</td>
<td>-1.225</td>
</tr>
<tr>
<td>18-M</td>
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<td>1.030</td>
<td>0.481</td>
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<td>0.071</td>
<td>1.978</td>
<td>7.979**</td>
<td>-1.207</td>
</tr>
<tr>
<td>24-M</td>
<td>0.739</td>
<td>1.010</td>
<td>0.479</td>
<td>0.149</td>
<td>0.058</td>
<td>2.003</td>
<td>7.557**</td>
<td>-1.215</td>
</tr>
</tbody>
</table>

**Note:** 1-M, 2-M, ...,24-M denote futures contracts with maturities of one, two, ...,twenty–four months. SD denotes the standard deviation, skew the skewness and kurt the kurtosis of the considered time series. J-B is the Jarque–Bera test statistics for normality that is approximately chi-squared distributed with two degrees of freedom, rejection of the null hypothesis at the 10%, 5% and 1% level of significance is indicated by *, **, *** respectively. ADF (level) and ADF (diff) reports the test statistic of the Augmented Dickey–Fuller test for stationarity for the original spot and futures rates in their level and the first differences. The considered sample period is from January 1999 to December 2013.

Figure 1 presents the movements of the spot exchange rate and the forward rates of maturity 1–M, 6–M, 12–M and 24–M for the considered time period. We observe that the Australian dollar has initially depreciated against the US dollar during the time January 1999 to March 2001. However, from April 2001 to December 2007, the Australian dollar has appreciated against the US dollar. One important observation to be noticed is that during the 2007–08 financial turmoil, the US dollar has appreciated against Australian dollar. For example, US dollar started appreciating against Australian dollar from June 2008 (0.96 USD per AUD) to January 2009 (0.64 USD per AUD). After the crisis, the Australian dollar again started appreciating against the US dollar. The reason behind the appreciation of AUD against USD may be that actually the US dollar has been depreciating due to weak economic fundamentals.
The United States have been accumulating huge current account balances, which is causing the dollar depreciation. For example, Blanchard et al. (2005) state that the US appears to have entered the depreciating phases. Using a simple model of the exchange rate and current account determination, they conclude that the path of adjustment is likely to be associated primarily with Asian currencies’ appreciation, but also with a further appreciation of the euro against the dollar.

![Figure 1](image-url)  

**Figure 1.** Movement of spot and forward prices for US dollar versus Australian dollar for forward contracts with maturity of 1, 6, 12 and 24 months from January 1999 to December 2013.

The figure further shows that at any point in time, for all maturities, forward quotes are lower than the spot rate what is caused by the lower interest rates for the US dollar in comparison to the Australian dollar. Moreover, the difference between spot and forward quotes becomes larger with increasing maturity of the forward contract.

### 4.3. Examination of Realized Risk Premiums
In this section, we focus on the examination of the ex–post or realized risk premiums in the AUD–USD currency market. We aim at examining the significance and nature of the time–varying realized risk premium. Figure 2 provides a three–dimensional view of the risk premium series for the Australian dollar. The large amount of temporal variation is visually apparent. Table 2 presents the descriptive statistics of realized risk premiums for all maturities.

Table 2 shows that the mean of the risk premiums is positive and increases as the maturity of the forward contract increases\(^6\). Our results are similar to what is reported by other studies like Canova and Ito (1991), Tai (2003) and Kumar and Trück (2014). Canova and Ito (1991) study the time–series properties of the risk premium for the Japanese yen US dollar exchange rate. They report a positive average risk premium for three out of four sub–samples of their study period. Tai (2003) find positive monthly forward exchange risk premiums for several currencies, e.g. a monthly premium of 0.006% for the Hongkong dollar, 0.143% for Singapore dollar and 0.031% for Malaysian ringgit. Similarly, Kumar and Trück (2014) also report negative risk premiums for the Indian rupee US dollar exchange rate and conclude that the risk premiums increase with the maturity of the contract. However, they have taken the US dollar as the base rate and we have considered Australian dollar as the base rate against the US dollar. Therefore, the sign of the risk premium needs to be interpreted with care: negative risk premium

\(^6\) We believe the ‘overlapping’ effect would be significant while reporting the means and descriptive statistics of the risk premium series. Therefore, the premiums are not independent such that the application of a standard \(t\)-test is not correct. Therefore, we have corrected the same by using the Newey-West heteroskedasticity and autocorrelation adjusted standard errors and \(t\)-stats for the estimates due to the high autocorrelation in the data.
reported by Kumar and Trück (2014) are equivalent to the positive risk premium reported in this study.

Table 2
Descriptive Statistics of the Risk Premium Series

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>SD</th>
<th>Skew</th>
<th>Kurt</th>
<th>J–B</th>
<th>ADF</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-M</td>
<td>0.005</td>
<td>0.101</td>
<td>-0.170</td>
<td>0.039</td>
<td>-0.718</td>
<td>5.174</td>
<td>44.1***</td>
<td>-11.6***</td>
<td>156</td>
</tr>
<tr>
<td>2-M</td>
<td>0.011</td>
<td>0.152</td>
<td>-0.243</td>
<td>0.058</td>
<td>-1.019</td>
<td>5.688</td>
<td>73.9***</td>
<td>-4.99***</td>
<td>156</td>
</tr>
<tr>
<td>3-M</td>
<td>0.016*</td>
<td>0.232</td>
<td>-0.332</td>
<td>0.070</td>
<td>-1.126</td>
<td>7.675</td>
<td>175***</td>
<td>-4.87***</td>
<td>156</td>
</tr>
<tr>
<td>4-M</td>
<td>0.021*</td>
<td>0.240</td>
<td>-0.348</td>
<td>0.084</td>
<td>-1.042</td>
<td>6.991</td>
<td>131***</td>
<td>-4.01***</td>
<td>156</td>
</tr>
<tr>
<td>5-M</td>
<td>0.025*</td>
<td>0.277</td>
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<td>0.096</td>
<td>-0.944</td>
<td>5.851</td>
<td>76.0***</td>
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<td>156</td>
</tr>
<tr>
<td>6-M</td>
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<td>0.106</td>
<td>-0.853</td>
<td>5.479</td>
<td>58.9***</td>
<td>-3.17**</td>
<td>156</td>
</tr>
<tr>
<td>7-M</td>
<td>0.035*</td>
<td>0.334</td>
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<td>0.115</td>
<td>-0.764</td>
<td>4.905</td>
<td>38.8***</td>
<td>-2.92**</td>
<td>156</td>
</tr>
<tr>
<td>8-M</td>
<td>0.040**</td>
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<td>-0.666</td>
<td>4.461</td>
<td>25.4***</td>
<td>-2.534*</td>
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<tr>
<td>9-M</td>
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<td>-0.367</td>
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<td>3.999</td>
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<tr>
<td>10-M</td>
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<td>0.375</td>
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<td>-0.462</td>
<td>3.451</td>
<td>6.87**</td>
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<tr>
<td>11-M</td>
<td>0.055**</td>
<td>0.356</td>
<td>-0.329</td>
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<td>-0.400</td>
<td>3.071</td>
<td>4.186**</td>
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<tr>
<td>12-M</td>
<td>0.059**</td>
<td>0.355</td>
<td>-0.323</td>
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<td>-0.392</td>
<td>2.927</td>
<td>4.0245 **</td>
<td>-2.2724</td>
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<tr>
<td>18-M</td>
<td>0.090***</td>
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<tr>
<td>24-M</td>
<td>0.119***</td>
<td>0.502</td>
<td>-0.288</td>
<td>0.187</td>
<td>-0.227</td>
<td>2.424</td>
<td>3.4935 **</td>
<td>-2.2812</td>
<td>156</td>
</tr>
</tbody>
</table>

Note: Table 2 presents the descriptive statistics for the risk premium series for AUD-USD for different maturities. Risk premium has been calculated as \(s_{t+j} - f_t^j\), where \(s_{t+j}\) is the natural logarithm of the realized spot rate at time \(j\) and \(f_t^j\) is the natural logarithm of the futures spot rate at time \(t\) for delivery at time \(j\). 1-M, 2-M, ...,24-M denote futures contracts with maturities of one, two, ..., twenty four months. SD denotes the standard deviation, skew the skewness and kurt the kurtosis of the considered time series. J–B is the Jarque–Bera test statistics for normality that is approximately chi-squared distributed with two degrees of freedom, rejection of the null hypothesis at the 10%, 5% and 1% level of significance is indicated by *, **, *** respectively. ADF reports the test statistic of the Augmented Dickey–Fuller test for stationarity. The considered sample period is from January 1999 to December 2013.

These results are highly coherent with the literature that the risk premium becomes more significant for contracts with longer maturity. For example, see Poghosyan, Kocenda and Zemcik (2006) and Inci and Lu (2007). Further, Peresetsky and de Roon (1997) show that the risk premium is larger for longer maturity contracts compared to shorter maturity contracts. The reason for maturity effects in the risk premium may be attributed to the fact that for shorter maturities, bank credits are easily available, what is not the case for contracts with longer maturities. This reasoning implies that interest rate risk becomes highly important for longer maturity contracts.

Table 2 also reports the results of conducted augmented Dickey–Fuller (ADF) tests for the realized risk premiums for all maturities. The results confirm that the observed risk premiums are stationary and reject the null hypothesis of a unit root at least at 5% significance level.
Another interesting fact is that the standard deviation is the lowest for 1–month risk premium and highest for the 24–month risk premium i.e. risk premiums for exhibited by contract with longer maturities are far more volatile and dynamic (Figure 3). In other words, the longer the forecast horizon, the further we expect our forward quote to be away from the realized spot rate. This is in contrast to the term structure of interest rates. For example, Diebold and Li (2006) and Koopman and Van der Wel (2011) report that the standard deviations for longer maturities are lower in comparison to shorter maturities. However, their study is focussed towards the modelling of interest rates.

In Figure 3, we illustrate the dynamic behaviour of the risk premiums through time. We plot the risk premiums for 3–months, 6–months and 24–months maturity. Risk premiums of 24–months maturity are more volatile and dynamic than 6–months maturity which in turn are more volatile than 3–months risk premiums. The fact that observed risk premiums in our study are typically positive, can also be related to carry trade strategies\(^7\). Since in recent times, the AUD has generally offered higher interest rates than the USD or also many other major currencies, therefore, carry trader have taken long position in the AUD and short position in the USD (Brunnermeier, 2009). Further, Christiansen (2011) show that risk exposures of carry trade returns are highly regime–dependent and the returns have a direct exposure to volatility.

![Figure 3: Dynamic behaviour of risk premiums for 3–months, 6–months and 24–months maturity.](image)

4.4. Realized Risk Premiums and Explanatory Variables

\(^7\) A currency carry trade is defined as borrowing a low-yielding asset and buying a higher-yielding asset denominated in another currency (Christiansen et al. (2011))
In this section, we try to establish an empirical relationship between the risk premiums and the explanatory variables, for example, currency spot returns, realized variance, skewness and the kurtosis of the currency returns and the forward premium. A number of variables have been suggested in the literature which plays a crucial role in explaining the realized risk premiums. For example, Peresetsky and de Roon (1997) suggest that the spot rate, futures rate and basis explain the risk premium while Jiang and Chiang (2000) establish a significant influence of currency and stock market volatility on the risk premium. Christiansen (2011) states that realized variance and skewness of the spot rate are possible determinants of the risk premium. However, as shown in Table 1, the spot rate is found to be a non-stationary variable and using the spot rate directly as a determinant of the risk premium might lead to spurious results. Therefore, we have considered the first difference of the log of spot exchange rate (spot currency returns) in the analysis. This variable turns out to be stationary as evident in Table 1.

We investigate equation (1) and results are presented in Table 3. We find that for the shorter maturity forward contracts (1–M, 2–M and 3–M); none of the explanatory variables is significant. However, for contracts with a longer maturity (from 9–M up to 24–M) spot currency returns are significant in explaining the realized risk premiums. Also note that the magnitude of the estimated coefficient increases with maturity, from -0.25 for 9–months (at the 5% level of significance) contracts to −0.60 for contracts with a maturity of 24–months (at 1% level of significance). The sign of the coefficients suggests that the higher the return on the spot rate, the more negative will generally be the realized risk premium, i.e. the more will the AUD–USD forward rate overestimate the realized spot rate.

In other words, since interest rates during the considered time period are higher in Australia than in the US, the Australian dollar depreciates less against the US dollar than suggested by the forward rate due to a positive risk premium in the market. Peresetsky and de Roon (1997) investigate the risk premiums in the ruble/dollar futures market and suggest that the current spot rate appears to have some predictive power for the risk premium. They report a maturity effect of the spot rate as an explanatory variable, while we also find that for longer maturities the magnitude of the coefficient for spot currency returns increases.

Another important variable explaining the risk premium is the forward premium. Similar to the currency spot returns, the explanatory power of forward premium increases as the maturity of the contract increases. For forward contracts with a maturity of 9–months and more, the forward premium seems to have a significant negative impact on the risk premium. Interestingly, for all models the sign of the coefficient for the forward premium is negative, suggesting that the higher the forward premium at time t, the more negative will be the realized risk premium. Recall that the forward premium is always negative during the considered time period, due to the higher interest rates in Australia. Therefore, our results suggest that the appreciation of the Australian dollar is even more pronounced than suggested by the forward premium. Since the risk premium on average is positive for the considered time period, the results also suggest that during times when the forward premium is low, the Australian dollar generally appreciated less than what had been suggested by the forward premium.
### Table 3

Results for the regression of the realized risk premium \((s_{t+j} - f_t^j)\) on currency spot returns \((\tau_t)\), the forward premium \((f_t^j - s_t)\), realized variance \((Var_t)\), skewness \((Skew_t)\) and kurtosis \((Kurt_t)\) of daily currency spot returns.

\[
s_{t+j} - f_t^j = \alpha + \beta_1 \tau_t + \beta_2 (f_t^j - s_t) + \beta_3 Var_t + \beta_4 Skew_t + \beta_5 Kurt_t + \epsilon_{t+j}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>1-M</th>
<th>2-M</th>
<th>3-M</th>
<th>4-M</th>
<th>5-M</th>
<th>6-M</th>
<th>7-M</th>
<th>8-M</th>
<th>9-M</th>
<th>10-M</th>
<th>11-M</th>
<th>12-M</th>
<th>18-M</th>
<th>24-M</th>
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<td>(0.45)</td>
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<td>(0.12)</td>
<td>(0.26)</td>
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<td>(0.77)</td>
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<td>(\tau_t)</td>
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<td>0.23</td>
<td>0.16</td>
<td>0.00</td>
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<td>-0.32**</td>
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<td></td>
<td>(0.28)</td>
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<td>(1.79)</td>
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<tr>
<td>(f_t^j - s_t)</td>
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<td>-0.80</td>
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<td>-4.08**</td>
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<td>500***</td>
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<td>(4.45)</td>
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<td>(2.82)</td>
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<td>(1.45)</td>
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<td>-0.04**</td>
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<td>(0.99)</td>
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<td>(1.70)</td>
<td>(1.83)</td>
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<td>(2.55)</td>
<td>(0.09)</td>
<td>(-4.00)</td>
</tr>
<tr>
<td>(Kurt_t)</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.03**</td>
<td>-0.04**</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(-0.07)</td>
<td>(-1.12)</td>
<td>(-0.43)</td>
<td>(0.72)</td>
<td>(0.35)</td>
<td>(0.00)</td>
<td>(-0.47)</td>
<td>(-0.89)</td>
<td>(-1.26)</td>
<td>(-1.49)</td>
<td>(-2.29)</td>
<td>(-3.59)</td>
<td>(-1.73)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.01</td>
<td>0.04</td>
<td>0.06</td>
<td>0.12</td>
<td>0.17</td>
<td>0.19</td>
<td>0.19</td>
<td>0.21</td>
<td>0.23</td>
<td>0.25</td>
<td>0.28</td>
<td>0.34</td>
<td>0.53</td>
<td>0.49</td>
</tr>
<tr>
<td>(F)-stat</td>
<td>0.45</td>
<td>1.21</td>
<td>1.81</td>
<td>3.99***</td>
<td>6.24***</td>
<td>7.18***</td>
<td>7.15***</td>
<td>7.81***</td>
<td>8.96***</td>
<td>9.88***</td>
<td>11.5***</td>
<td>15.5***</td>
<td>34.3***</td>
<td>29.2***</td>
</tr>
<tr>
<td>(N)</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
</tr>
</tbody>
</table>

**Note:** \(s_{t+j}\) indicates the natural logarithm of the realized spot rate at \(t+j\); \(s_t\) is the natural logarithm of the spot rate at \(t\); \(f_t^j\) indicates the natural logarithm of the forward quote for period \(t+j\) at time \(t\); \(\tau_t\) denotes the monthly spot currency log return \((s_t - s_{t-1})\). \((f_t^j - s_t)\) is the forward premium or the so called basis, \(Var_t, Skew_t\) and the \(Kurt_t\) denote the realized variance, skewness and kurtosis of daily spot currency returns based on a time period corresponding to the previous \(j\) months, i.e. the last month of daily spot currency returns for \(j=1\), the last two months of daily spot currency returns for \(j=2\), and so on. \(\epsilon_{t+j}\) is the \(j\)th period prediction error. Figures in the parenthesis indicate \(t\)-statistics for the estimated coefficients. The sample period is from September 2008 to January 2013. * Indicates significance of the coefficient at the 10% level, ** at the 5% level and *** at the 1% level.
Next, we find that the volatility of the currency spot return, measured by the realized variance of daily spot returns during the previous months for the respective maturities, plays an important role in the determination of the realized risk premium. The coefficient for the realized variance turns out to be significant for the middle month maturity contracts (4–M to 9–M) and then for the long maturity contracts (18–M and 24–M). Further, the sign of the coefficient $\beta_3$ is found to be positive, suggesting that the risk premium increases as the volatility of the currency spot returns increases. Our results are thus similar to Jiang and Chiang (2000) who also report a positive relationship between the risk premium and the variance of the spot exchange rate. For equity markets, Guo and Whitelaw (2006) also report a positive relationship between the risk premium and volatility.

We further find the realized skewness to be significant for the longer maturity contracts. The magnitude and significance of the coefficient for skewness increase with the maturity of the contract. However, the positive coefficients for the skewness in all regression models may suggest that the more skewed (to the right) daily spot currency returns have been before time $t$, the more will the futures rate overestimate the realized spot rate $s_{t+j}$. As far as the realized kurtosis is concerned, the coefficient is significant only for the longer maturity contracts (eg. 12–M, 18–M and 24–M).

Our finding that the risk premium is less significant for contracts with shorter maturity can probably be explained by two reasons. The first explanation can be the hedging theory, which states that hedgers play a big role in the longer maturity contracts and they demand more premium for their exposure in the market as the maturity of the contract gets longer. This is because the hedgers’ positions in the cash market are almost never for short–term, hence longer–maturity contracts serve their needs well as they will incur less rolling–over costs. According to Ederington and Lee (2002), hedging activities become more prevalent on contracts with longer maturity and decline for the shorter maturity contracts. However, though the speculative transactions take place for all the maturities, they dominate the short–maturity contracts mainly due to lack of hedging. If hedging activities in the market are more than speculative activities, we would observe more significant risk premiums in currency futures markets. As the pricing of longer maturity contracts will be more affected by risk premiums, therefore, the risk premium can be expected to be more pronounced as the maturity of the contract gets longer.

Second explanation can be the covered interest parity (CIP). The CIP theory states that the futures–spot basis is equivalent to the interest rate differential between two countries’ currencies. The short–end of the term structure of interest rates is not affected by the risk premium as it is set by the central banks. As we go to the far ends of the yield curves, the market risk premium starts playing more important role in the determination of the interest rates, as these rates are set up by the market forces. Therefore, risk premiums can be expected to be more significant for longer maturity contracts.

These results are also supported by Inci and Lu (2007) and Kumar and Trück (2014) who argue that for longer maturities risk premiums in the futures market start to play a more important role and lead to a rejection of the unbiasedness hypothesis. Since the economic rationale for
the existence of futures markets is the possibility to transfer the exchange rate risk from risk–averse investors to those most willing or able to take it, see e.g. Bessembinder (1993), the risk premium could be interpreted as a compensation for taking on these risks.

Overall, we find evidence for significant positive realized risk premiums in the AUD–USD forward market which increases as the maturity of the contracts increases. We also find a significant relationship between realized risk premiums and spot currency returns, the forward premium as well as the realized variance, skewness and kurtosis of the spot currency returns which becomes even more significant with the increasing maturity of the contract.

4.5. Factor Analysis Results – Term Structure of Time–Varying Realized Risk Premiums

Next, we apply principal component analysis in an attempt to measure and interpret the common factors that determine the term structure of the high dimensional realized risk premium series in the AUD–USD currency market. Table 4 shows the results of variance decomposition obtained from a PCA for the realized risk premiums. The term structure of risk premiums on a currency measures the relationship among the high–dimensional risk premium series that differs only in their term to maturity. In this regard, since only a small number of sources of systematic risk appear to underlie the pricing of a number of tradable financial assets, nearly all currency price information can be summarized with just a few constructed variables or factors.

It has already been established in the literature for the term structure of interest rates that three factors – level, slope and curvature – typically explain almost the entire movement of the yield curve. However, this study is the first one to explore the dynamics of the risk premium series over different maturities in currency forward markets, so a clear identification of the factors may be a difficult task. Risk premiums in currency forward markets may well behave very different to the term structure of interest rates. Even in the yield curve modelling literature, Knez et al. (1994) estimate their factor model and found no evidence against a model with 10 factors. However, they restrict their attention primarily to the three– and four–factor analysis only to align their results to the theoretical development in the field. Further, they state that though three– and four–factor models are statistically rejected, still they explain, on average, about 86 and 90% of the variations in the term structure of interest rates.

In our study, a four–factor model accounts for almost 93% of the total variation in the risk premiums. In Table 4, we present the variance decomposition for our four–factor model applied to realized risk premiums in the AUD–USD forward market. The decomposition shows the relative importance of each of the factors in explaining the variation in the risk premiums for each maturity level. The first factor explains approximately 73% of the total variation, and, therefore, by far the highest proportion of all factors. As illustrated by Table 4 the factor is also highly related to risk premiums at all maturity levels such that it can be labelled as a ‘level’ factor. However, Table 4 also illustrates that the first factor explains less for realized risk premiums of contracts with very short (one and two–month) maturities and contracts with relatively long maturities (18– and 24–months).
Table 4: Variance Decomposition – Realized Risk Premiums

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Total Variance Explained (Percent)</th>
<th>Proportion of the Total Variance Accounted for by (Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Factor1</td>
<td>Factor2</td>
</tr>
<tr>
<td>1-M</td>
<td>92.62</td>
<td>26.35</td>
</tr>
<tr>
<td>2-M</td>
<td>87.23</td>
<td>50.55</td>
</tr>
<tr>
<td>3-M</td>
<td>88.79</td>
<td>65.91</td>
</tr>
<tr>
<td>4-M</td>
<td>92.02</td>
<td>76.68</td>
</tr>
<tr>
<td>5-M</td>
<td>92.99</td>
<td>85.18</td>
</tr>
<tr>
<td>6-M</td>
<td>93.05</td>
<td>91.16</td>
</tr>
<tr>
<td>7-M</td>
<td>93.12</td>
<td>94.69</td>
</tr>
<tr>
<td>8-M</td>
<td>94.30</td>
<td>93.70</td>
</tr>
<tr>
<td>9-M</td>
<td>95.43</td>
<td>90.05</td>
</tr>
<tr>
<td>10-M</td>
<td>96.09</td>
<td>84.67</td>
</tr>
<tr>
<td>11-M</td>
<td>95.92</td>
<td>80.18</td>
</tr>
<tr>
<td>12-M</td>
<td>93.57</td>
<td>77.22</td>
</tr>
<tr>
<td>18-M</td>
<td>91.65</td>
<td>53.92</td>
</tr>
<tr>
<td>24-M</td>
<td>91.60</td>
<td>50.48</td>
</tr>
</tbody>
</table>

Average 92.74 72.91 15.98 6.835 4.279

Note: Table 4 shows the total variance explained by the four factor model for each maturity of the realized risk premium and the proportion of the total explained variance accounted for by each factor for each maturity. The data period is from January, 1999 to December, 2013.

The second factor explains approximately 16% of the variation in the realized risk premiums for the sample period. However, the number drops to 9.55% if we exclude the observations for the first four maturities. Thus, the contribution of second factor to the explained variance is much larger at the very short end of the risk premium curve which leads us to interpret the second factor as a ‘slope’ factor. The third factor explains approximately 7% of the total variation not accounted for by the first two factors. One important point to be observed here is that the explained variation by factor three is the highest for short maturities of the forward contracts, then decreases and after increasing in the middle of the curve, decreases again finally is the highest at for contracts with maturities of 18 and 24 months. For example, the variation explained by the factor for the realized risk premium of one–month contracts is 16.09% out of the total 92.62% variation explained by the four factors. The explained variance decreases for the 2–M, 3–M and 4–M risk premium which again increases for the 5–M, 6–M and 7–M risk premium being 2.68%, 4.97% and 5.2% respectively. Again, the explained variance after decreasing for the 8–M, 9–M, 10–M and 11–M risk premiums, increases for the risk premiums of 12–M, 18–M and 24–M respectively with 0.59%, 24.21% and 24.84% explained variance.
Figure 4: Loadings for the Four–Factor Model. Figure displays the loadings in the risk premium for factor 1, 2, 3 and 4. The plot of each factor represents the change that would be caused by a one standard deviation shock to that factor. The sample period is from January, 1999 to December 2013.

As far as fourth factor is concerned, it explains almost 4.3% of the remaining variation not accounted for by the first three factors. An important observation about factor four is that it is similar to factor three in that it explains the maximum variation in the risk premium at the shortest and the longest end of the risk premium. However, the contribution of the fourth factor becomes highly significant at the farthest end of the risk premium curve. For example, it explains 9.9% variation in the risk premium for contracts with a maturity of 24 months.

Figure 4 depicts the factor loadings for the risk premium of all maturities on the four factors. The graph of each factor represents the change that would be caused by a one standard deviation shock to that factor. As seen from Figure 3, the effect of the first factor is relatively stable for maturities between four and 24 months. Thus a one standard deviation movement in first factor causes almost a “parallel” shift in the risk premium curve for maturities between four and 24 months. Further, notice that the loadings are ordered almost uniformly in magnitude across maturities. For this reason, similar to Knez et al. (1994), we call the first factor a ‘level’ factor. Another measure of assigning the first factor as a level factor is provided by Diebold et al. (2006)\(^8\). They suggest that an increase in the first factor increases all the yields equally, as the loadings are identical at all maturities, thereby changing the level of the yield curve. Similarly, we define the level factor as the average of the 1–month, 9–month and 24–month risk premiums and find that the first factor and the level of the risk premiums are approximately 89.4% correlated. Therefore, we present strong evidence of factor one to be labelled as a ‘level’ factor.

As shown in Figure 4, we label the second factor as the ‘slope’ factor. A unit shock to the short–end of the curve causes the risk premiums to change by a proportionately larger amount than does a shock to the long–end of the curve. Alternatively, notice that an increase in the second factor

\(^8\) Diebold et al. (2006) define the ‘level’ factor as the average of the 3-month, 24-month and 120-month yields; the ‘slope’ factor as the 120-month yield minus the 3-month yield and the ‘curvature’ factor as twice the two year yield minus the sum of the ten-year and three-month yields.
factor increases short–term risk premiums more than long–term risk premiums because the short–term risk premiums load more heavily on the second factor, thereby changing the slope of the risk premium curve. The net effect is a steepening of the risk premium curve. This means that for a unit shock to second factor, currency risk premium curves steepen uniformly across maturities. That is why we call the second factor as the slope factor. The slope factor loadings decrease almost linearly with maturity in the 1 to 12–month range. Further, the correlation between the second factor and the empirically calculated slope of the risk premiums (24–month minus one–month risk premium) is also relatively high yields a coefficient of correlation of almost 0.5.

We have defined the third factor as a ‘curvature’ factor, as shown in Figure 4. It represents the movements in the currency risk premium series that captures the curvature effect. As suggested earlier, the loadings for third factor are the highest at the very short–end and the very long–end of the curve giving the factor the shape of a curve. Further, the correlation between the third factor and the empirical curvature measure suggested in Diebold et al. (2006) is very high (approximately 75%). As far as fourth factor is concerned, it shows a behavior similar to the third factor and explains the highest variation in the risk premium curve at the shortest and the longest end. The fourth factor also shows some correlation with the empirical curvature factor (approximately 33%). Therefore, we label the fourth factor as a ‘second curvature’ factor.

4.6. In–Sample Fit for the Four–Factor Model

In order to evaluate the fit of the suggested four–factor model to actually observed realized risk premiums, we plot the actual and estimated risk premiums for some selected maturities. The estimated risk premiums are calculated by multiplying each factor score with their respective factor loading for a particular maturity and then adding them up. In Figure 5, we provide a plot of the estimated risk premiums for six–month and 12–month contracts based on the proposed factor model together with the actual realized risk premium for the AUD–USD contracts for these maturities. The figure shows that the estimated and realized are almost identical or nearly overlap for the sample period, January, 1999 to December 2013. In a similar manner, the estimated and actual risk premiums for other maturities also coincide for other maturities due to the high explanatory power of the model that is greater than 90%.
Figure 5: Performance evaluation of the model. We show the plots of the actual and the estimated risk premium series for 6–month and 12–month maturity for the Australian dollar for the sample period, January, 1999 to December 2013. The solid line shows the actual risk premium and the dotted line is estimated (model–based) risk premium.

For a robustness check of the in–sample fit of the proposed model, we present the residuals of the fitted versus the actual risk premiums for the AUD–USD forward contracts in Table 5. The table illustrates that the estimated mean of residual errors is zero for all maturities and the estimated standard deviations are very small. We also report the mean absolute error and the root mean square error: both are very small and indicate a good in–sample fit. Moreover, similar to Gürkaynak, Sack, and Wright (2007) and Christensen, Diebold, and Rudebusch (2009), we observe largest fitting errors at the longest maturities. The last three columns of the table show the autocorrelations in the residual errors at lags 1, 12 and 30. The autocorrelation functions are very low and they decline with the number of lags which indicates that there is a quite low or no persistence in the errors. Our results contrast with the results of Diebold and Li.
(2006) and Medeiros and Rodriguez (2011) who report very high and persistent correlations in the residual errors for fitting factor models to the term structure of the yield curve. Overall, our results suggest that the proposed factor model is statistically significant in explaining and estimating the term–structure of risk premiums for currency forward contracts.

Table 5:
Descriptive Statistics for AUD–USD Risk Premium Residual Errors

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>SD</th>
<th>MAE</th>
<th>RMSE</th>
<th>(\rho(1))</th>
<th>(\rho(12))</th>
<th>(\rho(30))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-M</td>
<td>0.000</td>
<td>0.035</td>
<td>-0.28</td>
<td>0.111</td>
<td>0.008</td>
<td>0.011</td>
<td>-0.013</td>
<td>-0.051</td>
<td>-0.041</td>
</tr>
<tr>
<td>2-M</td>
<td>0.000</td>
<td>0.062</td>
<td>-0.061</td>
<td>0.021</td>
<td>0.016</td>
<td>0.021</td>
<td>-0.406</td>
<td>-0.013</td>
<td>-0.099</td>
</tr>
<tr>
<td>3-M</td>
<td>0.000</td>
<td>0.067</td>
<td>-0.069</td>
<td>0.024</td>
<td>0.018</td>
<td>0.023</td>
<td>0.066</td>
<td>-0.063</td>
<td>-0.050</td>
</tr>
<tr>
<td>4-M</td>
<td>0.000</td>
<td>0.083</td>
<td>-0.075</td>
<td>0.024</td>
<td>0.018</td>
<td>0.024</td>
<td>-0.052</td>
<td>-0.088</td>
<td>-0.059</td>
</tr>
<tr>
<td>5-M</td>
<td>0.000</td>
<td>0.086</td>
<td>-0.080</td>
<td>0.025</td>
<td>0.019</td>
<td>0.025</td>
<td>-0.003</td>
<td>-0.108</td>
<td>-0.058</td>
</tr>
<tr>
<td>6-M</td>
<td>0.000</td>
<td>0.080</td>
<td>-0.092</td>
<td>0.028</td>
<td>0.020</td>
<td>0.028</td>
<td>0.138</td>
<td>-0.082</td>
<td>-0.026</td>
</tr>
<tr>
<td>7-M</td>
<td>0.000</td>
<td>0.086</td>
<td>-0.094</td>
<td>0.030</td>
<td>0.023</td>
<td>0.030</td>
<td>0.238</td>
<td>-0.031</td>
<td>-0.006</td>
</tr>
<tr>
<td>8-M</td>
<td>0.000</td>
<td>0.093</td>
<td>-0.089</td>
<td>0.029</td>
<td>0.022</td>
<td>0.029</td>
<td>0.250</td>
<td>-0.039</td>
<td>0.008</td>
</tr>
<tr>
<td>9-M</td>
<td>0.000</td>
<td>0.075</td>
<td>-0.084</td>
<td>0.027</td>
<td>0.021</td>
<td>0.027</td>
<td>0.161</td>
<td>-0.064</td>
<td>-0.006</td>
</tr>
<tr>
<td>10-M</td>
<td>0.000</td>
<td>0.074</td>
<td>-0.099</td>
<td>0.026</td>
<td>0.020</td>
<td>0.026</td>
<td>0.061</td>
<td>-0.064</td>
<td>-0.023</td>
</tr>
<tr>
<td>11-M</td>
<td>0.000</td>
<td>0.065</td>
<td>-0.113</td>
<td>0.028</td>
<td>0.021</td>
<td>0.028</td>
<td>0.191</td>
<td>0.023</td>
<td>-0.029</td>
</tr>
<tr>
<td>12-M</td>
<td>0.000</td>
<td>0.073</td>
<td>-0.163</td>
<td>0.036</td>
<td>0.027</td>
<td>0.036</td>
<td>0.472</td>
<td>0.002</td>
<td>0.009</td>
</tr>
<tr>
<td>18-M</td>
<td>0.000</td>
<td>0.177</td>
<td>-0.174</td>
<td>0.048</td>
<td>0.032</td>
<td>0.048</td>
<td>0.818</td>
<td>0.019</td>
<td>-0.003</td>
</tr>
<tr>
<td>24-M</td>
<td>0.000</td>
<td>0.176</td>
<td>-0.201</td>
<td>0.054</td>
<td>0.038</td>
<td>0.054</td>
<td>0.793</td>
<td>-0.153</td>
<td>0.073</td>
</tr>
</tbody>
</table>

Note: Table 5 presents the descriptive statistics for the risk premium residuals of different maturities for AUD–USD. The last three columns of the table show the sample autocorrelations at lag 1, 12 and 30. SD, MAE, RMSE and N stand for standard deviation, mean absolute error, root mean square error and number of observations respectively. The data covers the period from January, 1999 to December 2013.

4.7. Relating the Factors to Financial Variables

In a next step, we try to relate the common factors obtained from the PCA of the term structure of risk premiums to fundamental financial or macroeconomic variables. Many models have used latent factor models to explain term structure movements, however, unlike financial or macroeconomic variables; the factors are not directly observable. For example, in the interest rate term structure modelling, Pearson and Sun (1994) label their factors as “short rate” and “inflation,” however, their estimation does not use inflation data. These terms are just the convenient names for the unobserved or latent factors. Similarly, Litterman and Scheinkman (1991) call their factors as “level,” “slope” and “curvature.” Further, Dai and Singleton (2000) use the terms “level,” “slope” and “butterfly” to describe their factors. These labels simply stand for the effect of the factors on the yield curve rather than describing the economic sources of risk. Diebold at al. (2006) have explicitly incorporated macroeconomic variables into their multi–factor yield curve models. Diebold et al. (2006) apply factor models to the interest rates in the US and obtain three common factors which they label as level, slope and curvature. Further, they relate their first factor (level factor) to a measure of inflation (the 12–month% change in the price deflator for personal consumption expenditures) and find that the two are 43% correlated which is consistent with a link between the level of yield curves and inflationary
expectations. Same support is found in the studies of Kozicki and Tinsley (2001), Rudebusch and Wu (2003) and Dewachter and Lyrio (2006). They find a 39% correlation between the second factor (slope factor) and the capacity utilization which suggests that the yield curve slope is intimately connected to the cyclical dynamics of the economy. However, they could not find a suitable macroeconomic proxy for their third factor (the curvature factor).

In the following, we investigate the relation between the derived factors and financial variables. We attempt to relate the identified factors for the risk premiums to a number of independent variables such as forward premiums, realized variance, skewness, kurtosis of spot currency returns. Recall that the realized measures have been calculated from spot returns of the exchange rate based on different periods ranging from one month up to 24 months. Therefore, we also have a vector of realized variance, skewness, kurtosis of spot currency returns and forward premiums similar to the matrix of risk premiums at each point in time. We apply PCA on these variables and extract common factors which in turn could be related to the four primary factors explaining the realized risk premiums. We find that the first factor of the realized variances explains the movements of the first factor (level factor) of the realized risk premiums, yielding a coefficient of correlation of approximately 0.42. In Figure 6–8, we plot the respective factors with their statistical measures and empirical proxies. Figure 6 shows the estimated first factor and two closely linked series: an empirical proxy for level factor as explained above and a realized variance factor extracted from the realized variance of the AUD–USD spot exchange rate.

![Graph showing the first factor of the risk premium, the level measure, and the first factor of the realized variance.](image)

**Figure 6:** The figure shows the plot of the first factor of the risk premium, the level measure and the first factor of the realized variance.

We find that a factor extracted from the realized kurtosis helps to explain 22% of the variation for the slope factor of the realized risk premium. In Figure 7, we provide a plot for second risk premium factor with the slope measure (as defined above) and the factor extracted from the realized kurtosis of the spot exchange rate.
Figure 7: The figure shows the plot of the second factor of the risk premium and its slope measure with a factor extracted from the time series of realized kurtosis.

Figure 8: The figure shows the plot of the third factor of the risk premium and its curvature measure with the second factor of realized skewness.

Finally, in Figure 8, we plot the third factor of the realized risk premiums with that of the empirical proxy of the curvature as defined above. The third factor and the curvature measure are approximately 75% correlated to each other. Moreover, the curvature factor is found to be related to the second factor extracted from the realized skewness series with a correlation of approximately 0.35. Further, we find that the first factor of the forward premium also helps to explain much of the variation in the curvature of the term structure of risk premiums with a coefficient of correlation of 0.31. Unfortunately, we are not able to link any suitable proxy derived from the considered financial variables to the fourth factor of realized risk premiums.
Overall, our results provide some strong indication that the dynamics of the term structure of realized risk premiums in the AUD–USD forward market can be described by the proposed four-factor model. The observed term structure dynamics of the risk premiums can also be related to empirically observable financial variables based on the behavior of the spot exchange rate.

5. Conclusion

In this paper, we explore the term structure of the time-varying realized risk premiums in the AUD–USD currency forward market. To the best of our knowledge, this is the first study ever to have undertaken such an analysis for the term structure of risk premiums in any currency market. We analyze data for the AUD–USD spot exchange rate as well as forward rates for 14 different maturities ranging from one month to 24 months. Using monthly observations, we study the term–structure of risk premiums over a sample period from January 1999 to December 2013.

We find positive and statistically significant realized risk premiums in the AUD–USD currency market that increase in the maturity of the forward contract. Our results are similar to earlier studies by Canova and Ito (1991), Tai (2003), Inci and Lu (2007) and Kumar and Trück (2014). Possible reasons for the increased risk premiums with maturity of the contracts may be the following: the first explanation can be related to hedging theory, which states that hedgers play a big role for longer maturity contracts and they demand a higher premium for their exposure in the market as the maturity of the contract increases. Hedgers’ positions in the cash market are almost never for short-term contracts, hence longer-maturity contracts serve their needs well as they will incur less rolling-over costs. According to Ederington and Lee (2002), hedging activities become more prevalent on contracts with longer maturity and decline for shorter maturity contracts.

The second explanation can be related to covered interest parity (CIP). The CIP theory states that the futures–spot basis is equivalent to the interest rate differential between two countries’ currencies. The short end of the interest rate term structure is typically less affected by risk premium as it is set by the central banks. As we go to the far end of the yield curve, market risk premiums start playing a more important role in the determination of interest rates, as these rates are set up by market forces. Moreover, the positive risk premiums are can be explained through carry trade strategy. Since in recent times, interest rates in Australia are higher than the US, therefore, carry traders must have taken long position in AUD and short position in USD (Brunnermeier, 2009) and earn a positive risk premium.

We also study the empirical relationship between observed risk premiums in the AUD–USD forward market and financial variables, for example, currency spot returns, realized variance, skewness and the kurtosis of currency returns as well as the forward premium. We find that for shorter maturity forward contracts, none of the explanatory variables is significant. However, for contracts with longer maturities, the explanatory power of the considered variables increases. While observed risk premiums are positively related to realized variance and
skewness in the spot exchange rate, they are found to be negatively related to currency spot returns, the forward premium and realized kurtosis of spot currency returns. These results may also point towards the predictive power of these variables for future exchange rate movements what could be investigated in future studies.

Next, we apply principal component analysis in an attempt to measure and interpret the common factors that determine the term structure of the high dimensional realized risk premium series in the AUD–USD currency forward market. We find that a four–factor model accounts for almost 93% of the total variation in the risk premiums for all maturities. The first factor explains on average the largest proportion of the total explained variation and can be regarded as a ‘level’ factor, since the risk premiums of all maturities load heavily and on the factor. Moreover, we find that the first factor and the level (the average of the one–month, nine–month and 24–month risk premiums) of the risk premiums are highly correlated. We further find that the first identified risk premium factor can be related to the realized variance in the spot exchange rate. We show that the contribution of the second factor to the explained variation in realized risk premiums is much larger at the very short end of the curve; therefore, the second factor is labelled as the ‘slope’ factor. Interestingly, the factor can also be related to the realized kurtosis of AUD–USD currency spot returns. The explanatory power of the third factor is the highest at for very short maturities as well as at the furthest end of the risk premium term structure curve and can be considered as a curvature factor. Moreover, the factor is found to be related to the forward premium and the realized skewness of currency spot returns.

Overall, our results provide important insights on the nature and dynamics of risk premiums for participants in currency forward or futures markets. We provide a pioneer study to analyse the significance and behaviour of the premiums for a large set of maturities ranging from one month to 24 months. Our findings provide strong evidence that the dynamics of the term structure of realized risk premiums in the AUD–USD forward market can be described by a model with a relatively small number of factors. Interestingly, the observed term structure dynamics of the risk premiums can also be related to empirically observable financial variables based on the behavior of the spot exchange rate. We strongly recommend further analysis of the predictive relationship between these variables in future work.
References


