On the Inference of Risk-neutral Market Beta

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Abstract

This paper pertains to the idea of using options data to infer risk neutral market beta. We investigate the validity of hypotheses of the zero skewness for the idiosyncratic component of the market model and the independence between market return and idiosyncratic component under the risk-neutral measure by an existing methodology. We find that, for about 30% of the stocks, the risk-neutral betas obtained from the methodology imply negative risk-neutral variances or fourth central moments of the idiosyncratic components. Results of sensitivity analysis show that zero skewness assumption changes the order of risk-neutral betas drastically. The implications of the zero skewness and independence hypotheses are strongly rejected for the majority of the stocks with traded options.

Keywords: option-implied market beta, option-implied moments, zero skewness hypothesis, independence hypothesis.

JEL Classification: G12

1 Introduction

Market beta, which is standardized covariance between an individual stock return and the market return, is very important in asset pricing and corporate finance. In the well-known Capital Asset Pricing Model (CAPM), market beta is the only relevant risk measure for asset pricing purposes. Although versions of the CAPM have been rejected by numerous tests, the importance of market beta remains, as the market return is still the most important factor driving returns on individual stocks. One of the issues in this literature is the estimation of the market betas. While the theory allows the conditional market betas to be time-varying, depending on other state variables at the time, the most common method of beta estimation is to run regressions of individual returns on market returns over a historic period. Such backward looking estimates certainly cannot capture the changes in the market beta quickly enough, so the usefulness of the market beta in describing stock returns has not been adequately assessed. Accurate measurement of the market beta has always been a hot topic in both academia and industry.

A relatively new idea is to use the information contained in the options prices to infer the conditional market beta. The benefit of using options data is that the betas estimated that way are forward looking, rather backward looking, as options are priced based on the information available at the time regarding the future risk-neutral return distributions. Unlike inferences about the characteristics of the distribution of individual stocks, the market beta involve the joint distribution of an individual stock and the market return. Since options written on joint events do not exist, the risk neutral market beta cannot be inferred without making additional assumptions. Chang, Christofferson, Jacobs and Vainberg (2011, hereafter CCJV) make an assumption that the risk-neutral skewnesses of the idiosyncratic component of individual stock returns are all zero, in addition to the assumption that the idiosyncratic component is independent of the market return. They report that their estimated risk-neutral market betas possess certain desired features and forecast future physical market betas better than historically estimated market betas.

The current paper is a follow-up study of CCJV. We first point out certain undesired features of the CCJV betas. In particular, we report that for about 30% of the stocks that have options traded on them, the CCJV betas imply negative risk-neutral variances and fourth moments of the idiosyncratic components. Sensitivity analysis further confirms that estimated risk-neutral skewness is sensitive to the skewness hypothesis and, as a result, the CCJV methodology does not generate accurate and reliable risk-neutral market betas. We formally test the implications of the zero skewness hypothesis and the independence hypothesis using the generalized method of moments. The results show that for majority of the stocks with options, the hypotheses are strongly rejected.

The rest of the paper is organized as follows. Section 2 describes the moment relations under risk neutral measure, the assumptions made by CCJV, and a methodology to calculate unconditional risk-neutral expectation. Section 3 describes the data used in this paper. Section 4 presents the results. Section 5 concludes.

2 The Potential Problem

2.1 Risk Neutral Moments

Corresponding to the market model under the physical measure, the market model under the risk-neutral measure is as follows,

$$R_{i,t} = \widetilde{\alpha}_i + \widetilde{\beta}_i R_{m,t} + \widetilde{\epsilon}_{i,t} \tag{1}$$

where $R_{i,t}$ and $R_{m,t}$ are the log return of stock i and the market index in period t, respectively. $\widetilde{\epsilon}_{i,t}$ captures the idiosyncratic shock of stock i and is uncorrelated with market return under the risk-neutral measure, with $\widetilde{E}[\widetilde{\epsilon}_{i,t}] = 0$. A tilde indicates a quantity under the risk-neutral measure. For example, $\widetilde{\beta}_i$ is the risk-neutral beta and $\widetilde{\epsilon}_{i,t}$ is the idiosyncratic component of the return on stock i, which is supposed to satisfy $\tilde{E}[R_{m,t}\widetilde{\epsilon}_{i,t}] =$

0.

According to the market model for log return under the risk-neutral measure, the following equations for risk-neutral conditional moments under the additional assumption that $R_{m,t}$ and $\widetilde{\epsilon}_{i,t}$ are independent under the risk-neutral measure:

$$\widetilde{Var}_{i,t,\tau} = \beta_{i,t,\tau}^2 \widetilde{Var}_{m,t,\tau} + \widetilde{Var}_{\epsilon_i,t,\tau} \tag{2}$$

$$\widetilde{Skew}_{i,t,\tau} = \frac{\beta_{i,t,\tau}^3 \widetilde{Skew}_{m,t,\tau} \widetilde{Var}_{m,t,\tau}^{3/2} + \widetilde{Skew}_{\epsilon_i,t,\tau} \widetilde{Var}_{\epsilon_i,t,\tau}^{3/2}}{\widetilde{Var}_{i,t,\tau}^{3/2}} \tag{3}$$

$$\widetilde{Kurt}_{i,t,\tau} = \frac{\beta_{i,t,\tau}^4 \widetilde{Kurt}_{m,t,\tau} \widetilde{Var}_{m,t,\tau}^2 + 6\beta_{i,t,\tau}^2 \widetilde{Var}_{m,t,\tau} \widetilde{Var}_{\epsilon_i,t,\tau} + \widetilde{Kurt}_{\epsilon_i,t,\tau} \widetilde{Var}_{\epsilon_i,t,\tau}^2}{\widetilde{Var}_{i,t,\tau}^2} \tag{4}$$

where Var, $Skew$ and $Kurt$ represent variance, skewness and Kurtosis. The detail derivation of these equations are given in Appendix A.

Given the market model of log return under the risk-neutral measure and an independence assumption which states that $R_{m,t}$ and $\widetilde{\epsilon}_{i,t}$ are independent under the risk-neutral measure, CCJV propose a method for beta calculation by assuming that the skewness/third central moment of the idiosyncratic shock is zero, i.e., $\widetilde{Skew}_{\epsilon_i,t,\tau} = 0$. Substituting this into (4), we have

$$\widetilde{\beta}_i = \left(\frac{\widetilde{Skew}_i}{\widetilde{Skew}_m} \right)^{1/3} \left(\frac{\widetilde{Var}_i}{\widetilde{Var}_m} \right)^{1/2} = \left(\frac{\widetilde{\mu}_3(R_i)}{\widetilde{\mu}_3(R_m)} \right)^{1/3}$$

where $\widetilde{\mu}_n(X)$ is the n-th central moments of X under the risk-neutral measure. Using methods for estimating option-implied moments such as **?**, one can easily calculate the third option-implied moments of both individual stock returns and market index returns, therefore obtain the option-implied beta.

CCJV obtain the risk-neutral market betas using (4) only. However, substituting $\beta_{i,t,\tau}^{\mathrm{CCJV}}$ into equations (2) and (4), one cannot guarantee that $\widetilde{Var}_{\epsilon_i,t,\tau}$ and $\widetilde{Kurt}_{\epsilon_i,t,\tau}$ are positive, which is contradictory with the definition of variance and kurtosis. We can easily verify this once we calculate the risk-neutral moments of the individual stock and the market index.

2.2 Option-implied moments

We calculate the option-implied moments (volatility, skewness and kurtosis) using the method proposed by **?** (BKM hereafter). BKM show that the conditional annualized variance, skewness and kurtosis of $tau-$period log return of stock i under the risk-neutral measure at time t can be calculated as:

$$\widetilde{Var}_{i,t,\tau}^{BKM} = \frac{e^{r\tau}V_{i,t,\tau} - \mu_{i,t,\tau}^2}{\tau} \tag{5}$$

$$\widetilde{Skew}_{i,t,\tau}^{BKM} = \frac{e^{r\tau}W_{i,t,\tau} - 3\mu_{i,t,\tau}e^{r\tau}V_{i,t,\tau} + 2\mu_{i,t,\tau}^3}{(e^{r\tau}V_{i,t,\tau} - \mu_{i,t,\tau}^2)^{3/2}} \tag{6}$$

$$\widetilde{Kurt}_{i,t,\tau}^{BKM} = \frac{e^{r\tau}X_{i,t,\tau} - 4\mu_{i,t,\tau}e^{r\tau}W_{i,t,\tau} + 6\mu_{i,t,\tau}^2 e^{r\tau}V_{i,t,\tau} - 3\mu_{i,t,\tau}^4}{(e^{r\tau}V_{i,t,\tau} - \mu_{i,t,\tau}^2)^2} \tag{7}$$

where

$$\begin{aligned}
\mu_{i,t,\tau} &= e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V_{i,t,\tau} - \frac{e^{r\tau}}{6}W_{i,t,\tau} - \frac{e^{r\tau}}{24}W_{i,t,\tau}, \\
V_{i,t,\tau} &= \int_{S_{i,t}}^{\infty} \frac{2(1-\ln[\frac{K}{S_{i,t}}])}{K^2} C_{i,t}(\tau;K)dK + \int_0^{S_{i,t}} \frac{2(1+\ln[\frac{S_{i,t}}{K}])}{K^2} P_{i,t}(\tau;K)dK, \\
W_{i,t,\tau} &= \int_{S_{i,t}}^{\infty} \frac{6\ln[\frac{K}{S_{i,t}}] - 3(\ln[\frac{K}{S_{i,t}}])^2}{K^2} C_{i,t}(\tau;K)dK \\
&\quad - \int_0^{S_{i,t}} \frac{6\ln[\frac{S_{i,t}}{K}] + 3(\ln[\frac{S_{i,t}}{K}])^2}{K^2} P_{i,t}(\tau;K)dK, \\
X_{i,t,\tau} &= \int_{S_{i,t}}^{\infty} \frac{12(\ln[\frac{K}{S_{i,t}}])^2 - 4(\ln[\frac{K}{S_{i,t}}])^3}{K^2} C_{i,t}(\tau;K)dK \\
&\quad + \int_0^{S_{i,t}} \frac{12(\ln[\frac{S_{i,t}}{K}])^2 + 4(\ln[\frac{S_{i,t}}{K}])^3}{K^2} P_{i,t}(\tau;K)dK,
\end{aligned}$$

where $V_{i,t,\tau}$, $W_{i,t,\tau}$ and $X_{i,t,\tau}$ are the prices of the squared, cubed, and quartic contracts formulated through a portfolio of options indexed by the strikes.

Following **?**, **?**, **?** and **?**, we use a trapezoidal method to estimate $V_{i,t,\tau}$, $W_{i,t,\tau}$ and $X_{i,t,\tau}$. The detail trapezoidal approach to estimating the integrals are introduced in Appendix B.

2.3 Unconditional risk-neutral moments of idiosyncratic shock

Even for stocks that positivity of variance and kurtosis is not violated, the CCJV beta may not be right in the sense that the CCJV beta generates a sequence of idiosyncratic component of the stock return, which does not satisfy the usual orthogonality condition $\tilde{E}[R_{m,t}\widetilde{\epsilon}_{i,t}] = 0$, nor the zero skewness assumption $\widetilde{Skew}_{\epsilon_i,t,\tau} = 0$. We can check these conditions by the implied unconditional moments. To do this, we need a stochastic discount factor.

To verify whether CCJV's assumption of zero skewness of idiosyncratic shock under the risk-neutral measure is right, we have to calculate the unconditional risk-neutral third central moments, $\widetilde{E}[\widetilde{\epsilon}^3_{i,t,t+\tau}]$. According to the relation between the physical measure and the risk-neutral measure, we have

$$\widetilde{E}[\widetilde{\epsilon}^3_{i,t,t+\tau}] = E[\tilde{M}_{i,t,t+\tau}\widetilde{\epsilon}^3_{i,t,t+\tau}] \tag{8}$$

where $\tilde{M}_{i,t,t+\tau} = E_t[M_{t,t+\tau}|R_{i,t,t+\tau}]$ is the projected stochastic discount factor (SDF hereafter), which is the general SDF $M_{t,t+\tau}$ projected on the space spanned by stock return $R_{i,t,t+\tau}$ conditional on time$-t$ information. Given $\widetilde{\beta}^{\text{CCJV}}_{i,t,\tau}$, we can obtain a time-series of residual under the risk-neutral measure according to equation (**??**), $\widetilde{e}^{\text{CCJV}}_{i,t,\tau} = r_{i,t,t+\tau} - \widetilde{\beta}^{\text{CCJV}}_{i,t,\tau} r_{m,t,t+\tau}$ for $t = 1, \cdots, T$. Since $\widetilde{\alpha}_{i,\tau}$ is assumed to be constant, $\widetilde{E}[\widetilde{\epsilon}^3_{i,t,t+\tau}] = \widetilde{E}[(\widetilde{e}^{\text{CCJV}}_{i,t,\tau} - \widetilde{E}[\widetilde{e}^{\text{CCJV}}_{i,t,\tau}])^3]$. Thus the core question to calculate $\widetilde{E}[\widetilde{\epsilon}^3_{i,t,t+\tau}]$ is to estimate the projected SDF $\tilde{M}_{i,t,t+\tau}$.

Following **?**, we propose a nonparametric approach for estimating the projected SDF. Let $\mathbf{R}_{t,t+\tau}$ be the $N \times 1$ vector of $\tau-$period gross return of N stocks. We apply principle component analysis on $R_{t,t+\tau}$ regarding the extracted $r \times 1$ vector of factors as $\mathbf{R}^B_{t,t+\tau}$,

$$\mathbf{R}_{t,t+\tau} = \Lambda \mathbf{R}^B_{t,t+\tau} + \mathbf{v}_t \tag{9}$$

where Λ is $N \times r$ matrix, $\Lambda'\Lambda = I_r$. Then we can estimate that

$$\widehat{\mathbf{R}}^B_{t,t+\tau} = \widehat{\Lambda}' \mathbf{R}_{t,t+\tau} \tag{10}$$

where $\widehat{\Lambda}$ is the first r eigenvectors of $T^{-1}\sum_{t=1}^{T} \mathbf{R}_{t,t+\tau}\mathbf{R}'_{t,t+\tau}$.

Consider the Euler equation

$$E_t[M_{t,t+\tau}\mathbf{R}'_{t,t+\tau}] = 1'_N \tag{11}$$

where E_t is the expectation conditional on time-t information set $\mathcal{F}$ is generated by a state vector X_t. Assume $M_{t,t+\tau} = c'_r \widehat{\mathbf{R}}^B_{t,t+\tau}$, then we have

$$E_t[c'_r \widehat{\mathbf{R}}^B_{t,t+\tau}\mathbf{R}'_{t,t+\tau}] = E_t[c'_r \widehat{\mathbf{R}}^B_{t,t+\tau}\widehat{\mathbf{R}}^{B\prime}_{t,t+\tau}\widehat{\Lambda}'] = 1'_N \tag{12}$$

We solve that

$$c'_r = 1'_N \widehat{\Lambda} E_t[\widehat{\mathbf{R}}^B_{t,t+\tau}\widehat{\mathbf{R}}^{B\prime}_{t,t+\tau}]^{-1} \tag{13}$$

Thus, the projected SDF for stock i

$$\begin{aligned}
\tilde{M}^i_{t,t+\tau} &= E_t[M_{t,t+\tau}|R^i_{t,t+\tau}] \\
&= E_t[c'_r \widehat{\mathbf{R}}^B_{t,t+\tau}|R^i_{t,t+\tau}] \\
&= 1'_N \widehat{\Lambda} E_t[\widehat{\mathbf{R}}^B_{t,t+\tau}\widehat{\mathbf{R}}^{B\prime}_{t,t+\tau}]^{-1} E_t[\widehat{\mathbf{R}}^B_{t,t+\tau}|R^i_{t,t+\tau}] \\
&= 1'_N \widehat{\Lambda} E[\widehat{\mathbf{R}}^B_{t,t+\tau}\widehat{\mathbf{R}}^{B\prime}_{t,t+\tau}|X_t]^{-1} E[\widehat{\mathbf{R}}^B_{t,t+\tau}|R^i_{t,t+\tau}, X_t]
\end{aligned} \tag{14}$$

. Let the state vector X_t contain the VIX index and 1-m risk-free rate at time t, we use locally weighted regression to estimate each element in $E[\widehat{\mathbf{R}}^{B}_{t,t+\tau}\widehat{\mathbf{R}}^{B\prime}_{t,t+\tau}|X_t]$ and $E[\widehat{\mathbf{R}}^{B}_{t,t+\tau}|R^{i}_{t,t+\tau}, X_t]$ referring to **?**, then we obtain the estimate of $\tilde{M}^{i}_{t,t+\tau}$.

3 Data Description

We study the common stocks with options in the sample period from the end of Jan. 1996 to the end of Dec. 2015 in this paper. The data frequency is monthly and our study focus on 1-m and 6-m investment horizon of returns. The stock and option data is briefly described in Section 3.1. The option-implied moments estimation is discussed in Section 3.2.

3.1 Stock and option data

The individual stock return data are available in CRSP database while the option data are obtained from OptionMetrics. The risk-free rate and market return are from French's data library. At the last trading day of each month, we extract the security ID, implied volatility, implied strike from the Volatility_surface file at delta of 0.2,0.25,0.3,0.35,0.4,0.45 and 0.5 (-0.2,-0.25,-0.3,-0.35,-0.4,-0.45 and -0.5) for call (put) options with expiration of 30 and 182 calendar days. We apply the local linear regression model for fitting the implied volatility curve then transform it into option price using Black-Shore formula. When fitting the implied volatility curve, we restrict the moneyness between 70% of the moneyness corresponding to delta of -0.2 and 130% of the moneyness corresponding to delta of 0.2. We climate those stocks with less than 12 time-series observations during Jan. 1996 to Dec. 2015 and finally obtain data for over 1900 stocks on average for both 1-m and 6-m horizons.

Summary statistics of the gross return of stocks, annualized implied volatility, and minimum (delta = -0.2) and maximum ((delta = 0.2)) moneyness of options for 1-m and 6-m horizons are presented in Table 1. The corresponding time-series mean of each variable are plotted in Figure 1. It shows that the spread between the moneyness of out-of call option and out-of put option is large when the option-implied volatility is high, which indicates a flat risk-neutral density function appears in period of high option-implied volatility such as the financial crisis in 2008.

3.2 Option-implied moments estimation

As introduced in Section 2.2, we estimate the option-implied variance, skewness, and kurtosis for log returns of individual stocks and S&P 500 index for $\tau = 1$ month and 6 months using BKM method and report the characteristics of each variable in this section.

Table 2 presents the time-series means for the cross-sectional statistics of option-implied variance, skewness and kurtosis of individual log returns. It shows that the average option-implied skewness of log return of individual stock is negative. The 1-m option-implied skewness is less than 6-m option implied skewness while the 1-m option-implied kurtosis is larger than 6-m option-implied kurtosis.

We compare the time-series of cross-sectional mean of option-implied individual variance,skewness and kurtosis with the time-series of option-implied market index variance, skewness and kurtosis in Figure 2. The correlation between each average moment of individual stocks and moment of market index are positive with coefficient ranging from 0.48 to 0.82. The cross-sectional mean of individual variance is much higher than that of the market index, indicating that the idiosyncratic shock accounts a remarkable proportion of the total variance of individual stock. The market index option-implied skewness is negative and much lower than the cross-sectional mean of individual option-implied skewness at almost all the time. Opposite to the skewness, market index shows a higher kurtosis than the average kurtosis of individual stocks.

4 Empirical Results

In this section, we first discuss the characteristics of the estimates of risk-neutral following CCJV's idea. Then we provide empirical evidence for the violation of positive variance and kurtosis of idiosyncratic shocks for $\widetilde{\beta}^{\text{CCJV}}$. Finally, we construct hypothesis test to testing whether the zero skewness assumption of idiosyncratic shocks is empirically supported.

4.1 Option-implied betas

For each stock i at time t, given the option-implied variance and skewness of stock i and the market index log returns with maturity τ calculated by BKM method, we calculate the option-implied beta with zero idiosyncratic skewness assumption,

$$\widetilde{\beta}_{i,t,\tau}^{\text{CCJV}} = \left(\frac{\widetilde{Skew}_{i,t,\tau}^{BKM}}{\widetilde{Skew}_{m,t,\tau}^{BKM}} \right)^{1/3} \left(\frac{\widetilde{Var}_{i,t,\tau}^{BKM}}{\widetilde{Var}_{m,t,\tau}^{BKM}} \right)^{1/2} .$$

Figure 3 shows the histogram of the calculated $\widetilde{\beta}_{i,t,\tau}^{\text{CCJV}}$ for $\tau = 1m$ and $\tau = 6m$, about 4.78×10^5 values of betas for each τ. Since the skewness of market index log return is mostly negative, the negative betas in Figure 3 correspond to positive skewness of individual stock log returns. Table 3 presents the summary statistics of option-implied betas by CCJV's zero idiosyncratic skewness assumption with 1-m and 6-m option maturities during Jan 1996-Dec 2015. It shows that the mean, median and standard deviation values of 1-m option-implied beta are higher than those of 6-m option-implied beta. The time-series mean of 1-m and 6-m option-implied betas are plotted in Figure 4. The more negative mean of beta corresponds to the higher positive mean of individual skewness in the sample period.

Problems of betas calculated by zero idiosyncratic skewness assumption are discussed in the following subsections.

4.2 Violation of idiosyncratic positive variance and kurtosis

Given the option-implied beta with zero idiosyncratic skewness assumption, $\tilde{\beta}_{i,t,\tau}^{\text{CCJV}}$, we can calculate the conditional idiosyncratic variance and fourth central moment according to equations (2) and (4) as follows,

$$
\begin{aligned}
\widetilde{Var}_{\epsilon_i,t,\tau}^{\text{CCJV}} &= \widetilde{Var}_{i,t,\tau}^{BKM} - \tilde{\beta}_{i,t,\tau}^{\text{CCJV}^2}\widetilde{Var}_{m,t,\tau}^{BKM} \\
\widetilde{\mu}_{4\epsilon_i,t,\tau}^{\text{CCJV}} &= \widetilde{Kurt}_{i,t,\tau}^{BKM}\widetilde{Var}_{i,t,\tau}^{BKM^2} - \tilde{\beta}_{i,t,\tau}^{\text{CCJV}^4}\widetilde{Kurt}_{m,t,\tau}^{BKM}\widetilde{Var}_{m,t,\tau}^{BKM^2} - 6\tilde{\beta}_{i,t,\tau}^{\text{CCJV}^2}\widetilde{Var}_{m,t,\tau}^{BKM}\widetilde{Vol}_{\epsilon_i,t,\tau}^{BKM^2}
\end{aligned}
$$

According to the definition of variance and fourth central moment, $\widetilde{Var}_{\epsilon_i,t,\tau}^{\text{CCJV}}$ and $\widetilde{\mu}_{4\epsilon_i,t,\tau}^{\text{CCJV}}$ should be positive for all i, t and τ.

We plot the histograms of conditional variance and fourth central moment of idiosyncratic shock implied by option-implied betas given zero idiosyncratic skewness assumption with $\tau = 1m$ and $\tau = 6m$ in Figure 5. It shows that not all $\widetilde{Var}_{\epsilon_i,t,\tau}^{\text{CCJV}}$ and $\widetilde{\mu}_{4\epsilon_i,t,\tau}^{\text{CCJV}}$ are larger than zero, which means CCJV's zero idiosyncratic skewness assumption could generate negative conditional idiosyncratic variance and forth central moment.

The summary statistics of the valid and invalid conditional idiosyncratic variance and forth central moment are presented in Table 4. We find that the proportional of invalid conditional idiosyncratic variance is less than 10%, while the proportional of invalid conditional idiosyncratic forth central moment is more than 25%. There is total about 30% of the estimated $\tilde{\beta}_{i,t,\tau}^{\text{CCJV}}$ generate invalid conditional idiosyncratic variance and forth central moment.

We discuss the unconditional idiosyncratic risk-neutral skewness of the left 70% betas that generate positive idiosyncratic variance and forth central moment in next subsection.

4.3 Violation of zero idiosyncratic skewness assumption

As we discussed in Section 2.3, given the estimated $\widetilde{\beta}_{i,t,\tau}^{\text{CCJV}}$ and $\tilde{M}_{t,t+\tau}^{i}$, we can calculate the unconditional risk-neutral idiosyncratic skewness,

$$\widetilde{Skew}_{\epsilon_i,\tau}^{M} = \frac{\widetilde{E}[(\widetilde{e}_{i,t,\tau}^{\text{CCJV}} - \widetilde{E}[\widetilde{e}_{i,t,\tau}^{\text{CCJV}}])^3]}{(\widetilde{E}[(\widetilde{e}_{i,t,\tau}^{\text{CCJV}} - \widetilde{E}[\widetilde{e}_{i,t,\tau}^{\text{CCJV}}])^2])^{3/2}} \tag{15}$$

where $\widetilde{e}_{i,t,\tau}^{\text{CCJV}} = r_{i,t,t+\tau} - \widetilde{\beta}_{i,t,\tau}^{\text{CCJV}} r_{m,t,t+\tau}$.

Figure 6 plots the histograms of unconditional idiosyncratic skewness implied by option-implied betas given zero idiosyncratic skewness assumption for $\tau = 1m$ and $\tau = 6m$. The summary statistics of $\widetilde{Skew}_{\epsilon_i,1m}^{M}$ and $\widetilde{Skew}_{\epsilon_i,6m}^{M}$ are presented in Table 5. It shows that the mean and median values of the unconditional idiosyncratic skewness are negative, indicating that more individual stocks demonstrating negative unconditional idiosyncratic skewness given zero idiosyncratic skewness assumption. A simulation study in CCJV shows that the bias amounts to 10% of the true beta when the absolute value of idiosyncratic skewness is close to 0.5. In our sample, we find that the absolute value of idiosyncratic skewness of 40% of the individual stocks is larger than 0.5.

Assuming the conditional risk-neutral idiosyncratic third central moment equals to the uncondition skewness calculated by the residual time-series $\widetilde{e}_{i,t,\tau}^{\text{CCJV}}$ at each time, we simulate a new measure of betas,

$$\tilde{\beta}_{i,t,\tau}^{Simulate} = \left(\frac{\widetilde{Skew}_{i,t,\tau}^{BKM} \widetilde{Var}_{i,t,\tau}^{BKM^{3/2}} - \frac{1}{T_i}\sum_{t=1}^{T_i} \widetilde{M}_{i,t,\tau}(\widetilde{e}_{i,t,\tau}^{\text{CCJV}} - \frac{1}{T_i}\sum_{j=1}^{T_i} \widetilde{M}_{i,t,\tau}\widetilde{e}_{i,j,\tau}^{\text{CCJV}})^3}{\widetilde{Skew}_{m,t,\tau}^{BKM} \widetilde{Var}_{m,t,\tau}^{BKM^{3/2}}} \right)^{1/3}.$$

We plot $\tilde{\beta}_{i,t,\tau}^{Simulate}$ versus $\widetilde{\beta}_{i,t,\tau}^{\text{CCJV}}$ in Figure 7 and find significant difference between these two betas. Lot's of points are badly deviated from the 45 degree line in the figure on both sides, which indicates that the zero skewness idiosyncratic skewness assumption seriously twists the sequence of risk-neutral betas.

4.4 Zero idiosyncratic skewness and independence assumption test

In this section, we construct a statistic to test whether the zero idiosyncratic skewness and independence assumption hold for each individual stock i. In particular, we test the moments $\widetilde{E}[(\widetilde{e}_{i,t,\tau}^{\text{CCJV}} - \widetilde{E}[\widetilde{e}_{i,t,\tau}^{\text{CCJV}}])^3] = 0$, $\widetilde{E}[r_{m,t,\tau}(\widetilde{e}_{i,t,\tau}^{\text{CCJV}} - \widetilde{E}[\widetilde{e}_{i,t,\tau}^{\text{CCJV}}])] = 0$ and $\tilde{E}[r_{m,t,\tau}^2(\widetilde{e}_{i,t,\tau}^{\text{CCJV}} - \widetilde{E}[\widetilde{e}_{i,t,\tau}^{\text{CCJV}}])] = 0$. GMM formula is applied to test these moments. We first test each moment separately for each individual stock, than we construct joint test for three moments. The GMM formula for moments test is as following,

$$J_T = Tg_T' S^{-1} g_T \rightarrow \chi^2(n) \tag{16}$$

where g_T is the vector of sample means corresponding to the expectations of tested variables, S is the covariance matrix of tested variables, and n is the number of tested variables.

For $\tau = 1m$ and $\tau = 6m$, we calculate the p-values corresponding to the statistic J_T of univariate tests and joint tests and plot the histograms in Figures 8 and 9, respectively. The summary statistics of the p-values are presented in Tables 6 and 7, respectively. The univariate tests show that less than 10% of the individual stocks reject the null hypothesis of zero idiosyncratic skewness at 10% significant level, while about 50% of the individual stocks reject the null hypothesis of independence of market log return and idiosyncratic shocks at 10% significant level. The joint tests also confirm that large proportion of the firms reject the zero idiosyncratic skewness and independence assumption.

5 Conclusion

Motivated by the research of CCJV about measuring risk-neutral beta, this paper further investigates whether CCJV's zero idiosyncratic risk-neutral skewness assumption is reasonable in a large sample containing almost all common stocks having options during Jan. 1996- Dec. 2015.

Our empirical results show that about 30% of the estimated betas by zero idiosyncratic risk-neutral skewness assumption generate negative idiosyncratic risk-neutral variance or fourth central moment. For the rest betas, we summarize the unconditional idiosyncratic risk-neutral skewness implied by these betas and find that 40% of the individual stocks have the absolute value of idiosyncratic skewness larger than 0.5, which would lead a bias more than 10% of the true beta. We also simulate a group of betas considering the conditional idiosyncratic risk-neutral central third moment equal to the CCJV's beta-implied unconditional one. Comparing the simulated beta with CCJV's beta, we find that the zero skewness idiosyncratic skewness assumption seriously twists the sequence of risk-neutral betas.

The statistics constructed to test the null hypothesis of zero idiosyncratic skewness and independence of market log return and idiosyncratic shocks show that large proportion of firms reject the null at 10% significant level. Our further research will focus on developing a measure of risk-neutral beta without imposing the zero idiosyncratic risk-neutral skewness constrain for all individual stocks and considering the dependence of market return and idiosyncratic shocks.

Appendices

A Risk-neutral Moments Equations

Given the one-factor model of the log return under the risk-neutral measure in equation (??), we have

$$\begin{aligned}
\widetilde{Var}_{i,t,\tau} &= \widetilde{E}_t[(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}])^2] \\
&= \widetilde{E}_t[((\widetilde{\alpha}_i + \widetilde{\beta}_{i,t,\tau} r_{m,t+1} + \epsilon_{i,t+1}) - (\widetilde{\alpha}_i + \widetilde{\beta}_{i,t,\tau}\widetilde{E}_t[r_{m,t+1}]))^2] \\
&= \widetilde{E}_t[(\widetilde{\beta}_{i,t,\tau}(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}]) + \epsilon_{i,t+1})^2] \\
&= \widetilde{\beta}^2_{i,t,\tau}\widetilde{E}_t[(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}])^2] + \widetilde{E}_t[\epsilon^2_{i,t+1}] \\
&= \widetilde{\beta}^2_{i,t,\tau}\widetilde{Var}_{m,t,\tau} + \widetilde{Var}_{\epsilon_i,t,\tau} \qquad (17)
\end{aligned}$$

$$\begin{aligned}
\widetilde{Skew}_{i,t,\tau} &= \frac{\widetilde{E}_t[(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}])^3]}{\widetilde{Var}^{3/2}_{i,t,\tau}} \\
&= \frac{\widetilde{E}_t[((\widetilde{\alpha}_i + \widetilde{\beta}_{i,t,\tau} r_{m,t+1} + \epsilon_{i,t+1}) - (\widetilde{\alpha}_i + \widetilde{\beta}_{i,t,\tau}\widetilde{E}_t[r_{m,t+1}]))^3]}{\widetilde{Var}^{3/2}_{i,t,\tau}} \\
&= \frac{\widetilde{E}_t[(\widetilde{\beta}_{i,t,\tau}(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}]) + \epsilon_{i,t+1})^3]}{\widetilde{Var}^{3/2}_{i,t,\tau}} \\
&= \frac{\widetilde{\beta}^3_{i,t,\tau}\widetilde{E}_t[(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}])^3] + \widetilde{E}_t[\epsilon^3_{i,t+1}]}{\widetilde{Var}^{3/2}_{i,t,\tau}} \\
&= \frac{\widetilde{\beta}^3_{i,t,\tau}\widetilde{Skew}_{m,t,\tau}\widetilde{Var}^{3/2}_{m,t,\tau} + \widetilde{Skew}_{\epsilon_i,t,\tau}\widetilde{Var}^{3/2}_{\epsilon_i,t,\tau}}{\widetilde{Var}^{3/2}_{i,t,\tau}} \qquad (18)
\end{aligned}$$

$$
\begin{aligned}
\widetilde{Kurt}_{i,t,\tau} &= \frac{\widetilde{E}_t[(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}])^4]}{\widetilde{Var}^2_{i,t,\tau}} \\
&= \frac{\widetilde{E}_t[((\widetilde{\alpha}_i + \widetilde{\beta}_{i,t,\tau} r_{m,t+1} + \epsilon_{i,t+1}) - (\widetilde{\alpha}_i + \widetilde{\beta}_{i,t,\tau}\widetilde{E}_t[r_{m,t+1}]))^4]}{\widetilde{Var}^2_{i,t,\tau}} \\
&= \frac{\widetilde{E}_t[(\widetilde{\beta}_{i,t,\tau}(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}]) + \epsilon_{i,t+1})^4]}{\widetilde{Var}^2_{i,t,\tau}} \\
&= \frac{\widetilde{\beta}^4_{i,t,\tau}\widetilde{E}_t[(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}])^4] + 6\widetilde{\beta}^2_{i,t,\tau}\widetilde{E}_t[(r_{i,t+1} - \widetilde{E}_t[r_{i,t+1}])^2]\widetilde{E}_t[\epsilon^2_{i,t+1}] + \widetilde{E}_t[\epsilon^4_{i,t+1}]}{\widetilde{Var}^2_{i,t,\tau}} \\
&= \frac{\widetilde{\beta}^4_{i,t,\tau}\widetilde{Kurt}_{m,t,\tau}\widetilde{Var}^2_{m,t,\tau} + 6\widetilde{\beta}^2_{i,t,\tau}\widetilde{Var}_{m,t,\tau}\widetilde{Var}_{\epsilon_i,t,\tau} + \widetilde{Kurt}_{\epsilon_i,t,\tau}\widetilde{Var}^2_{\epsilon_i,t,\tau}}{\widetilde{Var}^2_{i,t,\tau}} \qquad (19)
\end{aligned}
$$

B Risk-neutral Moments Calculation

We introduce how to estimate the integrals for the prices of the squared ($V_{i,t,\tau}$), cubed ($W_{i,t,\tau}$), and quartic ($X_{i,t,\tau}$) contracts from the prices of option with discrete strikes in this section.

Define the strike differences for call (put) options as $\Delta K^C_j = K^C_j - K^C_{j-1}$ ($\Delta K^P_j = K^P_{j-1} - K^P_j$) for $i = 2, \cdots, n_C$ ($i = 2, \cdots, n_P$) and $\Delta K^C_1 = K^C_1 - S_{i,t}$ ($\Delta K^P_1 = S_{i,t} - K^P_1$). Then we can approximate the BKM integrals for $V_{i,t,\tau}$, $W_{i,t,\tau}$ and $X_{i,t,\tau}$ as follows,

$$
\begin{aligned}
V_{i,t,\tau} &= v_C(K^C_1)C_{i,t}\Delta K^C_1 + \sum_{j=2}^{n_C}\frac{1}{2}[v_C(K^C_j)C_{i,t,j} + v_C(K^C_{j-1})C_{i,t,j-1}]\Delta K^C_i \\
&\quad + v_P(K^P_1)P_{i,t}\Delta K^P_1 + \sum_{j=2}^{n_P}\frac{1}{2}[v_P(K^P_j)P_{i,t,j} + v_P(K^P_{j-1})P_{i,t,j-1}]\Delta K^P_i \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
W_{i,t,\tau} &= w_C(K^C_1)C_{i,t}\Delta K^C_1 + \sum_{j=2}^{n_C}\frac{1}{2}[w_C(K^C_j)C_{i,t,j} + w_C(K^C_{j-1})C_{i,t,j-1}]\Delta K^C_i \\
&\quad - w_P(K^P_1)P_{i,t}\Delta K^P_1 + \sum_{j=2}^{n_P}\frac{1}{2}[w_P(K^P_j)P_{i,t,j} + w_P(K^P_{j-1})P_{i,t,j-1}]\Delta K^P_i \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
X_{i,t,\tau} &= x_C(K_1^C)C_{i,t}\Delta K_1^C + \sum_{j=2}^{n_C}\frac{1}{2}[x_C(K_j^C)C_{i,t,j} + x_C(K_{j-1}^C)C_{i,t,j-1}]\Delta K_i^C \\
&\quad + x_P(K_1^P)P_{i,t}\Delta K_1^P + \sum_{j=2}^{n_P}\frac{1}{2}[x_P(K_j^P)P_{i,t,j} + x_P(K_{j-1}^P)P_{i,t,j-1}]\Delta K_i^P \qquad (22)
\end{aligned}
$$

where

$$v_C(K) = \frac{2(1-\ln[\frac{K}{S_{i,t}}])}{K^2}$$

$$v_P(K) = \frac{2(1+\ln[\frac{S_{i,t}}{K}])}{K^2}$$

$$w_C(K) = \frac{6\ln[\frac{K}{S_{i,t}}] - 3(\ln[\frac{K}{S_{i,t}}])^2}{K^2}$$

$$w_P(K) = \frac{6\ln[\frac{S_{i,t}}{K}] + 3(\ln[\frac{S_{i,t}}{K}])^2}{K^2}$$

$$x_C(K) = \frac{12(\ln[\frac{K}{S_{i,t}}])^2 - 4(\ln[\frac{K}{S_{i,t}}])^3}{K^2}$$

$$x_P(K) = \frac{12(\ln[\frac{S_{i,t}}{K}])^2 + 4(\ln[\frac{S_{i,t}}{K}])^3}{K^2}$$

References

Andrews, D. W. (1991). Cross-validation in regression with heteroskedastic errors. Journal of Econometrics, 47, 359-377.

Bakshi, G., Kapadia, N., and Madan, D. (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. The Review of Financial Studies, 16, 101-143.

Bali, T. G., Hu, J., and Murray, S. (2016). Option implied volatility, skewness, and kurtosis and the cross-section of expected stock returns. Working paper.

Bali, T. G. and Murray, S. (2013). Does risk-neutral skewness predict the cross-section of equity option portfolio returns? Journal of Financial and Quantitative Analysis, 48, 1145-1171.

Chang, B.-Y., Christoffersen, P., Jacobs, K., and Vainberg, G. (2011). Option-implied measures of equity risk. Review of Finance, 16, 385-428.

Dennis, P. and Mayhew, S. (2002). Risk-neutral skewness: Evidence from stock options. Journal of Financial and Quantitative Analysis, 37, 471-493.

Duan, J.-C. and Wei, J. (2009). Systematic risk and the price structure of individual equity options. The Review of Financial Studies, 22, 1981-2006.

Zhang, C. and Zhou, T. (2015). The peso problem: Evidence from the S & P 500 options market, Working paper.

Table 1: Summary statistics of return, implied volatility and moneyness

	Mean	Std	5% quantile	Median	95% quantile	Firm
$R_{i,1m}$	1.008	0.132	0.814	1.004	1.214	1989
Implied $Vol_{i,1m}$	0.504	0.230	0.238	0.454	0.935	1989
$k_{i,1m}^{min}$	0.892	0.038	0.822	0.897	0.941	1989
$k_{i,1m}^{max}$	1.154	0.095	1.061	1.130	1.319	1989
$R_{i,6m}$	1.050	0.337	0.582	1.025	1.590	1956
Implied $Vol_{i,6m}$	0.474	0.208	0.231	0.428	0.873	1956
$k_{i,6m}^{min}$	0.807	0.047	0.727	0.811	0.876	1956
$k_{i,6m}^{max}$	1.458	0.383	1.154	1.354	2.084	1956

Note: This table presents summary statistics of gross return of individual stocks, implied volatility, minimum and maximum moneyness of individual stock options for horizons of 1 and 6 months during Jan 1996-Dec 2015. Each month, the mean (Mean), standard deviation (Std), fifth percentile (5% quantile), median (Median), and 95th percentile (95% quantile) values of the cross-sectional distribution of each variable are calculated. The table presents the time-series means for each cross-sectional value.

Table 2: Summary statistics of option-impled moments

	Mean	Std	5% quantile	Median	95% quantile	Firm
$\widetilde{Var}_{i,1m}^{BKM}$	0.371	0.355	0.076	0.271	0.982	1989
$\widetilde{Skew}_{i,1m}^{BKM}$	-0.447	0.981	-2.144	-0.389	0.970	1989
$\widetilde{Kurt}_{i,1m}^{BKM}$	5.983	4.182	2.646	4.589	14.129	1989
$\widetilde{Var}_{i,6m}^{BKM}$	0.269	0.205	0.064	0.207	0.688	1956
$\widetilde{Skew}_{i,6m}^{BKM}$	-0.204	0.485	-0.863	-0.227	0.522	1956
$\widetilde{Kurt}_{i,6m}^{BKM}$	2.947	1.657	1.920	2.734	4.475	1956

Note: This table presents summary statistics of option-implied variance, skewness and kurtosis calculated by BKM method for $\tau = 30, 182$ days during Jan 1996-Dec 2015. Each month, the mean (Mean), standard deviation (Std), fifth percentile (5% quantile), median (Median), and 95th percentile (95% quantile) values of the cross-sectional distribution of each variable are calculated. The table presents the time-series means for each cross-sectional value.

Table 3: Summary statistics of option-implied betas by CCJV's zero idiosyncratic skewness assumption

	Mean	Std	5% quantile	Median	95% quantile	Firm
$\widetilde{\beta}_{1m}^{\text{CCJV}}$	0.783	1.608	-2.135	1.165	2.919	1989
$\widetilde{\beta}_{6m}^{\text{CCJV}}$	0.239	1.232	-2.316	0.616	1.460	1956

Note: This table presents summary statistics of option-implied betas by CCJV's zero idiosyncratic skewness assumption with 30-day and 182-day option maturities during Jan 1996-Dec 2015. Each month, the mean (Mean), standard deviation (Std), fifth percentile (5% quantile), median (Median), and 95th percentile (95% quantile) values of the cross-sectional distribution of $\widetilde{\beta}_{1m}^{\text{CCJV}}$ and $\widetilde{\beta}_{6m}^{\text{CCJV}}$ are calculated. The table presents the time-series means for each cross-sectional value.

Table 4: Summary statistics of conditional variance and fourth central moment of idiosyncratic shock

	Mean	Std	5% quantile	Median	95% quantile	Firm	
	All	0.227	0.281	-0.014	0.147	0.721	1989
$\widetilde{Var}^{CCJV}_{\epsilon_i,t,1m}$	Valid	0.246	0.278	0.024	0.160	0.739	1866
	Invalid	-0.053	0.071	-0.168	-0.032	-0.002	123
	All	0.165	0.135	0.019	0.132	0.421	1956
$\widetilde{Var}^{CCJV}_{\epsilon_i,t,6m}$	Valid	0.169	0.129	0.026	0.134	0.422	1933
	Invalid	-0.092	0.136	-0.389	-0.039	-0.003	24
	All	0.086	1.788	-0.645	0.058	1.247	1989
$\widetilde{\mu}^{CCJV}_{4\epsilon_i,t,1m}$	Valid	0.383	0.816	0.005	0.130	1.517	1452
	Invalid	-0.699	2.824	-2.802	-0.092	-0.002	537
	All	0.002	0.312	-0.255	0.014	0.227	1956
$\widetilde{\mu}^{CCJV}_{4\epsilon_i,t,6m}$	Valid	0.078	0.180	0.002	0.032	0.273	1506
	Invalid	-0.222	0.471	-0.996	-0.066	-0.002	451

Note: This table presents summary statistics of conditional variance and fourth central moment of idiosyncratic shock implied by option-implied betas given CCJV's zero idiosyncratic skewness assumption with 30-day and 182-day option maturities during Jan 1996-Dec 2015. Each month, the mean (Mean), standard deviation (Std), fifth percentile (5% quantile), median (Median), and 95th percentile (95% quantile) values of the cross-sectional distribution of $\widetilde{Var}^{CCJV}_{\epsilon_i,t,1m}$, $\widetilde{Var}^{CCJV}_{\epsilon_i,t,6m}$, $\widetilde{\mu}^{CCJV}_{4\epsilon_i,t,1m}$, and $\widetilde{\mu}^{CCJV}_{4\epsilon_i,t,6m}$ are calculated. The table presents the time-series means for each cross-sectional value. The 'Valid' and 'Invalid' represent positive and negative beta-implied conditional variance and fourth central moment of idiosyncratic shock, respectively.

Table 5: Summary statistics of unconditional idiosyncratic skewness implied by option-implied betas given CCJV's zero idiosyncratic skewness assumption

	Mean	Std	5% quantile	Median	95% quantile	Firm
$\widetilde{Skew}^{M}_{\epsilon_i,1m}$	-0.284	0.820	-1.806	-0.180	0.827	4434
$\widetilde{Skew}^{M}_{\epsilon_i,6m}$	-0.235	0.708	-1.440	-0.171	0.770	4181

Note: This table presents summary statistics of unconditional idiosyncratic skewness implied by option-implied betas given CCJV's zero idiosyncratic skewness assumption with 30-day and 182-day option maturities.The mean (Mean), standard deviation (Std), fifth percentile (5% quantile), median (Median), and 95th percentile (95% quantile) values of the cross-sectional distribution of $\widetilde{Skew}^{M}_{\epsilon_i,1m}$ and $\widetilde{Skew}^{M}_{\epsilon_i,6m}$ are calculated.

Table 6: Summary statistics of p-values of p-values of univariate test

H_0	Mean	Std	5% quantile	Median	95% quantile	Proportion of $p \leq 0.1$
$\tilde{E}[\tilde{\epsilon}^3_{i,t,1m}] = 0$	0.548	0.236	0.194	0.523	0.947	0.005
$\tilde{E}[r_{m,t,1m}\tilde{\epsilon}_{i,t,1m}] = 0$	0.278	0.294	0.001	0.159	0.888	0.416
$\tilde{E}[r^2_{m,t,1m}\tilde{\epsilon}_{i,t,1m}] = 0$	0.502	0.241	0.161	0.461	0.933	0.016
$\tilde{E}[\tilde{\epsilon}^3_{i,t,6m}] = 0$	0.463	0.270	0.077	0.430	0.933	0.077
$\tilde{E}[r_{m,t,6m}\tilde{\epsilon}_{i,t,6m}] = 0$	0.235	0.287	0.000	0.094	0.855	0.511
$\tilde{E}[r^2_{m,t,6m}\tilde{\epsilon}_{i,t,6m}] = 0$	0.380	0.278	0.049	0.309	0.917	0.167

Note: This table presents summary statistics of p-values of univariate test. The mean (Mean), standard deviation (Std), fifth percentile (5% quantile), median (Median), and 95th percentile (95% quantile) values of the cross-sectional distribution of p-values as well as the proportion of firms with p-values less equal than 0.1 for each test are calculated.

Table 7: Summary statistics of p-values of p-values of joint test

H_0: $\tilde{E}[\tilde{\epsilon}^3_{i,t,1m}] = 0$ and $\tilde{E}[r_{m,t}\tilde{\epsilon}_{i,t,1m}] = 0$					
Mean	Std	5% quantile	Median	95% quantile	Proportion of $p \leq 0.1$
0.321	0.292	0.001	0.242	0.875	0.333
H_0: $\tilde{E}[\tilde{\epsilon}^3_{i,t,1m}] = 0$ and $\tilde{E}[r^2_{m,t}\tilde{\epsilon}_{i,t,1m}] = 0$					
Mean	Std	5% quantile	Median	95% quantile	Proportion of $p \leq 0.1$
0.511	0.241	0.116	0.512	0.918	0.038
H_0: $\tilde{E}[\tilde{\epsilon}^3_{i,t,1m}] = 0$, $\tilde{E}[r_{m,t,1m}\tilde{\epsilon}_{i,t,1m}] = 0$ and $\tilde{E}[r^2_{m,t,1m}\tilde{\epsilon}_{i,t,1m}] = 0$					
Mean	Std	5% quantile	Median	95% quantile	Proportion of $p \leq 0.1$
0.312	0.281	0.001	0.235	0.841	0.327
H_0: $\tilde{E}[\tilde{\epsilon}^3_{i,t,6m}] = 0$ and $\tilde{E}[r_{m,t,6m}\tilde{\epsilon}_{i,t,6m}] = 0$					
Mean	Std	5% quantile	Median	95% quantile	Proportion of $p \leq 0.1$
0.212	0.267	0.000	0.081	0.811	0.526
H_0: $\tilde{E}[\tilde{\epsilon}^3_{i,t,6m}] = 0$ and $\tilde{E}[r^2_{m,t,6m}\tilde{\epsilon}_{i,t,6m}] = 0$					
Mean	Std	5% quantile	Median	95% quantile	Proportion of $p \leq 0.1$
0.337	0.273	0.008	0.276	0.873	0.245
H_0: $\tilde{E}[\tilde{\epsilon}^3_{i,t,6m}] = 0$, $\tilde{E}[r_{m,t,6m}\tilde{\epsilon}_{i,t,6m}] = 0$ and $\tilde{E}[r^2_{m,t,6m}\tilde{\epsilon}_{i,t,6m}] = 0$					
Mean	Std	5% quantile	Median	95% quantile	Proportion of $p \leq 0.1$
0.177	0.242	0.000	0.054	0.743	0.579

Note: This table presents summary statistics of p-values of joint test. The mean (Mean), standard deviation (Std), fifth percentile (5% quantile), median (Median), and 95th percentile (95% quantile) values of the cross-sectional distribution of p-values as well as the proportion of firms with p-values less equal than 0.1 for each test are calculated.

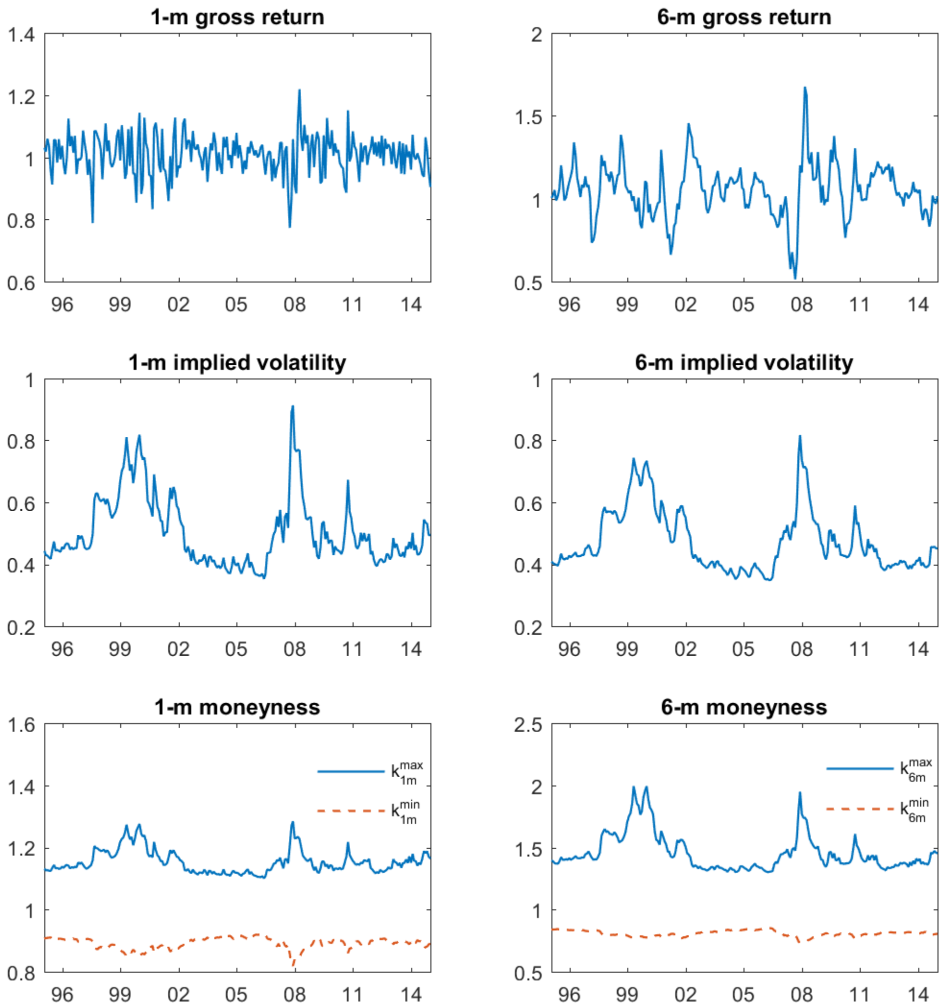

Figure 1: Time series of cross-sectional mean of gross return of individual stocks, implied volatility, minimum and maximum moneyness of individual stock options for horizons of 1 and 6 months

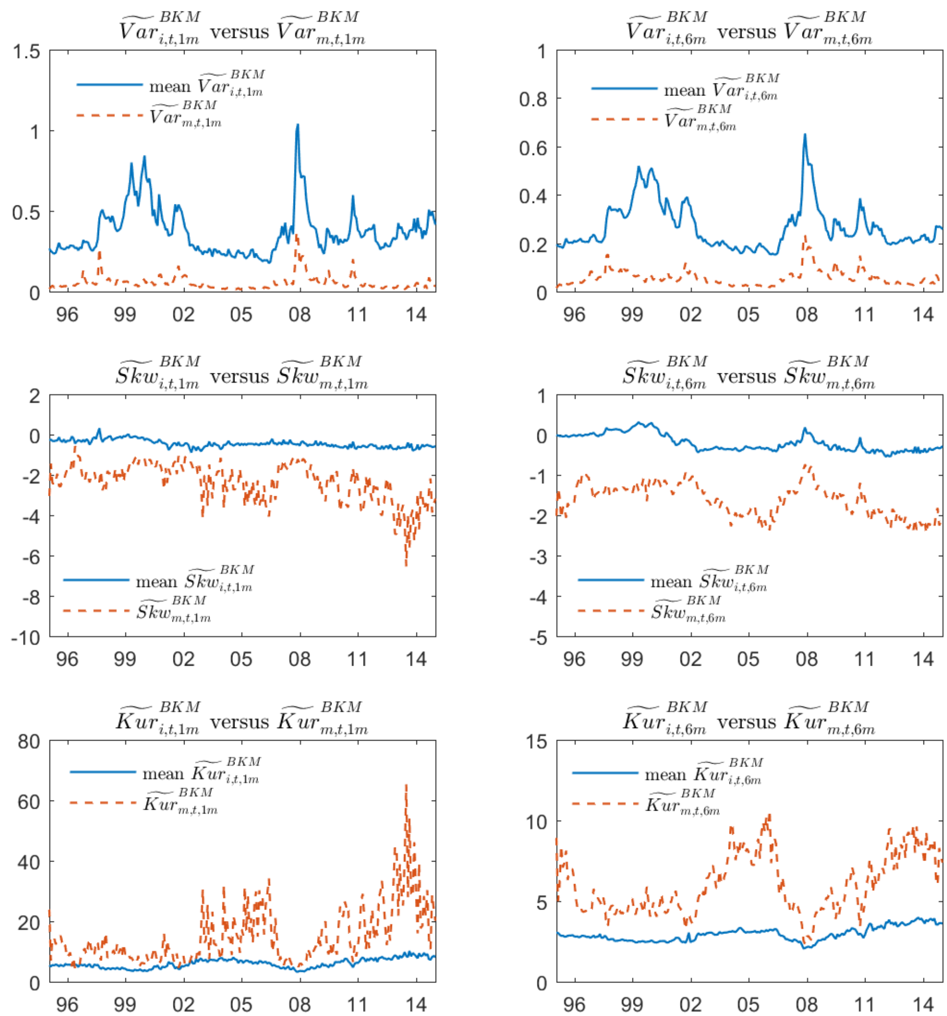

Figure 2: Time series of cross-sectional mean of individual option-implied moments versus index option-implied moments

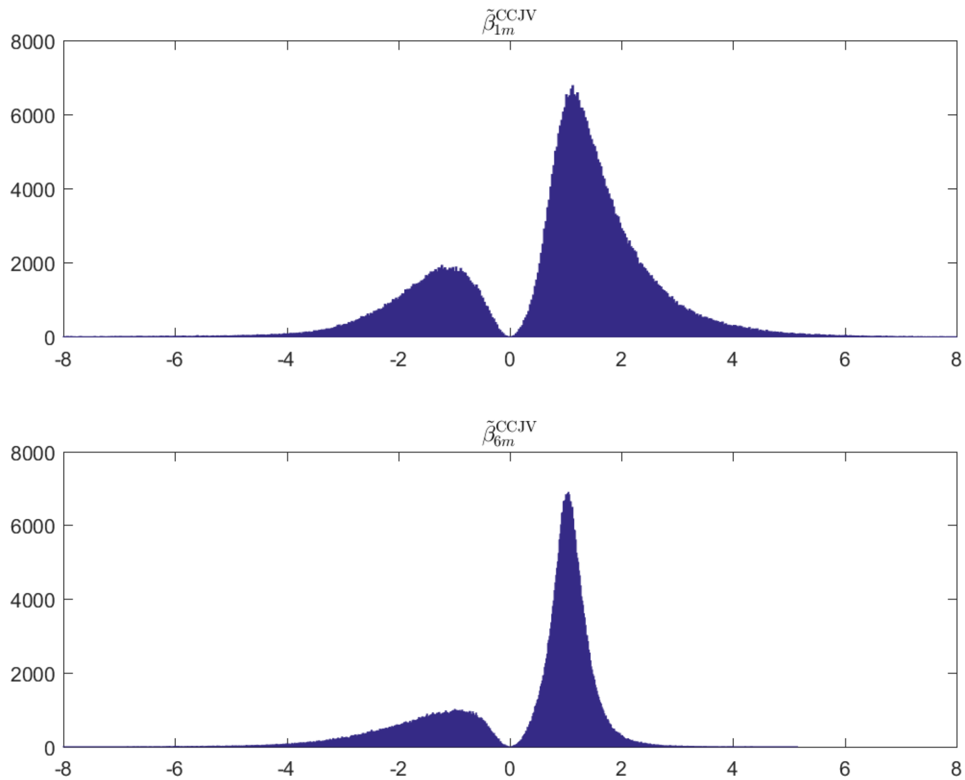

Figure 3: Histogram of estimated option-implied betas by CCJV's zero idiosyncratic skewness assumption with 30-day and 182-day option maturities

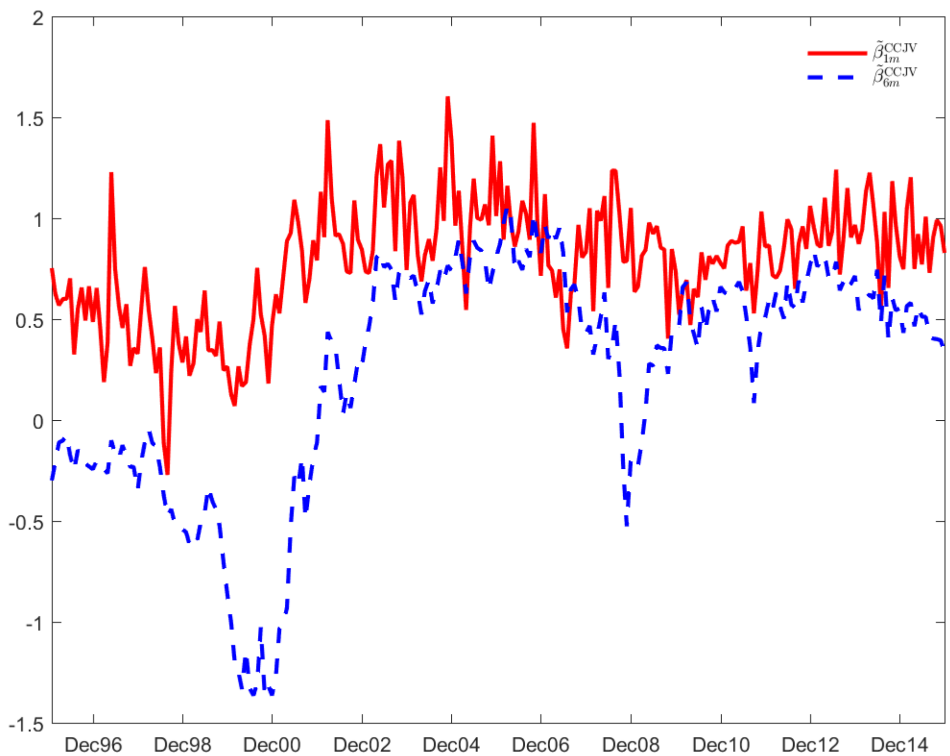

Figure 4: Time series of cross-sectional mean of estimated option-implied betas by CCJV's zero idiosyncratic skewness assumption with 30-day and 182-day option maturities

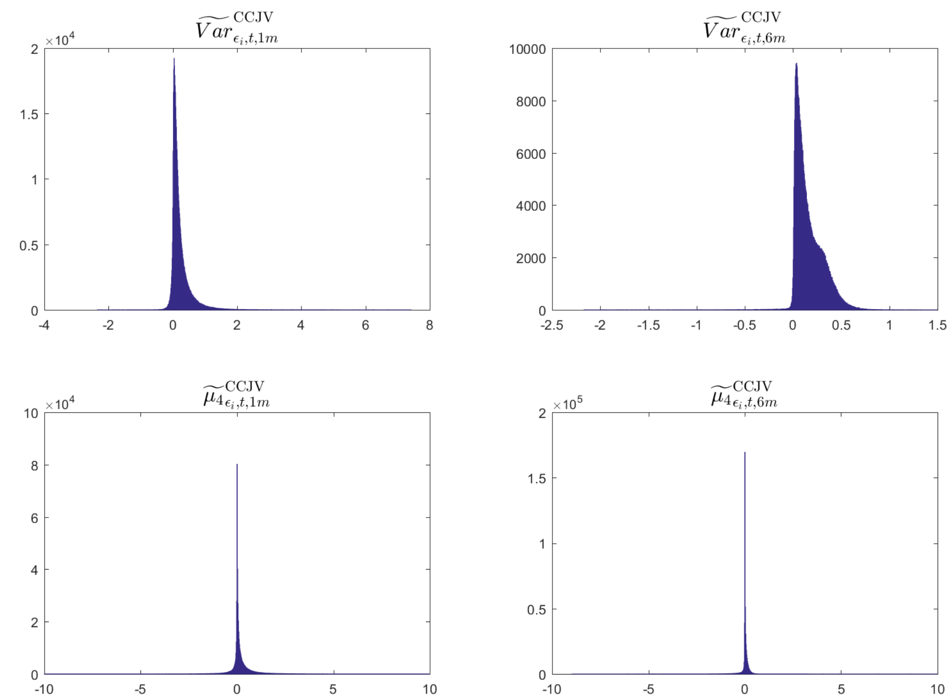

Figure 5: Histogram of conditional variance and fourth central moment of idiosyncratic shock implied by option-implied betas given CCJV's zero idiosyncratic skewness assumption with 30-day and 182-day option maturities

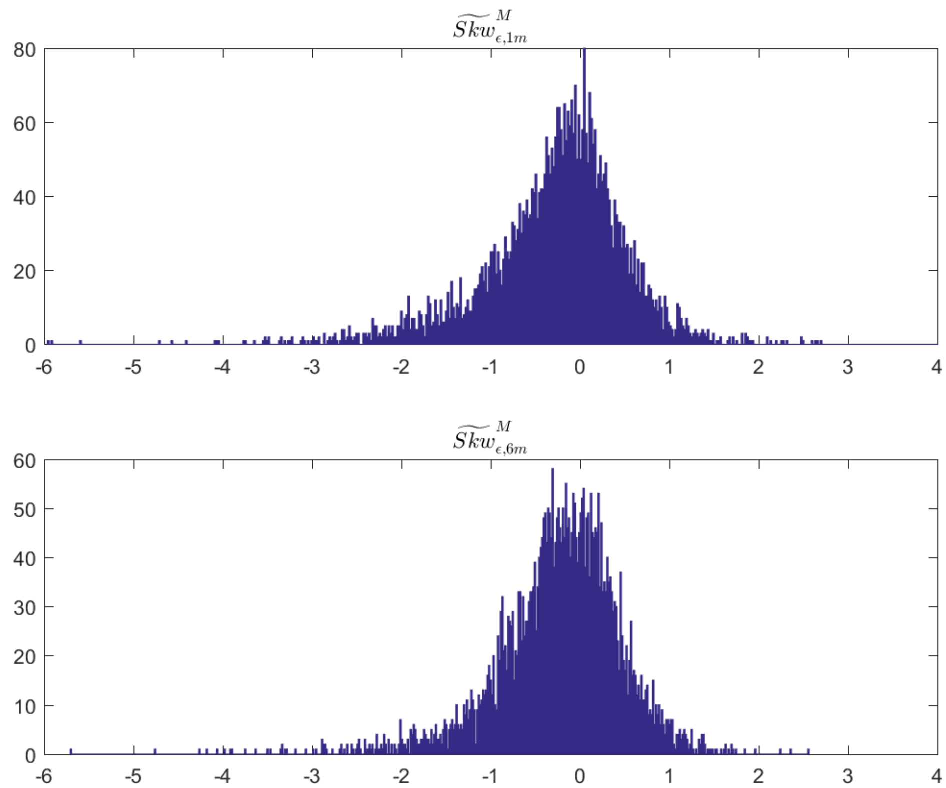

Figure 6: Histogram of unconditional skewness implied by option-implied betas given CCJV's zero idiosyncratic skewness assumption with 30-day and 182-day option maturities

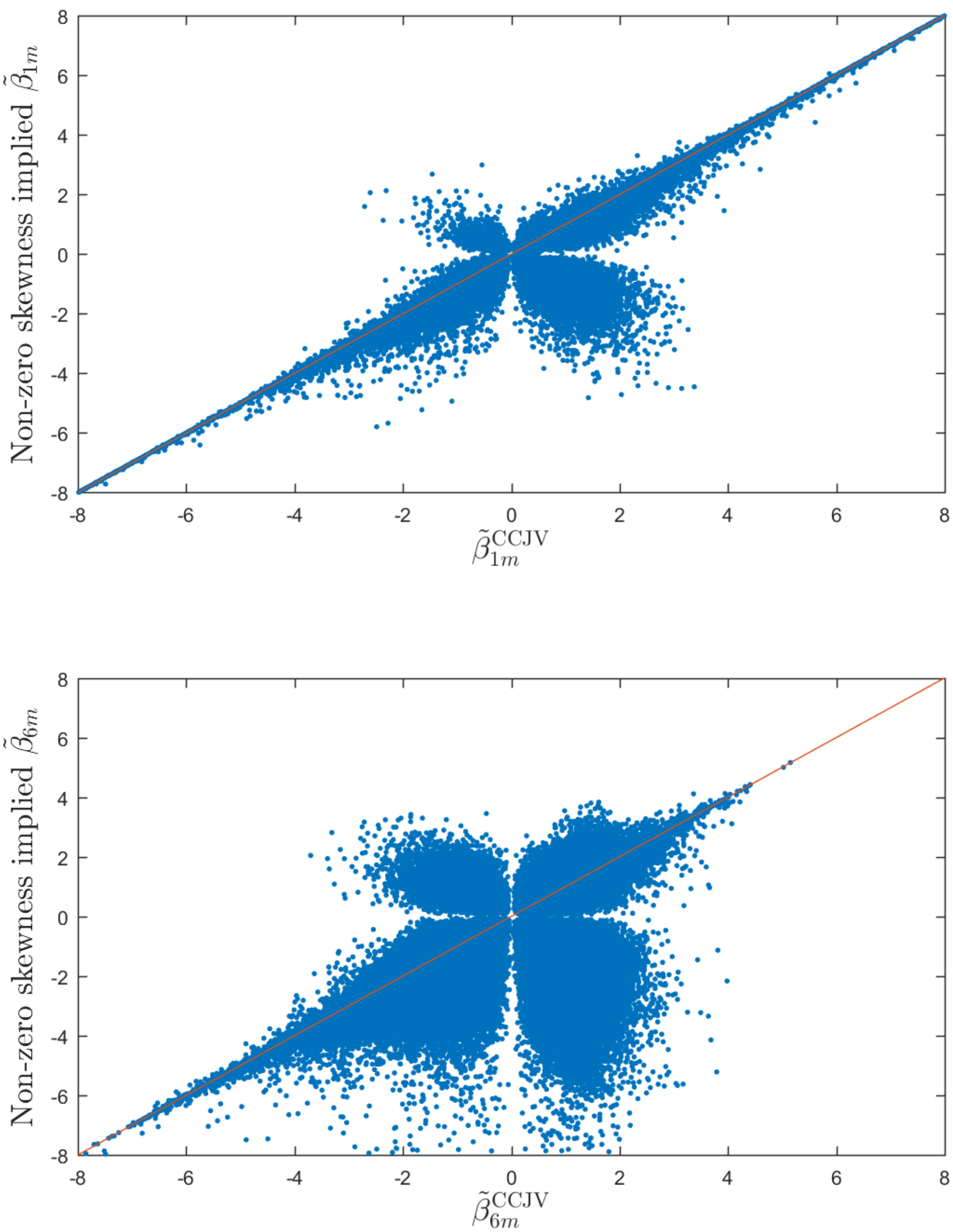

Figure 7: Option-implied beta with zero-skewness assumption versus option-implied beta without zero-skewness assumption

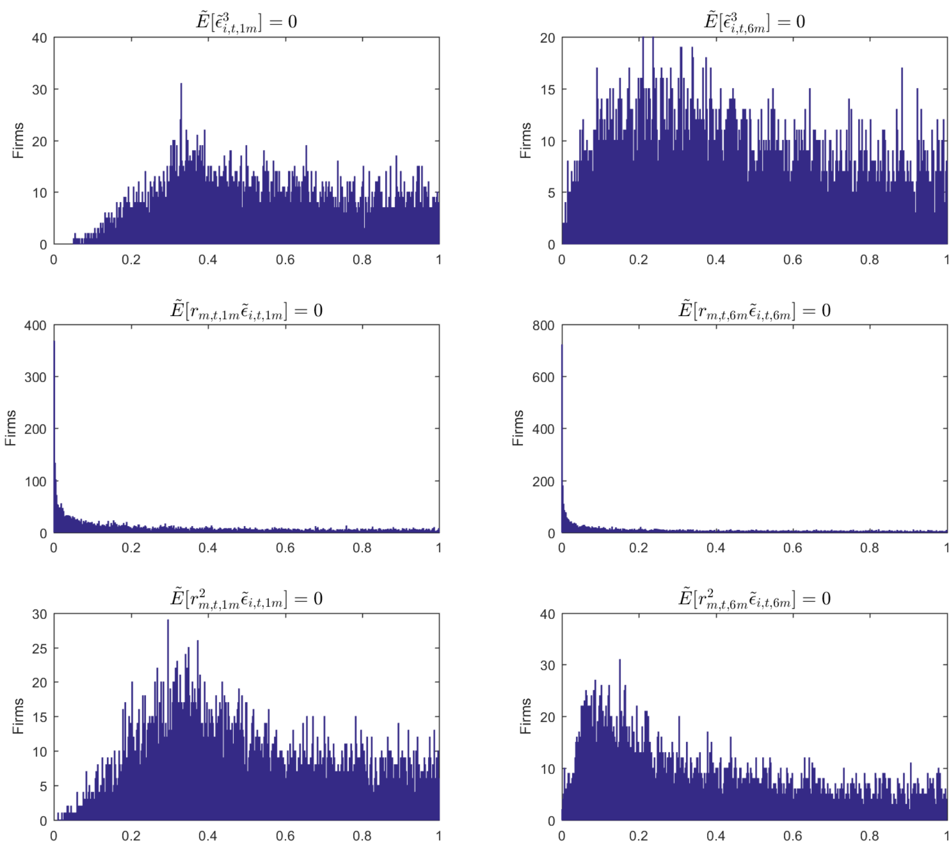

Figure 8: Histogram of p-values for univariate test

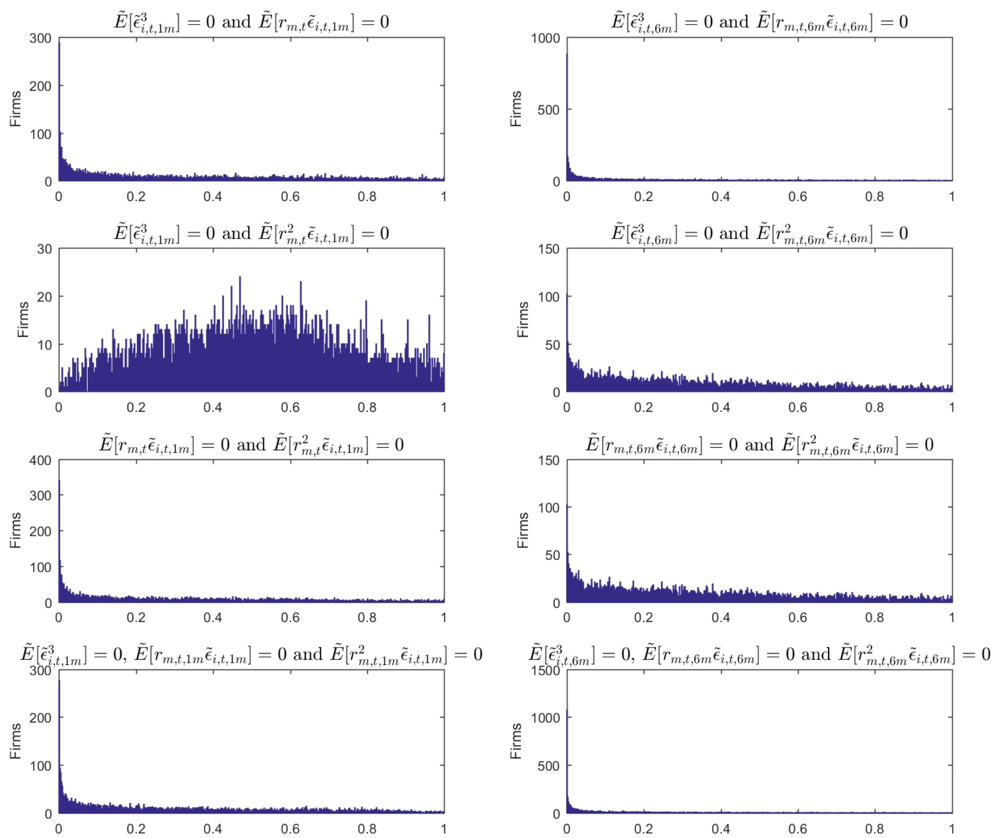

Figure 9: Histogram of p-values for joint test