# How Do Firms Resolve Patent Disputes? Insights from Competitive Dynamics and Market Uncertainty[*]

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**Abstract**

We develop a real options model to examine the determinants of patent dispute outcomes between two producing firms. By focusing on the dynamic strategic interactions between a patent-owning incumbent and an allegedly infringing challenger, we find that competitive pressure from new market entry plays a crucial role in determining whether firms settle their disputes, the timing of settlements, and the terms of any royalties. Greater competition, higher market volatility, and a larger divergence in the firms' willingness to continue paying litigation costs reduce the likelihood of settlement. Our model provides insights into litigation and settlement patterns and post-dispute market structures, illustrating how innovation-driven competition, market uncertainty, patent protection, and legal frameworks collectively shape the landscape of patent disputes.

**Keywords:** *Patent dispute; competition; uncertainty; litigation; settlement*

**JEL Classification:** *O34; L10; D81; C73*

[*]We thank Xuelin Li, Benoıt Chevalier-Roignant, Zhongyu Wang, Walter Zimmerman, Brendan Costello, Jiaqi Qian, Filippo Mezzanotti, Song Ma, Cecilia Parlatore, Per Stromberg, Matthieu Bouvard, Isil Erel, Anthony Neuberger, Kai Li, Stefan Kupfer, John Thanassoulis, Cecilia Bustemante, Richmond Mathews, Claudia Custodio, Philipp Krüger, Andreas Fuster, Barney Hartman-Glaser, Johan Hombert, Bruno Biais, Paul Ehling, David Skeie, Paulo J. Pereira, Sebastian Gryglewicz, Pete Kyle, and participants at the Twelfth Annual Northwestern/USPTO Conference on Innovation Economics, the 23rd, 24th, and 27th Annual International Real Options Conferences, WBS brownbag seminar, Junior Entrepreneurial Finance and Innovation virtual seminar, 33rd Australasian Finance and Banking Conference, 11th Portuguese Finance Network Conference, 2021 FMA European Conference, the 2021 AEA poster session, 2021 FMA Annual meeting, 2023 AFFECT mentor workshop at AFA, 2024 UK Women in Finance workshop for their comments and suggestions. Previous versions of different parts of the paper had been circulated with the titles of "The Impact of Product Market Characteristics on Firms' Strategies in Patent Litigation", "Innovation and Patent Litigation with Financial Constraints: American versus English Rule", and "A Real Options Model of Patent Dispute". All errors are our own. Du Liu acknowledges financial support from Pujiang Talent program [grant number 2023PJC033].

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## 1. Introduction

Patents are essential for protecting intellectual property (IP) generated through innovation, granting patent holders the right to litigate against infringement. However, patent litigation is both costly and time-consuming, often taking 2 to 5 years to resolve and costing firms millions of dollars (Bessen and Meurer, 2012; Bessen et al., 2018).[1] Furthermore, litigation outcomes are highly uncertain (Lemley and Shapiro, 2005), with patent holders facing only a 50% chance of success when asserting their rights in court (Moore, 2000). Recent finance literature highlights the importance of patent litigation, linking it to firms' innovation incentives, financing decisions, and firm value (e.g., Lerner, 2002; Claessens and Laeven, 2003; Galasso and Schankerman, 2018; Caskurlu, 2019; Lee, Oh, and Suh, 2021; Mezzanotti, 2021; Caskurlu, 2022; Acikalin et al., 2023; Giebel, 2023; Suh, 2023). Despite these advances, a fundamental question remains underexplored: how do firms resolve patent disputes? Specifically, what factors drive the outcomes of patent disputes?

Patent disputes are not always resolved through litigation. Faced with the high financial burden and uncertain of litigation, many patent-holding firms abandon patent lawsuits after initiating them (Jeon, 2015), while some alleged infringers may even go out of business in the process (Bessen and Meurer, 2012).[2] Settlements are the most common means of resolving patent disputes once litigation begins (Kesan and Ball, 2006), but firms also use royalty agreements without litigation (Choi and Gerlach, 2017) or leverage the threat of legal action to force competitors out of the market. These alternative dispute resolutions suggest the thousands of patent lawsuits filed annually[3] represent only a fraction of all patent-related disputes (National Research Council, 2003). Understanding firms' trade-offs in patent disputes is essential to recognizing the limitations of relying solely on observed lawsuits for studying patent conflicts.

The dynamic and strategic nature of firms' interactions in patent disputes makes this a complex problem that benefits from a theoretical perspective. In this paper, we employ a real options model to examine the determinants of patent dispute outcomes between two operating firms, taking into account the uncertainty of future profits and the irreversibility of decisions. We explore the conditions under which value-maximizing firms choose to litigate, settle, or

[1] Bessen and Meurer (2012) report a mean loss of \$75.9 million (2010 dollars) and a median loss of \$6.5 million for alleged public infringers between 1984 and 1999, while Bessen et al. (2018) document an aggregate loss of \$308 billion (2010 dollars) for publicly-traded alleged infringers from 1986 to 2009. A 2015 survey by the American Intellectual Property Law Association found that the median cost to litigate a single-patent infringement case through trial was \$5 million for cases with more than \$25 million at risk. Litigation costs case can be prohibitively high for many firms, especially small and medium enterprises, limiting their ability to benefit from patent protection (Lanjouw and Schankerman, 2001; Hu, Yoshioka-Kobayashi, and Watanabe, 2017).

[2] For example, Akimbo Systems, a startup providing video-on-demand services, faced a patent infringement lawsuit from competitor ReplayTV, contributing to its shutdown. The legal expenses associated with the patent dispute, combined with other business challenges, led to the company's closure.

[3] Between 2000 and 2020, over 70,000 patent infringement lawsuits were filed in U.S. District Courts, involving approximately 68,000 plaintiff firms and 105,000 defendants firms. Data source: Lex Machina. Majority litigants mentioned were not non-practicing entities.

exit the market.

Patterns of patent lawsuits suggest that industry dynamics and market uncertainties can play important roles in patent dispute resolution. The proportion of litigated patents (relative to all granted ones) varies significantly across industries (Lanjouw and Schankerman, 2001; Lemley and Shapiro, 2005).[4]. Patent-owning incumbents are more likely to initiate patent litigation during favorable demand conditions (Schliessler, 2015), whereas firms are more likely to seek settlement when market demand deteriorates (Shih et al., 2020). Higher litigation risks are also associated with more volatile cash flows (Lowry and Shu, 2002). These findings indicate that the tradeoffs in patent litigation are closely linked to market dynamics, where competition and volatility play critical roles.

Our real options model considers a patent-holding *incumbent* firm ("I") whose profits have decreased from monopoly to duopoly levels due to the entry of a *challenger* firm ("C") that potentially uses IP protected by the patent. Both firms face uncertain future profits. If the incumbent exercises its option to sue the challenger (*I-litigate*), both firms incur ongoing litigation costs; the court ruling happens at an exponentially-distributed random time $\tau$. With probability $p$ which is common knowledge, the ruling favors the incumbent, restoring its monopoly; with probability $1-p$, the lawsuit is dismissed, and the duopoly persists. We allow for legal proceedings to end before a court ruling if the incumbent withdraws the lawsuit (*I-withdraw*) or the challenger exits the market (*C-exit*). Firms can also settle without initiating litigation (*ex-ante* settlement) or after litigation starts (*ex-post settlement*). Under settlement, the challenger remains in the market but pays the incumbent a royalty rate on its future profits. Settlement is less costly than litigation, and each firm maximizes its value by employing threshold strategies based on market demand, factoring in its own and the competitor's alternative strategies and potential future decisions.

Central to our analysis is the idea that the option to abandon litigation is valuable, not only for the firm that exercises it (the *leaving firm*), but even more so for the other *remaining firm*. Abandonment occurs when it is no longer optimal for the leaving firm to continue bearing litigation costs in pursuit of the expected court ruling payoff. By ceasing litigation, both firms avoid further litigation costs and forego the expected judgement payoff, but the remaining firm effectively secures a favorable court ruling immediately. Firms will settle their dispute only if, for each, the value of settling exceeds their non-settlement value–the value associated with the leaving firm's abandonment option. These conditions place constraints on the range of royalty rates that enable ex-post settlement. Consequently, factors that increase the *non-settlement value* relative to the settlement value reduce the feasibility of ex-post settlement. Additionally, we recognize the compound option feature of the *I-litigate* option, as exercising

[4]For example, among patents granted between 1980 and 1999, 0.53% in the field of drugs and medical technology were litigated, compared to 0.25% in chemicals and 0.38% in mechanicals (Galasso and Schankerman, 2015)

it activates subsequent options of non-settlement and ex-post settlement. By influencing the option values and determining the identities of the leaving and remaining firms if litigation proceeds, industry dynamics and market uncertainty also shape firms' choice between *I-litigate* and *ex-ante settlement*.

This leads to our first key result: the competition introduced by the challenger's market entry is a determining factor for patent dispute outcomes and settlement terms. Our model generates a novel inverse measure of competition between two firms relying on the same intellectual property, which we call the *gain-to-loss ratio* $\Phi$: the ratio of the challenger's profit increase from market entry to the incumbent's (patent holder's) profit loss. Higher values of $\Phi$ correspond to lower competition.

The model demonstrates that as competition increases (or as $\Phi$ decreases), the potential dispute outcome[5] shifts from ex-ante settlement to ex-post settlement, and ultimately to litigation without settlement. In other words, settlement is feasible only if the competition from the challenger's entry is not overly severe, indicating that the challenger's products are sufficiently "complementary". When competition is intense (low $\Phi$), where the incumbent's loss significantly outweighs the challenger's gain, the value of non-settlement outweighs each firm's settlement option. In such cases, abandoning litigation becomes more appealing, as it offers complete litigation cost savings, rather than the partial savings from settlement. Consequently, there exists a lower bound on $\Phi$, below which firms would not pursue settlement. As is standard in the real options literature, the relative timing flexibility associated with ex-post settlement compared to ex-ante settlement adds value to the ex-post settlement option and leads to a greater scope for it. We find that ex-ante settlement is feasible over a narrower range of parameter values than ex-post settlement, generally only when ex-post settlement is likely to occur immediately or shortly after litigation begins. This is more probably at high values of $\Phi$; as $\Phi$ increases, settlement option payoffs grow, making it worthwhile exercising across a broader range of market demand levels. The ex-post settlement threshold rises, advancing the timing of settlement and enhancing the feasibility of ex-ante settlement. The model also shows when settlement is feasible, the royalty rate in ex-post settlement rises with increasing competition.

Furthermore, our model underscores the role of market uncertainty in shaping firms' resolution of patent disputes. We find that settlement becomes less likely as market demand volatility increases; specifically, the minimum gain-to-loss ratio required for feasible settlement rises with volatility. This result stems from the standard real options effect, where volatility raises option values, combined with the relative sensitivity of the alternative options in patent dispute. Higher volatility has the greatest impact on increasing the non-settlement values, which set constraints on settlement. While the option values of ex-post and ex-ante settlement also

[5] The model addresses the *potential* rather than *realised* outcome, as firms' actions depend on whether market demand reaches the action threshold.

increase, they do so to a lesser extent than the non-settlement values, primarily because they are closer to being exercised.

Our second main finding is that patent dispute outcomes also depend on firms' relative willingness to finance ("WtF") in the attrition game during litigation. We summarize each firm's abandonment incentive using its *reservation threshold* (Lambrecht, 2001), which captures the lowest level of market demand, down to which a firm would be willing to continue financing litigation, assuming the other firm abandons litigation first. The firm with the lower reservation threshold–indicating a stronger remaining incentive– wins the attrition game; we refer to this firm as the *remaining* firm. Our model shows that when firms have similar WtF, the range of competition levels under which settlement is feasible broadens. The identity of the remaining firm determines each firm's non-settlement value, thereby setting the bounds on feasible settlement royalty rates. A wider gap in firms' WtF raises the remaining firm's non-settlement value, reducing the scope for settlement. Consequently, as the difference in WtF between the two firms increases, it is less likely for firms to settle as the minimum $\Phi$ required for settlement rises.

The identity of the *remaining firm* is particularly important in understanding how the patent holder's probability of winning the lawsuit or the strength of patent protection, $p$, influences the outcomes of patent disputes. The probability $p$ directly affect each firm's incentive to continue paying the litigation costs. As $p$ increases, the incumbent's expected payoff from judgement rises, while the challenger's expected payoff decreases, thereby increasing the incumbent's relative WtF. As a result, the incumbent is more likely to become the remaining firm, and the impact of $p$ depends on which firm remains in the attrition game: when the incumbent is the remaining firm (*C-exit*), an increase in $p$ widens the gap in firms' WtF, reducing the likelihood of settlement. Otherwise, when the challenger is the remaining firm (*I-withdraw*), the effect of $p$ is muted.

In the model extension, we delve further into the role of firms' WtF, as well as the influence of competition and market uncertainty, by examining patent dispute outcomes under the English Rule–where the losing party covers both sides' litigation costs–compared to the baseline American Rule, in which each party pays its own costs. While most of the baseline model results hold, the English Rule increases the incumbent's effective litigation costs while decreasing those of the challenger. This adjustment significantly raises the incumbent's incentive to abandon litigation, often shifting the remaining firm in the attrition game from the incumbent to the challenger across a range of realistic parameters. Under the English Rule, a higher $p$ increases the scope for settlement, making it feasible even in fiercely competitive market environment (low $\Phi$). In contrast, under the American Rule, a higher $p$ may decrease the likelihood of settlement scope if the incumbent's WtF is comparatively low. Additionally, our model demonstrates that the ex-post settlement royalty rate is lower under the English Rule.

Overall, our model shows that patent dispute outcomes are driven by the combined effects of the competitive impact of the potentially infringing challenger's entry, market volatility, and relative incentive to remain in the attrition game of litigation. Our model has wider implications, particularly for post-dispute market structure: a reversion to monopoly after the dispute is more likely if competition is intense (low $\Phi$), market volatility is high, and the incumbent's incentive to continue bearing litigation costs outweighs the challenger's– such as when the challenger faces higher litigation costs and the likelihood of a favorable court ruling for the patent holder is high. Conversely, stronger product complementarity, lower market volatility and similar WtF lead to settlement and thus a duopoly market structure. Additionally, our model generates testable implications for litigation and settlement rates, defined as the probability of litigation or settlement occuring within a specified amount of time after the challenger's market entry, based on a set of model parameters.

Our study contributes to several strands of literature. Our work provides a complementary explanation for settlement failure to the law and economics literature, particularly in general lawsuits (e.g., Landes, 1971; Gould, 1973; Shavell, 1982; P'ng, 1983; Bebchuk, 1984; Nalebuff, 1987; Bebchuk, 1996). Traditional models explain settlement failures through factors such as divergent beliefs or information asymmetries regarding conviction probability, as well as risk aversion and different expectations of judgement amounts relative to legal costs. In contrast, our model shows that settlement failure can occur even when both parties share identical information and views, expanding the scope of factors contributing to non-settlement.

Our research also relates to the specialized literature on patent litigation (e.g., Meurer, 1989; Choi, 1998; Hughes and Snyder, 1995; Aoki and Hu, 1999b; Bessen and Meurer, 2006; Galasso and Schankerman, 2010; Spier, 2007). Patent litigation often involves firms with shared uncertain market demand, and where judgement can significantly alter future industry cash flows, so the value implications of the court-ruling are much more than an inter-party transfer, as in general litigation. In particular, our paper highlights the role of product market competition based on the same new technology and firms' willingness to bear litigation costs, both of which are first-order factors in determining firms' litigation strategies under market uncertainty. This paper also adds to the recent discussion of how financing considerations affect patent litigation (e.g. Aoki and Hu, 1999a; Cohen, Gurun, and Kominers, 2016; Choi and Spier, 2018; Antill et al., 2024; Emery and Woeppel, 2024).

Several studies have modeled either generic litigation and patent enforcement using real options frameworks (e.g., Grundfest and Huang, 2005; Marco, 2005; Jeon, 2015). Distinctively, our model incorporates the possibility of the defendant exiting litigation due to high ongoing costs. While related to finance literature on litigation risk (e.g., Hassan, Houston, and Karim, 2021; Guan et al., 2021; Liu, Si, and Miao, 2022), we focus on the strategic interaction between competing product firms. Although third-party litigation financing is not modeled (e.g., Antill et al., 2024), our approach underscores the importance of financial constraints in shaping litigation and settlement incentives.

This study is among the first to analyze strategic interactions in patent litigation from a corporate finance perspective. Previous game theory models (e.g., Meurer, 1989; Crampes and Langinier, 2002; Choi and Gerlach, 2015) often overlook market uncertainty and profit volatility. Using a real options framework allows us to capture decision-making under uncertainty and to highlight the compound nature of litigation options, in contrast to prior models that treat litigation as isolated decisions (e.g., Marco, 2005; Jeon, 2015). Our work integrates real options and game theory (e.g., Grenadier, 2002; Lambrecht, 2004; Antill et al., 2024) and contributes to the broader finance literature on real options (e.g., Bernile, Lyandres, and Zhdanov, 2012; Bustamante, 2012; Bolton, Chen, and Wang, 2013; Lambrecht, Pawlina, and Teixeira, 2016; Garlappi, Giammarino, and Lazrak, 2022) by applying it to patent litigation. In doing so, we reveal new determinants of settlement failure, including market competition from the challenger, product volatility, and differences in financial resilience. By uncovering the key theoretical factors that determine whether and when firms choose to settle patent disputes, our work also contributes to the recent literature on the intersection of finance and industrial organization (e.g., Phillips and Zhdanov, 2013; Bustamante and Frésard, 2021; Lin, 2023).

This paper proceeds as follows. Section 2. introduces the model. Section 3. presents and discusses the model solutions. In Section 4., we provide comparative statics on patent dispute outcomes. Section 5. explores model extensions and implications. Section 6. concludes.

## 2. The model

We consider two value-maximizing firms, both of which are all-equity financed. The incumbent firm (denoted as "$I$") owns a patent uses its patented technology in production.[6] The market demand for the product, denoted by $x_t$, follows a geometric Brownian motion, where $\mu$ represents the growth rate, $\sigma$ is the market demand volatility, and $W_t$ is a standard Wiener process:

$$dx_t = \mu x_t dt + \sigma x_t dW_t. \tag{1}$$

The incumbent earns a monopoly profit flow of $\pi_1^I x_t dt$ until a challenger (denoted by "$C$") enters the market with a product based on similar technology, potentially infringing the incumbent's patent.[7] Once the challenger enters, the two firms earn duopoly profit flows of $\pi_2^I x_t dt$ and $\pi_2^C x_t dt$, respectively.[8] Total market profit may increase or decrease after the alleged infringement, with $\pi_2 \equiv (\pi_2^I + \pi_2^C) \gtrless \pi_1 \equiv \pi_1^I$. We denote the change in profit driven by the

[6] Industry leaders are often firms highly proficient in generating intangible assets (Crouzet and Eberly, 2018).

[7] Such infringements are typically inadvertent rather than deliberate, as unintentional patent infringement is more common in practice (Bessen and Meurer, 2013).

[8] The term "duopoly" here broadly includes cases where the challenger derives profit from a product significantly different from the incumbent's, akin to Type 2 infringement described in Antill et al. (2024).

challenger's entry as $\Delta\pi$:

$$\Delta\pi^I \equiv \pi_2^I - \pi_1^I < 0, \quad \Delta\pi^C \equiv \pi_2^C - 0 > 0, \quad \Delta\pi \equiv \pi_2 - \pi_1 = \Delta\pi^I + \Delta\pi^C \gtrless 0. \tag{2}$$

Figure 1 illustrates the sequence of potential events, starting with the market entry of the allegedly infringing challenger. The model is divided into two stages: *pre-litigation* and *during litigation*. Firm strategies are represented as optimal timing decisions, which, under standard assumptions, follow threshold strategies – meaning that firms act when market demand reaches a specific level.

[Insert Figure 1 here]

**Pre-litigation** At this stage, the firms' actions can be viewed as exercising a call option. Potential strategic actions include: (1) *I-litigate*: The incumbent initiates an infringement lawsuit against the challenger once demand exceeds a litigation threshold $x_l$, occurring at time $\inf\{t : x_t \geq x_l\}$. This action activates the real options associated with the subsequent *during-litigation* stage. For simplicity, we assume negligible costs for initiating litigation. (2) *Ex-ante settlement*: To avoid costly litigation, firms can settle by agreeing on a royalty license agreement (Meurer, 1989). Following the negotiation framework in Lambrecht (2004) and Lukas and Welling (2012), the incumbent proposes a royalty rate $\theta_a$, representing the fraction of the challenger's future profit payable to the incumbent. The challenger sets a settlement threshold $x_a$ and agrees to pay the royalty starting at time $\inf\{t : x_t \geq x_a\}$.[9] Each firm incurs a one-time settlement cost $C_s^j$, where $j \in \{I, C\}$. The change in profit due to ex-ante settlement is denoted by $\Delta\pi_a^j$, with the post-settlement profit flow for Firm $j$ given by $(\pi_2^j + \Delta\pi_a^j)x_t dt$.

$$\Delta\pi_a^I \equiv \theta_a \pi_2^C > 0, \quad \Delta\pi_a^C \equiv -\theta_a \pi_2^C < 0. \tag{3}$$

(3) *forcing-out*: the incumbent can issue a litigation threat when demand falls to a sufficiently low level, making it unprofitable for the challenger to remain in the market. This threat forces the challenger to exit, restoring the incumbent's monopoly position. Upon the challenger's exit, profit flows return to $(\pi_1 x_t dt, 0)$. This option, discussed in Section 3.2, is relevant only if market demand follows trajectories that make staying in the market unsustainable for the challenger.

[9] Several models exit for joint options, each with limitations (Lambrecht, 2004; Morellec and Zhdanov, 2005; Lukas and Welling, 2012; Banerjee, Güçbilmez, and Pawlina, 2014): (1) One firm makes a take-it-or-leave-it offer on both the sharing rule and threshold, but the credibility of committing to a fixed threshold is questionable. (2) The firms negotiate the sharing rule and threshold via a Nash bargaining game, though bargaining power is assumed exogenous. (3) Firms agree on a global action threshold and sharing rule, but this fails to capture the non-cooperative dynamics of patent disputes.

**During-litigation** Once the incumbent initiates litigation, each firm incurs ongoing litigation costs of $c^j dt$, where $j \in \{I, C\}$.[10] A court decision occurs at a random time $\tau$, modeled as the first arrival in a Poisson process with rate $\lambda$ (Antill and Grenadier, 2023). The expected total litigation cost for Firm $j$ is $C_l^j \equiv \frac{c^j}{r+\lambda}$, where $j \in \{C, I\}$. The ratio of the challenger's litigation cost to the incumbent's is termed the *relative litigation cost* $\Lambda = \frac{C_l^C}{C_l^I}$. With probability $p$, known to both firms (Lemley and Shapiro, 2005), the court rules in favor of the incumbent, forcing the challenger to exit and restoring the incumbent's monopoly. [11] With probability $1-p$, the court rules no infringement, allowing the challenger to remain, thus preserving the duopoly.

Taking action to terminate the ongoing lawsuit is equivalent to exercising a put option. The firms have three such options: (1) *I-withdraw*: The incumbent can withdraw from the litigation when demand falls to the withdrawal threshold $x_w$, occurring at time $\inf\{t : x_t \le x_w\}$. This allows the incumbent to avoid further litigation costs, and the market continues as a duopoly thereafter. (2) *C-exit*: The challenger can exit the market at its exit threshold $x_e$, occurring at time $\inf\{t : x_t \le x_e\}$. After the challenger exits, the incumbent's monopoly is restored. Practically, challengers may file for bankruptcy or propose settlement by leaving the market if their expected payoff from continuing the litigation turns negative.[12] (3) *Ex-post settlement*: The firms may settle through a joint option, where the incumbent proposes a royalty rate $\theta_p$, and the challenger sets a settlement threshold $x_p$, leading to settlement at $\inf\{t : x_t \le x_p\}$. The profit parameter changes post-settlement are:

$$\Delta\pi_p^I \equiv \theta_p \pi_2^C, \quad \Delta\pi_p^C \equiv -\theta_p \pi_2^C. \tag{4}$$

This results in new profit flows of $\left((\pi_2^I + \Delta\pi_p^I)x_t dt, (\pi_2^C + \Delta\pi_p^C)x_t dt\right)$. Given that litigation is costlier than settlement ($C_l^j > C_s^j$, where $C_s^j$ is the cost of either ex-ante or ex-post settlement), the cost savings $\Delta C^j \equiv C_l^j - C_s^j > 0$ incentive firms to settle rather than continue litigation.[13]

Our model naturally generates a *gain-to-loss ratio* $\Phi$, capturing the competitive dynamics between the incumbent (patent holder) and the challenger firm that enters the market with

[10] As noted by Bebchuk (1996), patent litigation costs accumulate over time, including both direct costs (e.g., attorney and administrative fees) and indirect costs (e.g., management time, business disruption) (Bessen and Meurer, 2012). Litigation can strain relationships, hinder collaborations, and damage reputations. Financially weak firms may also face higher credit costs due to bankruptcy risks, and alleged infringers might experience production shutdowns or lose customers if injunctions are imposed. However, we assume that litigation costs do not impact the probability of winning the case (Friedman, 1969).

[11] $p$ reflects the probability that the court finds infringement or does not invalidate the patent, as discussed by Choi and Gerlach (2015). Following Marco (2005), we do not distinguish between the two. After the America Invents Act in 2011, U.S. Federal district courts are unlikely to grant permanent injunctions to non-practicing entities but may do so if the claimant practices its invention, directly competes with the defendant, and relies on the patented technology as core to its business.

[12] The challenger may exit due to financial strain, particularly if the incumbent seeks an injunction rather than damages, as discussed by Bessen and Meurer (2012).

[13] If ex-ante and ex-post settlement costs differ, the main results hold, though royalty rates would adjust.

products based on the same intellectual property (IP). This ratio, defined as

$$\text{The gain-to-loss ratio:} \quad \Phi \equiv \frac{\Delta\pi^C}{|\Delta\pi^I|} = \frac{\pi_2^C}{\pi_1^I - \pi_2^I} > 0. \tag{5}$$

represents the challenger's profit gain relative to the incumbent's loss of profit due to the challenger's entry. The ratio $\Phi$ inversely reflects the intensity of competition induced by the challenger's entry and use of innovative technology. A lower $\Phi$ indicates stronger competition, as the incumbent's profit erosion outweighs the challenger's profit increase. We focus on the nontrivial case of profit or producer surplus contraction ($\pi_2 < \pi_1$, or equivalently $\Phi \leq 1$) to analyze patent dispute outcomes.[14]

## 3. Model Solution

We solve the model using backward induction, first examining firms' *during-litigation* strategies (Section 3.1), and then analyzing their *pre-litigation* strategies (Section 3.2).

### 3.1 During-litigation Solutions

We begin by deriving the general form of firm values during litigation (Proposition 1). Next, we examine the options of incumbent withdrawal (*I-withdraw*) and challenger exit (*C-exit*) (Corollaries 1 – 2) to identify the non-settlement outcome during litigation $s_{ns}$ (Lemma 2), as an alternative to ex-post settlement.[15] Following this, we examine firms' ex-post settlement strategies (Corollary 3 and Theorem 1), and assess whether firms settle after the litigation begins (Theorem 2).

To facilitate our propositions, we define $\delta$ as the deferred perpetual factor for the demand stream starting at the court ruling time $\tau$,[16] and $\omega$ denotes the corresponding equivalent perpetual cash flow rate:

$$\delta \equiv \frac{1}{r-\mu} - \frac{1}{r+\lambda-\mu}, \quad \omega \equiv \delta(r-\mu) \in (0,1). \tag{6}$$

The present value of a perpetual stream $\{x_t\}_{t=\tau}^{\infty}$ starting at the court ruling $\tau$ is $\delta x_0$, where $x_0$ is the current demand level. A perpetual stream $\{\omega x_t\}_{t=0}^{\infty}$ starting immediately has the same present value of $\delta x_0$.

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[14]When $\Phi \leq 1$, we have $\Delta\pi \leq 0$: $\Phi = \frac{\Delta\pi^C}{-\Delta\pi^I} = \frac{\Delta\pi - \Delta\pi^I}{-\Delta\pi^I} = \frac{\Delta\pi}{-\Delta\pi^I} + 1$. If $\Phi > 1$, market expands with the challenger's entry, making non-settlement outcomes less relevant, as firms have strong incentives to settle and maintain higher total market profits through profit transfer. Our model does not directly apply to patent trolls, whose business model depends on extracting settlement payments by threatening litigation (Cohen, Gurun, and Kominers, 2016): in such cases, alleged infringement significantly boosts total market profits, implying $\Phi \gg 1$.

[15]The *likely outcome during litigation* refers to the refers to the anticipated outcome during the litigation process, assuming no court ruling occurs before firms take action.

[16]$\frac{1}{r-\mu}$ is the perpetuity factor for $\{x_t\}_{t=0}^{\infty}$ from $t=0$ to $t=\infty$. $\frac{1}{r+\lambda-\mu}$ is the annuity factor for $\{x_t\}_{t=0}^{\tau}$ from $t=0$ to $t=\tau$.

**The general form of during-litigation firm values** The during-litigation firm value $V^j(x)$, where $j \in \{I, C\}$, satisfies the following ordinary differential equation (*ODE*), with $\mathbb{E}_t dV^j(x) = \left(\mu x \frac{\partial V^j}{\partial x} + \frac{1}{2}\frac{\partial^2 V^j}{\partial x^2}x^2\sigma^2\right)dt$ and $\Delta\pi^j$ as defined in Equation (2):

$$\mathbb{E}_t dV^j + (\pi_2^j x - c^j)dt + \lambda\Big(\frac{\pi_2^j - p\Delta\pi^j}{r-\mu}x - V^j\Big)dt = rV^j dt, \quad \text{for } j \in \{I, C\}. \tag{7}$$

The first term on the left-hand side of Equation (7) captures the expected instantaneous change in firm value due to market uncertainties. The next two terms represent the firm's expected net profit flow during litigation, and the final term reflects the expected change in firm value due to a potential court ruling. A boundary condition for $V^j$ is derived from the notion that it is not optimal for firms to take any actions when demand is sufficiently high, expressed as $\lim_{x\to\infty} V^j(x) = \left(\frac{\pi_2^j}{r-\mu} - p\delta\Delta\pi^j\right)x - C_l^j$, for $j \in \{I, C\}$. We present $V^j$ in Proposition 1, with the proof in Appendix A1.

**Proposition 1.** *After the litigation starts and before any further actions are taken or the court rules, the* ***during-litigation firm value*** $V^j(x)$ *is*

$$V^j(x) = \Big(\frac{\pi_2^j}{r-\mu} - p\delta\Delta\pi^j\Big)x - C_l^j + B^j x^{\beta_\lambda}, \quad j \in \{I, C\}, \tag{8}$$

*where* $\beta_\lambda = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(r+\lambda)}{\sigma^2}}$, $C_l^j = \frac{c^j}{r+\lambda}$, *the constants* $B^I$ *and* $B^C$ *depend on the respective strategies.*

In Equation (8), the term $B^j x^{\beta_\lambda}$ represents Firm $j$'s option value during litigation. The remaining terms describe the firm's value if no further action is taken: $\frac{\pi_2^j}{r-\mu}x$ is the present value of future profits under the duopoly status quo, and $-p\delta\Delta\pi^j x$ is the present value of the expected payoff from judgement: only if the court rules in favor of the incumbent, Firm j's profit changes, and by $-\Delta\pi^j x$ from judgement onward. $C_l^j$ represents the present value of litigation costs. Provided that the challenger's market entry leads to a decline in overall market profits ($\Delta\pi < 0$), the present value of the two firms' **total judgement payoff** ($-p\Delta\pi x$) is positive ($-p\delta\Delta\pi x > 0$) and increases with market demand, .

### 3.1.1 Non-settlement During Litigation ($s_{ns}$)

Both firms have the option to abandon litigation, with each firm's abandonment strategy taking into account the optimal abandonment strategy of its rival. In this attrition game during litigation, the firm that abandons first is the *leaving firm*, while the *remaining firm* has a stronger incentive to continue bearing litigation costs in order to obtain the expected judgement payoff.

We analyze two scenarios: when the incumbent is the leaving firm (*I-withdraw*) and when the challenger is the leaving firm (*C-exit*). We denote the firms' payoffs at I-withdraw as $\hat{V}_w^j(x)$, for $j \in \{I, C\}$, after which the market remains a duopoly, and their payoffs at C-exit as $\hat{V}_e^j(x)$, after which the incumbent's monopoly is restored.

$$\hat{V}_w^j(x) = \frac{\pi_2^j x}{r-\mu}, \quad \hat{V}_e^j(x) = \frac{\pi_2^j - \Delta\pi^j}{r-\mu} x, \quad j \in \{I, C\}. \tag{9}$$

**Corollary 1.** ***(I-withdraw)*** *The firm value with the I-withdraw option, $V_w^j$, follows Equation (8) in Proposition 1, with $B_w^j = [C_l^j + p\delta\Delta\pi^j x_w]x_w^{-\beta_\lambda}$, $j \in \{I, C\}$. The I-withdraw threshold is $x_w = \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{C_l^I}{p\delta|\Delta\pi^I|}$.*

Corollary 1 implies that the option value of *I-withdraw* for the incumbent ($B_w^I x^{\beta_\lambda}$) is the product of the net present value of withdrawal and a stochastic discount factor, expressed as $\big(C_l^I + p\delta\Delta\pi^I x_w\big)(\frac{x}{x_w})^{\beta_\lambda}$. By withdrawing, the incumbent avoids all future litigation costs but gives up the potential to revert to monopoly, resulting in the net payoff of $C_l^I + p\delta\Delta\pi^I x_w$. The stochastic discount factor $(\frac{x}{x_w})^{\beta_\lambda}$ values these future payoffs at the time of I-withdraw, taking into account that a court ruling kills the I-withdraw option. Interestingly, the I-withdraw option is even more valuable for the challenger (since $\Delta\pi^C > 0 > \Delta\pi^I$), as reflected in the challenger's value associated with this option:

$$B_w^C x^{\beta_\lambda} = \overbrace{\Big(C_l^C + p\delta\Delta\pi^C x_w\Big)}^{\text{saved litigation cost + avoided profit loss from judgment}}(\frac{x}{x_w})^{\beta_\lambda}. \tag{10}$$

When the incumbent withdraws, the challenger not only saves on future litigation costs but also prevents the potential loss in an adverse court ruling. Corollary 1 suggests that the incumbent delays withdraw until its cost of remaining in the litigation outweighs the benefits by a sufficient margin ( $\frac{C_l^I}{p\delta|\Delta\pi^I|x_w} = \frac{\beta_\lambda - 1}{\beta_\lambda} > 1$ with $\beta_\lambda < 0$), driven by the irreversibility of action and market uncertainty.

**Corollary 2.** ***(C-exit)*** *The firm value with the C-exit option, $V_e^j$, follows Equation (8) in Proposition 1, with $B_e^j = [C_l^j + (p\delta - \frac{1}{r-\mu})\Delta\pi^j x_e]x_e^{-\beta_\lambda}$, $j \in \{I, C\}$. The C-exit threshold is $x_e = \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{C_l^C}{(\frac{1}{r-\mu} - p\delta)\Delta\pi^C}$.*

By exiting, the challenger saves on future litigation cost but forfeits its duopoly profits, yielding a present value of $C_l^C + (p\delta - \frac{1}{r-\mu})\Delta\pi^C x_e < C_l^C$. This option is even more valuable for the incumbent, who, in addition to saving litigation costs, regains monopoly profits immediately, resulting in a present value of $C_l^I + (p\delta - \frac{1}{r-\mu})\Delta\pi^I x_e > C_l^I$. For both firms, the option values of C-exit ($B_e^j x^{\beta_\lambda}$) is the product of the aforementioned present value at the C-exit and the stochastic discount factor $(\frac{x}{x_e})^{\beta_\lambda}$.

**Non-settlement: *I-withdraw* or *C-exit*?** To determine the non-settlement outcome–either *I-withdraw* or *C-exit*–we compare the firms' *reservation thresholds* (Lambrecht, 2001), defined as the lowest demand level at which a firm would continue paying litigation costs, assuming the other firm abandons first. We argue that the leaving firm is the one with the higher reservation threshold,[17] and obtain the following results with proofs provided in Appendix A3:

**Lemma 1.** *The reservation threshold of the incumbent $x_I$ satisfies $\frac{1}{1-\beta_\lambda}\left(\frac{x_I}{x_w}\right)^{\beta_\lambda} - \frac{\beta_\lambda}{\beta_\lambda-1}\frac{(1-p\omega)}{p\omega}\frac{x_I}{x_w} = 1$. The reservation threshold of the challenger $x_C$ satisfies $\frac{1}{1-\beta_\lambda}\left(\frac{x_C}{x_e}\right)^{\beta_\lambda} - \frac{\beta_\lambda}{\beta_\lambda-1}\frac{p\omega}{(1-p\omega)}\frac{x_C}{x_e} = 1$. The non-settlement outcome during litigation is $s_{ns} = \{$I-withdraw$\}$ if $x_I > x_C$, and $s_{ns} = \{$C-exit$\}$ if $x_I < x_C$.*

**Lemma 2.** *$s_{ns} = \{$I-withdraw$\}$ if $p\omega > 0.5$ and $\Phi > \frac{p\omega}{1-p\omega}\Lambda$. $s_{ns} = \{$C-exit$\}$ if $p\omega < 0.5$ and $\Phi < \frac{p\omega}{1-p\omega}\Lambda$. If $p\omega = 0.5$, then $s_{ns} = \{$I-withdraw$\}$ if $\Phi > \Lambda$ and $s_{ns} = \{$C-exit$\}$ if $\Phi < \Lambda$.*

Lemma 2 relates to the economics and finance literature on exit order in duopolies, emphasizing the real options value of exiting when there is possibility of exogenous termination (here, by the court). Instead of relying solely on the relative benefit and costs between the two firms, the exit order also depends on the uncertainty of when and how the court will rule (via the terms $p\omega$ and $1 - p\omega$). The incumbent may abandon litigation first, even when it receives high cash flows from staying in the litigation (i.e., $p\omega > 0.5$), as long as the gain-to-loss ratio $\Phi$ sufficiently exceeds the litigation cost ratio $\Lambda$ (with a multiplier of $\frac{p\omega}{1-p\omega} > 1$).[18] Similarly, *C-exit* is the non-settlement outcome even if the incumbent has low cash flows in litigation (i.e., $p\omega < 0.5$), as long as $\Phi$ is sufficiently lower than $\Gamma$.

**Summary of non-settlement** Based on Corollaries 1–2 and Equation (9), we summarize the firm values $V^j_{ns}$, payoffs $\hat{V}^j_{ns}$, arbitrary constants in the option values $B^j_{ns}$, and the thresholds $x^j_{ns}$, for $j \in \{I, C\}$, all associated with non-settlement as follows:

$$\left(V^j_{ns}(x)^j, \hat{V}^j_{ns}(x), B^j_{ns}, x_{ns}\right) = \begin{cases} \left(V^j_w(x), \hat{V}^j_w(x), B^j_w, x_w\right), & s_{ns} = \{\text{I-withdraw}\} \\ \left(V^j_e(x), \hat{V}^j_e(x), B^j_e, x_e\right), & s_{ns} = \{\text{C-exit}\} \end{cases} \quad j \in \{I, C\}. \quad (11)$$

The non-settlement value represents the value of optimally abandoning litigation for the leaving firm, and the value given its rival's optimal abandonment for the remaining firm.

### 3.1.2 Settlement During Litigation (*Ex-post settlement*)

An ex-post settlement occurs when the incumbent drops the patent infringement claim in exchange for a royalty payment from the challenger. We model the ex-post settlement as a

[17] This subgame perfect equilibrium corresponds to the knife-edge case in Lambrecht (2001).

[18] The condition $\Phi > \frac{p\omega}{1-p\omega}\Lambda$ is equivalent to $\frac{\Phi}{\Lambda} = \frac{\Delta\pi^C}{C^C_l}\left(\frac{-\Delta\pi^I}{C^I_l}\right)^{-1} > \frac{p\omega}{1-p\omega}$, which is greater than 1 when $p\omega > 0.5$.

Stackelberg leader-follower game (Lambrecht, 2004), where the incumbent proposes a licensing agreement with the royalty rate $\theta_p$.[19] The challenger then decides when to settle, effectively choosing the settlement threshold $x_p$. Equation (12) represents the ex-post settlement payoffs $\hat{V}_p^j(x;\theta_p)$, after the firms incur settlement costs $C_s^j$.

$$\hat{V}_p^j(x;\theta_p) = \frac{\pi_2^j + \Delta\pi_p^j}{r-\mu}x, \quad j \in \{I, C\}. \tag{12}$$

**Corollary 3.** ***(Ex-post settlement)*** *The firm values with the ex-post settlement option, $V_p^I$ and $V_p^C$, follow Equation (8). Given a royalty rate $\theta_p$, the arbitrary constants are $B_p^I(\theta_p) = \left[\Delta C^I + \left(p\delta\Delta\pi^I + \frac{\theta_p\Delta\pi^C}{r-\mu}\right)x_p\right]x_p^{-\beta_\lambda}$ and $B_p^C(\theta_p) = \left[\Delta C^C + \left(p\delta - \frac{\theta_p}{r-\mu}\right)\Delta\pi^C x_p\right]x_p^{-\beta_\lambda}$. The settlement threshold is $x_p(\theta_p) = \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{\Delta C^C}{(\frac{\theta_p}{r-\mu} - p\delta)\Delta\pi^C}$.*

The option value of settlement, $B_p^j x^{\beta_\lambda}$, is intuitive and can also be expressed as $B_p^j(\theta_p) = \left[\Delta C^j + \left(p\delta\Delta\pi^j + \frac{\Delta\pi_p^j}{r-\mu}\right)x_p\right]x_p^{-\beta_\lambda}$ where $\Delta\pi_p^j$ is defined in Equation (4). For each firm, the option value of settlement equals its surplus from settlement multiplied by a stochastic discount factor that values the future payoffs at the time of settlement, factoring in that the court ruling kills the settlement option. Besides the cost savings (litigation cost minus settlement cost), valued as $\Delta C^j$, the incumbent gives up potential recovery of monopoly profits but gains guaranteed royalty payments, whilst the challenger avoids losing duopoly profits in an adverse court rules and pays the royalty. Consistent with Banerjee, Güçbilmez, and Pawlina (2014), our model shows that the incumbent trades off the benefit of higher royalty rates against the delay in settlement ($\frac{\partial x_p}{\partial \theta_p} = -\frac{\beta_\lambda}{\beta_\lambda - 1}\frac{\Delta C^C(r-\mu)}{\Delta\pi^C}\frac{1}{(\theta_p - p\omega)^2} < 0$) when determining the optimal royalty rate $\theta_p$ to maximize $V_p^I(x_p, \theta_p)$. Define the *relative-cost-saving* as:[20]

$$\text{The relative-cost-saving:} \quad \Gamma \equiv \frac{\Delta C^C}{\Delta C^I} = \frac{C_l^C - C_s^C}{C_l^I - C_s^I}. \tag{13}$$

Theorem 1 provides the subgame-perfect Nash equilibrium strategy in the ex-post settlement negotiations, with proof in Appendix A5.

**Theorem 1.** *In ex-post settlement, the incumbent proposes $\theta_p^* = p\omega\left(1 - g(\Gamma)\right) + \frac{p\omega}{\Phi}g(\Gamma) \in (0,1)$, and the challenger sets the settlement threshold $x_p^* = \frac{\beta_\lambda}{\beta_\lambda - 1} \cdot \frac{\Delta C^C}{p\delta g(\Gamma)(-\Delta\pi)}$, where $g(\Gamma) = (\frac{\beta_\lambda}{\beta_\lambda - 1} + \frac{1}{\Gamma})^{-1}$.*

[19] We assume the incumbent makes the offer once litigation begins, as this timing is optimal for the incumbent; the outcome remains unchanged as long as the offer precedes the challenger's settlement acceptance threshold.

[20] When litigation costs greatly exceed settlement costs, or when settlement costs are proportional to litigation costs, the relative-cost-saving approximates the relative litigation cost $\Lambda$. Appendix A5 provides conditions for a higher settlement threshold than the non-settlement threshold ($x_p > x_{ns}$), which is always satisfied if settlement costs are negligible compared to litigation costs.

The optimal royalty rate $\theta_p^*$ is a weighted average of two rates ($p\omega$ and $\frac{p\omega}{\Phi}$). The rate $p\omega$ makes the challenger indifferent between settling and continuing litigation without considering costs: $p\omega = \{\theta : \frac{-\pi_2^C \theta x}{r-\mu} = -p\delta\Delta\pi^C x\}$. The second rate, $\frac{p\omega}{\Phi}$, makes the incumbent indifferent between settling (with a royalty stream) and continuing litigation (to gain a judgement payoff), expressed as $\frac{p\omega}{\Phi} = \{\theta : \frac{\theta\Delta\pi^C x}{r-\mu} = -p\delta\Delta\pi^I x\}$. This rate decreases with the gain-to-loss ratio. The weights $(1 - g(\Gamma), g(\Gamma))$ adjust based on cost considerations. Upon settling, the two firms share the **overall settlement surplus** ($p\delta\Delta\pi x_p^* + \Delta C^C + \Delta C^I$, with $\Delta\pi < 0$), which includes the value of the foregone judgement payoff and total cost savings. Theorem 1 implies this surplus is split proportionally: $\frac{1}{1+g(\Gamma)}$ for the incumbent and $\frac{g(\Gamma)}{1+g(\Gamma)}$ for the challenger.[21]

Corollary 4 presents the comparative statics for the ex-post settlement threshold and terms (proof in Appendix A6), assuming feasibility as discussed in Section 3.1.3. Firms accelerate ex-post settlement when cost savings are greater but delay as market uncertainty rises or the incumbent's winning probability increases. Notably, higher levels of competition from the challenger's entry (corresponding to a lower $\Phi$) raise the ex-post settlement royalty rates, as stronger competition the means the incumbent can regain more profits from a favorable judgement, relative to the challenger's profits that are preserved in settlement. Consequently, the incumbent demands a higher royalty rate to settle rather than await judgement.

**Corollary 4.** *$\frac{\partial x_p^*}{\partial \Delta C^j} > 0$ for $j \in \{I, C\}$, $\frac{\partial x_p^*}{\partial \sigma^2} < 0$, $\frac{\partial x_p^*}{\partial \Delta\pi} > 0$, $\frac{\partial x_p^*}{\partial p} < 0$. $\frac{\partial \theta_p^*}{\partial \Phi} < 0$, $\frac{\partial \theta_p^*}{\partial \sigma^2} > 0$, $\frac{\partial \theta_p^*}{\partial \Gamma} > 0$, $\frac{\partial \theta_p^*}{\partial p} > 0$.*

### 3.1.3 During Litigation: *Ex-post settlement* or *Non-settlement*?

Non-settlement and settlement are alternative options during litigation. The firms only settle (and at $x_p$) if, for each firm, the value with the settlement option (discussed in Section 3.1.2) is at least as high as the value with the non-settlement option (Section 3.1.1):

$$\hat{V}_p^I(x_p, \theta_p) = V_p^I(x_p, \theta_p) \geq V_{ns}^I(x_p) \quad \text{and} \quad \hat{V}_p^C(x_p, \theta_p) = V_p^C(x_p, \theta_p^C) \geq V_{ns}^C(x_p). \tag{14}$$

The conditions in Expression (14) narrow down the range of royalty rates for feasible ex-post settlement, resulting in both a lower bound ($\theta_p^{Imin}$) and an upper bound ($\theta_p^{Imax}$) on the royalty rate from the incumbent's consideration. The lower bound ensures the incumbent is compensated for not continuing with the litigation, while the upper bound prevents excessive settlement delay, which is suboptimal. At low levels of $\theta_p$, the positive effect of $\theta_p$ on $V_p^I(x_p, \theta_p)$ (from gaining a higher share of the challenger's profits) dominates, whereas at high levels of $\theta_p$, the negative effect (from settlement delay) becomes more pronounced. Expression (14) also im-

[21]The total settlement surplus also equals $\Delta C^C + \Delta C^I - \frac{\Delta C^C \beta_\lambda}{g(\Gamma)(\beta_\lambda - 1)}$ (using Theorem 1). The incumbent's share is $\Delta C^I + \frac{(\theta_p^* \pi_2^C - p\omega(\pi_1 - \pi_2^I))}{r-\mu} x_p^*$, simplifying to $\Delta C^I - \frac{1-g(\Gamma)}{g(\Gamma)} \frac{\beta_\lambda}{\beta_\lambda - 1} \Delta C^C$, and the challenger's share is $\Delta C^C + \frac{\pi_2^C(p\omega - \theta_p^*)}{r-\mu} x_p^*$, which simplifies to $\Delta C^C - \Delta C^C \frac{\beta_\lambda}{\beta_\lambda - 1}$. Thus, the proportions thus follow.

plies an upper bound $\theta_p^{Cmax}$ for the challenger, above which its retained profits are insufficient to make settlement worthwhile. With proof in Appendix A7:[22]

**Theorem 2.** *An ex-post settlement is the likely outcome during litigation, as opposed to non-settlement, if and only if* $\theta_p^* \in \left[\theta_p^{Imin}, \min\{\theta_p^{Cmax}, \theta_p^{Imax}\}\right]$*, with* $\theta_p^{Cmax}$*,* $\theta_p^{Imin}$*, and* $\theta_p^{Imax}$ *specified in Equations* (A.6) *and* (A.7) *in the appendix.*

**Summary of during-litigation values and strategies** Based on Corollary 3, Theorem 1, Equations (11) and (12), we summarize the firm values $V^j$, payoffs after exercising an option $\hat{V}^j$, the arbitrary constants in option values $B^j$, $j \in \{I, C\}$, and the lawsuit termination threshold $X$ during litigation as follows:

$$\left(V^j(x), \hat{V}^j(x), B^j, X\right) = \begin{cases} \left(V_p^j(x), \hat{V}_p^j(x), B_p^j, x_p\right) & \text{if the condition in Theorem 2 holds,} \\ \left(V_{ns}^j(x), \hat{V}_{ns}^j(x), B_{ns}^j, x_{ns}\right) & \text{otherwise.} \quad j \in \{I, C\}. \end{cases} \tag{15}$$

### 3.2 Pre-litigation Solutions

We present the general form of firm values at this stage (Proposition 2) and briefly discuss the incumbent's option to force the challenger out of the market. Net, we examine the *I-litigate* decision (Corollary 5), considering firm values during litigation (Equation (15)). We then explore firms' *ex-ante settlement* (Corollary 6) before determining their strategies at this stage (Theorem 3).

**The general form of pre-litigation value functions** After the challenger's entry with the alleged infringement, firms make duopoly profits. With the proof in Appendix A8, we show:

**Proposition 2.** *After the challenger's entry and before any actions are taken, the* ***pre-litigation firm value*** *is given by*

$$V_0^j = \begin{cases} \frac{\pi_2^j x}{r-\mu} + A^j x^\alpha, & \text{if} \quad s_{ns} = \{\textit{I-withdraw}\} \\ \frac{\pi_2^j x}{r-\mu} + a^j x^\alpha + b^j x^\beta. & \text{if} \quad s_{ns} = \{\textit{C-exit}\} \end{cases}, \quad j \in \{I, C\} \tag{16}$$

*where* $\alpha, \beta = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}}$ *(with* $\alpha > 1$ *and* $\beta < 0$*). The constants* $A^j$*,* $a^j$*, and* $b^j$ *depend on the respective options.*

[22] Our numerical analysis suggests that $\theta_p^{Imax} \leq \theta_p^{Cmax}$ if $s_{ns} = \{\text{C-exit}\}$, but $\theta_p^{Imax} > \theta_p^{Cmax}$ if $s_{ns} = \{\text{I-withdraw}\}$. The challenger's strong willingness to finance the royalty fee when $s_{ns} = \{\text{C-exit}\}$, as indicated by $\theta_p^{Imax} \leq \theta_p^{Cmax}$, stems from the challenger's motivation to avoid exiting. Conversely, when $s_{ns} = \{\text{I-withdraw}\}$, the challenger prefers the outcome over its own exit and is therfore unwilling to pay a royalty fee to avoid *I-withdraw*, as reflected by $\theta_p^{Cmax} < \theta_p^{Imax}$.

The firm value $V_0^j$ includes an additional term $b^j x^\beta$ if $s_{ns}$ = {C-exit}, reflecting the incumbent's extra option to force the challenger out of the market (*forcing-out*). The incumbent exercises the *forcing-out* option optimally if market demand falls to the C-exit threshold $x_e$ after the challenger's entry but before reaching the *I-litigate* threshold. Recognizing the *forcing-out* option is both economically important and technically necessary to solve the model.[23] The *Forcing-out* option is effective in driving the challenger to exit if $s_{ns}$ = {C-exit}, as the challenger would optimally leave at $x_e$ after litigation starts. Meanwhile, the incumbent can credibly commit not to offer ex-post settlement if $s_{ns}$ = {C-exit}. By exercising the *forcing-out* option at $x_e$, the incumbent immediately regains the monopoly profits without incurring litigation costs.

### 3.2.1 Litigation by the Incumbent (*I-litigate*)

Once the incumbent initiates litigation, firm values transition to their during-litigation values $V^j$, as shown in Equation (15). With proof in Appendix A9, we have the following corollary:

**Corollary 5.** ***(I-litigate)*** *The firm value with the I-litigate option, $V_l^j$, for $j \in \{I, C\}$, follows Equation (16), with $A_l^j = \left[-C_l^j - p\delta\Delta\pi^j x_l + B^j x_l^{\beta_\lambda}\right] x_l^{-\alpha}$, and $a_l^j$ and $b_l^j$ specified in Equation* (A.12). *The litigation threshold $x_l$ satisfies $(\alpha - 1)p\delta\Delta\pi^I x_l + (\beta_\lambda - \alpha)B^I x_l^{\beta_\lambda} + \alpha C_l^I + \frac{\partial B^I}{\partial x_l} x_l^{\beta_\lambda+1} = 0$ if $s_{ns}$ = {I-withdraw}, where $B^I$ is defined in Equation* (15)*. If $s_{ns}$ = {C-exit}, $x_l$ satisfies Equation* (A.13) .

By exercising the *I-litigate* option, the incumbent expects to incur litigation costs $C_l^I$. However, with probability $p$, it restores the profit loss caused by the challenger's entry, $|\Delta\pi^I x|$, through a court ruling, valued at $-p\delta\Delta\pi^I x_l$). Additionally, the incumbent gains follow-on options during litigation, worth $B^I x_l^{\beta_\lambda}$, where $B^I$ is defined in Equation (15). The challenger's option value for *I-litigate* can be interpreted similarly.

### 3.2.2 Settlement without Litigation (*Ex-ante settlement*)

Equation (17) shows the ex-ante settlement payoffs $\hat{V}_a^j(x;\theta_a)$, after firms pay the settlement cost $C_s^j$, with a given royalty rate $\theta_a$. Corollary 6 is presented below, with proof in Appendix A10.

$$\hat{V}_a^j(x;\theta_a) = \frac{\pi_2^j + \Delta\pi_a^j(\theta_a)}{r-\mu} x, \quad j \in \{I, C\}, \quad \Delta\pi_a^j \text{ is defined in Equation (3)} \tag{17}$$

**Corollary 6.** ***(Ex-ante settlement)*** *The firm value with the ex-ante settlement option, $V_a^j$ for $j \in \{I, C\}$, follows Equation (16), with $A_a^j = (\frac{\Delta\pi_a^j}{r-\mu} x_a - C_s^j) x_a^{-\alpha}$ and $a_a^j$ and $b_a^j$ expressed in Equation* (A.14).

[23] Technically, the forcing-out option imposing value matching conditions for both firms at $x_e$ before any potential litigation. See Equation (A.11) in the appendix.

*The ex-ante settlement threshold is $x_a = x_l$, where the I-litigate threshold $x_l$ is expressed in Corollary 5.*

The challenger's value of ex-ante settlement option is negative because the incumbent's exercise of the option results in royalty payments and settlement fees, without generating positive cash flows for the challenger. Corollary 6 also reveals that ex-ante settlement offers less flexibility in settlement timing compared to ex-post settlement. Firms settle ex-ante just before the incumbent would otherwise initiate litigation. This occurs because, by settling ex-ante, the challenger agrees to pay a portion of its future profits to the incumbent. This is a significant cost, so the challenger will only agree if the alternative – facing litigation– proves at least as costly, and if settlement cannot be delayed any further.

#### 3.2.3 Pre-litigation: *I-litigate* or *Ex-ante settlement*?

Settlement can occur only if it is optimal for both firms. Specifically, firms will settle ex-ante if each firm's value from settling exceeds its value from litigation:

$$\hat{V}_a^C(\theta_a, x_a) - C_s^C \geq V_l^C(x_a) \Rightarrow \theta_a \leq \theta_a^{Cmax}, \quad \hat{V}_a^I(\theta_a, x_a) - C_s^I \geq V_l^I(x_a) \Rightarrow \theta_a \geq \theta_a^{Imin}. \tag{18}$$

**Theorem 3.** *Following the challenger's market entry, firms settle ex-ante with the royalty rate $\theta_a = \theta_a^{Cmax}$ if and only if $\theta_a^{Imin} \leq \theta_a^{Cmax}$, where $\theta_a^{Imin}$ and $\theta_a^{Cmax}$ are specified in Equation (A.15). Otherwise, the incumbent will proceed with I-litigate at $x_l$, as specified in Corollary 5.*

The challenger will prefer litigation over ex-ante settlement if the royalty rate exceeds its maximum acceptable level $\theta_a^{Cmax}$. The incumbent's minimum required rate $\theta_a^{Imin}$ can be interpreted similarly. Unlike ex-post settlement, the incumbent does not set an upper bound on the royalty rate in ex-ante settlement, as the timing inflexibility eliminates concerns about settlement delays (see the discussion after Corollary 6). Given the alternative of the *I-litigate* option, the incumbent proposes the highest royalty rate acceptable to the challenger, $\theta_a^{Cmax}$.

**Summary of pre-litigation strategies and values** Based on Corollaries 5 and 6, we represent the pre-litigation firm values $V_0^j$, the payoff functions $\hat{V}_0^j$ ,$j \in \{I, C\}$, the relevant arbitrary constants in the firm values, and the strategy threshold $X_0$ as follows:

$$\left(V_0^j(x), \hat{V}_0^j(x), A^j, a^j, b^j, X_0\right) = \begin{cases} \left(V_a^j(x), \hat{V}_a^j(x), A_a^j, a_a^j, b_a^j, x_a\right) & \text{if } \theta_a^{Imin} \leq \theta_a^{Cmax} \text{ in Theorem 3,} \\ \left(V_l^j(x), \hat{V}_l^j(x), A_l^j, a_l^j, b_l^j, x_l\right) & \text{otherwise.} \end{cases} \tag{19}$$

### 3.3 An Example of Model Solution: Patent Dispute Outcome at the Benchmark

Figure 2 illustrates an example of a market demand path, showing the firms' sequential actions under benchmark parameter values (details in Section 4.). Following the steps outlined in Sections 3.1–3.2, we find that the likely outcome in this example is *I-litigate* (at $x_l$) followed by *ex-post settlement* (at $x_p$) with $x_l = 1.72 > x_p = 0.76$. As shown in the figure, the incumbent initiates litigation as market demand rises after the challenger's entry, indicated by the red-dot. Firms then settle ex-post as demand declines, marked by the green-dot, assuming a court ruling has not yet occurred.

[Insert Figure 2 here.]

### 3.4 Discussion on Pre-litigation and During-litigation Thresholds

The pre-litigation call option is exercised as demand rises from below whilst the during-litigation put option is exercised as demand drops from above. Typically, the pre-litigation threshold exceeds the during-litigation threshold, i.e., $X_0 = \{x_a, x_l\} \geq X = \{x_{ns}, x_p\}$. However, new complications arise if $X_0 < X$. For instance, if the model solution indicates an *I-litigate* followed by *ex-post settlement*, but $x_p \geq x_l$, this implies firms would settle immediately upon the start of litigation, a scenario we refer to as "*immediate settlement*". Recognizing the possibility of immediate settlement, the firms re-optimize. Appendix A13 provides the technical details for incorporating immediate settlement into our model solutions.[24] With immediate settlement, ex-ante settlement always occurs beforehand, as we assume equal costs for ex-ante and ex-post settlement.

## 4. Quantitative Analysis on Patent Dispute Outcomes

We use quantitative analysis to examine the effects of competitive dynamics and market uncertainty on patent dispute outcomes. The benchmark parameter values in Table 1 largely follow the literature (e.g., Jeon, 2015). Market demand is characterized by $\mu = 2\%$ and $\sigma = 30\%$. The expected time to court ruling ($\frac{1}{\lambda} = 2.5$ years) aligns with empirical evidence on U.S. patent litigation cases, and the probability of the incumbent winning the infringement case is set at $p = 0.5$.

[Insert Table 1 here.]

[24]Technically, there are two types of immediate settlement. In a *constrained immediate settlement*, the optimal ex-post settlement royalty rate exceeds the challenger's maximum acceptable rate ($\theta_p^* > \theta_p^{Cmax}$), so the royalty rate is set at $\theta_p^{Cmax}$. In *unconstrained immediate settlement*, $\theta_p^* < \theta_p^{Cmax}$, allowing the royalty rate to be $\theta_p^*$. While immediate settlement is rarely modeled in the literature, the practice of settling soon after patent litigation begins is documented in empirical studies (Kessler, 1996; Spier, 1992; Fanning, 2016; Vasserman and Yildiz, 2019). We apply the approach of Décamps, Mariotti, and Villeneuve (2006) to solve immediate settlement. Our findings show that the ex-ante settlement royalty rate aligns with the rate in immediate settlement.

Given the parameter values of the product market ($\mu, \sigma, \pi_1^I, \pi_2^I, \pi_2^C$), the legal system ($p, \lambda, c_l^I, c_l^C, C_s^I, C_s^C$), the risk free rate $r$, we solve the model numerically as outlined in Section 3.. This approach yields the likely outcome for resolving the patent dispute, assuming firms follow optimal strategies and market demand reaches their action thresholds before the court ruling. We repeat the numerical procedure by varying one model parameter value at a time, mapping likely patent dispute outcome to two factors suggested by Theorem 1: the gain-to-loss ratio $\Phi$ (our new measure of competition) and the relative-cost-saving $\Gamma$.

### 4.1 Baseline Patent Dispute Outcomes

We use Figure 3 to illustrate the likely patent dispute outcomes around the benchmark parameter values. Consistent with Lemma 2, above (or below) this line, the incumbent (or the challenger) is the remaining firm during litigation, with the non-settlement outcome being *C-exit* (or *I-withdraw*). The potential likely outcomes include *ex-ante settlement* (blue area), *I-litigate* followed by *ex-post settlement* (green area), or *I-litigate* followed by non-settlement (*I-withdraw* or *C-exit*), represented by the white area.[25]

[Insert Figure 3 here.]

Figure 3 (along with additional unreported analyses) highlights two primary determinants of patent dispute outcomes: the competitive dynamics from the challenger's entry and the firms' relative willingness to finance (WtF). As competition intensifies (i.e., as $\Phi$ decreases), disputes are more likely to proceed from settlement (first ex-ante, then ex-post) to litigation without settlement. Additionally, a smaller gap in firms' WtF increases the likelihood of settling, as indicated by the larger colored area near the dashed line.

We find that firms never settle their dispute when competition from the challenger is too intense, meaning the gain-to-loss ratio $\Phi$ is sufficiently low. For example, if the challenger introduces substitute products based on the same new technology as the incumbent's, the firms are likely to engage in patent litigation and remain in legal proceedings until the end. This is because settling would require forfeiting the possibility of a judgement, which could restore overall market profits to pre-entry monopoly level with probability $p$. The *total judgement payoff* (discussed after Proposition 1), representing the incumbent's expected gain minus the challenger's expected loss from judgement, is thus positive and increases with market demand. In settlement, the two firms would instead share the *overall settlement surplus* (discussed after Theorem 1), which decreases with market demand, with the decline rate reflecting the (negative) impact on overall market profits. When product substitutability is high (low $\Phi$), the impact on market profits from the challenger's entry is substantial, leading to a small settlement

[25] The challenger being forced out of the market without actual litigation is also a possible outcome. However, since the *forcing-out* option depends on whether $x_e$ is reached before $x_l$, we cannot plot this outcome directly in the graphs. *Forcing-out* can potentially occur anywhere in the C-exit region, while court-ruling is only realized in the non-blue area.

surplus, which is only positive when market demand is low. In such cases, each firm's abandonment option is more valuable than its settlement option at any royalty rate, making settlement infeasible. As $\Phi$ increases, the range of market demand levels for which the settlement option payoffs are in-the-money increases at a much faster rate than for the abandonment options.[26] For high enough $\Phi$, the value of settling thus exceeds the non-settlement value for both firms, making settlement feasible. This suggests a lower bound for $\Phi$ below which firms do not settlement.

Furthermore, we find that lower levels of competition (higher $\Phi$) brings settlement forward: shifting from ex-post settlement–when demand falls sufficiently after litigation begins but before a court ruling–to ex-ante settlement, which occurs just before litigation would otherwise commence. This shift arises because, unlike ex-post settlement, ex-ante settlement offers less flexibility in the option exercise timing. Once litigation begins, the challenger's value declines significantly due to litigation costs and potential loss of future profits in an advert court ruling; however, these costs only apply once litigation starts. Before litigation begins, since royalty payments would be an immediate cost, the challenger prefers to delay ex-ante settlement until litigation is imminent. In contrast, the timing flexibility of the ex-post settlement option brings additional value to settling and thus greater scope for ex-post settlement. Our computational analysis indicates that feasible ex-ante settlement occurs only for a subset of feasible ex-post settlement parameters, and typically when ex-post settlement would happen immediately or shortly after litigation.[27] This is relevant for high $\Phi$, because it raises the settlement option payoffs and ex-post settlement thresholds, which, in turn, brings forward the expected timing of potential ex-post settlement and makes ex-ante settlement feasible.[28]

Regarding the second determinant, settlement becomes feasible over a wider range of competition levels (including lower $\Phi$) when firms have closer willingness to finance (WtF). The identity of the remaining firm (the one with the greater WtF) not only establishes each firm's non-settlement value and hence the bounds of feasible settlement royalty rates, but it also affects the relative sensitivity of settlement versus non-settlement option values to changes in WtF. Specifically, changes in the remaining firm's WtF impact settlement values more strongly

[26]This is because settlement only requires covering a portion of the judgement payoff, whereas the abandonment must cover the firm's expected benefit of winning relative to losing the lawsuit, with payoffs equivalent to a court ruling against them.

[27]Intuitively, as long as ex-ante settlement costs do not exceed those of ex-post settlement, ex-ante settlement (weakly) dominates ex-post settlement if ex-post settlement would occur immediately after litigation commences. When the litigation threshold is significantly above the ex-post settlement threshold, there is substantial value in waiting rather than settling immediately at the ex-post royalty rate $\theta_p^*$. Consequently, the incumbent's minimum ex-ante royalty rate exceeds $\theta_p^*$ whilst the challenger's acceptable rate falls below $\theta_p^*$, making ex-ante settlement infeasible if the expected time from litigation start to ex-post settlement is sufficiently long.

[28]For instance, as $\Phi$ approaches 1, the incumbent's profitability reduction from challenger's entry is only marginally larger than the challenger's gains, the expected net payoff from judgement becomes small and is dominated by the cost savings from settling, so the settlement option becomes sufficiently in-the-money for immediate ex-post settlement to be optimal.

than non-settlement values, whereas changes in the leaving firm's WtF have the opposite effect. Because either firm's option values decrease with higher WtF, a wider gap between the firms' WtF–whether from an increase in the remaining firm's WtF or a decrease in the leaving firm's WtF–reduces the likelihood of settlement. To elaborate the intuition, suppose one firm's WtF increases due to reduced litigation costs. As a result, both settlement and non-settlement option values decrease: the firm's expected value from judgement (net of litigation costs) increases, lowering its abandonment incentive. This also diminishes the payoff from settling, reducing both firms' settlement option values since the settlement surplus is shared (see the discussion after Theorem 1). The effect on settlement feasibility then depends on the identity of the firms: increases in the leaving firm's WtF make settlement more likely, while increases in the remaining firm's WtF decrease settlement likelihood. Increases in the leaving firm's WtF reduce the non-settlement threshold, lowering both its own and the remaining firm's non-settlementment value. This reduction occurs partly because the remaining firm expects a longer wait before its rival leaves, and partly due to the non-settlement payoff being positive and linear in demand and thus has a lower value when the abandonment threshold is lower. Overall, the reduction in non-settlement option values exceeds that in settlement option, facilitating settlement. Conversely, when the remaining firm's WtF increases, the abandonment threshold of the leaving firm remains unchanged, so its non-settlement value is unaffected. Although the remaining firm's non-settlement option value declines (due to a lower abandonment payoff), this decrease is less than pronounced than the drop in settlement option value, as the abandonment threshold is lower than the settlement threshold and has a reduced likelihood of being triggered before judgement. Thus, the settlement option values are more sensitive to changes in the remaining firm's WtF, reducing the range of $\Phi$ for which settlement is feasible.

Our analysis also shows that when the incumbent has a higher willingness to finance (WtF) than the challenger (*C-exit*), making it the remaining firm, the range of competition levels where ex-ante settlement is feasible narrows and shifting to higher $\Phi$ values compared to the *I-withdraw* scenario. This narrowing occurs because the incumbent decides whether to initiate litigation or offer ex-ante settlement, and as the remaining firm, it also retains the valuable option to push the challenger out of the market (as described after Equation (3)). Since this forcing-out option holds considerable value but has to be given up once litigation begins, the incumbent raises its litigation threshold, opting to delay litigation to preserve this option. This delay reduces the likelihood of ex-ante settlement, which would otherwise occur just before litigation.

### 4.2 The Impact of $\sigma$, $p$, and $\lambda$ on Patent Dispute Outcomes

Figure 4 illustrates the representative impact of market uncertainty, the probability of the incumbent wins the lawsuit, and the expected duration of litigation process on patent dispute outcomes.

[Insert Figure 4 here.]

Panel (a) shows that market uncertainty, represented by demand volatility ($\sigma$), reduces the likelihood of patent disputes being resolved through settlement. As demand volatility increases, the minimum gain-to-loss ratio $\Phi$ required for settlement rises, and the settlement area shrinks. This effect arises due to differing sensitivities of option values to volatility. As is standard in the real options theory, increased volatility raises the value of options, making the non-settlement particularly valuable because it is farther from being exercised compared to the alternatives. Ex-post settlement option value follows, as it provides more flexibility in exercise timing, and finally ex-ante settlement. Consequently, both the colored area (indicating settlement of any type) and the blue area (indicating ex-ante settlement) shrink as market uncertainty rises. This finding is consistent with empirical research showing that higher litigation risks are associated with more volatile cash flows (Lowry and Shu, 2002).

Panel (b) demonstrates a negative impact of the incumbent's winning probability, or patent strength ($p$), on the likelihood of settlement: as $p$ increases, the settlement area (both ex-ante and ex-post) shrinks. This effect is driven by two forces. First, a higher $p$ boosts the incumbent's relative willingness to finance (WtF): as $p$ rises, the incumbent's expected judgement payoff increases, enhancing its willingness to continue litigation, while simultaneously reducing the challenger's WtF. As a result, the parameter range for *C-exit* expands relative to *I-withdraw*, shifting the equal-WtF boundary downward. Second, increases in $p$ affect the tradeoff between settlement and non-settlement options. When the challenger is the leaving firm in the attrition game (*C-exit*), higher $p$ increases the non-settlement option value for both firms, while decreasing their settlement values, making settlement less feasible. However, if the incumbent is the leaving firm (*I-withdraw*), an increase in $p$ reduces both settlement and non-settlement option values, resulting in an ambiguous effect on settlement feasibility in *I-withdraw*. Overall, a higher $p$ tends to reduce the likelihood of settlement in many realistic parameter ranges.[29] The finding contrasts with previous literature (Lemley, 2001; Lemley and Shapiro, 2005; Jeon, 2015), which suggests that a higher probability of patent validity encourages settlement by strengthening the incumbent's litigation position and thereby increasing its threat credibility, which should theoretically prompt the challenger to accept an ex-ante settlement. Our model, however, considers the value of non-settlement options and the compound feature of the *I-litigate* option–elements not accounted for in prior studies. This perspective reveals that as $p$

[29] If the challenger is the leaving firm (*C-exit*), a higher $p$ raises the exit threshold by reducing the challenger's expected judgment payoff, which increases its non-settlement option value and makes C-exit before judgment more likely. This, in turn, boosts the incumbent's non-settlement value. In contrast, if the incumbent is the leaving firm (*I-withdraw*), a higher $p$ lowers the withdrawal threshold by increasing the incumbent's judgment payoff, which reduces its abandonment option value and delays the challenger's opportunity to have the litigation threat lifted, thus decreasing the likelihood of *I-withdraw* before judgment. Overall, the increase in $p$ narrows the WtF gap for feasible settlement, especially by increasing the minimum $\Phi$ required for feasible settlement in *C-exit*, while having a minimal effect on *I-withdraw*.

increases, the value of the *I-litigate* option rises, leading the incumbent to demand a higher royalty rate in ex-ante settlement, which reduces firms' incentive to settle outside the legal system as $p$ grows.

# 5. A Model Extension and Discussions

## 5.1 Legal cost allocation rules: English Rule vs. American Rule

We extend the baseline model to examine how firms' relative WtF influences patent dispute outcomes under different legal cost allocation rules. In contrast to the *American Rule* where each firm typically bears its own legal costs (as in our baseline model), the *English Rule* requires the losing litigant to cover both parties' legal expenses. The English Rule primarily increases the incumbent's effective litigation costs relative to the challenger, due to the asymmetry in their financial capabilities to cover the opponent's costs in the event of an adverse ruling. This asymmetry provides the challenger with an advantage under the English Rule, as its litigation costs are partially or fully covered if it wins the case, while limited liability protects it from paying the incumbent's costs if it loses. Under the English Rule, the incumbent also faces the risk of liquidation if its value falls below the required amount to cover the challenger's litigation costs in the event of a loss. In such cases, the challenger assumes the role of a new monopoly (and earn $\pi_1^C x_t dt$ thereafter. The risk of liquidation further reduces the incumbent's WtF relative to the challenger.

We separate the analysis into two cases: Case A where *the incumbent remains a going-concern*, and Case B, where *the incumbent may liquidate*. The analysis for Case A parallels that in Section 3., with generalized expected litigation costs defined as $\bar{C}_l^I = C_l^I + \frac{C_l^C(1-p)\lambda}{r+\lambda}\mathbb{1}_{\text{E}}$ for the incumbent and $\bar{C}_l^C = C_l^C - \frac{C_l^C(1-p)\lambda}{r+\lambda}\mathbb{1}_{\text{E}}$ for the challenger, where $\mathbb{1}_{\text{E}}$ is the indicator variable for the English Rule. We define the generalized cost saving and relatively-cost-saving as $\Delta\bar{C}^j = \bar{C}_l^j - C_s^j$ and $\bar{\Gamma} \equiv \frac{\Delta\bar{C}^C}{\Delta\bar{C}^I}$, respectively. Using Theorem 1 and the obviously relationship $\bar{\Gamma}_{\text{English}} < \bar{\Gamma}_{\text{American}}$, we derive Theorem 4.[30]

**Theorem 4.** *Ceteris paribus, if the incumbent remains a going-concern under the English Rule, the ex-post settlement royalty rate $\theta_p$ is lower than under the American Rule, i.e., $\theta_{p,\text{English}} < \theta_{p,\text{American}}$.*

Case B, which applies only under the English Rule, introduces the incumbent's liquidation threshold, defined as $\bar{x} = \frac{C_l^C(r-\mu)}{\pi_2^I}$. This threshold represents the demand level at which the incumbent's firm value, in the event of losing the lawsuit, equals the challenger's expected litigation costs (i.e., $\frac{\pi_2^I\bar{x}}{r-\mu} = C_l^C$). Due to the complexity of Case B, the detailed model derivation is provided in Appendix A13.

[30]Specifically, $\Delta\bar{C}^C(\mathbb{1}_{\text{E}} = 1) < \Delta\bar{C}^C(\mathbb{1}_{\text{E}} = 0)$ and $\Delta\bar{C}^I(\mathbb{1}_{\text{E}} = 1) > \Delta\bar{C}^I(\mathbb{1}_{\text{E}} = 0)$, leading to $\bar{\Gamma}(\mathbb{1}_{\text{E}} = 1) < \bar{\Gamma}(\mathbb{1}_{\text{E}} = 0)$.

**Quantitative analysis** Figure 5 illustrates potential patent dispute outcomes under the English Rule, with each panel depicting variations in $\sigma$, $p$ or $\lambda$, and Plot 5 (a).ii serving as the benchmark.

[Insert Figure 5 here.]

Most baseline findings from the American Rule hold under the English Rule, indicating consistent effects of competition and market uncertainty on patent dispute outcomes across legal systems. Under both rules, firms settle only if the gain-to-loss ratio $\Phi$ is sufficiently high, a wider gaps in their relative WtF and market uncertainty dampening settlement incentives.

Panel (b) reveals a positive impact of $p$ on the likelihood of settlement under the English Rule, in contrast to the negative effect observed under the American Rule. This divergence arsies from two forces related to firms' relative WtF. First, under the English Rule, the incumbent's effective litigation costs increase while the challenger's decrease, which reduces the incumbent's relative WtF. As a result, the English Rule often shifts the identity of the remaining firm in litigation from the incumbent to the challenger, expanding the *I-withdraw* area in Figure 5 Plot a.(ii), compared to the American rule in Figure 3. This shift amplifies the influence of $p$ $p$ in *I-withdraw* relative to *C-exit*. Second, beyond the effects of $p$ under the American Rule (see Page 22 in Section 4.2), the English Rule introduces an additional effect: as $p$ increases, the incumbent's potential liability to pay the challenger's litigation costs decreases, effectively reducing its litigation costs and increasing those of the challenger. This change raises the incumbent's WtF while lowers the challenger's, narrowing the WtF gap in *I-withdraw* (where the challenger has a higher WtF) and making settlement more feasible. Our analysis builds on previous work examining the impact of cost allocation rules on settlement and litigation outcomes (Bebchuk, 1984; Meurer, 1989; Bebchuk, 1996; Aoki and Hu, 1999b) by considering firms' WtF.

### 5.2 Discussion on Model Assumptions

Our baseline model makes several simplifying assumptions, which we argue do not drive the main results. One assumption is that firms' cash flows – particularly the challenger's–are based solely on selling products associated with the patented technology, without the use of debt or external litigation financing (Antill et al., 2024). Another assumption is the absence of damage rulings in cases of convicted infringement, as modeled in Bebchuk (1984) and Lanjouw and Lerner (1998).[31] Relaxing either assumption individually does not alter the baseline findings, because the challenger's optimal decision would remain unchanged if it had additional cash flows unrelated to profits from the patented technology. Similarly, if an adverse ruling were issued, the challenger's value would still be insufficient to cover damages without supplementary cash flows. Only when both assumptions are relaxed simultaneously might our results

[31] This assumption is justified by recognizing that patent litigation differs from other types of litigation: a court ruling directly impacts firms' future sales, which often have much larger stakes than any potential trial award, including damages or compensation for lost profits.

differ: in that scenario, the incumbent's WtF would increase relative to the challenger's, as the challenger could then partially or fully cover damages. However, we expect this effect to be the opposite to the impact of the English Rule discussed in Section 5.1, which increases the challenger's relative WtF.

### 5.3 Model Predictions on Litigation and Settlement Rates

Our model predicts patent litigation rates – the probability of litigation initiation within $T$ years after the challenger's market entry–and settlement rates, defined as the probability that firms settle by Year $T$, based on a set of parameters and the demand level at which the challenger enters the market with infringement concerns. The online appendix provides these rates as functions of model parameters. Our analysis shows that litigation rates increase with intensified competition from the challenger, greater market volatility, and shorter expected litigation duration. Meanwhile, settlement rates decline with higher market uncertainty and larger WtF gaps between firms, though they increase with litigation duration. With stronger patent protection, litigation rates decrease and settlement rates increase under the English Rule, whereas these effects vary under the American Rule. Our model serves as a valuable tool for understanding cross-country differences in the litigation and settlement patterns (Hughes and Snyder, 1995; Helmers and McDonagh, 2013; Cremers et al., 2017). Additionally, a patent with broader scope implies that the patented technology can be applied across more diverse product lines, making alleged infringement more likely to correspond with a higher $\Phi$ value, as the incumbent faces less profit erosion from the new, distinct products. Our model predicts a higher likelihood for settlement in these cases. In contrast, alleged infringement on a narrow-scoped patent is more likely to lead to litigation that proceeds to judgement.

### 5.4 Model implications on post-dispute market structure

Our model has implications for market structure following a patent dispute: IP-based competition can, counter-intuitively, result in sustained monopoly and higher market concentration. The model suggests that intense competition from a challenger offering similar IP-based products to those of the patent-owning incumbent is more likely to trigger litigation that proceeds to judgement. Consequently, the post-dispute market structure is more likely to revert back from duopoly to monopoly, as the incumbent regains its dominant position either through a favorable court-ruling, or through the challenger's market exit, with or without actual litigation (corresponding to the C-exit and forcing-out options). On the contrary, low competition levels encourage firms to settle patent dispute, accommodating the new entrant. Our study connects with the *Hypothesis of Intangible Assets*, which attributes rising market concentration in the U.S. (Grullon, Larkin, and Michaely, 2019; Kwon, Ma, and Zimmermann, 2024) to the growth of intangible assets. Prior research on this hypothesis suggests that intellectual property creates entry barrier that confer market power, reduce competition, and increase concentration. We offer a new perspective, demonstrating that high competition levels in innovation-driven markets can paradoxically lead to increased market concentration, especially when considering the imperfect nature of IP protection and the strategic interaction between firms in patent dispute.

## 6. Concluding Remarks

In this paper, we analyze the strategic interactions between two firms engaged in a patent dispute, and examine the likely outcomes of the resolution. Using a real options approach on a sequential game with time-varying market demand, we find that the competitive pressure introduced by the challenger's entry significantly affect settlement feasibility, terms, and timing. Our theoretical contribution includes showing that firms do not use settlement to resolve their IP-based dispute if the gain-to-loss ratio– a new inverse measure of competition introduced in our framework–is low. When this ratio is high, settlement not only becomes feasible but also motivates early resolution without resorting to litigation. However, wider gaps in firms' relative willingness to finance litigation and higher product market volatility reduces settlement feasibility and increases the likelihood of litigation. An extended model demonstrates that the English Rule (lose pays) shifts the relative WtF from the incumbent to the challenger, resulting in a lower royalty rate in ex-post settlements compared to the American Rule (each party pays).

Our model generates testable implication regarding litigation rates, settlement rates, and post-dispute market structure, given different characteristics of the product market, patented technology, and legal environment. Several directions for future research appear promising. One area of interest is firms' market entry decisions and their innovation incentives. Another potential extension is to study industry equilibrium with multiple entrants and to examine the welfare implications of patent litigation.

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## Figure 1: Timeline of events

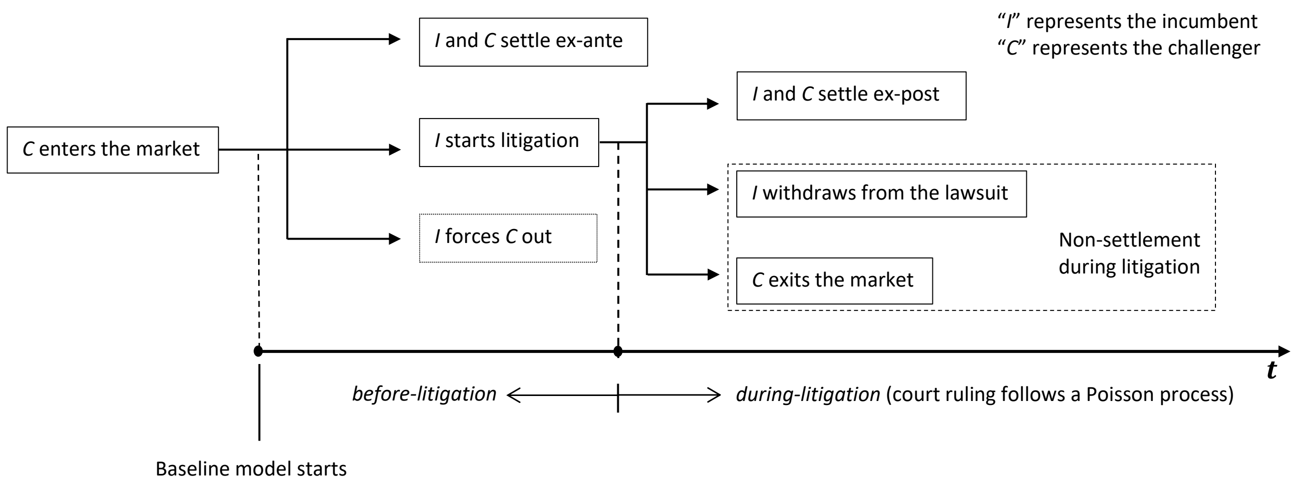


**Figure 2: An example of a realized demand path and the strategies**

This figure illustrates a scenario of the likely outcome being the incumbent litigates and the firms then settle ex-post. The parameter values are set at the benchmark in Table 1. The blue line is a realized demand path $x_t$ that starts at $x_0 = 1$ and follows a stochastic process specified in Equation (1). The litigation threshold is $x_l = 1.76$ and the ex-post settlement threshold is $x_p = 0.72$. The red dot represents the time of litigation, which is when the demand reaches $x_l$ from below for the first time. The green dot represents the ex-post settlement time, which is when the demand reaches $x_p$ from above for the first time, assuming the court has not yet ruled then.

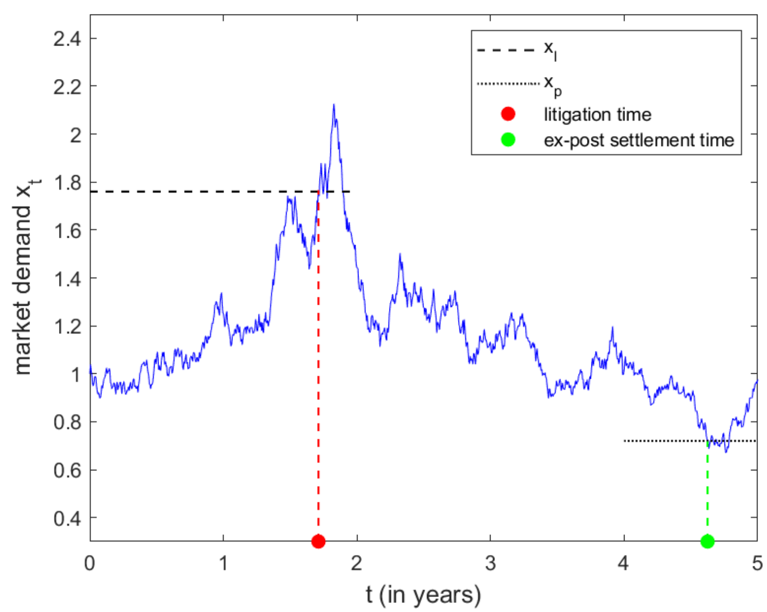


**Figure 3: The likely outcome of a patent dispute**

The graph describes the likely outcome of a patent dispute after the challenger's market entry. The white area marks *I-litigate* followed by *C-exit* ("C-e") as $x \leq x_e$ or *I-withdraw* ("I-w") as $x \leq x_w$. The green area marks *I-litigate* followed by *ex-post settlement* as $x \leq x_p$. The blue area marks *ex-ante settlement* as $x \geq x_a$. The black dashed line represents situations in which firms have equal willingness to finance for the litigation. $\Phi = \frac{\Delta\pi^C}{|\Delta\pi^I|}$ and $\Gamma = \frac{\Delta C^C}{\Delta C^I}$. We vary $\Phi$ and $\Gamma$ by changing $\pi_2^I$ and $c_l^I$), respectively. Table 1 lists the benchmark parameter values.

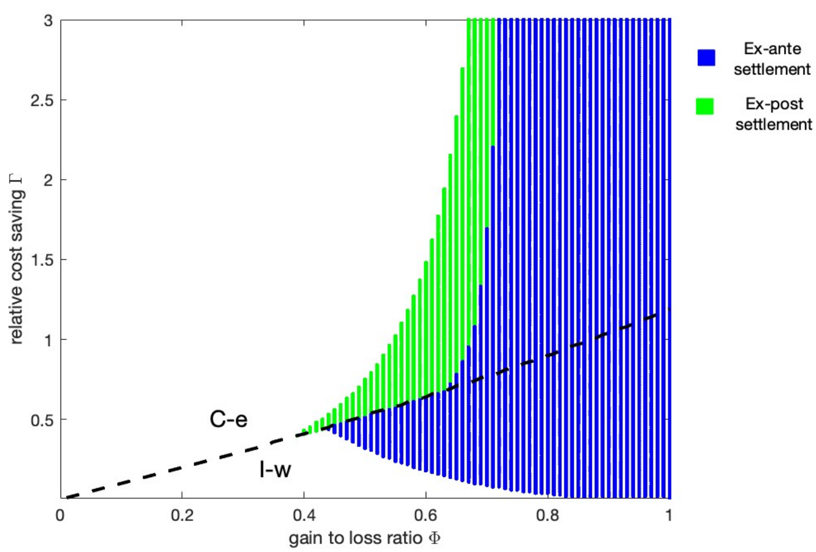
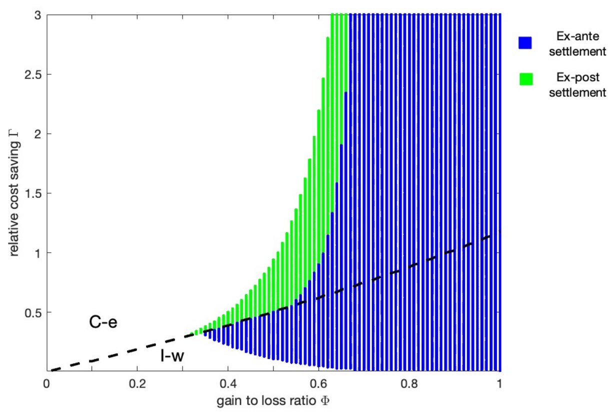
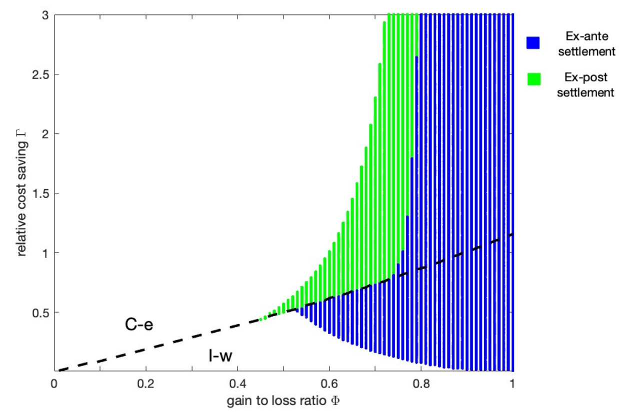


**Figure 4: The effect of $\sigma$, $p$, or $\lambda$ on the likely outcome of a patent dispute**

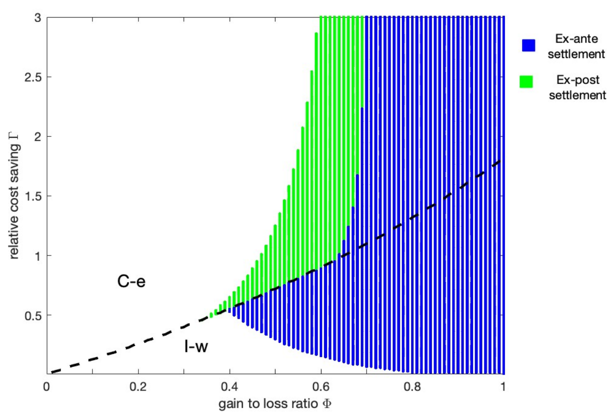
The graphs illustrate the likely litigation outcomes at the benchmark (Table 1), as we change $\sigma$ in Panel (a), $p$ in Panel (b), or $\lambda$ in Panel (c). The white area marks *I-litigate* followed by *C-exit* or *I-withdraw*, the green area marks the *ex-post settlement* and the blue area marks *ex-ante settlement*. The black dashed line represents the situations in which firms have equal willingness to finance for the litigation. $\Phi = \frac{\Delta\pi^C}{|\Delta\pi^I|}$, $\Gamma = \frac{\Delta C^C}{\Delta C^I}$. Section 4.2 discusses the details.



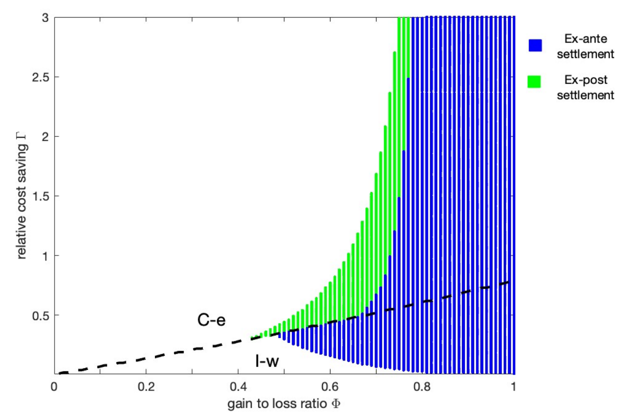

(a).i. low $\sigma$ ($\sigma = 0.2$)



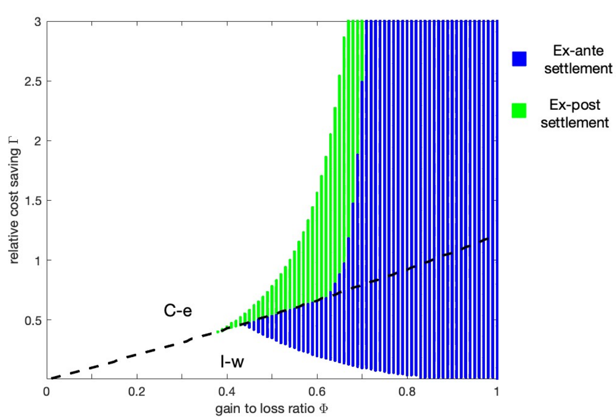

(a).ii. high $\sigma$ ($\sigma = 0.4$)



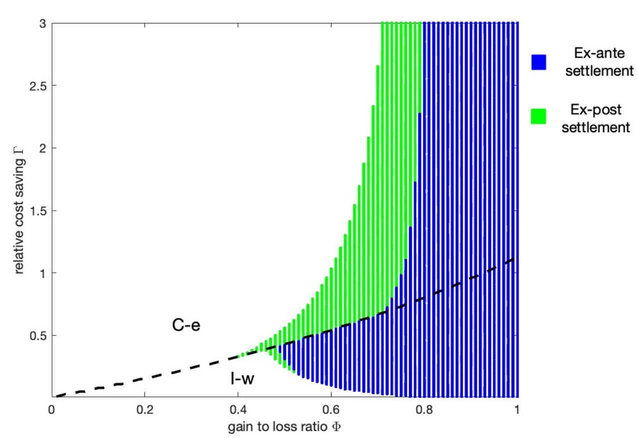

(b).i. low p ($p = 0.4$)




(b).ii. high p ($p = 0.6$)



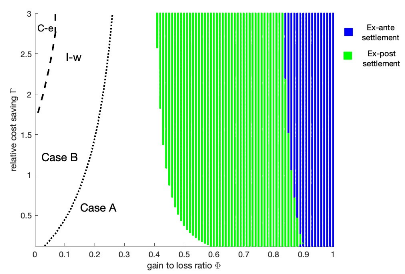

(c).i. low $\lambda$ ($\lambda = \frac{1}{3.5}$)



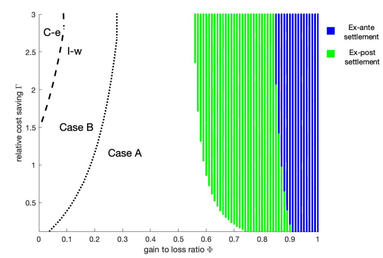

(c).ii. high $\lambda$ ($\lambda = \frac{1}{1.5}$)

**Figure 5: Likely outcomes under the English Rule**

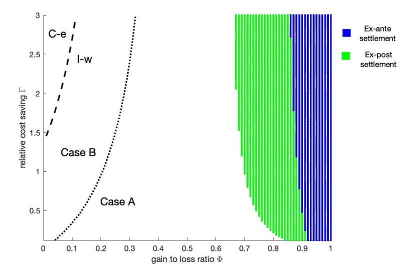
This graph illustrates the likely patent dispute outcomes under the English Rule around the benchmark (Table 1), as we change $\sigma$ in Panel (a), or $p$ in Panel (b). The white area marks *I-litigate* followed by *C-exit* or *I-withdraw*, the green area marks *ex-post settlement* and the blue area marks *ex-ante settlement*. The black dashed line marks the boundary of $s_{ns}$ = (*I-withdraw*) vs. $s_{ns}$ = (*C-exit*). The dotted line separates Case A (the incumbent remains a going-concern) and Case B (the incumbent may liquidate). $\Phi = \frac{\Delta\pi^C}{|\Delta\pi^I|}$, $\Gamma = \frac{\Delta C^C}{\Delta C^I}$. Section 5.1 discusses the details.



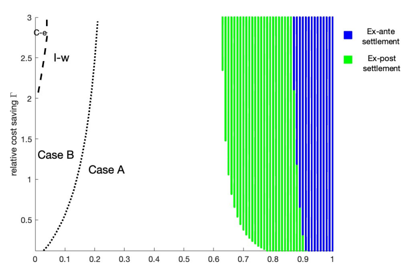

(a).i low $\sigma$ ($\sigma = 0.2$)



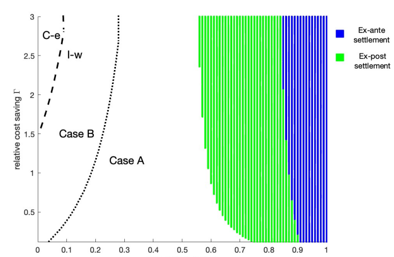

(a).ii baseline $\sigma$ ($\sigma = 0.3$)



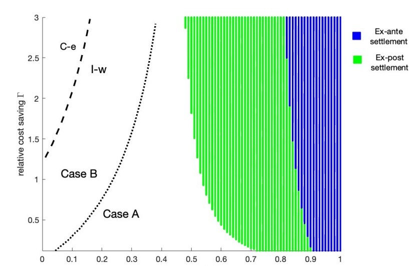

(a).iii high $\sigma$ ($\sigma = 0.4$)



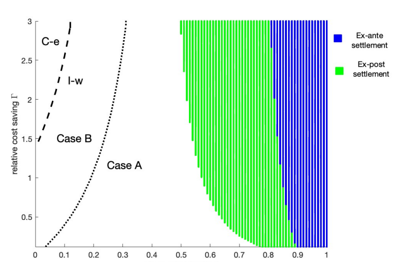

(b).i low p ($p = 0.4$)



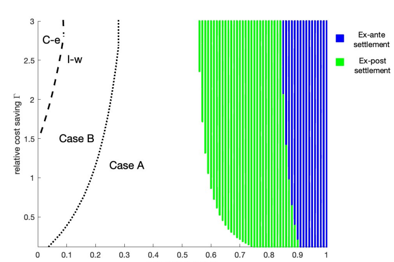

(b).ii baseline p ($p = 0.5$)



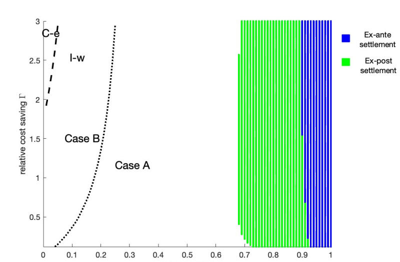

(b).iii high p ($p = 0.6$)




iii.(a) low $\lambda$ ($\lambda = \frac{1}{3.5}$)




iii.(b) baseline $\lambda$ ($\lambda = \frac{1}{2.5}$)




iii.(c) high $\lambda$ ($\lambda = \frac{1}{1.5}$)

**Table 1: List of parameters and their benchmark values**

| Parameters | Symbol and value |
|---|---|
| *Basics* | |
| Risk-free rate | $r = 0.05$ |
| Arrival rate of court ruling | $\lambda = \frac{1}{2.5}$ |
| Arrival rate of R&D success | $\epsilon = \frac{1}{5}$ |
| Probability of patent validity | $p = 0.5$ |
| Growth rate/volatility of the demand shock | $\mu = 0.02, \sigma = 0.3$ |
| Profit multipliers (profit = $\pi x$) | $\pi_1 = 1.2, \pi_2^I = 0.7, \pi_2^C = 0.3$ |
| Flow(Expected total) litigation cost litigation cost | $c_l^j = 1$ ($C_l^j \equiv \frac{c^j}{r+\lambda} = 2.22$) |
| One-time settlement costs | $C_s^j = 0.5$ |
| | |
| *Ratios* | |
| Gain-to-loss ratio | $\Phi = \frac{\Delta\pi^C}{\lvert\Delta\pi^I\rvert} = \frac{\pi_2^C}{\pi_1 - \pi_2^I} = 0.6$ |
| Relative-cost-saving | $\Gamma = \frac{\Delta C^C}{\Delta C^I} = \frac{\frac{c_l^C}{r+\lambda} - C_s^C}{\frac{c_l^I}{r+\lambda} - C_s^I} = 1$ |
| Relative litigation cost | $\Lambda = \frac{C_l^C}{C_l^I} = 1$ |
| *Other Greeks* | |
| Deferred perpetual factor | $\delta = \frac{1}{r-\mu} - \frac{1}{r+\lambda-\mu} = 31.01$ |
| Equivalent perpetual rate | $\omega = \delta(r - \mu) = 0.93$ |
| | |
| *Thresholds on demand x* | |
| Litigation/ex-ante/expost settlement thresholds | $x_l/x_a = 1.76, \quad x_p = 0.72$ |
| C-exit/I-withdraw thresholds | $x_e = 0.31, \quad x_w = 0.21$ |
| Reservation threshold for C/I | $x_C = 0.17, \quad x_I = 0.12$ |
| (extension) Liquidation threshold for I | $\bar{x} = \frac{C_l^C(r-\mu)}{\pi_2^I} = 0.095$ |

**Table 2: Notations for model solutions**

| **Notation** | **Interpretation (I - incumbent; C - challenger)** |
|---|---|
| *Thresholds* | |
| $x_l$ / $x_a$ | I-litigate / ex-ante settlement threshold |
| $x_e$ / $x_w$/$x_p$ | C-exit / I-withdraw / ex-post settlement threshold |
| *Royalty rates* | |
| $\theta_a$ / $\theta_p$ | ex-ante / ex-post settlement royalty rate |
| *Value functions* | |
| $V_0$ | before-litigation firm value that includes option values |
| $V_l$ / $V_a$ | with I-litigate / ex-ante settlement option |
| $V$ | during-litigation firm value which includes option values |
| $V_p$ / $V_{ns}$ | with /without the ex-post settlement option |
| $V_e$ / $V_w$ | with C-exit / I-withdraw option |
| $\hat{V}$ | payoff after option exercising |
| *Other* | |
| $\Delta\pi^j$ | change in $\pi$ due to C's entry ($\Delta\pi^I = \pi_2^I - \pi_1^I, \Delta\pi^C = \pi_2^C$) |
| $\Delta\pi_p^j$ | change in $\pi$ due to ex-post settlement ($\Delta\pi_p^I = \theta_p\pi_2^C, \Delta\pi_p^C = -\theta_p\pi_2^C$) |
| $\Delta\pi_a^j$ | change in $\pi$ due to ex-ante settlement ($\Delta\pi_a^I = \theta_a\pi_2^C, \Delta\pi_p^C = -\theta_a\pi_2^C$) |
| $\Delta C^j$ | saved litigation cost by settling ($\Delta C^j = C_l^j - C_s^j > 0$) |
| $\Pi^j$ | used in extension ($\Pi^I = p\pi_1^I, \Pi^C = (1-p)\pi_1^C$) |

# Appendices

## Appendix A Proofs

### A1 Proof of Proposition 1

*Proof.* The general solution of the ODE Equation (7) is as follows:

$$V^j(x) = A^j x^{\alpha_\lambda} + B^j x^{\beta_\lambda} + V_p^j,$$

where

$$\beta_\lambda = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(r+\lambda)}{\sigma^2}} < 0, \quad \alpha_\lambda = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(r+\lambda)}{\sigma^2}} > 1$$

are the solutions to $\frac{1}{2}x(x-1)\sigma^2 + \mu x - (r+\lambda) = 0$. As mentioned in the main text, a boundary condition is

$$\lim_{x\to\infty} V^j(x) = \Big(\frac{\pi_2^i}{r-\mu} - p\delta\Delta\pi^j\Big)x - C_l^j.$$

Thus $A^j = 0$ and $V^j(x) = B^j x^{\beta_\lambda} + V_p^j$. A linear particular solution for $V_p^j(x)$ is $V_p^j = \Big(\frac{\pi_2^i}{r-\mu} - p\delta\Delta\pi^j\Big)x - C_l^j$, from which we obtain Equation (8) in the proposition. □

### A2 Proof of Corollaries 1 and 2

*Proof.* $\boldsymbol{s_{ns}}$ **=** {I-withdraw}: The incumbent maximizes its firm value by choosing $x_w$. We apply the value-matching conditions (VM) on both firms' value functions during litigation with respect to their payoff upon I-withdraw, and the smooth-pasting condition (SP) on the incumbent's value function at $x_w$. That is, for $j \in \{I, C\}$:

$$\text{VM:} \quad V_w^j(x_w) = \hat{V}_w^j(x_w) \quad \Rightarrow \Big(\frac{\pi_2^i}{r-\mu} - p\delta\Delta\pi^j\Big)x_w - C_l^j + B_w^i x_w^{\beta_\lambda} = \frac{\pi_2^j}{r-\mu}x_w, \quad j \in \{I, C\}.$$

$$\text{SP:} \quad \frac{\partial V_w^I(x)}{\partial x}\bigg|_{x_w} = \frac{\partial \hat{V}_w^I(x)}{\partial x}\bigg|_{x_w} \quad \Rightarrow \frac{\pi_2^I}{r-\mu} - p\delta\Delta\pi^I + \beta_\lambda B_w^I x_w^{\beta_\lambda - 1} = \frac{\pi_2^I}{r-\mu}.$$

We solve the three unknowns $B_w^I$, $B_w^C$ and $x_w$, as expressed in Corollary 1 from the above three equations.

$\boldsymbol{s_{ns}}$ **=** {C-exit}: The challenger maximizes its firm value during litigation by choosing $x_e$. Similar to *I-withdraw*, we use the following condition to get the expressions of $B_e^I$, $B_e^C$ and $x_e$ in Corollary 2.

$$\text{VM:} \quad V_e^j(x_e) = \hat{V}_e^j(x_e) \quad \Rightarrow \Big(\frac{\pi_2^i}{r-\mu} - p\delta\Delta\pi^j\Big)x_e - C_l^j + B_e^i x_e^{\beta_\lambda} = \frac{\pi_2^j - \Delta\pi^j}{r-\mu}x_e, \quad j \in \{I, C\}.$$

$$\text{SP:} \quad \frac{\partial V_e^C(x)}{\partial x}\bigg|_{x_e} = \frac{\partial \hat{V}_e^C(x)}{\partial x}\bigg|_{x_e} \quad \Rightarrow \frac{\pi_2^C}{r-\mu} - p\delta\Delta\pi^C + \beta_\lambda B_e^C x_e^{\beta_\lambda - 1} = \frac{\pi_2^C}{r-\mu}.$$

□

## A3 Proofs of Lemmas 1 and 2

Both firms act strategically knowing the other firm also has the option to leave the litigation, so a direct comparison of the two firms' leaving thresholds $x_w$ vs. $x_e$ from Corollaries 1 and 2 is not appropriate to determine $s_{ns}$. Instead, we compare the two firms' *reservation thresholds* (Lambrecht, 2001) during litigation, that is, the demand level for which a firm is "indifferent between leaving first at their optimal exit/withdrawal threshold and waiting until the rival leaves". The incumbent's reservation threshold $x_I$, is the demand level at which its during-litigation firm value including the option to withdraw at $x_w$ equals its firm value if the challenger exits the market earlier, in which case the incumbent would lose its withdrawal option but restore its monopoly. Likewise, the challenger's reservation threshold $x_C$ is the demand level at which its during-litigation firm value including its exit option at $x_e$ equals its firm value if the incumbent withdraws earlier and the challenger loses its exit option but remains in a duopoly. As the text after Lemma 1 argues, $s_{ns} = \{\text{I-withdraw}\}$ if $x_I > x_C$ and $s_{ns} = \{\text{C-exit}\}$ otherwise.

*Proof.* For the incumbent, the condition of its reservation threshold $x_I$, with $\omega = p\delta(r-\mu) < 1$ defined, can be written and rearranged as

$$V_w^I(x = x_I; x_w) = \hat{V}_e^I(x = x_I) \;\Rightarrow\; \Big(\frac{\pi_2^I}{r-\mu} - p\delta\Delta\pi^I\Big)x_I - C_l^I + (C_l^I + p\delta\Delta\pi^I x_w)\Big(\frac{x_I}{x_w}\Big)^{\beta_\lambda} = \frac{\pi_1 x_I}{r-\mu}$$

$$\Rightarrow\; \Delta\pi^I(\frac{1}{r-\mu} - p\delta)x_I + \frac{C_l^I}{1-\beta_\lambda}\Big(\frac{x_I}{x_w}\Big)^{\beta_\lambda} - C_l^I = 0$$

Divide by $C_l^I$, replace $x_I$ by $x_I \cdot \dfrac{x_w}{x_w}$ $\Rightarrow$ $z_1 = \dfrac{x_I}{x_w}$ satisfies the following condition, as shown in the lemma:

$$y_1(z_1) = 1 - \frac{1}{1-\beta_\lambda}z_1^{\beta_\lambda} + \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{(1-p\omega)}{p\omega}z_1 = 0. \tag{A.1}$$

Note that $\beta_\lambda < 0$, $p\omega \in (0, p)$ and $\frac{(1-p\omega)}{p\omega} > 0$, thus the first-order derivative of $y_1(z_1)$ with respect to $z_1$ is negative, and the solution of $y_1(z_1) = 0$ is unique, which means there is one and only one solution of $x_I$. For the challenger, the condition of its reservation threshold $x_C$ can be written and rearranged as

$$V_e^C(x = x_C; x_e) = \hat{V}_w^C(x = x_C) \;\Rightarrow\; \Big(\frac{\pi_2^C}{r-\mu} - p\delta\Delta\pi^C\Big)x_C - C_l^C + \Big(C_l^C + \Delta\pi^C x_e(p\delta - \frac{1}{r-\mu})\Big)\Big(\frac{x_C}{x_e}\Big)^{\beta_\lambda} = \frac{\pi_2 x_C}{r-\mu}$$

$$\Rightarrow\; -\Delta\pi^C p\delta x_C + \frac{C_l^C}{1-\beta_\lambda}\Big(\frac{x_C}{x_e}\Big)^{\beta_\lambda} - C_l^C = 0$$

Divide by $C_l^C$, replace $x_C$ by $x_C \cdot \dfrac{x_e}{x_e}$ $\Rightarrow$ $z_2 = \dfrac{x_C}{x_e}$ is the unique solution of the following condition:

$$y_2(z_2) = 1 - \frac{1}{1-\beta_\lambda}z_2^{\beta_\lambda} + \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{p\omega}{(1-p\omega)}z_2 = 0. \tag{A.2}$$

Equations (A.1) and (A.2) are equivalent to the conditions in Lemma 1. They can be written more generally, that is, $z_1$ and $z_2$ are the solutions to $y(z; M) = 0$ where

$$y = 1 - \frac{1}{1-\beta_\lambda} z^{\beta_\lambda} + \frac{\beta_\lambda}{\beta_\lambda - 1} M \cdot z, \text{ with } M = \begin{cases} \dfrac{1-p\omega}{p\omega}, & \text{for } z_1 \\ \dfrac{p\omega}{1-p\omega}, & \text{for } z_2 \end{cases} \tag{A.3}$$

Because $\beta_\lambda < 0$, $\dfrac{\partial y}{\partial z} = \frac{\beta_\lambda}{\beta_\lambda - 1} z^{\beta_\lambda - 1} + \frac{\beta_\lambda}{\beta_\lambda - 1} M > 0$, $\lim_{z \to 0} y(z) = -\infty$, and $\lim_{z\to 1} y(z) > 0$, a unique solution of $z < 1$ is guaranteed. Thus $\frac{x_I}{x_w} < 1$ and $\frac{x_C}{x_e} < 1$, which implies $x_I < x_w$ and $x_C < x_e$. Apply the implicit function theorem on Equation (A.3), we get

$$\frac{\partial z}{\partial M} = -\frac{\dfrac{\partial y}{\partial M}}{\dfrac{\partial y}{\partial z}} = -\frac{z}{z^{\beta_\lambda - 1} + M} < 0.$$

Next, we compare $z_1$ and $z_2$ in order to compare $x_C$ vs. $x_I$. There are three possibilities:

1. If $p\omega > 1/2$, then $M(z_1) < 1 < M(z_2)$. This means $\frac{x_I}{x_w} > \frac{x_C}{x_e} \Rightarrow \frac{x_I}{x_C} > \frac{x_w}{x_e} = \frac{1-p\omega}{p\omega}\frac{\Phi}{\Lambda}$. With an added condition of $\frac{1-p\omega}{p\omega}\frac{\Phi}{\Lambda} > 1$, we can have $x_I > x_C$, and $s_{ns} = \{\text{I-withdraw}\}$.
2. If $p\omega < 1/2$, then $M(z_1) > 1 > M(z_2)$. This means $\frac{x_I}{x_w} < \frac{x_C}{x_e} \Rightarrow \frac{x_I}{x_C} < \frac{x_w}{x_e}$. With an added condition of $\frac{1-p\omega}{p\omega}\frac{\Phi}{\Lambda} < 1$, we can have $x_I < x_C$, and $s_{ns} = \{\text{C-exit}\}$.
3. If $p\omega = 1/2$, then $M(z_1) = M(z_2) = 1$. This means $\frac{x_I}{x_w} = \frac{x_C}{x_e} \Rightarrow \frac{x_I}{x_C} = \frac{x_w}{x_e}$.
   (a) If $\frac{\Phi}{\Lambda} < 1$, then $x_I < x_C$, and $s_{ns} = \{\text{C-exit}\}$.
   (b) If $\frac{\Phi}{\Lambda} > 1$, then $x_I > x_C$, and $s_{ns} = \{\text{I-withdraw}\}$.
   (c) If $\frac{\Phi}{\Lambda} = 1$, then $x_I = x_C$.

□

## A4 Proof of Corollary 3

*Proof.* Given any proposed royalty rate $\theta_p$, the challenger chooses the settlement threshold $x_p$ to maximize its value with the settlement option. Thus, we use the value-matching conditions on both firms' values and the smooth-pasting condition on the challenger's firm value at $x_p$. That is, for $j \in \{I, C\}$:

$$\text{VM:} \quad V_p^j(x_p) = \hat{V}_p^j(x_p) - C_s^j \quad \Rightarrow \left(\frac{\pi_2^i}{r-\mu} - p\delta\Delta\pi^j\right) x_p - C_l^j + B_p^i x_p^{\beta_\lambda} = \frac{\pi_2^j + \Delta\pi_p^j}{r-\mu} x_p - C_s^j \tag{A.4}$$

$$\text{SP:} \quad \left.\frac{\partial V_p^C(x)}{\partial x}\right|_{x_p} = \left.\frac{\partial(\hat{V}_p^C(x) - C_s^C)}{\partial x}\right|_{x_p} \quad \Rightarrow \frac{\pi_2^C}{r-\mu} - p\delta\Delta\pi^C + \beta_\lambda B_p^I x_p^{\beta_\lambda - 1} = \frac{\pi_2^C + \Delta\pi_p^C}{r-\mu}. \tag{A.5}$$

We can then solve $x_p$ and the arbitrary constants $B_p^j$ as shown in the corollary. □

## A5 Proof of Theorem 1

*Proof.* From Corollary 3, the first-order derivative $\dfrac{dx_p}{d\theta_p^*} = \dfrac{-x_p}{\theta_p^* - p\delta(r-\mu)}$. The first-order condition for the incumbent's optimal royalty rate is

$$\frac{dV_p^I}{d\theta_p^*} = \frac{dB_p^I}{d\theta_p^*} = \frac{\partial B_p^I}{\partial \theta_p^*} + \frac{\partial B_p^I}{\partial x_p}\frac{dx_p}{d\theta_p^*} = 0.$$

That is, $\beta_\lambda(\Delta C^I + \frac{\pi_2^C \theta_p}{r-\mu}x_p) - p\delta x_p(\Delta\pi^C + \Delta\pi^I(1-\beta_\lambda)) = 0$. For the simple case of $x_l > x_p$ (the litigation threshold is higher than the ex-post settlement threshold), the challenger optimally accepts the ex-post settlement offer at $x_p$, and the royalty rate choosen by the incumbent is the one that maximizes $V_p^I$. Thus,

$$\theta_p^* = \frac{p\delta(r-\mu)[\Delta C^I(\beta_\lambda - 1) + \Delta C^C(1 + \frac{\beta_\lambda - 1}{\Phi})]}{\Delta C^I(\beta_\lambda - 1) + \beta_\lambda \Delta C^C}.$$

Simplification leads to the expression in the theorem. $\theta_p^* > 0$ is guaranteed under $\Phi < 1$ and $\theta_p^* < 1$ leads to the condition of the model parameters: $p\omega[1 + g(\Gamma)(\frac{1}{\Phi} - 1)] < 1$.

We can plug the expression of $\theta_p = \theta_p^*$ in $x_p(\theta_p) = \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{\Delta C^C}{(-p\delta + \frac{\theta_p}{r-\mu})\Delta\pi^C}$ (implied by Corollary 3), and get the expression of $x_p^*$ in the theorem. The sufficient conditions for $x_p > x_{ns}$ are (i) $x_p > x_e$ and (ii) $x_p > x_w$. Using the expressions of $x_p$ from the theorem and $x_e$ from Corollary 2, $x_p > x_e$ is equivalent to $\frac{\Delta C^C}{g(\Gamma)} > \frac{C_l^C p\omega}{1-p\omega}(\frac{1}{\Phi} - 1)$, which is further equivalent to $\frac{\Delta C^C}{C_l^C} > \frac{\theta_p - p\omega}{1-p\omega}$. If we have negligible settlement cost, thus $\frac{\Delta C^C}{C_l^C} \longrightarrow 1$, then the condition is guaranteed to be satisfied. Similarly, $x_p > x_w$ is equivalent to $\frac{\Delta C^C}{C_l^I} > g(\Gamma)(1-\Phi)$ or $\frac{\Delta C^I}{C_l^I}[\Gamma \times \frac{\beta_\lambda}{\beta_\lambda} + 1] > 1 - \Phi$, and if we have negligible settlement cost, which indicates $\frac{\Delta C^I}{C_l^I} \longrightarrow 1$, then it is always satisfied. □

## A6 Proof of Corollary 4

*Proof.* $\frac{\partial \beta_\lambda}{\partial \sigma^2} = \mu\sigma^{-4} \times \Big(1 - (\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{r+\lambda}{\mu})\big((\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(r+\lambda)}{\sigma^2}\big)^{-1/2}\Big) > 0$. $\frac{\partial \beta_\lambda}{\partial \lambda} = -\frac{\big((\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(r+\lambda)}{\sigma^2}\big)^{-1/2}}{\sigma^2} < 0$. $\frac{\partial g(\Gamma)}{\partial \Gamma} = (\frac{\beta_\lambda}{\beta_\lambda}\Gamma + 1)^{-2} > 0$, $\frac{\partial g(\Gamma)}{\partial \beta_\lambda} = (\beta_\lambda + \frac{\beta_\lambda - 1}{\Gamma})^{-2} > 0$. Using the expression of $\theta_p^*$ from Theorem 1 with $\Phi < 1$ and $g(\Gamma) > 0$:

$$\frac{\partial \theta_p^*}{\partial \Phi} = -\frac{p\omega g(\Gamma)}{\Phi^2} < 0, \quad \frac{\partial \theta_p^*}{\partial p} = \omega\Big(1 + g(\Gamma)(\frac{1}{\Phi} - 1)\Big) > 0,$$

$$\frac{\partial \theta_p^*}{\partial \Gamma} = \frac{\partial \theta_p^*}{\partial g(\Gamma)} \times \frac{\partial g(\Gamma)}{\partial \Gamma} = p\omega(\frac{1}{\Phi} - 1) \times \big(\frac{\beta_\lambda \Gamma}{\beta_\lambda - 1} + 1\big)^{-2} > 0,$$

$$\frac{\partial \theta_p^*}{\partial \sigma^2} = \frac{\partial \theta_p^*}{\partial g(\Gamma)} \times \frac{\partial g(\Gamma)}{\partial \beta_\lambda} \times \frac{\partial \beta_\lambda}{\partial \sigma^2} = p\omega(\frac{1}{\Phi} - 1)\frac{\partial g(\Gamma)}{\partial \beta_\lambda} \times \frac{\partial \beta_\lambda}{\partial \sigma^2} > 0.$$

We can write $x_p^*$ in Theorem 1 as $x_p^* = \frac{\beta_\lambda}{\beta_\lambda-1}\cdot\frac{-\Delta C^C}{p\delta g(\Gamma)\Delta\pi} = \frac{\beta_\lambda}{\beta_\lambda-1}\cdot\frac{\Delta C^C}{-p\delta\Delta\pi}(\frac{\beta_\lambda}{\beta_\lambda-1}+\frac{1}{\Gamma}) = \frac{\beta_\lambda}{\beta_\lambda-1}\cdot\frac{1}{-p\delta\Delta\pi}\big(\frac{\beta_\lambda}{\beta_\lambda-1}\Delta C^C+\Delta C^I\big)$.
Thus,

$$\frac{\partial x_p^*}{\partial\beta_\lambda} = -\frac{1}{p\delta|\Delta\pi|(\beta_\lambda-1)^2}\big(\frac{2\beta_\lambda}{\beta_\lambda-1}\Delta C^C+\Delta C^I\big) < 0 \Rightarrow \frac{\partial x_p^*}{\partial\sigma^2} = \frac{\partial x_p^*}{\partial\beta_\lambda}\times\frac{\partial\beta_\lambda}{\partial\sigma^2} < 0.$$

And $\frac{\partial x_p^*}{\partial\Delta\pi} > 0$, $\frac{\partial x_p^*}{\partial p} < 0$, $\frac{\partial x_p^*}{\partial\Delta C^j} > 0$. □

## A7 Proof of Theorem 2

*Proof.* From Equation (14), we use Corollary 3 for the expression of $V_p^j$, use Corollaries 1 and 2 and Expression (11) for the expressions of $V_{ns}^j$. If $\boldsymbol{s_{ns}} = \{\text{I-withdraw}\}$:

$$\begin{aligned} V_p^I > V_w^I \quad &\Rightarrow \quad B_p^I > B_w^I \Rightarrow \big[\Delta C^I + \big(p\delta\Delta\pi^I + \frac{\theta_p\Delta\pi^C}{r-\mu}\big)x_p\big]x_p^{-\beta_\lambda} > \frac{C_l^I}{1-\beta_\lambda}x_w^{-\beta_\lambda} \\ &\Rightarrow \quad \frac{1-\beta_\lambda}{\Gamma} - \beta_\lambda\frac{\theta_p - p\omega/\Phi}{\theta_p - p\omega} > \big(\frac{\Delta C^C}{C_l^I}\big)^{\beta_\lambda-1}\big(\frac{p\omega}{\Phi(\theta_p-p\omega)}\big)^{\beta_\lambda} \\ &\Rightarrow \quad \text{the implicit expression for } \theta_p^I \in [\theta_p^{Imin}, \theta_p^{Imax}] \end{aligned}$$

$$\begin{aligned} V_p^C > V_w^C \quad &\Rightarrow \quad B_p^C > B_w^C \Rightarrow \frac{\Delta C^C}{1-\beta_\lambda}x_p^{-\beta_\lambda} > (C_l^C + p\delta\Delta\pi^C x_w)x_w^{-\beta_\lambda} \\ &\Rightarrow \quad \theta_p < \Big[\Big(\frac{(1-\beta_\lambda)\Lambda}{\Phi} - \beta_\lambda\Big)^{\frac{1}{\beta_\lambda}}\big(\frac{\Delta C^C}{C_l^I}\big)^{1-\frac{1}{\beta_\lambda}} + 1\Big]p\omega = \theta_p^{Cmax}. \end{aligned}$$

If $\boldsymbol{s_{ns}} = \{\text{C-exit}\}$:

$$\begin{aligned} V_p^I > V_e^I \quad &\Rightarrow \quad B_p^I > B_e^I \Rightarrow \big[\Delta C^I + \big(p\delta\Delta\pi^I + \frac{\theta_p\Delta\pi^C}{r-\mu}\big)x_p\big]x_p^{-\beta_\lambda} > [C_l^I + (p\delta - \frac{1}{r-\mu})\Delta\pi^I x_e]x_e^{-\beta_\lambda} \\ &\Rightarrow \quad \frac{1-\beta_\lambda}{\Gamma} - \beta_\lambda\frac{\theta_p - p\omega/\Phi}{\theta_p - p\omega} > \big(\frac{\Delta C^C}{C_l^C}\big)^{\beta_\lambda-1}\big(\frac{1-p\omega}{\theta_p-p\omega}\big)^{\beta_\lambda}\big(\frac{1-\beta_\lambda}{\Lambda} - \frac{\beta_\lambda}{\Phi}\big) \\ &\Rightarrow \quad \text{the implicit expression for } \theta_p^I \in [\theta_p^{Imin}, \theta_p^{Imax}] \end{aligned}$$

$$\begin{aligned} V_p^C > V_e^C \quad &\Rightarrow \quad B_p^C > B_e^C \Rightarrow \frac{\Delta C^C}{1-\beta_\lambda}x_p^{-\beta_\lambda} > \frac{C_l^C}{1-\beta_\lambda}x_e^{-\beta_\lambda} \Rightarrow \frac{x_e}{x_p} < \Big(\frac{C_l^C}{\Delta C^C}\Big)^{\frac{1}{\beta_\lambda}} \\ &\Rightarrow \quad \theta_p < \Big(\frac{C_l^C}{\Delta C^C}\Big)^{\frac{1}{\beta_\lambda}-1}(1-p\omega) + p\omega \equiv \theta_p^{Cmax} \end{aligned}$$

We summarize $\theta_p^{Cmax}, \theta_p^{Imin}, \theta_p^{Imax}$ as follows:

$$\theta_p^{Cmax} = \begin{cases} \Big[\Big(\frac{(1-\beta_\lambda)\Lambda}{\Phi} - \beta_\lambda\Big)^{\frac{1}{\beta_\lambda}}\big(\frac{\Delta C^C}{C_l^I}\big)^{1-\frac{1}{\beta_\lambda}} + 1\Big]p\omega, & s_{ns} = \{\text{I-withdraw}\} \\ \Big(\frac{\Delta C^C}{C_l^C}\Big)^{1-\frac{1}{\beta_\lambda}}(1-p\omega) + p\omega, & s_{ns} = \{\text{C-exit}\}. \end{cases} \tag{A.6}$$

$\theta_p^I \in [\theta_p^{Imin}, \theta_p^{Imax}]$ satisfies

$$f(\theta_p) = A(\theta_p - p\omega)^{-\beta_\lambda} - \frac{1-\beta_\lambda}{\Gamma} + \beta_\lambda \frac{\theta_p - p\omega/\Phi}{\theta_p - p\omega} \leq 0, \tag{A.7}$$

where

$$A = \begin{cases} \left(\frac{\Delta C^C}{C_l^I}\right)^{\beta_\lambda - 1}\left(\frac{p\omega}{\Phi}\right)^{\beta_\lambda}, & s_{ns} = \{\text{I-withdraw}\} \\ \left(\frac{\Delta C^C}{C_l^C}\right)^{\beta_\lambda - 1}\left(\frac{1-\beta_\lambda}{\Lambda} - \frac{\beta_\lambda}{\Phi}\right)(1-p\omega)^{\beta_\lambda}, & s_{ns} = \{\text{C-exit}\}. \end{cases}$$

The first-order derivative can be represented as

$$f'(\theta_p) = \frac{\beta_\lambda}{\theta_p - p\omega}\left(1 - A(\theta_p - p\omega)^{-\beta_\lambda} - \frac{\theta_p - p\omega/\Phi}{\theta_p - p\omega}\right). \tag{A.8}$$

□

## A8 Proof of Proposition 2

*Proof.* After the challenger's market entry with the alleged infringement and before firms take any action(s), the firm value $V_0^j$ follows the ODE:

$$\text{For } j \in \{I, C\}: \quad \mathbb{E}_t dV_0^j + \pi_2^j x dt = rV_0^j dt, \text{ where } \mathbb{E}_t dV_0^j = \left(\mu x \frac{\partial V_0^j}{\partial x} + \frac{1}{2}\frac{\partial^2 V_0^j}{\partial x^2} x^2 \sigma^2\right) dt \tag{A.9}$$

The general form of the solutions of the ODE is as follows, with $\alpha$ and $\beta$ specified in Proposition 2:

$$V_0^j(x) = \frac{\pi_2^j}{r-\mu} x + A_1^j x^\alpha + A_2^j x^\beta.$$

If $s_{ns} = \{\text{I-withdraw}\}$, firms take no action when the market demand is low provided that any action is equivalent to exercising a call option, so we can impose the boundary conditions at zero demand:

$$\lim_{x \to 0} V_0^j(x) = 0 \Rightarrow A_2^j = 0 \tag{A.10}$$

With $s_{ns} = \{\text{C-exit}\}$, if the market demand decreases to exit threshold $x_e$ before the incumbent starts litigation, then the incumbent forces the challenger to exit at time $\inf\{t : x_t \leq x_e\}$. Thus, we use the value-matching conditions at $x_e$ as follows

$$V_0^I(x_e) = \frac{\pi_1 x_e}{r-\mu}, V_0^C(x_e) = 0$$

from which $A_2^j \neq 0$. To separate from I-withdraw, we use $a^j$ and $b^j$ to represent the arbitrary constants in C-exit.

□

## A9 Proof of Corollary 5

*Proof.* $s_{ns}$ = {I-withdraw}: We apply (1) the value matching and smooth-pasting conditions on the incumbent's firm value at $x_l$, and (2) the challenger's value matching condition at $x_l$, where $V_l^j(x)$ is specified in Equation (16) of Proposition 2 and $V^j(x)$ is specified in Equation (8) of Proposition 1:

$$\text{VM:} \qquad V_l^j(x_l) = V^j(x_l) \qquad \Rightarrow \frac{\pi_2^j x_l}{r-\mu} + A_l^j x_l^\alpha = \Big(\frac{\pi_2^i}{r-\mu} - p\delta\Delta\pi^j\Big)x_l - C_l^j + B^i x_l^{\beta_\lambda}, \quad j \in \{I, C\}.$$

$$\text{SP:} \qquad \frac{\partial V_l^I(x)}{\partial x}\bigg|_{x_l} = \frac{\partial V^I(x)}{\partial x}\bigg|_{x_l} \qquad \Rightarrow \frac{\pi_2^I}{r-\mu} + \alpha A_l^I x_l^{\alpha-1} = \frac{\pi_2^I}{r-\mu} - p\delta\Delta\pi^I + \beta_\lambda B^I x_l^{\beta_\lambda - 1} + \frac{\partial B^I}{\partial x_l} x_l^{\beta_\lambda}.$$

We can then solve the arbitrary constants $A_l^I$, $A_l^C$ and the litigation threshold $x_l$, as expressed in the corollary.

$s_{ns}$ = {C-exit}: Besides the two VM and one SP conditions as follows

$$\text{VM:} \qquad V_l^j(x_l) = V^j(x_l) \qquad \Rightarrow \frac{\pi_2^j x_l}{r-\mu} + a_l^j x_l^\alpha + b_l^j x_l^\beta = \Big(\frac{\pi_2^i}{r-\mu} - p\delta\Delta\pi^j\Big)x_l - C_l^j + B^i x_l^{\beta_\lambda}, \quad j \in \{I, C\}$$

$$\text{SP:} \qquad \frac{\partial V_l^I(x)}{\partial x}\bigg|_{x_l} = \frac{\partial V^I(x)}{\partial x}\bigg|_{x_l} \qquad \Rightarrow \frac{\pi_2^I}{r-\mu} + \alpha a_l^I x_l^{\alpha-1} + \beta b_l^I x_l^{\beta-1} = \frac{\pi_2^I}{r-\mu} - p\delta\Delta\pi^I + \beta_\lambda B^I x_l^{\beta_\lambda - 1} + \frac{\partial B^I}{\partial x_l} x_l^{\beta_\lambda},$$

there is one additional value-matching condition for each firm, due to the incumbent's forcing-out option:

$$\left.\begin{aligned} V_l^I(x_e) = \tfrac{\pi_1 x_e}{r-\mu} &\Rightarrow \tfrac{\pi_2^I x_e}{r-\mu} + a_l^I x_e^\alpha + b_l^I x_e^\beta = \tfrac{\pi_1 x_e}{r-\mu} \\ V_l^C(x_e) = 0 &\Rightarrow \tfrac{\pi_2^C x_e}{r-\mu} + a_l^C x_e^\alpha + b_l^C x_e^\beta = 0 \end{aligned}\right\} a_l^j x_e^\alpha + b_l^j x_e^\beta = \frac{-\Delta\pi^j x_e}{r-\mu} \tag{A.11}$$

With the five equations, the arbitrary constants can be solved as:

$$a_l^j = \frac{-p\delta\Delta\pi^j x_l^{1-\beta} + \frac{\Delta\pi^j}{r-\mu} x_e^{1-\beta} + B^i x_l^{\beta_\lambda - \beta} - C_l^j x_l^{-\beta}}{x_l^{\alpha-\beta} - x_e^{\alpha-\beta}}, \quad b_l^j = \frac{-p\delta\Delta\pi^j x_l^{1-\alpha} + \frac{\Delta\pi^j}{r-\mu} x_e^{1-\alpha} + B^i x_l^{\beta_\lambda - \alpha} - C_l^j x_l^{-\alpha}}{x_l^{\beta-\alpha} - x_e^{\beta-\alpha}}, \tag{A.12}$$

and the litigation threshold $x_l$ satisfies

$$\begin{aligned} &\big[(\alpha-1)(\frac{x_e}{x_l})^\beta - (\beta-1)(\frac{x_e}{x_l})^\alpha\big]p\delta\Delta\pi^I x_l + \big[(\beta_\lambda-\alpha)(\frac{x_e}{x_l})^\beta - (\beta_\lambda-\beta)(\frac{x_e}{x_l})^\alpha\big]B^I x_l^{\beta_\lambda} \\ &= (\alpha-\beta)\frac{\Delta\pi^I}{r-\mu}x_e - \Big(\alpha(\frac{x_e}{x_l})^\beta - \beta(\frac{x_e}{x_l})^\alpha\Big)C_l^I - \big[(\frac{x_e}{x_l})^\beta - (\frac{x_e}{x_l})^\alpha x_l^{\alpha-\beta}\big]\frac{\partial B^I}{\partial x_l}x_l^{\beta_\lambda+1}. \end{aligned} \tag{A.13}$$

□

## A10 Proof of Corollary 6

*Proof.* $s_{ns}$ = {I-withdraw}: We first list the value-matching conditions for both firms at $x_a$:

$$\text{VM:} \quad V_a^j(x_a) = \hat{V}_a^j(x_a) - C_s^j \Rightarrow \frac{\pi_2^j x_a}{r-\mu} + A_a^j x_a^\alpha = \frac{\pi_2^j + \Delta\pi_a^j}{r-\mu}x_a - C_s^j, \quad j \in \{I, C\},$$

Thus, $A_a^C(x_a, \theta_a) = -(\frac{\theta_a \pi_2^C}{r-\mu} x_a + C_s^C) x_a^{-\alpha} < 0$. We know $x < x_a$ before firms possibly settle ex-ante, and $\frac{\partial}{\partial x_a} A_a^C(x_a|\theta_a) = (\alpha - 1)\frac{\theta_a \pi_2^C}{r-\mu} x_a^{-\alpha} + \alpha C_s^C x_a^{-\alpha-1} > 0$ indicates the challenger waits as long as possible before settling ex-ante (or higher $x_a$). However, ex-ante settlement has to happen, if ever, before I-litigate. Thus, $x_a = x_l$.

$\boldsymbol{s_{ns}} = \{\text{C-exit}\}$:

$$\text{VM at } x_a: \quad V_a^j(x_a) = \hat{V}_a^j(x_a) - C_s^j \Rightarrow \frac{\pi_2^j x_a}{r-\mu} + a_a^j x_a^\alpha + b_a^j x_a^\beta = \frac{\pi_2^j + \Delta\pi_a^j}{r-\mu} x_a - C_s^j, \quad j \in \{I, C\},$$

$$\text{VM at } x_a: \quad V_a^I(x_e) = \frac{\pi_1 x_e}{r-\mu} \text{ and } V_a^C(x_e) = 0 \Rightarrow a_a^j x_e^\alpha + b_a^j x_e^\beta = \frac{-\Delta\pi^j x_e}{r-\mu}.$$

From the three equations above, we get the arbitrary constants $a_a^j$ and $b_a^j$.

$$a_a^j = \frac{\frac{\Delta\pi_a^j}{r-\mu} x_a^{1-\beta} + \frac{\Delta\pi^j}{r-\mu} x_e^{1-\beta} - C_s^j x_a^{-\beta}}{x_a^{\alpha-\beta} - x_e^{\alpha-\beta}}, \quad b_a^j = \frac{\frac{\Delta\pi_a^j}{r-\mu} x_a^{1-\alpha} + \frac{\Delta\pi^j}{r-\mu} x_e^{1-\alpha} - C_s^j x_a^{-\alpha}}{x_a^{\beta-\alpha} - x_e^{\beta-\alpha}} \tag{A.14}$$

For economically meaningful parameters, our numerical method always show that $x_a = x_l$. □

## A11 Proof of Theorem 3

*Proof.* For ex-ante settlement to be feasible, both firms must have higher values with the ex-ante settlement option than with the I-litigation option. Thus, if $\boldsymbol{s_{ns}} = \{\text{I-withdraw}\}$:

$$\begin{aligned} V_a^I \geq V_l^I \quad &\Rightarrow \quad \frac{\pi_2^I x}{r-\mu} + A_a^I x^\alpha \geq \frac{\pi_2^I x}{r-\mu} + A_l^I x^\alpha \Rightarrow A_a^I \geq A_l^I \Rightarrow (\frac{\Delta\pi_a^I}{r-\mu} x_a - C_s^I) x_a^{-\alpha} \geq A_l^I \\ &\Rightarrow \quad (\frac{\theta_a \pi_2^C}{r-\mu} x_a - C_s^I) x_a^{-\alpha} \geq A_l^I \Rightarrow \theta_a \geq \frac{(A_l^I x_a^\alpha + C_s^I)(r-\mu)}{\pi_2^C x_a} = \theta_a^{Imin} \end{aligned}$$

$$\begin{aligned} V_a^C \geq V_l^C \quad &\Rightarrow \quad \frac{\pi_2^C x}{r-\mu} + A_a^C x^\alpha \geq \frac{\pi_2^C x}{r-\mu} + A_l^C x^\alpha \Rightarrow A_a^C \geq A_l^C \Rightarrow (\frac{\Delta\pi_a^C}{r-\mu} x_a - C_s^C) x_a^{-\alpha} \geq A_l^C \\ &\Rightarrow \quad (\frac{-\theta_a \pi_2^C}{r-\mu} x_a - C_s^C) x_a^{-\alpha} \geq A_l^C \Rightarrow \theta_a \leq -\frac{(A_l^C x_a^\alpha + C_s^C)(r-\mu)}{\pi_2^C x_a} = \theta_a^{Cmax} \end{aligned}$$

If $\boldsymbol{s_{ns}} = \{\text{C-exit}\}$:

$$\begin{aligned} V_a^I \geq V_l^I \quad &\Rightarrow \quad \frac{\pi_2^I x_a}{r-\mu} + a_a^I x_a^\alpha + b_a^I x_a^\beta \geq \frac{\pi_2^I x_a}{r-\mu} + a_l^I x_a^\alpha + b_l^I x_a^\beta \Rightarrow \frac{\theta_a \pi_2^C}{r-\mu} x_a - C_s^I \geq a_l^I x_a^\alpha + b_l^I x_a^\beta \\ &\Rightarrow \quad \theta_a \geq (a_l^I x_a^\alpha + b_l^I x_a^\beta + C_s^I) \frac{r-\mu}{\pi_2^C x_a} = \theta_a^{Imin} \end{aligned}$$

$$V_a^C \geq V_l^C \quad \Rightarrow \quad \frac{\pi_2^C x_a}{r-\mu} + a_a^C x_a^\alpha + b_a^C x_a^\beta \geq \frac{\pi_2^C x_a}{r-\mu} + a_l^C x_a^\alpha + b_l^C x_a^\beta \Rightarrow \frac{-\theta_a \pi_2^C}{r-\mu} x_a - C_s^C \geq a_l^C x_a^\alpha + b_l^C x_a^\beta$$
$$\Rightarrow \quad \theta_a \leq (a_l^C x_a^\alpha + b_l^C x_a^\beta + C_s^C) \frac{r-\mu}{-\pi_2^C x_a} = \theta_a^{Cmax}$$

Together, the feasible range of ex-ante settlement can be written as

$$\theta_a \in [\theta_a^{Imin}, \theta_a^{Cmax}] \begin{cases} \left[\frac{(A_l^I x_a^\alpha + C_s^I)(r-\mu)}{\pi_2^C x_a}, -\frac{(A_l^C x_a^\alpha + C_s^C)(r-\mu)}{\pi_2^C x_a}\right], & s_{ns} = \{\text{I-withdraw}\} \\ \left[\frac{r-\mu}{\pi_2^C x_a}(a_l^I x_a^\alpha + b_l^I x_a^\beta + C_s^I), -\frac{r-\mu}{\pi_2^C x_a}(a_l^C x_a^\alpha + b_l^C x_a^\beta + C_s^C)\right], & s_{ns} = \{\text{C-exit}\}. \end{cases} \tag{A.15}$$

Regarding $\theta_a$, we have $\frac{\partial A_a^I}{\partial \theta_a} > 0$ if $s_{ns} = \{\text{I-withdraw}\}$ and $\frac{\partial V_a^I}{\partial \theta_a} = \frac{\partial (a_a^I x_a^\alpha + b_a^I x_a^\beta)}{\partial \theta_a} = \frac{\partial a_a^I}{\partial \theta_a} x^\alpha + \frac{\partial b_a^I}{\partial \theta_a} x^\beta$ if $s_{ns} = \{\text{C-exit}\}$. Because $\frac{\partial x_a}{\partial \theta_a} = 0$ as proved in Appendix A10, we can show that $\frac{\partial V_a^I}{\partial \theta_a}\Big|_{x=x_a} = \frac{\pi_2^C}{r-\mu} x_a > 0$. Hence, $\theta_a^* = \theta_a^{Cmax}$ regardless of the $s_{ns}$. When $s_{ns} = \{\text{I-withdraw}\}$, we have

$$\theta_a = -\frac{(A_l^C x_a^\alpha + C_s^C)(r-\mu)}{\pi_2^C x_a} = -\frac{(-p\delta\pi_2^C x_l + B_p^C x_l^{\beta_\lambda} - \Delta C^C)(r-\mu)}{\pi_2^C x_l}$$
$$= p\delta(r-\mu)(1-(\frac{x_l}{x_p})^{\beta_\lambda - 1}) + \frac{r-\mu}{\pi_2^C x_l}\Delta C^C (1-(\frac{x_l}{x_p})^{\beta_\lambda}) + \theta_p^*(\frac{x_l}{x_p})^{\beta_\lambda - 1}$$

$$\theta_p - \theta_a \Rightarrow p\omega g(\Gamma)(\frac{1}{\Phi} - 1) - \frac{r-\mu}{\pi_2^C x_l}\Delta C^C \frac{1-(\frac{x_l}{x_p})^{\beta_\lambda}}{1-(\frac{x_l}{x_p})^{\beta_\lambda - 1}}$$

The last expression is equivalent to the time value at $x_l$ of the option to settle ex-post that is exercised at $x_p$, which is greater than zero. Thus, $\theta_p > \theta_a$. □

## A12 Proof of Theorem 4

*Proof.* From Theorem 1, the incumbent's optimal royalty rate in an ex-post settlement is $\theta_p^* = p\omega(1 - g(\Gamma)) + \frac{p\omega}{\Phi} g(\Gamma)$, where $g(\Gamma) = (\frac{\beta_\lambda}{\beta_\lambda - 1} + \frac{1}{\Gamma})^{-1} > 0$. Because $\bar{\Gamma}(\mathbb{1}_E = 1) < \bar{\Gamma}(\mathbb{1}_E = 0)$, we then have $g(\bar{\Gamma}(\mathbb{1}_E = 1)) < g(\bar{\Gamma}(\mathbb{1}_E = 0))$. Given $\Phi \leq 1$ in ex-post settlement, we get $\theta_p^*(\mathbb{1}_E = 1) < \theta_p^*(\mathbb{1}_E = 0)$. □

## A13 Model analyses for immediate settlement

Immediate settlement is relevant if the baseline module solution suggests $x_p > x_l$. Then we examine all scenarios of ex-post settlement by splitting the possible $x_l$ into four regions, depending on the approach we use to derive the royalty rate. We need to resolve the litigation threshold given ex-post settlement should occur immediately ($x_p = x_l$), and check the modified feasibility conditions for ex-post settlement. Intuitively, because we assume zero-cost of starting the litigation, immediate settlement all fall into cases of ex-ante settlement. See the online appendix here for the full details.

## A14 Model analyses for the English Rule

Under the English Rule, the analysis of Case A (*I remains a going-concern*) resembles Section 3., with modified litigation costs of $(\bar{C}_l^I, \bar{C}_l^C)$: $\bar{C}_l^I = C_l^I + \frac{C_l^C(1-p)\lambda}{r+\lambda}\mathbb{1}_E$ and $\bar{C}_l^C = C_l^C - \frac{C_l^C(1-p)\lambda}{r+\lambda}\mathbb{1}_E$, where $\mathbb{1}_E$ is the indicator for the English Rule. The expression of $\bar{C}_l^j$ can be derived from $c_l^I + \lambda[(1-p)C_l^C - \bar{C}_l^I] = r\bar{C}_l^I$, and $c_l^C - \lambda[(1-p)C_l^C - \bar{C}_l^C] = r\bar{C}_l^C$. More generally, we can use a cost parameter to capture two realistic considerations: (1) some other cost allocation rules (Bebchuk, 1984), (2) the indirect cost of diverting internal resources to deal with litigation is not paid for by the losing party under the English Rule. Our results should not be affected qualitatively.

Case B (*I may liquidate*) is more complex, depending on whether the market demand is higher or lower than the incumbent's liquidation threshold at the time of having the relevant options. We have to solve the extended model using backward induction step-by-step. See the online appendix here for the full details.

The condition for Case A is $x_d \geq \bar{x}$, where $d = \{w, e, p\}$ represents *I-withdraw*, *C-exit*, and *ex-post settlement*, whichever is the likely outcome during litigation, and the condition for Case B is $x_d < \bar{x}$. The court rules either (1) before firms take actions ($\tau \leq t_d$) or (2) after ( $\tau > t_d$). $x_d \geq \bar{x}$ is the condition for Case A, because it implies $x_\tau > x_d \geq \bar{x}$ in (2), that is, the incumbent's value at court ruling is high enough to cover the challenger's litigation cost. It implies in (2) that the lawsuit ends before court rules and liquidation is thus irrelevant. $x_d < \bar{x}$ is the condition for Case B, as (1) is further split into (1).i at a demand lower than the liquidation cutoff ( $\tau < t_d$ and $x_\tau < \bar{x}$), or (1).ii at a demand higher than the liquidation cutoff ( $\tau < t_d$ and $x_\tau \geq \bar{x}$). With $x_d < \bar{x}$, the incumbent liquidates in Scenario (1).i if the court rules against the incumbent. Note the incumbent still remains a going-concern in the other two scenarios during litigation.