Do Different Measures of Stock Market Volatility Risks Have the Same Price? *

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August 5, 2023

Abstract

Common measures of aggregate stock market volatility are priced differently in the cross-section of stock returns. Stocks with high sensitivities to changes in realized and expected volatilities have significantly low average returns, while option implied volatility is not priced. The differential pricing of market volatility risks is hard to reconcile with existing theories, but potentially consistent with partial segmentation between index options and equity markets. I argue the comovement between option implied and actual stock volatilities contains valuable information about time varying segmentation between equity and options markets. The two markets appear to have become more integrated in recent years.

JEL classification: G12

*I would like to thank Oleg Chuprinin, Mathieu Fournier, Andrew Grant, Jianlei Han, Mark Humphery-Jenner, Alexandre Jeanneret, Amy Kwan, Jonathan Lewellen, Hamish Malloch, Ronald Masulis, Terry Pan, Neil Pearson, Xinfeng Ruan, Elizabeth Sheedy, Lei Shi, Danjue Shang, and seminar participants at the University of Sydney, UNSW, Macquarie University, 2022 Derivative Markets Conference, 2022 FMA Asia Pacific and 16th Annual Risk Management Conference for helpful comments. Please send correspondence to Guanglian Hu, Discipline of Finance, Business School, University of Sydney; telephone: +61 2 8627 9431. E-mail: guanglian.hu@sydney.edu.au.

1 Introduction

The behavior of stock market volatility is difficult to explain. Shiller (1981) shows that stock prices appear to fluctuate too much relative to the ex-post variability of dividends. Closely related, LeRoy and Porter (1981) confirm the existence of "excess volatility" using tests based on implied variance bounds. Furthermore, Schwert (1989) finds that the dynamics in stock volatility are also puzzling in that standard measures of economic activities can only explain a small part of movements in stock volatility, especially during the Great Depression.

This paper contributes to the literature by documenting that common measures of stock market volatility are priced differently and this differential pricing of volatility risks cannot be explained by existing theories of volatility. Stock market volatility is inherently latent and can be measured in various ways. Figure 1 plots three measures of stock market volatility over the sample period 1986–2020. The first one is realized volatility, which is calculated from intraday data and reflects return variation that has been realized. The second one is the implied volatility extracted from the index options market. Option implied volatility is attractive because it is forward looking and does not rely on any particular volatility forecasting model. The third measure is the expected volatility computed from a leading volatility forecasting model. Figure 1 shows that while the three measures of stock market volatility share common patterns and respond similarly to changes in underlying economic conditions, they are not perfectly correlated with noticeable differences.

My paper focuses on the pricing of realized, implied, and expected market volatilities in the cross-section of stock returns. The empirical strategy closely follows Cremers, Halling, and Weinbaum (2015) and Ang, Chen, and Xing (2006) by examining whether stocks with different exposures to market volatility have different average returns contemporaneously. A contemporaneous relation between factor loadings and average returns is the foundation of a cross-sectional risk-return relation. Using both portfolio sorts and Fama-MacBeth regressions, I find that realized and expected market volatilities are negatively priced in the cross-section of equity returns. Stocks that perform well when realized or expected market volatilities are high earn significantly lower average returns. This is consistent with the economic intuition that aggregate volatility shocks are viewed as bad and investors pay a premium for hedging against increases in volatility. In contrast, option implied market volatility is not priced in the cross-section of stock returns. There is no statistically significant relationship between average returns and exposure to option implied volatility. This result is somewhat surprising as it suggests that equity investors do not care about fluctuations in option implied volatility and are not willing to pay a premium to hedge against increases in implied volatility. The empirical findings are robust to different implementations.

The differential pricing of stock market volatility risks is hard to reconcile with existing theories. In intertemporal asset pricing models (e.g., Merton, 1973; Campbell, 1993, 1996; Campbell, Giglio, Polk, and Turley, 2018) in which market volatility is a state variable that drives the investment opportunity set, investors pay a premium to hedge against changes in market volatility because increasing volatility represents a deterioration in the future investment opportunity set. Realized, implied, and expected market volatilities could carry different risk premiums if they contain different information about future investment opportunities. In the data, however, the predictive power of option implied volatility is very similar to that of realized and expected volatilities, which suggests that investors would equally care about fluctuations in implied volatility and be willing to pay a premium to hedge against increases in implied volatility.

Another theory of the volatility risk premium relates the pricing of volatility risk to downside protection (e.g., Bakshi and Kapadia, 2003), as periods of high volatility tend to coincide with downward market movements. However, this theory cannot explain why exposure to realized or expected market volatilities earns a negative risk premium but exposure to implied volatility does not. In the data, implied volatility comoves more negatively with index returns than both realized and expected volatilities, and therefore one would expect, if anything, implied volatility should be priced more strongly.

Option implied volatility, under perfect market assumptions, measures the expected volatility under the risk neutral measure and therefore it reflects not only conditional volatility but also time varying risk aversion/premium. Implied volatility might not be priced if the risk premium component moves in a way that makes implied volatility a noisy measure of true risk. However, this explanation is unlikely to be plausible because it contradicts the prediction from virtually all models that risk premium is in proportion to the risk, and therefore risk neutral and physical volatilities should be highly correlated. Empirically, I show the lack of a significant risk premium for option implied volatility is not due to the time varying risk version by controlling for the risk aversion index constructed by Bekaert, Engstrom, and Xu (2022).

The differential pricing of realized, implied, and expected market volatilities is potentially consistent with friction and partial segmentation between options and equity markets, which may arise for several reasons. First of all, compared to equity investment, option trading involves significant constraints (e.g., margin requirement) that limit market participation. As such, option implied volatility may not reflect the preferences and beliefs of the average investor. In addition, existing studies suggest that financial intermediaries play an important role in the options market and option prices are affected by financial intermediaries' ability to bear risk and take on leverage. See, among others, Barras and Malkhozov (2016), Chen, Joslin, and Ni (2019), and Gârleanu, Pedersen, and Poteshman (2009). Lastly, option implied volatility may deviate from true stock market volatility because of the manipulation in the options market (Griffin and Shams, 2018), the inclusion of illiquid options in calculating implied volatility (Andersen, Bondarenko, and Gonzalez-Perez, 2015) or microstructure biases (Duarte, Jones, and Wang, 2022). In the presence of market segmentation, option implied volatility captures not only shocks to the economic fundamentals but also shocks that are specific to the options market. Option implied volatility is not priced in the cross-section of stock returns because equity investors may not want to hedge shocks that are specific to the options market.

Turning the reasoning around, I argue the comovement between option implied volatility and actual stock volatility measures contains useful information about time varying segmentation between equity and index options markets. In particular, a stronger comovement between stock volatility and option implied volatility would indicate a decreased level of market segmentation. Panel A of Figure 2 shows that the correlations of option implied volatility with realized and expected market volatilities are time-varying and exhibit a dramatic increase late in my sample period, suggesting that equity and index options markets have become more integrated in recent years. Consistent with the conjecture that market segmentation has declined, I find option implied volatility is indeed priced in the 2010–2020 sample.

Related Literature

The seminal articles by Shiller (1981) and LeRoy and Porter (1981) demonstrate that the level of stock market volatility is too high relative to standard models with rational expectation and constant discount rate. Their findings have motivated many subsequent studies.[1] My paper contributes to this literature by documenting that common measures of stock market volatility are priced differently, a puzzling result that current theories of

[1]For example, Gabaix, Gopikrishnan, Plerou, and Stanley (2006) develop a model in which excess volatility is caused by trades by large institutional investors. Adam, Marcet, and Nicolini (2016) show that consumption-based asset pricing models with time-separable preferences can generate realistic amounts of stock price volatility with small deviation from rational expectation.

volatility cannot explain.

This paper is closely related to Dew-Becker, Giglio, Le, and Rodriguez (2017) and Dew-Becker, Giglio, and Kelly (2021). The two papers exploit derivatives data and convincingly show that exposure to realized volatility earns a significant premium, whereas exposure to implied volatility does not, highlighting the importance of considering and distinguishing different forms of volatility. Consistent with their results, I find that stocks with high sensitivities to realized market volatility have significantly low average returns, whereas implied volatility is not priced. Moreover, I show that exposure to expected volatility is priced with a significantly negative risk premium and that the correlation between option implied and actual stock volatilities contains valuable information about market segmentation.[2]

My findings also have implications for the option pricing literature. The central issue in that literature is to reconcile the inconsistency between time series properties of index returns with those implied from the cross-section of index option prices (Bates, 2003). While existing studies have made significant progress by modeling additional risks and their pricing (see, among others, Broadie, Chernov, and Johannes, 2007; Pan, 2002; Eraker, 2004), the analysis of this paper indicates that the discrepancy may be in part due to the segmentation between equity and index options markets.[3] Closely related, my results also suggest that the variance risk premium (e.g., Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009), the difference between option implied volatility and a measure of actual stock volatility, needs to be interpreted with caution as option implied volatility is an imprecise measure of risk neutral expected volatility in the presence of market segmentation, complementing

[2]Related, Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018), and Bali, Brown, and Tang (2017) study the effect of economic uncertainty. My analysis suggests that the VIX index may be a noisy proxy for uncertainty as it also incorporates shocks that are specific to options market in the presence of market segmentation.

[3]Dew-Becker and Giglio (2022) analyze synthetic options and find no evidence of mispricing, and they relate the divergence in results between listed and synthetic options to segmentation in the derivatives markets.

the finding in Barras and Malkhozov (2016) that the option market implied variance risk premium is strongly affected by measures of the financial standing of intermediaries.

There are several previous studies that investigate the pricing of volatility risk in the cross-section of stock returns.[4] Cremers, Halling, and Weinbaum (2015) use a novel option trading strategy to isolate volatility exposure from jump exposure, finding that volatility risk is negatively priced in the cross-section. Adrian and Rosenberg (2008) fit a GARCH model with long run and short run volatility components to index returns and find that the price of risk is negative and significant for both components. Ang, Hodrick, Xing, and Zhang (2006) document that innovations in the VIX carry a statistically significant negative price of risk, while Chang, Christoffersen, and Jacobs (2013) find an insignificant price of risk for option implied volatility.[5] This paper contributes to the literature by examining the pricing of different measures of market volatility risk in a unified empirical framework and highlighting the implications of differential pricing of market volatility risks.

Lastly, this study contributes to the long standing literature that examines the time series relation between market volatility and the expected return on the market. The empirical evidence regarding the mean-variance relation is inconclusive.[6] This paper differs from prior studies by focusing on the cross-sectional pricing of market volatility risk.

[4]There is a separate literature that examines the pricing of volatility risk using derivative securities. See, among others, Coval and Shumway (2001), Bakshi and Kapadia (2003), Eraker and Wu (2014), Cheng (2019), and Egloff, Leippold, and Wu (2010). This study differs from this literature in that the focus here is on the pricing of volatility risk in the equity market. Using the cross-section of stock returns allows one to easily control for exposures to common risk factors that have been shown to be related to expected returns.

[5]My results with implied volatility are more consistent with Chang, Christoffersen, and Jacobs (2013). The differences could be driven by the differences in empirical strategy. For instance, Ang, Hodrick, Xing, and Zhang (2006) form portfolios sorted on past VIX betas and link the ex-post portfolio returns to their exposure to a volatility risk mimicking portfolio ($FVIX$) instead of post-formation VIX betas. See Section 5.2 for additional discussion.

[6]Some studies find a positive relation (e.g., Scruggs, 1998; Ghysels, Santa-Clara, and Valkanov, 2005; Guo and Whitelaw, 2006; Pastor, Sinha, and Swaminathan, 2008; Lundblad, 2007; Ludvigson and Ng, 2007), but others report a negative relation (e.g., Campbell, 1987; Turner, Startz, and Nelson, 1989; Campbell and Hentschel, 1992; Glosten, Jagannathan, and Runkle, 1993; Brandt and Kang, 2004) or find no statistically significant relation (e.g., French, Schwert, and Stambaugh, 1987).

The rest of the paper is organized as follows. Section 2 computes and compares realized, option implied, and expected market volatilities. Section 3 presents the main empirical results on the pricing of the three measures of market volatility in the cross-section of stock returns. Section 4 discusses the implications of these findings. Section 5 contains several extensions and robustness analysis that complement the main results, and Section 6 concludes the paper.

2 Measuring Stock Market Volatility

This section constructs and compares three common measures of daily stock market volatility over the sample period from January 1986 to December 2020. Throughout the paper, stock market volatility refers to the volatility of the S&P 500 index. The first measure is realized volatility (RV), which is backward looking and reflects historical volatility that has been realized. The recent realized volatility literature demonstrates that using high frequency data can lead to more precise estimates of ex-post realized volatility. The basic idea dates back to Merton (1980)'s insight: volatilities can be approximated arbitrarily well as the sampling frequency goes to infinity.[7] Motivated by this literature, I compute daily realized volatility of the S&P 500 index as the square root of the sum of the squared 5-min log index returns.[8] I obtain intraday price data on the S&P 500 index from the Thomson Reuters Tick History database.

The second volatility measure is option implied volatility (IV), which is inferred from the

[7]The theoretical properties of realized volatility are formally established by a number of studies including Andersen, Bollerslev, Diebold, and Ebens (2001), Andersen, Bollerslev, Diebold, and Labys (2003), and Barndorff-Nielsen and Shephard (2002).

[8]I focus on the 5-minute interval because Liu, Patton, and Sheppard (2015) show that it is difficult to outperform 5-minute realized variance even with more sophisticated sampling techniques. Moreover, following the literature (e.g., Bollerslev, Tauchen, and Zhou, 2009), overnight and weekend are treated as an additional 5-minute interval.

index options market. Option implied volatility is attractive because it is forward looking and does not rely on any particular volatility forecasting model. The first generation of option implied volatility measures is based on inverting a parametric option pricing model such as the Black-Scholes-Merton model (Black and Scholes, 1973; Merton, 1973). More recently, a number of papers demonstrate that option implied volatility can be calculated in an essentially model free way without relying on any option pricing model (see, among others, Dupire, 1994; Neuberger, 1994; Britten-Jones and Neuberger, 2000; Jiang and Tian, 2005). In this paper, I construct an extended time series of option implied volatility dating back to 1986 by combining the VIX index, which is based on the model-free approach, and the VXO index, which is based on inverting the Black-Scholes-Merton model. Specifically, I use the VIX index as a proxy for implied volatility over the sample period from 1990 to 2020. Prior to 1990 when the VIX data is not available, I use the VXO as a proxy for implied volatility.[9] I download the data on VIX and VXO from the Chicago Board Options Exchange (CBOE) website.

The third measure is the expected volatility (EV) estimated from a leading variance forecasting model. In particular, following Bekaert and Hoerova (2014), I consider the following model to estimate the conditional expectation of future realized volatility:

$$\log RV^2_{t\to t+22} = \delta_0 + \delta_1 \log RV^2_{t-22\to t} + \delta_2 \log RV^2_{t-5\to t} + \delta_3 \log RV^2_{t-1\to t} + \delta_4 \log VIX^2_t + \epsilon_{t\to t+22}, \quad (1)$$

[9]The CBOE introduced the original version of the VIX index (now called VXO) in 1993, which was based on the average BSM implied volatility of S&P 100 at-the-money options. The CBOE provides historical data on VXO going back to 1986. In 2003, the CBOE developed a new VIX index (VIX). The new VIX index is based on the model-free approach and calculated as the weighted average of out-of-the-money S&P 500 index option prices. The CBOE provides historical data on the VIX back to 1990. Despite the difference in methodology, VIX and VXO are highly correlated.

where $RV^2_{t-22\to t}$, $RV^2_{t-5\to t}$, and $RV^2_{t-1\to t}$ represent realized variances over the past month (22 trading days), week (5 trading days), and day, respectively.[10] I estimate the above regression based on full sample data and compute the expected variance as follows:

$$E_t\left[RV^2_{t\to t+22}\right] = \exp\left(\delta_0 + \delta_1 \log RV^2_{t-22\to t} + \delta_2 \log RV^2_{t-5\to t} + \delta_3 \log RV^2_{t-1\to t} + \delta_4 \log VIX^2_t + \frac{1}{2}\sigma^2_\epsilon\right). \quad (2)$$

Finally I take the square root of $E_t\left[RV^2_{t\to t+22}\right]$ as the expected volatilities. Note that as with the implied volatility, the expected volatility is measured over a horizon of one month. Section 5.1 conducts additional analysis with other volatility measures such as those implied from GARCH/EGARCH models.

Panel A of Table 1 reports the summary statistics of the three stock market volatility measures. Throughout the paper, I express market volatility in annualized volatility percentage term. Consistent with the findings in Carr and Wu (2009) and Bollerslev, Tauchen, and Zhou (2009), implied volatility is on average higher than realized and expected volatilities (19.92 v.s 12.45 and 13.83). Moreover, all three measures of stock market volatility exhibit positive skewness and large kurtosis.

Figure 1 plots the three stock volatility measures over the 1986–2020 sample period. Realized volatility, implied volatility, and expected volatility all exhibit mean-reverting behavior, and appear to follow common patterns and respond similarly to changes in underlying economic conditions. However, perhaps more importantly, Figure 1 also highlights that the three volatility measures are not perfectly correlated with noticeable differences. For example, realized volatility reached the all time high on March 16, 2020 at the beginning of the

[10] Because realized variances are approximately log normal, it is more appropriate to forecast logarithmic variance with linear models. The log specification also ensures volatility forecasts always remain positive.

COVID-19 pandemic, whereas implied volatility peaked on October 19, 1987 (Black Monday) when the S&P 500 index dropped by more than 20%. Moreover, expected volatility is smoother than implied and realized volatilities, which is consistent with the standard deviations reported in Table 1.

To further investigate the imperfect co-movements among different volatility measures, Panel B of Table 1 reports pairwise correlations of the changes in the three volatility measures. Realized volatility and expected volatility have a high correlation of 0.90, but their correlations with implied volatility are modest, 0.31 and 0.50 respectively. Lastly, Panel B shows that all three volatility measures are negatively correlated with index returns. The negative correlation between index returns and volatility has been well documented and is often referred to as the leverage effect in the literature (e.g., Black, 1976; Christie, 1982).

3 The Pricing of Realized, Implied, and Expected Market Volatilities

This section presents the main results on the pricing of realized, implied, and expected market volatility risks in the cross-section of stock returns. Section 3.1 discusses the empirical design. Section 3.2 considers portfolio sorts. Section 3.3 reports the results of Fama-MacBeth regressions.

3.1 Empirical Design

This paper conducts an investigation of different measures of stock market volatility as pricing factors in the cross-section of stock returns. The empirical design closely follows Ang, Chen, and Xing (2006) and Cremers, Halling, and Weinbaum (2015) by considering the contemporaneous relation between realized factor loadings and realized stock returns. Uncovering a contemporaneous effect is important because a contemporaneous relation between factor loadings and average returns is the foundation of a cross-sectional risk-return relation (e.g., Black, Jensen, and Scholes, 1972; Fama and French, 1993; Jagannathan and Wang, 1996).

Following Ang, Chen, and Xing (2006) and Cremers, Halling, and Weinbaum (2015), I estimate the following time series regressions at monthly frequency using daily data over rolling annual periods to measure a stock's sensitivity to market volatility risk:

$$R_t^i = \alpha^i + \beta_{MKT}^i * MKT_t + \beta_{RV}^i * \Delta RV_t + \epsilon \tag{3}$$

$$R_t^i = \alpha^i + \beta_{MKT}^i * MKT_t + \beta_{IV}^i * \Delta IV_t + \epsilon \tag{4}$$

$$R_t^i = \alpha^i + \beta_{MKT}^i * MKT_t + \beta_{EV}^i * \Delta EV_t + \epsilon \tag{5}$$

where R_t^i is the daily excess return of stock i, MKT_t is the daily excess return of the market, and ΔRV_t, ΔIV_t, and ΔEV_t denote changes in realized, implied, and expected market volatilities. As in Ang, Chen, and Xing (2006) and Cremers, Halling, and Weinbaum (2015), I consider a two-factor specification and only include the market factor when estimating volatility betas, but I control for exposures to other systematic risk factors extensively in the cross-sectional tests. The empirical analysis is based on the entire CRSP universe and excludes all securities with an average price below $3 during the previous year.

Panel C of Table 1 reports the time series averages of the cross-sectional mean, standard deviation, skewness, and kurtosis of β_{RV}, β_{IV}, and β_{EV}. Stocks on average have negative volatility exposures. The mean estimate is -0.011% for β_{RV}, -0.005% for β_{IV}, -0.069% for β_{EV}. Panel C also shows that the standard deviations are much larger than the means, suggesting that there is a substantial cross-sectional variation in volatility betas. Lastly, volatility betas exhibit small skewness but large kurtosis.

3.2 Portfolio Sorts

This section forms quintile portfolios with heterogeneous exposures to market volatility risk and compares the relative performance of these portfolios contemporaneously. If volatility risk is priced, stocks with different volatility betas would exhibit different returns on average. Specifically, for each month in my sample, I sort stocks into five quintile portfolios based on their sensitivities to market volatility risk, which are estimated from the time-series regressions in equations (2)–(4) using daily data over the past 12 months, so that portfolio L (H) contains stocks with the lowest (highest) volatility betas. I then compute the returns of these portfolios over the same 12-month period, using both equal-weighting and value-weighting schemes. Evaluating annual returns at the monthly frequency employs overlapping information and hence introduces autocorrelations in the residuals. Following Cremers, Halling, and Weinbaum (2015), I calculate t-statistics using Newey and West (1987) (NW) standard error with 12 lags.[11]

Table 2 reports the averages returns of quintile portfolios sorted on $\beta_{RV,t}$, $\beta_{IV,t}$, and $\beta_{EV,t}$, as well as the average returns and alphas of a hedge portfolio that is long the highest quintile portfolio and short the lowest quintile portfolio, denoted as "H-L". I consider several asset pricing models to compute alpha, including the CAPM (Sharpe, 1964), the Fama-French Three-Factor and Five-Factor models (Fama and French, 1993, 2015), and the q5 model (Hou, Mo, Xue, and Zhang, 2021). Panels A and C of Table 2 show that average stock returns exhibit a negative and statistically significant relationship with both realized and expected

[11] Because the NW standard error downweights higher-order autocorrelations, I also include more lags to be conservative. In this case, I find statistical significance increases for the H-L returns associated with β_{RV} and β_{EV}, and decreases for the H-L returns associated with β_{IV}.

volatility betas. For example, when sorting on $\beta_{RV,t}$ ($\beta_{EV,t}$), the average returns of equally weighted portfolios decrease monotonically from 20.4% (20.9%) per year for portfolio L to 12.1% (12.0%) per year for portfolio H. The return spreads are highly statistically significant with a Newey-West t-statistic of -5.149 and -5.371 respectively, and cannot be explained by existing asset pricing models. Panels A and C also confirm that the pricing of realized and expected market volatilities holds for value-weighted portfolios as well. A negative volatility risk premium is consistent with the economic intuition that volatility shocks are viewed as bad and investors are willing to pay a premium for hedging against increases in volatility.

In contrast, Panel B of Table 2 reports that there is no statistically significant relationship between stock returns and implied volatility betas. The average return first decreases and then increases with $\beta_{IV,t}$. The return differences between stocks with the highest and lowest sensitivities to changes in implied volatility are statistically insignificant. This result is somewhat surprising because it suggests that investors do not necessarily view increases in option implied volatility as being bad and do not pay a premium to hedge implied volatility risk. Section 4 contains a detailed discussion on the possible explanations and implications of the differential pricing of market volatility risks.

The focus of my analysis has been on the statistical significance of the differences in returns between the two extreme portfolios (H-L). However, the pricing of aggregate volatility risk would imply that expected returns should monotonically decrease with the exposure to volatility, and comparing only the average returns on the top and bottom portfolios is not sufficient to test for a monotonic relation between expected returns and volatility betas. I now formally test for monotonicity in average returns using the test in Patton and Timmermann (2010). The Patton and Timmermann test is non-parametric without specifying the functional form relating the sorting variable to expected returns, and easy to implement via bootstrap methods.[12]

The last column of Table 2 reports p-values for monotonicity tests on the average returns of quintile portfolios sorted by volatility betas. There is strong statistical evidence in favor of a monotonic return pattern for portfolios sorted by β_{RV} and β_{EV}, In contrast, the Patton and Timmermann test fails to find evidence in support of monotonicity in the average returns of portfolios sorted by β_{IV} with large p-values.

3.3 Fama-MacBeth Regressions

In this section, I estimate Fama and MacBeth (1973) cross-sectional regressions to control as comprehensively as possible for exposures to other systematic risk factors that have been shown in the existing literature to be related to expected stock returns. For each month in my sample, I estimate the following cross-sectional regressions:

$$R_t^i = \gamma_{0,t} + \gamma_{RV,t} * \beta_{RV,t}^i + \Phi_t * Z_t^i + \epsilon \tag{6}$$

$$R_t^i = \gamma_{0,t} + \gamma_{IV,t} * \beta_{IV,t}^i + \Phi_t * Z_t^i + \epsilon \tag{7}$$

$$R_t^i = \gamma_{0,t} + \gamma_{EV,t} * \beta_{EV,t}^i + \Phi_t * Z_t^i + \epsilon \tag{8}$$

where as before $\beta_{RV,t}^i$, $\beta_{IV,t}^i$, and $\beta_{EV,t}^i$ are the volatility betas with respect to realized, implied, and expected market volatilities, R_t^i is the contemporaneous 12-month stock excess return, and Z_t^i is a vector of factor loadings estimated over the same 12-month period.

Table 3 reports the time-series averages of the cross-sectional γ and Φ estimates, along with Newey and West (1987) t-statistics. Columns 1 to 3 report the results for the first

[12]To test a monotonically decreasing pattern, I first need to reverse the order of the portfolios. Following Patton and Timmermann (2010), let $\mathbf{\Delta} = [\Delta_4, \Delta_3, \Delta_2, \Delta_1]$ denote return differentials among the five quintile portfolios. For example, Δ_2 is the average return in the second quintile minus the average return in the third quintile. I follow Patton and Timmermann (2010) and test whether there is a monotonically increasing pattern under the null hypothesis of $\mathbf{\Delta} <= 0$.

specification where I estimate a univariate regression of stock returns against their volatility betas without any control. The slope coefficients on β_{RV} and β_{EV} are strongly negative and statistically significant, while the slope coefficient on β_{IV} is insignificant. For example, the slope coefficient on β_{RV} is -61.711 with a NW t-statistic of -4.945. The difference in β_{RV} between the two extreme value-weighted portfolios sorted by β_{RV} in Table 2 is 0.14%, which implies a decline of $0.14\% \times (-61.711) = 8.33\%$ per year in average returns if a stock were to move from the bottom portfolio to the top portfolio, other characteristics held constant. This estimate is very similar to the sorting result in Table 2.

Columns 4 to 6 show the results for the second specification in which I control for the exposure to the market factor in the CAPM. The coefficients on realized and expected volatility betas somewhat decrease in magnitude but are still highly significant. The coefficient on β_{IV} remains insignificant. Columns 7–9 consider the Fama-French Three-Factor Model and add exposures to size and value factors as additional controls. In this case, realized and expected volatility betas remain statistically significant, and interestingly implied volatility beta also becomes marginally significant. The final two specifications (columns 10–12 and 13–15) demonstrate that the strong negative relationship between stock returns and realized and expected volatility betas persists even after accounting for exposures to systematic risk factors in the Fama-French Five-Factor and the q-factor models. In contrast, there is no statistically significant relationship between stock returns and implied volatility beta.

Table 3 also shows that the slope coefficient on the market factor is positive and statistically significant, which is consistent with the CAPM prediction. Of course, this does not imply the CAPM holds because the variation in expected returns cannot be explained by the market beta alone.

In summary, the results in Table 3 confirm the sorting results in Table 2. Realized and expected volatilities are strongly and negatively priced in the cross-section of stock returns.

Stocks that perform well when realized or expected market volatility is high earn significantly lower average returns and this finding cannot be explained by exposure to other common risk factors. On the other hand, option implied volatility does not seem to be priced in the cross-section.

4 Discussion

Section 4.1 discusses a number of potential explanations for the differential pricing of volatility risks and concludes that it is likely to be due to partial segmentation between equity and options markets. Section 4.2 assesses the time varying integration of equity and options markets based on the correlation between option implied volatility and actual stock volatilities.

4.1 Explaining Differential Pricing of Volatility Risks

The previous section documents strong statistical evidence for the pricing of realized and expected market volatilities in the cross-section of stock returns, while finding no evidence that option implied market volatility risk is priced. These results suggest that investors view increases in realized and expected market volatilities as bad and pay a premium to hedge against them. On the other hand, shocks to option implied market volatility are not associated with high marginal utility.

The differential pricing of realized, implied, and expected market volatility risks is difficult to reconcile with standard theories of the volatility risk premium. In intertemporal asset pricing models (e.g., Merton, 1973; Campbell, 1993; Campbell, Giglio, Polk, and Turley, 2018), if market volatility is a state variable that drives the investment opportunity set, then investors would pay a premium to hedge against changes in market volatility because increasing volatility represents a deterioration in future investment opportunity set. Investment opportunities in the stock market can deteriorate either because expected stock returns decline or because the volatility of stock returns increases. As such, realized, implied, and expected market volatilities can carry different prices of risk if they relate to future returns and volatility differently. To test this hypothesis, I estimate predictive regressions in which I use three measures of market volatility to forecast future returns and volatility over different horizons (h) ranging from 1 month to 12 months:

$$r_{t \to t+h} = \alpha_{t+h} + \beta_{t+h} * x_t + \epsilon_{t+h}, \tag{9}$$

$$rv_{t \to t+h} = \alpha_{t+h} + \beta_{t+h} * x + \epsilon_{t+h}, \tag{10}$$

where $r_{t \to t+h}$ and $rv_{t \to t+h}$ are future returns and future realized volatility over the relevant horizons, and x is realized, option implied, or expected market volatility. I estimate the above predictive regressions at daily frequency and use Hansen and Hodrick (1980) standard errors to account for overlapping returns.

Table 4 reports the predictive regression results. First, realized, implied, and expected market volatilities are positively but insignificantly related to futures stock market returns with very low R^2, confirming the weak risk return trade off well documented in the literature (e.g., French, Schwert, and Stambaugh, 1987).[13] On the volatility forecasting side, all three measures of market volatility are strong predictors of future realized volatility. Overall Table 4 shows that realized, implied, and expected market volatilities are very similar in terms of forecasting future returns and volatility and therefore, according to intertemporal asset pricing models, should be priced similarly, which is inconsistent with the data.

Another theory of the volatility risk premium relates the pricing of volatility risk to

[13] While none of the volatility measures can predict future returns, Bollerslev, Tauchen, and Zhou (2009) show that the difference between option implied volatility and realized or expected volatility can predict quarterly index returns.

downside protection. Investors want to hedge changes in market volatility because of the so-called leverage effect: periods of high volatility tend to coincide with downward market movements. The leverage effect, however, cannot explain the differential pricing of realized, implied, and expected market volatilities. As Table 1 shows, option implied volatility has a more negative correlation with index returns than both realized and expected volatilities, and therefore we would expect, if anything, implied volatility should be more strongly priced with a larger risk premium.[14]

Option implied volatility, under perfect market assumptions, measures the expected volatility under the risk neutral measure so it reflects not only conditional volatility but also time varying risk aversion/premium. Option implied volatility might not be priced if the risk premium component moves in a way that makes implied volatility a noisy measure of true risk. However, this explanation is unlikely to be plausible because at the theoretical level virtually all models would predict that risk premium is in proportion to the risk, and risk neutral and physical volatilities should be highly correlated.[15] Empirically, I control for the effect of time varying risk version by projecting option implied volatility onto the risk aversion index (RA) in Bekaert, Engstrom, and Xu (2022) and focusing on the pricing of the residuals (e_t):

$$IV_t = a_t + b_t * RA_t + e_t. \tag{11}$$

If an insignificant price of risk for option implied volatility is caused by the noise associated with time varying risk aversion, then we would expect the residuals (e_t) to be priced after removing the risk aversion component. By construction, e_t is uncorrelated with risk aversion.

[14] Hu, Jacobs, and Seo (2019) demonstrate that covariance is a more relevant measure for the variance risk premium. In unreported results, I find that implied volatility also has a larger covariance (e.g., more negative) with index returns than realized and expected volatilities.

[15] See Dew-Becker, Giglio, Le, and Rodriguez (2017) for a related discussion.

Table A1 reports the average returns of quintile portfolios sorted by exposure to e_t. The results demonstrate that exposure to e_t does not earn a statistically significant risk premium. This implies that the lack of a significant risk premium for implied volatility is not due to the time varying risk aversion embedded in index options.

The differential pricing of market volatility risks is potentially consistent with partial segmentation between index options and equity markets. Friction and segmentation between equity and options markets may arise for a number of reasons. First, option implied volatility may not reflect the preferences and beliefs of the average investor because, unlike equity investment, option trading involves significant constraints (e.g., high transaction costs and margin requirement) that limit investor participation. Moreover, a growing literature (e.g., Barras and Malkhozov, 2016; Chen, Joslin, and Ni, 2019; Gârleanu, Pedersen, and Poteshman, 2009) suggests that financial intermediaries play a key role in the index options market and option prices are affected by financial intermediaries' ability to bear risk and take on leverage. Lastly, option implied volatility may deviate from true stock market volatility because of the manipulation in options market (Griffin and Shams, 2018), illiquidity or microstructure biases. For example, Andersen, Bondarenko, and Gonzalez-Perez (2015) identify large systematic biases in the VIX index due to the inclusion of illiquid options in calculating the index. Duarte, Jones, and Wang (2022) show that microstructure biases affect the inference regarding the volatility risk premium. In the presence of market segmentation, option implied volatility captures not only shocks to the economic fundamentals but also shocks that are specific to the options market. Option implied volatility is not priced in the cross-section of stock returns because equity investors may not want to hedge shocks that are specific to the options market.

4.2 Assessing Segmentation Between Equity and Options Markets

The previous section shows that the differential pricing of volatility risks cannot be explained by existing theories or time varying risk aversion, but is potentially consistent with partial segmentation between equity and index options markets. It follows that the degree of comovement between option implied volatility and volatility measures based on stock data may contain useful information about the extent to which equity and options markets are integrated. In particular, a stronger comovement between stock volatility and option implied volatility should be associated with a decreased level of segmentation between equity and options markets.

Figure 2 plots the 7-year rolling average of annual correlations of daily percentage changes in the three volatility measures (i.e., $corr(\frac{\Delta IV_{t+1}}{IV_t}, \frac{\Delta RV_{t+1}}{RV_t})$). I focus on percentage changes because the three volatility measures have different means. Results are similar using absolute changes. Panel A of Figure 2 shows the correlations of option implied volatility with both realized and expected volatilities. Not surprisingly, expected volatility has a higher correlation with option implied volatility than realized volatility because both implied and expected volatilities are forward looking. Moreover, although there is a difference in the magnitudes of the two correlations, their dynamics are remarkably similar. The correlation between option implied volatility and stock volatility is relatively stable in the 1990s. It then drops significantly in subsequent years, reaching the bottom around 2005. The most striking feature of the figure is that there is a dramatic increase in the correlation between option implied and stock volatilities late in the sample period, suggesting that equity and options markets have become more integrated in recent years. Dew-Becker and Giglio (2022) also draw similar conclusion that segmentation between equity and options markets has declined from comparing the returns of listed and synthetic options. Panel B of Figure 2 plots the correlation between realized and expected volatilities. The two measures are highly correlated with a correlation coefficient of around 0.9, and the correlation is quite stable over time.

Motivated by the correlation evidence, I revisit the pricing of option implied volatility in two subsamples: 1986 to 2009 and 2010 to 2020. Table 5 summarizes the results. Consistent with the conjecture that equity and options markets have become more integrated in recent years, I find option implied volatility is priced in the cross section of stock returns in the 2010–2020 sample. The return difference between stocks with the largest and smallest exposure to option implied volatility is −4.5% per year for equally weighted portfolios and −5.2% for value weighted portfolios with a Newey West t-statistic of −2.042 and −2.389, respectively.

5 Extensions and Robustness

This section contains several extensions and robustness results. Section 5.1 considers the pricing of several alternative measures of stock market volatility. Section 5.2 presents the results with sorting on past volatility betas. Section 5.3 formally tests whether the risk premiums associated the three volatility measures are the same. Section 5.4 discusses the variance risk premium. Section 5.5 investigates how the pricing of volatility risks varies across different stock characteristics. Section 5.6 investigates the robustness of the baseline results in Table 2 to a number of implementation choices.

5.1 Alternative Volatility Measures

Another popular approach to model financial market volatility is to use GARCH-type models. For important contributions, see Engle (1982), Bollerslev (1986), and Nelson (1991). While there are different specifications, for parsimony I consider a GARCH(1,1) model and an

EGARCH model that extends the GARCH(1,1) model to incorporate the leverage effect.[16] I estimate the two models using daily stock returns in my sample and plot the estimated conditional volatility in Panel A of Figure 3.

In the main analysis, estimates of expected volatilities are based on the model recommended in Bekaert and Hoerova (2014). A potential concern is that the model uses the VIX as a conditioning variable, which may lead to biased forecasts. To ensure the results with expected market volatilities are not driven by this, I also use the Heterogeneous Autoregressive (HAR) model of Corsi (2009) to estimate expected market volatilities. The HAR model relates future realized variance to past realized variances over the previous month, week, and day, and does not include the VIX as a predictor. The resulting estimates of expected volatilities implied from the HAR model are plotted in Panel B of Figure 3

Table 6 reports the sorting results for these alternative measures of stock market volatility. The three alternative volatility measures are all strongly and negatively priced in the cross-section of stock returns. Stocks with large exposure to market volatility risks earn significantly lower average returns. These results suggest that the pricing of market volatility risk is pervasive regardless of whether it is a measure of realized, expected or spot volatility. The analysis also highlights that the differential pricing of volatility risks has more to do with the data one uses to compute volatility. Measures that rely on stock return data are strongly priced whereas implied volatility, computed from option data, is not.

5.2 Portfolios Sorted on Past Volatility Betas

In the main analysis, I examine the pricing of volatility risk by focusing on the contemporaneous relation between realized volatility betas and realized average returns. I find higher

[16] Hansen and Lunde (2005) compare 330 GARCH-type models and find no evidence that more sophisticated models can outpferm a GARCH(1,1).

exposures to realized and expected market volatilities are contemporaneously correlated with significantly lower average returns, whereas the relationship between option implied volatility betas and average returns is statistically insignificant. This section investigates how the average returns vary as a function of lagged volatility betas. Using lagged volatility betas to form portfolios is attractive because it reflects an implementable trading strategy. In particular, at the end of each month t, I sort stocks into quintile portfolios according to their volatility betas measured over the past 12 months ($t-11$ to t) and then calculate both equal-weighted and value-weighted average returns of these portfolios over the subsequent 12-month period ($t+1$ to $t+12$). Table 7 reports the results. When sorting on past volatility betas, the return spreads are small and none of them is statistically significant. This suggests that past volatility betas are not informative about future returns.

The insignificant relationship between past volatility betas and future stock returns is due to the fact that simply sorting on past volatility betas does not generate enough variation in future volatility exposures to provide significant spreads in future returns. To see this point, Figure 4 compares the value-weighted average pre-formation volatility betas of the quintile portfolios sorted by the three volatility betas with the corresponding average post-formation volatility betas, which are measured over the subsequent 12-month period. By construction, there is a monotonically increasing pattern in the pre-formation betas (the blue bars) with large spreads. In contrast, post-formation betas exhibit very little dispersion. For example, Panel C shows that the difference in the pre-formation volatility beta between first and fifth quintile portfolios sorted by β_{EV} is 0.657% (0.260% $-$ ($-$0.397%)). However, the corresponding difference in the post-formation betas is only 0.068% ($-$0.024% $-$ ($-$0.092%), a decrease of 90%. Those results suggest that volatility betas are highly time varying and pre-formation betas are poor predictors of future volatility betas.[17]

[17]Focusing on 1-month window, Ang, Hodrick, Xing, and Zhang (2006) also find very small spread in

5.3 Testing Equality of Risk Premiums

The main analysis shows that the risk premium (i.e., the H-L returns) associated with exposures to realized and expected market volatilities are negative and significantly different from zero, whereas the risk premium associated with the exposure to option implied volatility is much smaller and insignificantly different from zero. This, of course, does not imply that the risk premia are significantly different from each other. In this section, I formally test whether the risk premia for different volatility measures are the same statistically.

Table 8 computes the differences in the H-L returns associated with portfolio sorts in Table 2, and test whether they are statistically significant. Panel A shows that for equal-weighted portfolios, the risk premium for realized volatility (expected volatility) is −8.3% (−8.9%) per year, whereas the risk premium for option implied volatility is −3% per year. The resulting differences are −5.3% and −5.9%, and highly significant (t-stat=−4.71 and −5.21 respectively). In contrast, the difference in the risk premium for realized and expected volatility is small and statistically insignificantly different from zero (t-stat=0.70). Panel B confirms that similar results obtain for value-weighted portfolios. In summary, the results suggest that the risk premium for option implied volatility is significantly different from that for realized and expected volatilities.

5.4 The Variance Risk Premium

The variance risk premium, defined as the difference between the risk-neutral and physical expectations of future return variance, has recently received considerable attention in the literature because it reveals investor preferences towards variance risk and provides additional insight about the pricing of variance risk (see, among others, Bollerslev, Tauchen, and

post-formation VIX betas.

Zhou, 2009; Carr and Wu, 2009; Drechsler and Yaron, 2011; Bekaert and Engstrom, 2017). Empirically, it is common to use option implied variance as a proxy for the risk neutral expected variance and either lagged realized variance or a forecast from some statistical model as a proxy for the corresponding physical expectation.

Figure 5 shows two measures of the volatility risk premium over the sample period from 1986 to 2020: Panel A plots the difference between option implied volatility and realized volatility, and Panel B plots the difference between option implied volatility and expected volatility. Both measures of the volatility risk premium vary substantially over time, which suggests that the compensation for bearing volatility risk (i.e., selling variance) is likely to be time varying. Consistent with Bekaert and Hoerova (2014) and Cheng (2018), Figure 5 also highlights that the behavior of the volatility risk premium is somewhat puzzling in that it becomes less positive or even turn negative when risk is high.[18]

My analysis suggests that the puzzling dynamics of the variance risk premium might be partly due to the fact that option implied volatility is an imprecise indicator of risk neutral volatility in the presence of market segmentation. When the options and equity markets are segmented, option implied volatility captures not only genuine risk neutral volatility but also incorporates shocks specific to the options market. Consequently, this leads to imprecise estimates of the volatility risk premium.

[18]This result is puzzling because leading models in Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) would predict that the volatility risk premium is always positive and becomes larger when variance of variance or jump risk is high.

5.5 How the Pricing of Volatility Risk Varies Across Characteristics

This section studies how the pricing of market volatility risk varies across different stock characteristics via double sorts. Each month I first form five quintile portfolios based on some stock characteristic, and then within each quintile stocks are further sorted into five quintile portfolios according to their exposures to realized and expected volatilities(β_{RV} and β_{EV}). I consider a variety of well known characteristics including size, book-to-market, total volatility, volume, liquidity, coskewness, and cokurtosis.[19] For brevity, I report double sorting results only for value-weighted portfolios. The results with equally weighted portfolios are very similar.

Panel A of Table A2 reports the difference in average returns between the two extreme portfolios with the highest and lowest β_{RV} within each quintile portfolio sorted by characteristics. The results suggest that the pricing of realized volatility is pervasive. Stocks with highest sensitivities to realized market volatility continue to earn significantly low average returns across most characteristic quintiles. Moreover, the effect seems stronger among stocks that are small and have high volume and volatility. For example, the H-L return difference is -15.7% per year among the smallest stocks whereas it drops, in magnitude, to -4.3% per year among stocks with the largest market cap. Pane B of Table A2 reports the corresponding results for expected market volatility. The pricing of expected volatility is overall similar to that of realized volatility.

[19] Size and book-to-market are measured at the beginning of each annual period while total volatility, volume, liquidity, coskewness, and cokurtosis are calculated contemporaneously over the same 12-month period.

5.6 Robustness

This section examines whether the main results reported in Table 2 are robust to a number of implementation choices. The main analysis estimates volatility betas and evaluates returns over rolling periods of 12 months. In this section, I consider an alternative estimation window of 6 months. Table 9 confirms that the empirical results are not sensitive to the estimation window. Realized and expected market volatilities are strongly priced in the cross-section of stock returns with a negative price of risk while implied volatility is not priced. For brevity, Table 9 only reports the findings for value-weighted portfolios. The results for equal-weighted portfolios are similar and contained in Table A3.

The main analysis excludes stocks with price below \$3. To further reduce the effect of penny stocks, this section repeats the sorting exercise by excluding stocks with an average price below \$5 over the previous year. Consistent with the benchmark results, I find that average stock returns exhibit a negative and statistically significant relationship with β_{RV} and β_{EV}, whereas the relationship between stock return and β_{IV} is not significant.

To ensure the empirical findings are not driven by a few potential outliers, this section repeats the sorting exercise by excluding the Great Recession (December 2007 to June 2009). Table 9 shows that removing the Great Recession from the sample does not affect the conclusion. Confirming the benchmark results, stocks with high β_{RV} and β_{EV} continue to earn significantly lower average returns, but those with high implied volatility betas do not. In unreported results, I also examine a sample without the 1987 market crash and the conclusion remains the same.

Moreover, I use non-overlapping annual observations by implementing portfolio sorts once a year instead of every month. In particular, at the end of each June, I sort stock into different quintile portfolios based on their volatility betas over the previous year and compute both equal and value weighted returns of those portfolios contemporaneously. Table 9 demonstrates that the empirical findings are robust to considering non-overlapping observations. Consistent with the baseline results, there is a negative and statistically significant relationship between exposures to realized and expected market volatilities and average stock returns, whereas the relationship between β_{IV} and average stock returns is not significant.

Lastly, I implement a VAR(1) to extract innovations to the three volatility measures. The VAR model takes into account the contemporaneous correlations among the three volatility measures and presumably can better identify the innovations that are unique to each of the three variables. The estimated VAR(1) model over the full sample is given as:

$$\begin{bmatrix} RV_{t+1} \\ IV_{t+1} \\ EV_{t+1} \end{bmatrix} = \begin{bmatrix} 0.43209 & 0.05049 & 0.45930 \\ -0.05569 & 0.90601 & 0.18177 \\ -0.03853 & 0.02985 & 0.99241 \end{bmatrix} \begin{bmatrix} RV_{t+1} \\ IV_{t+1} \\ EV_{t+1} \end{bmatrix} + \begin{bmatrix} \epsilon_{RV,t+1} \\ \epsilon_{IV,t+1} \\ \epsilon_{EV,t+1} \end{bmatrix}.$$

Table 9 shows the results are similar when using volatility innovations estimated from the VAR model.

6 Conclusion

Stock market volatility is one of the most important quantity in finance, playing a critical role in many applications such as calculating cost of capital, option valuation, and risk management, to name a few. Common measures of stock market volatility share similar movements, but they are not perfectly correlated. Over the sample period from 1986 to 2020, I show that realized and expected market volatilities are negatively priced in the cross-section of equity returns. Stocks that perform well when realized or expected market volatility is high earn significantly lower average returns contemporaneously. On the other hand, option implied volatility is not priced. Stocks with large exposure to implied volatility do not earn a significant risk premium.

I further show that the differential pricing of market volatility risks is difficult to reconcile with existing theories that relate the volatility risk premium to either hedging shifts in investment opportunities or downside protection. It is also not due to the time varying risk aversion embedded in options. Instead, the findings are more consistent with partial segmentation and friction between options and equity markets. Option implied volatility is not priced because it reflects not only shocks to fundamentals, but also shocks specific to the options market which equity investors may not want to hedge.

I also use the comovement between option implied and actual stock volatilities to shed light on the time varying integration of equity and options markets. I find a substantial increase in the correlation between option implied and actual stock volatilities towards the end of the sample, suggesting that market segmentation has declined in recent years.

The current paper can be extended in several ways. First, it is critically important to understand the economic forces driving the correlation between stock and option implied volatilities.[20] Second, I have so far focused on the second moment, and extensions to comparing the pricing of option implied and actual stock higher moments risks (e.g., skewness) would be useful. I plan to address these in future research.

[20]For example, it may be interesting to study how the correlation is related to cost of arbitrage, changes in the transaction costs in the options market (Muravyev and Pearson, 2020), or the rise of retail trading and wholesalers in the options market (Bryzgalova, Pavlova, and Sikorskaya, 2022).

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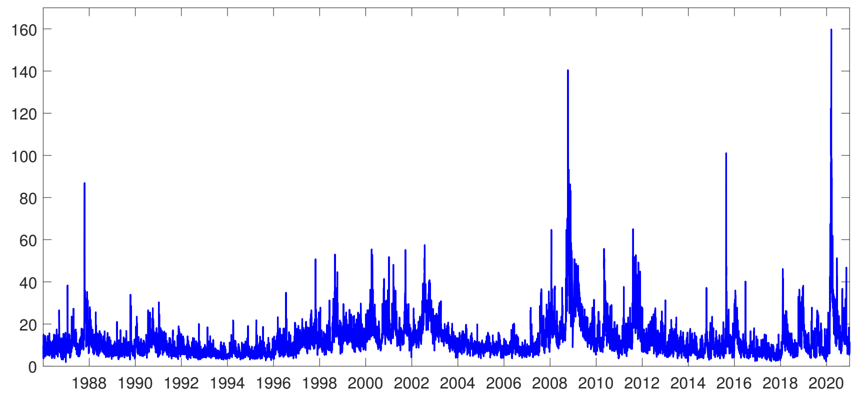
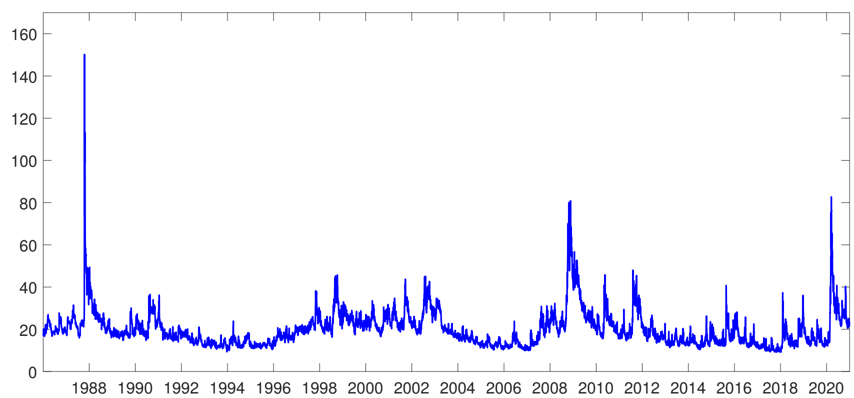
Figure 1: Time Series of Different Measures of Stock Market Volatility

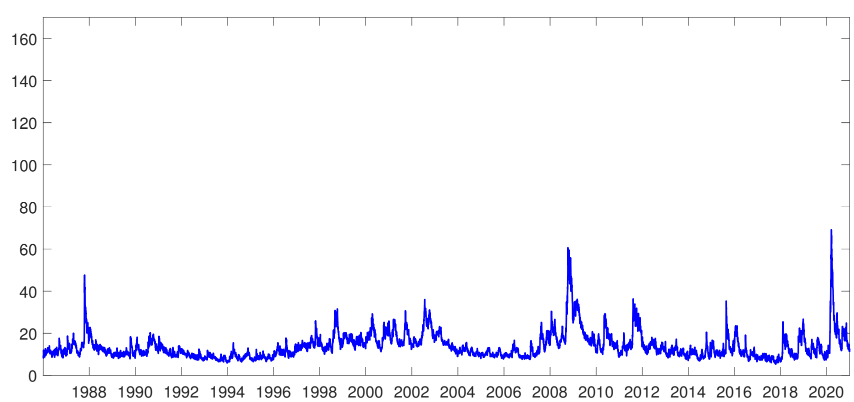
Panel A: *RV*



Panel B: *IV*



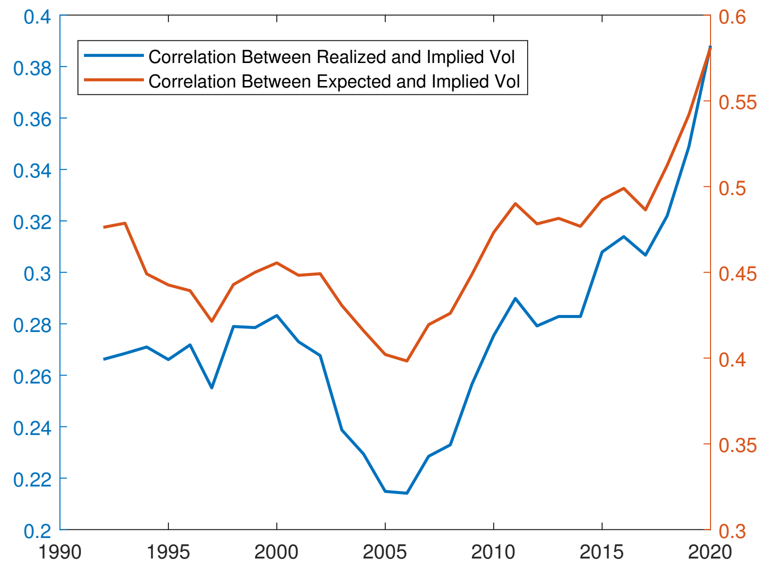
Panel C: *EV*



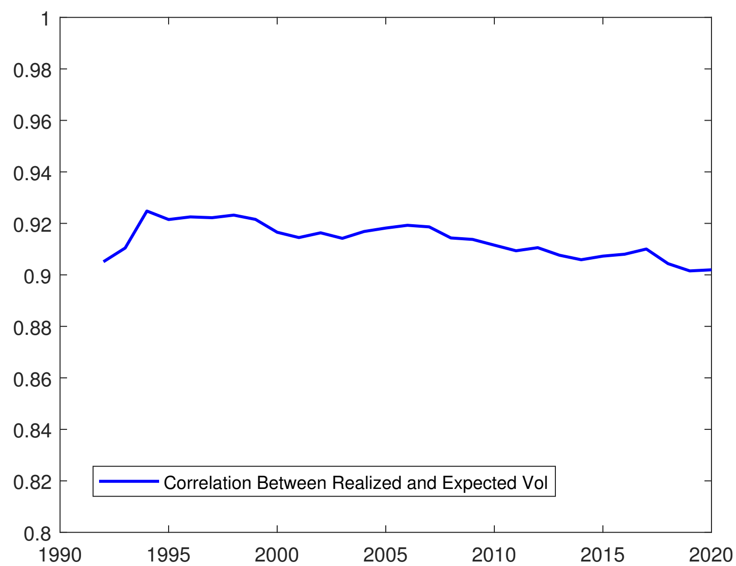
Notes: This figure plots daily realized (Panel A), option implied (Panel B), and expected volatilities (Panel C) of the S&P 500 index. Sample period: January 1986 to December 2020.

Figure 2: Time Series of Correlations of Different Stock Market Volatilities

Panel A: Correlation of IV with RV and EV



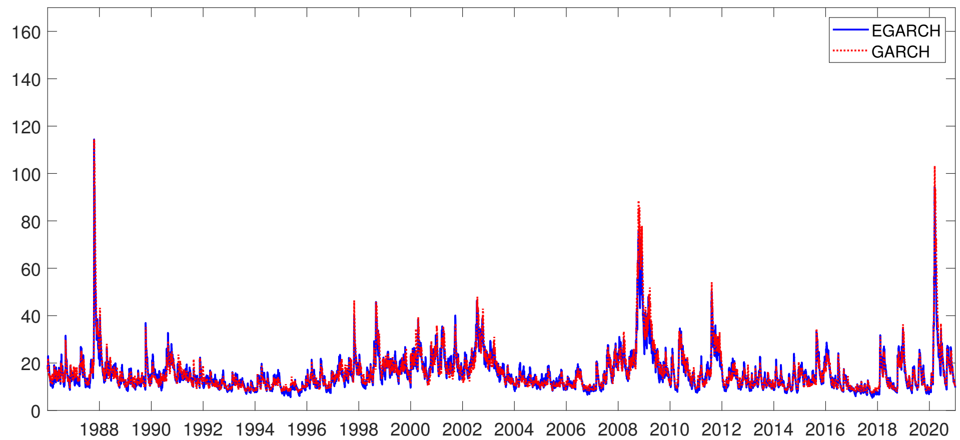
Panel B: Correlation Between RV and EV



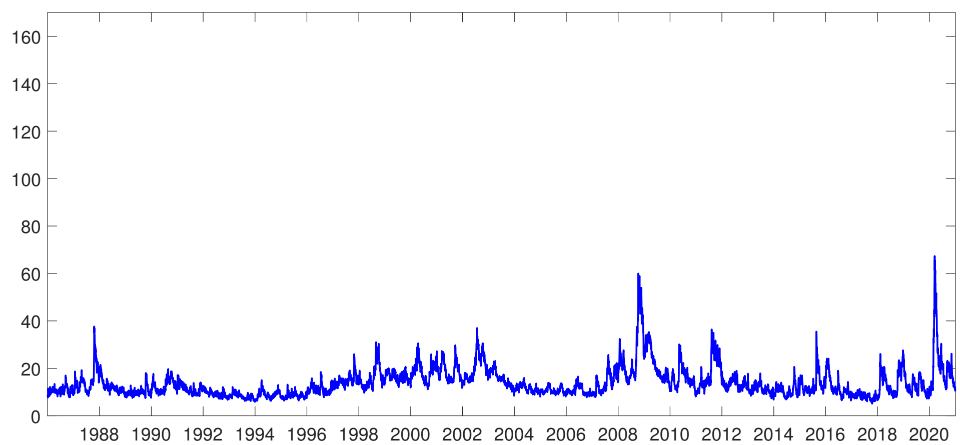
Notes: This figure plots 7-year rolling average of annual correlations of realized, option implied, and expected volatilities.

Figure 3: Alternative Measures of Stock Market Volatility

Panel A: *GARCH* and *EGARCH*

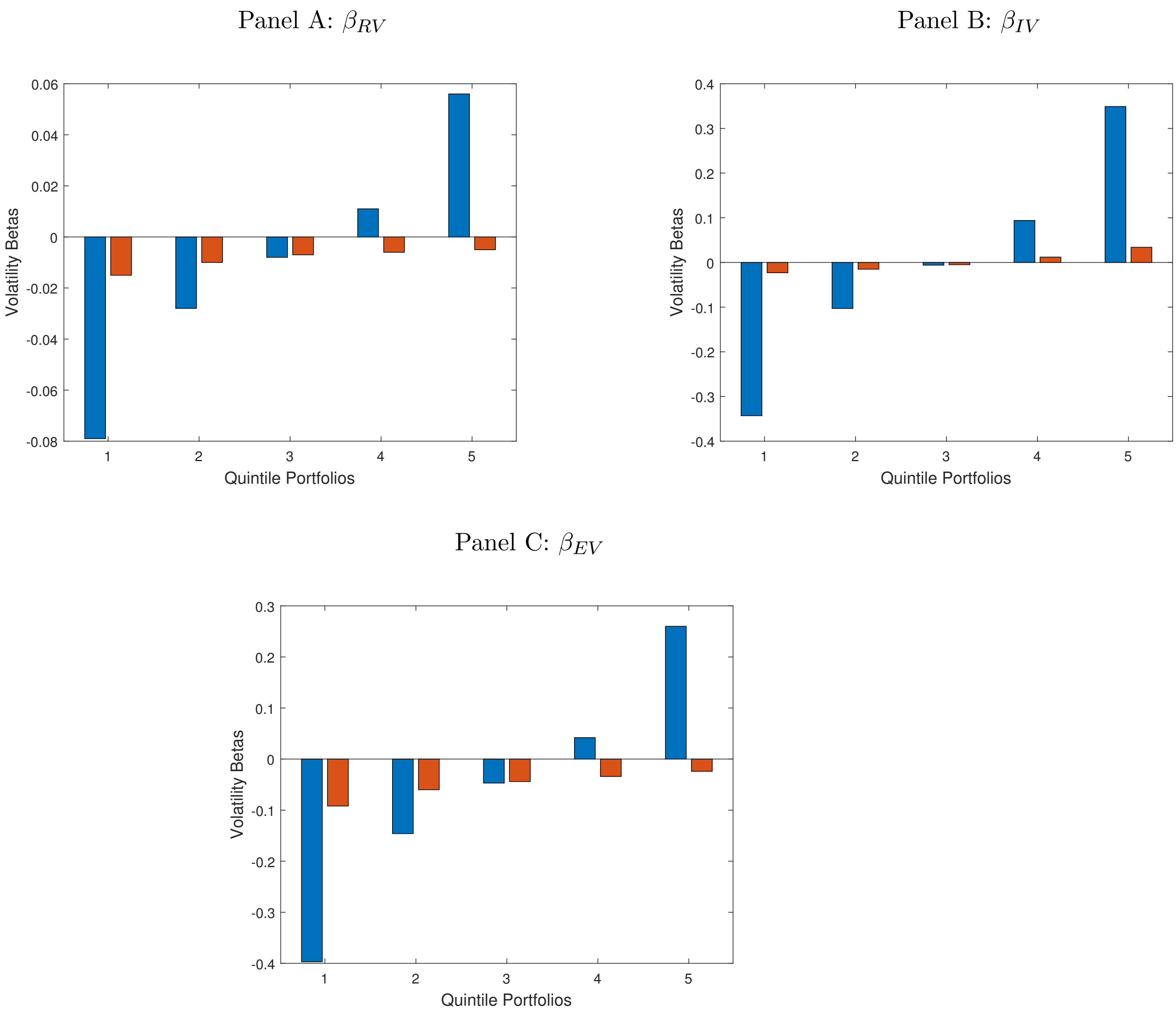


Panel B: *HAR*



Notes: Panel A plots volatility estimates computed from GARCH and EGARCH models. Panel B plots volatility estimates from the HAR model of Corsi (2009). Sample period: January 1986 to December 2020.

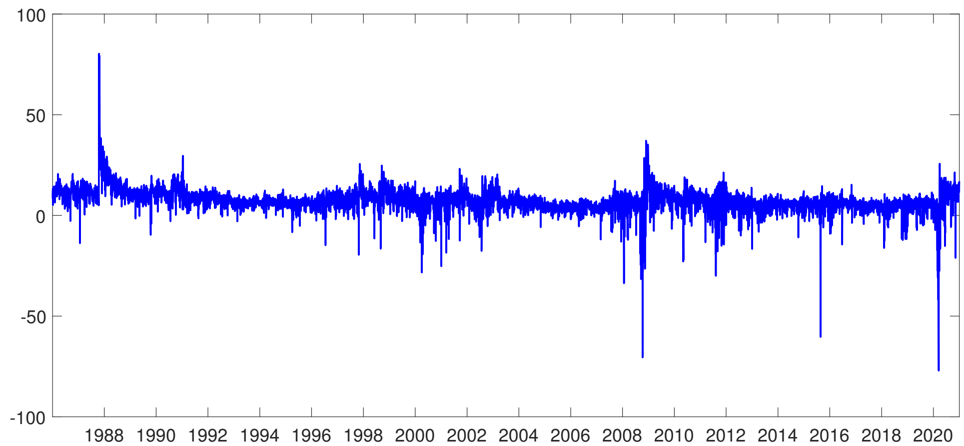
Figure 4: Pre- and Post-Formation Volatility Betas



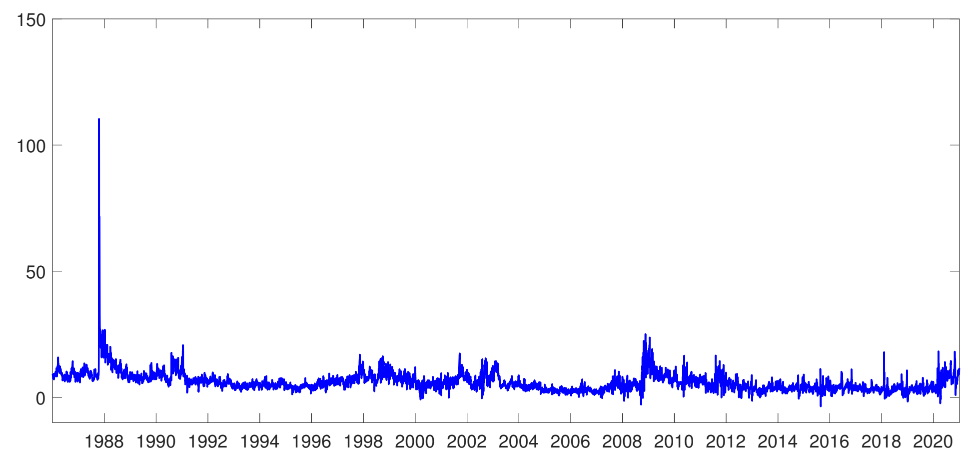
Notes: The blue (orange) bars plot the value-weighted average pre-formation (post-formation) volatility betas of the quintile portfolios sorted by β_{RV}, β_{IV} and β_{EV} in Table 2. The volatility betas are in percent.

Figure 5: Volatility Risk Premium

Panel A: $IV - RV$



Panel B: $IV - EV$



Notes: Panel A plots the difference between option implied volatility and realized volatility. Panel B plots the difference between option implied volatility and expected volatility. Sample period: January 1986 to December 2020.

Table 1: Descriptive Statistics

Panel A: Summary Stats of Market Volatilities				
	Mean	Std	Skew	Kurt
RV	12.447	8.970	4.022	31.197
IV	19.922	8.508	3.021	21.207
EV	13.834	6.253	2.642	11.699
Panel B: Correlations				
	ΔRV	ΔIV	ΔEV	SP
ΔRV	1.000			
ΔIV	0.313	1.000		
ΔEV	0.899	0.499	1.000	
SP	-0.246	-0.700	-0.400	1.000
Panel C: Summary Stats of Volatility Betas				
	Mean	Std	Skew	Kurt
β_{RV}	-0.011	0.001	-0.491	157.493
β_{IV}	-0.005	0.004	1.104	107.111
β_{EV}	-0.069	0.004	-0.461	182.040

Notes: Panel A reports mean, standard deviation (Std), skewness (Skew), and kurtosis (Kurt) of realized, option implied, and expected market volatilities. Panel B reports the correlations of changes in the three measures of stock market volatility as well as their correlations with index returns. Sample period: January 1986 to December 2020. Panel C reports the time series averages of the cross-sectional mean (in percent), standard deviation, skewness, and kurtosis of volatility betas, which are estimated at the monthly frequency based on daily data over the past 12 months. Sample period: December 1986 to December 2020.

Table 2: Average Returns of Portfolios Sorted on Volatility Betas

Panel A: Sorting on β_{RV}											
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α	Monotonicity Test
EW	0.204	0.147	0.130	0.125	0.121	-0.083***	-0.081***	-0.081***	-0.081***	-0.083***	$p = 0.007$
						(-5.149)	(-5.077)	(-5.116)	(-5.107)	(-5.423)	
VW	0.190	0.143	0.128	0.123	0.112	-0.078***	-0.077***	-0.077***	-0.076***	-0.079***	$p = 0.000$
						(-5.155)	(-5.081)	(-5.116)	(-5.133)	(-5.327)	
Panel B: Sorting on β_{IV}											
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α	Monotonicity Test
EW	0.178	0.142	0.129	0.130	0.148	-0.030	-0.031	-0.031	-0.029	-0.042*	$p = 0.479$
						(-1.189)	(-1.224)	(-1.231)	(-1.141)	(-1.905)	
VW	0.168	0.140	0.128	0.126	0.134	-0.034	-0.035	-0.035	-0.032	-0.045**	$p = 0.249$
						(-1.315)	(-1.352)	(-1.354)	(-1.249)	(-2.040)	
Panel C: Sorting on β_{EV}											
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α	Monotonicity Test
EW	0.209	0.148	0.127	0.124	0.120	-0.089***	-0.087***	-0.087***	-0.089***	-0.090***	$p = 0.004$
						(-5.371)	(-5.345)	(-5.408)	(-5.278)	(-5.516)	
VW	0.195	0.144	0.125	0.121	0.111	-0.084***	-0.082***	-0.082***	-0.083***	-0.085***	$p = 0.000$
						(-5.321)	(-5.288)	(-5.348)	(-5.242)	(-5.383)	

Notes: Each month I sort stocks into both equally-weighted (EW) and value-weighted (VW) quintile portfolios based on volatility betas (β_{RV}, β_{IV}, and β_{EV}) estimated over the past 12 months using daily data, and calculate the contemporaneous returns of these portfolios over the same 12-month period. The table reports the time series average returns of those portfolios as well as the return differences between the two extreme quintile portfolios, denoted as "H-L", and the corresponding alphas. The Newey-West t-statistics with 12 lags are reported in brackets. The last column reports the p-values of the monotonicity test in Patton and Timmermann (2010).

Table 3: Fama MacBeth Regressions

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
β_{RV}	-61.711***			-48.241***			-27.658***			-24.464***			-38.717***		
	(-4.945)			(-4.275)			(-2.709)			(-2.602)			(-4.781)		
β_{IV}		-3.752			-3.967			-3.818*			-2.609			-0.841	
		(-1.099)			(-1.328)			(-1.841)			(-1.304)			(-0.381)	
β_{EV}			-12.626***			-9.759***			-5.476**			-4.680**			-7.129***
			(-5.347)			(-4.333)			(-2.488)			(-2.281)			(-4.195)
β_{MKT}				0.053**	0.054**	0.053**	0.061**	0.063***	0.062**	0.082***	0.082***	0.082***	0.086***	0.085***	0.086***
				(2.347)	(2.504)	(2.339)	(2.584)	(2.629)	(2.584)	(3.221)	(3.205)	(3.221)	(2.922)	(2.909)	(2.926)
β_{SIZE}							0.033*	0.037*	0.032	0.017	0.021	0.017	0.026	0.028	0.026
							(1.668)	(1.931)	(1.632)	(1.006)	(1.243)	(0.994)	(1.311)	(1.473)	(1.294)
β_{HML}							-0.055**	-0.052**	-0.055**	-0.057**	-0.054**	-0.057**			
							(-2.211)	(-2.077)	(-2.174)	(-2.409)	(-2.271)	(-2.385)			
β_{RMW}										-0.019	-0.018	-0.018			
										(-0.763)	(-0.754)	(-0.748)			
β_{CMA}										-0.007	-0.006	-0.007			
										(-0.486)	(-0.391)	(-0.466)			
β_{IA}													-0.011	-0.010	-0.011
													(-0.715)	(-0.657)	(-0.702)
β_{ROE}													0.037**	0.035**	0.037**
													(2.403)	(2.229)	(2.400)
β_{RV}													0.024*	0.024*	0.024*
													(1.840)	(1.740)	(1.862)
$Adj.R^2$	0.70%	1.00%	0.70%	3.10%	3.20%	3.20%	7.60%	7.50%	7.70%	9.40%	9.40%	9.50%	9.30%	9.30%	9.40%

Notes: This table reports the results of the Fama-MacBeth regressions of stock returns against exposures to the three measures of stock market volatility risk as well as exposures to other systematic risk factors in leading asset pricing models including the CAPM, the Fama-French Three- and Five-Factor models, and the q5 model.

Table 4: Forecasting Future Index Returns and Volatility

Panel A: *RV*											
	Forecasting Future Returns						Forecasting Future Volatility				
Horizon	1	2	3	6	12	Horizon	1	2	3	6	12
Intercept	0.067	0.061	0.066	0.070	0.094	Intercept	16.964	15.450	14.470	12.521	9.721
t	(1.761)	(1.690)	(1.868)	(2.034)	(2.682)	t	(13.869)	(11.625)	(10.620)	(9.115)	(7.494)
Slope	0.002	0.002	0.002	0.002	0.000	Slope	2.206	1.331	0.962	0.534	0.332
t	(0.884)	(1.097)	(0.957)	(0.867)	(-0.149)	t	(29.814)	(17.089)	(12.364)	(7.266)	(4.641)
R^2	0.10%	0.31%	0.32%	0.44%	0.01%	R^2	53.11%	41.57%	34.53%	24.00%	18.86%
Panel B: *IV*											
	Forecasting Future Returns						Forecasting Future Volatility				
Horizon	1	2	3	6	12	Horizon	1	2	3	6	12
Intercept	-0.008	-0.007	0.006	0.021	0.064	Intercept	1.060	5.823	7.506	8.838	8.051
t	(-0.138)	(-0.124)	(0.099)	(0.383)	(1.191)	t	(0.407)	(2.308)	(3.062)	(3.837)	(3.841)
Slope	0.005	0.005	0.004	0.003	0.001	Slope	2.178	1.316	0.952	0.520	0.290
t	(1.828)	(1.865)	(1.665)	(1.447)	(0.558)	t	(18.352)	(11.531)	(8.629)	(5.117)	(3.133)
R^2	0.63%	1.24%	1.43%	1.91%	0.49%	R^2	46.59%	36.61%	30.41%	20.47%	14.39%
Panel C: *EV*											
	Forecasting Future Returns						Forecasting Future Volatility				
Horizon	1	2	3	6	12	Horizon	1	2	3	6	12
Intercept	0.035	0.029	0.043	0.049	0.094	Intercept	-2.070	3.539	5.641	7.451	6.540
t	(0.600)	(0.512)	(0.805)	(0.957)	(1.819)	t	(-1.071)	(1.840)	(2.942)	(3.989)	(3.699)
Slope	0.004	0.004	0.003	0.003	0.000	Slope	3.361	2.060	1.505	0.848	0.530
t	(1.089)	(1.239)	(0.976)	(0.918)	(-0.085)	t	(26.609)	(16.488)	(12.164)	(7.208)	(4.694)
R^2	0.22%	0.55%	0.48%	0.74%	0.00%	R^2	59.95%	48.43%	41.13%	29.50%	24.34%

Notes: This table reports the results for the predictive regressions of future returns and realized volatility over different horizons (ranging from 1-month to 12-month) against the three measures of stock market volatility. The t-statistics are based on Hansen and Hodrick (1980) standard error.

Table 5: Pricing of Option Implied Volatility: Subsample Analysis

Panel A: December 1986 to December 2009										
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α
EW	0.183	0.148	0.133	0.133	0.160	-0.023	-0.024	-0.025	-0.024	-0.039
						(-0.643)	(-0.676)	(-0.695)	(-0.663)	(-1.269)
VW	0.168	0.143	0.129	0.128	0.143	-0.025	-0.027	-0.027	-0.025	-0.041
						(-0.689)	(-0.726)	(-0.735)	(-0.685)	(-1.303)
Panel B: January 2010 to December 2020										
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α
EW	0.168	0.129	0.122	0.124	0.123	-0.045**	-0.045**	-0.044*	-0.041*	-0.045**
						(-2.042)	(-2.045)	(-1.966)	(-1.887)	(-2.048)
VW	0.167	0.132	0.123	0.124	0.114	-0.052**	-0.052**	-0.052**	-0.049**	-0.053**
						(-2.389)	(-2.390)	(-2.304)	(-2.238)	(-2.379)

Notes: This table reports the sorting results for option implied volatility in two subsamples: December 1986 to December 2009 and January 2010 to December 2020.

Table 6: Portfolio Sorts: Alternative Volatility Measures

Panel A: GARCH						
	L	2	3	4	H	H-L
EW	0.218	0.150	0.127	0.120	0.112	-0.106***
						(-4.143)
VW	0.204	0.147	0.125	0.118	0.102	-0.101***
						(-4.091)
Panel B: EGARCH						
	L	2	3	4	H	H-L
EW	0.196	0.142	0.128	0.130	0.131	-0.065***
						(-5.599)
VW	0.184	0.139	0.127	0.126	0.119	-0.064***
						(-5.554)
Panel C: HAR						
	L	2	3	4	H	H-L
EW	0.208	0.148	0.126	0.124	0.121	-0.087***
						(-4.686)
VW	0.193	0.144	0.124	0.121	0.112	-0.081***
						(-4.613)

Notes: This table reports the average returns of both equally-weighted (EW) and value-weighted (VW) quintile portfolios sorted on exposures to alternative volatility measures computed from GARCH, EGARCH, and HAR models.

Table 7: Portfolios Sorted on Past Volatility Betas

Panel A: Sorting on Past β_{RV}										
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α
EW	0.108	0.111	0.113	0.116	0.117	0.009	0.009	0.008	0.006	0.002
						(0.806)	(0.785)	(0.767)	(0.562)	(0.225)
VW	0.108	0.111	0.113	0.117	0.117	0.010	0.010	0.010	0.007	0.003
						(0.886)	(0.865)	(0.847)	(0.660)	(0.317)
Panel B: Sorting on Past β_{IV}										
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α
EW	0.107	0.111	0.111	0.117	0.119	0.011	0.011	0.011	0.015	0.016
						(0.900)	(0.855)	(0.938)	(1.208)	(1.362)
VW	0.108	0.112	0.112	0.117	0.118	0.011	0.010	0.011	0.014	0.015
						(0.812)	(0.767)	(0.847)	(1.122)	(1.276)
Panel C: Sorting on Past β_{EV}										
	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α
EW	0.110	0.113	0.113	0.115	0.113	0.003	0.003	0.002	0.001	-0.001
						(0.302)	(0.260)	(0.215)	(0.079)	(-0.139)
VW	0.110	0.113	0.113	0.116	0.114	0.004	0.004	0.003	0.002	-0.001
						(0.367)	(0.321)	(0.278)	(0.164)	(-0.064)

Notes: Each month I sort stocks into both equally-weighted (EW) and value-weighted (VW) quintile portfolios based on volatility betas (β_{RV}, β_{IV}, and β_{EV}) estimated over (t-11, t) and calculate the returns of these portfolios over the following 12-month period (t+1, t+12). The table reports the time series average returns of those portfolios as well as the return differences between the two extreme quintile portfolios, denoted as "H-L", and the corresponding alphas. The Newwey-West t-statistics with 12 lags are reported in brackets.

Table 8: Testing Equality of the Volatility Risk Premiums

Panel A: H-L Returns of EW Portfolios					
β_{RV}	-0.083	β_{EV}	-0.089	β_{RV}	-0.083
β_{IV}	-0.030	β_{IV}	-0.030	β_{EV}	-0.089
Diff	-0.053	Diff	-0.059	Diff	0.006
t	(-4.71)	t	(-5.21)	t	(0.70)
Panel B: H-L Returns of VW Portfolios					
β_{RV}	-0.078	β_{EV}	-0.084	β_{RV}	-0.078
β_{IV}	-0.034	β_{IV}	-0.034	β_{EV}	-0.084
Diff	-0.044	Diff	-0.050	Diff	0.006
t	(-3.95)	t	(-4.42)	t	(0.66)

Notes: This table compares the H-L returns from Table 2 and reports the t-statistics that test whether they are significantly different from each other.

Table 9: Robustness

Panel A: Sorting on β_{RV}						
	L	2	3	4	H	H-L
6-Month	0.091	0.069	0.063	0.065	0.062	-0.029***
						(-4.237)
Excluding Great Recession	0.212	0.160	0.143	0.139	0.130	-0.081***
						(-5.281)
Excluding Penny Stocks	0.206	0.148	0.131	0.127	0.123	-0.083***
						(-5.347)
Non-Overlapping Returns	0.178	0.133	0.122	0.107	0.089	-0.088***
						(-4.041)
VAR	0.192	0.144	0.125	0.123	0.112	-0.079***
						(-4.478)
Panel B: Sorting on β_{IV}						
	L	2	3	4	H	H-L
6-Month	0.083	0.069	0.063	0.063	0.071	-0.011
						(-1.153)
Excluding Great Recession	0.185	0.155	0.143	0.144	0.157	-0.028
						(-1.058)
Excluding Penny Stocks	0.183	0.144	0.132	0.132	0.146	-0.037
						(-1.361)
Non-Overlapping Returns	0.143	0.128	0.118	0.115	0.125	-0.019
						(-1.153)
VAR	0.171	0.142	0.126	0.125	0.131	-0.039
						(-1.565)
Panel C: Sorting on β_{EV}						
	L	2	3	4	H	H-L
6-Month	0.093	0.069	0.062	0.063	0.062	-0.030***
						(-4.247)
Excluding Great Recession	0.216	0.162	0.140	0.137	0.129	-0.087***
						(-5.461)
Excluding Penny Stocks	0.212	0.149	0.129	0.125	0.122	-0.090***
						(-5.507)
Non-Overlapping Returns	0.179	0.139	0.115	0.107	0.090	-0.089***
						(-4.352)
VAR	0.193	0.143	0.126	0.121	0.113	-0.080***
						(-5.139)

Notes: This table repeats the sorting exercise in Table 2 by estimating volatility betas over rolling 6-month period, excluding the Great Recession (December 2007 to June 2009), removing stocks with an average price below \$5 in the previous year, considering non-overlapping observations, and using a VAR(1) model to extract volatility shocks. Portfolio returns are based on value-weighting.

Online Appendix

Table A1: Controlling for Risk Aversion

	L	2	3	4	H	H-L	CAPM α	FF3 α	FF5 α	q5 α
EW	0.176	0.141	0.129	0.132	0.150	-0.025	-0.026	-0.026	-0.024	-0.036*
						(-1.014)	(-1.033)	(-1.047)	(-0.954)	(-1.653)
VW	0.165	0.138	0.127	0.128	0.136	-0.029	-0.030	-0.030	-0.027	-0.040*
						(-1.135)	(-1.156)	(-1.165)	(-1.055)	(-1.778)

Notes: This table reports average returns of both equally-weighted (EW) and value-weighted (VW) quintile portfolios sorted on exposure to the residuals from the regression of option implied volatility onto the time varying risk aversion index of Bekaert, Engstrom, and Xu (2022).

Table A2: Portfolios Double Sorted by Stock Characteristics and Volatility Betas

Panel A: Portfolios Double Sorted by Characteristics and β_{RV}					
	1	2	3	4	5
size	-0.157***	-0.085***	-0.068***	-0.063***	-0.043***
	(-4.122)	(-3.463)	(-3.870)	(-4.161)	(-2.717)
btm	-0.092***	-0.078***	-0.087***	-0.093***	-0.109***
	(-4.565)	(-4.072)	(-4.700)	(-5.435)	(-4.028)
vol	-0.009	-0.016*	-0.043***	-0.083***	-0.152***
	(-1.447)	(-1.936)	(-4.498)	(-5.918)	(-4.683)
volume	-0.010	-0.040***	-0.084***	-0.133***	-0.134***
	(-1.337)	(-3.562)	(-5.239)	(-5.316)	(-4.434)
iliq	-0.072***	-0.090***	-0.103***	-0.094***	-0.063***
	(-4.651)	(-4.886)	(-4.456)	(-4.884)	(-3.243)
coskew	-0.118***	-0.064***	-0.042***	-0.057***	-0.078***
	(-3.559)	(-4.837)	(-4.416)	(-5.438)	(-4.782)
cokurt	-0.112***	-0.062***	-0.050***	-0.065***	-0.094***
	(-3.906)	(-4.740)	(-4.322)	(-5.027)	(-5.272)
Panel B: Portfolios Double Sorted by Characteristics and β_{EV}					
	1	2	3	4	5
size	-0.167***	-0.089***	-0.072***	-0.067***	-0.041***
	(-4.573)	(-3.522)	(-4.068)	(-4.414)	(-2.651)
btm	-0.089***	-0.085***	-0.091***	-0.096***	-0.122***
	(-4.157)	(-4.660)	(-5.060)	(-5.238)	(-4.575)
vol	-0.013*	-0.021**	-0.049***	-0.085***	-0.142***
	(-1.958)	(-2.340)	(-4.552)	(-5.964)	(-5.153)
volume	-0.012	-0.049***	-0.094***	-0.150***	-0.132***
	(-1.393)	(-3.846)	(-5.567)	(-5.651)	(-4.866)
iliq	-0.071***	-0.101***	-0.116***	-0.107***	-0.067***
	(-4.029)	(-5.265)	(-4.871)	(-5.070)	(-3.297)
coskew	-0.117***	-0.073***	-0.052***	-0.063***	-0.080***
	(-3.906)	(-5.284)	(-4.791)	(-5.432)	(-4.704)
cokurt	-0.111***	-0.071***	-0.059***	-0.071***	-0.093***
	(-4.438)	(-5.094)	(-4.825)	(-5.132)	(-5.196)

Notes: Stocks are first sorted into five quintile portfolios based on stock characteristics and then further sorted into five quintile portfolios according to β_{RV} and β_{EV}. This table reports the difference in average returns between stocks that have highest and lowest volatility betas (i.e., "H-L") within each characteristic quintile.

Table A3: Robustness: Equal-Weighted Portfolios

Panel A: Sorting on β_{RV}						
	L	2	3	4	H	H-L
6-Month	0.097	0.070	0.063	0.066	0.067	-0.030***
						(-4.292)
Excluding Great Recession	0.226	0.165	0.145	0.142	0.140	-0.086***
						(-5.252)
Excluding Penny Stocks	0.220	0.152	0.133	0.130	0.132	-0.088***
						(-5.271)
Non-Overlapping Returns	0.190	0.136	0.124	0.109	0.097	-0.093***
						(-4.146)
VAR	0.206	0.148	0.127	0.125	0.12	-0.086***
						(-4.583)
Panel B: Sorting on β_{IV}						
	L	2	3	4	H	H-L
6-Month	0.087	0.07	0.063	0.065	0.079	-0.009
						(-0.876)
Excluding Great Recession	0.196	0.157	0.145	0.148	0.172	-0.024
						(-0.936)
Excluding Penny Stocks	0.193	0.147	0.134	0.135	0.159	-0.033
						(-1.241)
Non-Overlapping Returns	0.151	0.130	0.120	0.118	0.137	-0.015
						(-0.331)
VAR	0.182	0.146	0.128	0.128	0.144	-0.038
						(-1.487)
Panel C: Sorting on β_{EV}						
	L	2	3	4	H	H-L
6-Month	0.099	0.071	0.063	0.064	0.067	-0.032***
						(-4.334)
Excluding Great Recession	0.231	0.166	0.142	0.140	0.138	-0.093***
						(-5.494)
Excluding Penny Stocks	0.226	0.153	0.131	0.127	0.130	-0.096***
						(-5.471)
Non-Overlapping Returns	0.191	0.142	0.116	0.109	0.097	-0.094***
						(-4.580)
VAR	0.207	0.147	0.128	0.123	0.121	-0.086***
						(-5.238)

Notes: This table repeats Table 9 using equally-weighted portfolio returns.