A Model of Influencer Economy

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Abstract

In an influencer economy, consumers care about both product quality and their affinity to influencers with different "styles." Sellers thus compete for influencers as well as product markets. As technologies governing marketing outreach improve, influencer market concentration, payoffs, and income distribution exhibit non-monotonic changes. While influencer heterogeneity is a substitute to horizontal product differentiation, it can be either a complement or sugstitute to vertical product differentiation, depending on the style difference. Assortative matching between sellers and influencers occurs under endogenous influence-building, with the maximum horizontal differentiation principle recovered in the limit of costless style selection. Moreover, the sellers' bargaining power counteracts the influencers' tendency to overinvest in influence power and they jointly determine the direction and magnitude of influence building. Finally, which requiring balanced seller-influencer matching may encourage seller competition, uni-directional exclusivity contracts are welfare-improving for sufficiently differentiated products and uncrowded influencer markets.

JEL Classification:. M31. M37. L11. L20. L51

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1 Introduction

The past decade has witnessed the rise of the influencer economy (also known as "Wang Hong economy" and more recently dubbed by the media as the "creator economy") which prominently features social media marketing, testimonial endorsements, and product placements from people and organizations who have a purported expertise or social influence.[1] The phenomenal growth of the influencer economy is further accelerated by the recent COVID-19 pandemic (e.g., Sinha, 2021). In this new form of digital economy, influencers come in variety and from diverse background, and include content creators, celebrities and idols, and key opinion leaders (KOL) (Williams, 2016).[2] They enjoy their own fan base who are drawn to their talent, charisma, wisdom, appearance, etc., and profit by helping brand owners and service providers promote various products to the fans.[3] Influencers and (social) digital platforms coexist in a symbiotic relationship, with Multi-channel networks (MCNs) serving as bridge among these participants and helping linking the upstream content production and downstream e-commence. Understandably, existing studies have focused on the relationship between influencers and platforms or MCNs.

However, we have little theoretical understanding about the industrial organization of the influencer economy. How does technology affects the bargaining between sellers and influencers? How influencers shape product differentiation and pricing? How are influencers and brand owners matched and how to regulate the process? We answer these questions by developing a novel game-theoretic model in which sellers depend on influencers to acquire

[1] The influencer or creator economy generally refers to the independent businesses and side hustles launched by self-employed individuals who make money off of their knowledge, skills, or following. CBInsights (2021) provides an excellent introduction to the industry. According to a Benchmark survey (Geyser, 2021), from a mere $1.7 billion in 2016, influencer marketing has grown to a size of $9.7 billion in 2020, and is set to jump to approximately $13.8 Billion in 2021. There are approximately 50M creators today, according to SignalFire. In particular, in the Chinese market alone in 2019, there were already more than 6500 influencer-related companies and the total market value exceeded 10 billion CNY. Many influencers generate as much as seven-figure incomes. For example, Wei Ya, one famous influencer, made a fortune from her over 1 billion CNY in live streamed pre-sales on Single's day (Nov 11, the black Friday equivalent in China) alone in 2019, as reported at https://wk.askci.com/details/a0f1a24536ab46ac9da04fd494686476/. Famous Instagram influencers like Huda Kattan or Eleonora Pons net up to 6 figures per post. The top writers on Substack can rake in as much as $1M USD annually. Youtube paid out $30B to creators in the last 3 years (CBInsights, 2021).

[2] Content creators derive from "YouTube stars" marketed by YouTube in as early as 2011 (Lorenz, 2019). Now, it can be anyone who creates any form of content online, including TikTok videos and Clubhouse audios. For instance, on Twitch, daily users can watch live streams video games played by others via Streamlabs, and tips paid out on Twitch alone is estimated to be $141 million. Unlike live stream creators, Internet celebrities on Instagram and the like can post about or live stream special travel or dining experience, or simply routine daily lives. Many rely on physical attributes alone without actively creating content. For instance, Instagram enables brand owners to sell products through idols who attract consumers simply seeking to see them. Similarly, KOLs can target specific demographic in an interactive manner, making product sales more engaging by sharing their own thoughts and ideas.

[3] Influencers touch almost all aspects of life, including entertainment, fashion, food, movies, music, sports, etc., and increasingly utilize short videos (low cost and easy to spread). While there are many ways to monetize the influence, such as compensation for content creation or interaction with fans, influencers' largest income are still from commercials and e-commerce traffic direction.

customers and compete in both the product market and influencers' labor market.

Specifically, we model three important groups of agents in the influencer economy, sellers (who are also producers), influencers, and consumers, allowing pair-wise group interactions through the product market, the influencers' labor market, and the social media platforms (for influencers to connect with consumers). Consumers are uniformly located on the unit circle in $\mathbb{R}^2$ with consumption utilities determined by both the true quality of the product and the style, status, identity, etc.—things that draw people towards influencers on social media like Instagram, or Da Ren on Alibaba. Sellers or brand owners (not on the unit circle) depend on influencers to sell products to consumers. All agents interact in four sequential stages. First, influencers' type and power are set. Second, sellers make production decisions. Third, sellers decide which influencer(s) to hire in the labor market. Finally, consumers choose which influencer to follow and consume the products the influencer promotes.

We begin with a monopolist seller and abstract from seller competition and seller-influencer matching to highlight the impact of general purpose technologies such as digital social platforms on influencers' labor market. Interestingly, as technologies governing marketing outreach improve, the equilibrium features non-monotonicities in influencer market concentration, payoffs, and distributional inequality. These results are driven by two factors. First, the optimal hiring for joint profit maximization is non-monotonic. When the background technology is very costly, only the best influencer can break even and get picked and a sub-population gets the product. As the cost continues to decrease, the best influencer does not dominate. More people have local influence and others get hired too. At some point, all these influencers are in competition and again only the best becomes dominant. Thus, non-monotonic concentration in influencers' market follows when the seller's incentive is sufficiently aligned with the seller-influencer(s) group's joint profit.

Second, the seller's incentive for greater bargaining power distorts the hiring decision, causing the seller to hire the weak influencer even though it reduces their joint profit, whenever the seller's bargaining power is small. It naturally generates non-monotonicities in influencers' payoffs. Initially, when the technology cost is high, both influencers are paid very little because of the low joint profit. In contrast, when the technology cost becomes sufficiently small, both influencers are offered minimal wages because influencers are perfect substitutes, and thus the income gap is almost zero. Hence, we only see high payoffs for influencers and a big distributional inequality for intermediate technology cost range.

We then consider the setting in which two sellers compete in both labor market (two influencers) and the product market. Consistent with the current influencer market practice, we focus on "balanced matching" or mutual exclusivity contracts in the labor market, in which each seller can hire only one influencer. Under one-dimension heterogeneity, we fully characterize the price competition equilibrium with heterogeneity in either product quality, influencer power, or influencer style. Furthermore, to focus on pure strategy equilibrium that exist, we analyze two special cases of multiple-dimension heterogeneity. In the first case, sellers enjoy local monopoly power regardless of product quality when influencers' styles are sufficiently distinct. In the second case, it features market dominance, and to gain market dominance, the style difference between influencers needs to be suffciently small, and both the influencers' power gap and product quality gap must be sufficiently large simultaneously.

We move backward to endogenize sellers' production. Specifically, we investigate how influencer heterogeneity affects horizontal and vertical product differentiation, compared to traditional economies. We find that influencer heterogeneity and horizontal product differentiation are substitutes, mainly driven by the desire to avoid competition. When influencers' style difference is small, sellers differentiate products to reduce competition. When influencers' style difference is large, sellers hires influencers and have no incentive to differentiate products because more differentiated markets yield lower substitutability among products, therefore giving sellers less elastic demand for the products. This also implies that the well-established principle of maximum differentiation found in the literature (e.g., d'Aspremont et al., 1979; De Frutos et al., 1999) would no longer hold in an influencer economy.

Surprisingly, when it comes to vertical differentiation, small style differences complement while large differences substitute, mainly due to the incentive to grab the whole market and beat the competitor. To see it, note that when influencers' style difference increases, the return from investing in high quality also increases. When influencers' style difference is sufficiently large (small), both groups can (cannot) break even and choose high (low) quality and thus vertical differentiation is minimal. Only for intermediate style difference, can vertical differentiation be observed because the investment profit is only big enough to support one group investing to break even.

Next, we allow the influencers to endogenize their influence either in power or type. Under endogenous power acquisition, we show that socially inefficient under-investment and over-investment in influence can arise due to externality and endogenous bargaining power issue. In particular, influencers ignore the positive externality on consumer welfare in a uncongested influencer market, as well as the negative externality on other influencers in a congested influencer market. Similarly, a big/small bargaining power empowers/reduces the incentive for power acquisition. These two forces jointly determine the direction and magnitude of the sub-optimal acquisition. Furthermore, under endogenous style selection, assortative matching between sellers and influencers occur under endogenous influence, with the maximum horizontal differentiation principle recovered in the limit of costless style selection, mainly driven by the fact that the seller-influencer group's profit is supermodular in product quality and influencer power in a uncongested market.

Finally, to better understand the welfare implication of exclusivity contracts in this emerging industry and to guide regulatory policies geared towards balancing the power of influencers and sellers, we extend the analysis to "unbalanced matching". We find that regulations for balanced seller-influencer matching can encourage seller competition under single dimensional seller-influencer heterogeneity. But uni-directional exclusivity contracts are welfare-improving for sufficiently differentiated products and uncongested influencers' markets.

Literature— Previous research has focused on online platforms and has studied issues on revenue sharing rules (Bhargava, 2021; Jain and Qian, 2021), disclosure by internet influencers (Mitchell, 2021), search technology and advice transparency (Fainmesser and Galeotti, 2021), influencer cartels (Hinnosaar and Hinnosaar, 2021), and firms' optimal affiliation with influencers (Pei and Mayzlin, 2019). Our study adds foremost to the emerging literature on digital platforms and the influencer or creator economy. We abstract away from the intermediation by platforms, but focus on seller competition and seller-influencer matching, which in turn affects product differentiation and endogenous influence acquisition.

Our study is also related to the broad literature on marketing and industrial organization, including classical articles such as Salop (1979). We add by analyzing the interaction of the two in a fast emerging influencer economy. Studies on advertising, have focused on the aggregate and cross-sectional levels of advertising and its welfare implications (Becker and Murphy, 1993; Spence and Owen, 1977; Butters, 1978; Dixit and Norman, 1978; Grossman and Shapiro, 1984; Nichols, 1985; Stegeman, 1991; Nelson, 1974; Johnson and Myatt, 2006). Most models assume no media or only focus on the informational effects or nuisance costs on viewers of advertisements (e.g., Johnson, 2013). Moreover, most studies do not endogenize locations of media stations, and the ones that do (e.g., Gal-Or and Dukes, 2003; Dukes, 2004) typically take sellers' product differentiation as exogenous. We study influencers whose matching with sellers are affected by the consumer base, and analyze endogenous product differentiation and influencers' style choices simultaneously. We do not focus on the level of advertising or its informational role, but on the complementarity between the multiple dimensions of consumer utility from following influencers and consuming products.

More recently, Amaldoss and He (2010) study how firms strategically target consumers to avoid intense price competition. Several marketing articles analyze how how firms compete in the effort of hiring influencers, including advertising intensity, competitive targeting of influencers in a network, and the network structure and its influence on prices, firm profits, and consumer surplus (Galeotti and Goyal, 2009; Katona, 2018). In particular, Fainmesser and Galeotti (2021) analyze search quality, advice transparency, and influencer strategy in the market for online influence. We differ in our focus on the interaction of seller-influencer matching and product market competition. In addition, we add to the discussion on exclusivity contracting in that while earlier studies have analyzed the link between uni-directional exclusivity contracts and bargaining (e.g., Gal-Or, 1997; Dukes and Gal-Or, 2003), we contribute by contrasting uni-directional with mutual exclusivity contracts in their impact on welfare in the new influencer economy.

The rest of the paper is organized as follows: Section 2 lays out the baseline model. Section 3 discusses influencer-induced consumption and technology. Section 4 solves price competition equilibrium with endogenous influencer hiring. Section 5 endogenizes seller's production stage. Section 6 investigates endogenous influencer influencer and its welfare implications. Section 7 extends the analysis to unbalanced matching. The Online Appendix contains all the proofs.

2 Model Setup

There are $J \in \mathbb{Z}^+$ influencers, and the jth influencer's style type is denoted by $\theta_j \in \mathbf{S}^1$, where $\mathbf{S}^1 := \{s \in \mathbb{R}^2 : s_1^2 + s_2^2 = 1\}$ is the unit circle in $\mathbb{R}^2$. To capture distinctions among celebrities, macro-influencers, and micro-influencers, we allow them to differ in their own influence power $I_j \in \mathbb{R}_+$ which measures how easily consumers derive utility from affinity with them. An influencer's influence is also affected by a technology parameter, $c > 0$, that governs her outreach. c can be interpreted as how quickly the influence decreases on the circle away from the influencer. Traditional advertising channels through TVs, newspapers, etc., can be viewed as exerting the influnce with an extremely large c since the production of TV commercials, for example, would be tremendously costly due to limited airtime and labor- and capital-intensive outreach. Overall, the influencer-specific power and the common technology jointly determine the effective consumer base.

There are $K \in \mathbb{Z}^+$ risk-neutral sellers in the economy, and the kth seller sells a product of common consumption value y_k for consumers, with $k \in \{1, \cdots, K\}$. The sellers traditionally use advertisements to market their products but in an influencer economy, they work with influencers for marketing and outreach (and even direct sales). We denote the kth seller's utility or profit as U_k.

Consumer agents derive utilities from two sources. First, they enjoy having similar "style" as cerntain influencers. Style could refer to identity, fashion taste, and other things that draw people towards influencers on Instagram, Tiktok, Alibaba, etc., among the recent proliferation of social networks, digital platforms, and broadcasting channels. At the same time, they derive regular consumption value based on the quality of the goods.[4] Specifically, the ith consumer's style is given by $x_i \in \mathbb{R}^2$, which follows a uniform distribution on $\mathbf{S}^1$. Moreover, $\forall x_1, x_2 \in \mathbf{S}^1$, we define the norm $\|x_1 - x_2\|$ to be the distance along the short arc on the unit circle. The total mass of consumers is 2π.

[4]For further institutional background, see, e.g., `https://wearesocial.com/blog/2020/01/the-dawn-of-a-new-influencer-economy`.

Recall that x_i is the ith consumer's style type and y is the consumption value of the target product. Without consumption, the utility is normalized such that $u_i(x_i, y) = 0$. With a unit consumption (we assume that he faces a discrete choice regarding whether to buy a single unit of the product), consumer i gets:

$$u_i(x_i, y) = y * \left(1 - \frac{c}{I} * \|x_i - \theta\|\right) - p,$$

where p is the unit price charged for the consumption good and we assume $y \in \mathbb{R}^{++}$. Here, brand owners and product sellers depend on influencers to sell the goods, with only consumers having $\|x_i - \theta\| \leq \frac{I}{c}$ enter the demand function. In the baseline model, we assume a small fixed cost $\varepsilon > 0$ of hiring a new influencer and turning on the influence power. ε helps break an indifference to avoid the seller's hiring multiple influencers yet keeping them idle, and is taken to be infinitesimal later.

Importantly, a consumer's utility depends on the proximity to the influencer they follow (i.e., the distance between x_i and θ). Using products advertised by influencers closer by yields a higher utility. Specifically, the ith consumer's utility function is given by:

$$u_i(x_i, y) = \begin{cases} y * \left(1 - \frac{c}{I}\|x_i - \theta\|\right) - p, & \text{if a unit good is consumed,} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Our setting adequately captures the many reasons typically given for engaging celebrities in advertising campaigns: grabbing attention, persuasion through expertise, and global outreach (Moeran, 2003).[5] I can represent attention grabbing, either through vacuous "human pseudo-events" in the words of American historian Daniel Boorstin or through skills or performance unrelated to the products; expertise and global, cross-cultural outreach can manifest through the combination of location θ and power I.

Timeline. In an influencer economy, influencers' type and power are first set. The sellers then decide on the products and subsequently hire influencers. Finally, the consumers choose which influencer to follow and consume the products offered. In Sections 3 and 4, we take the influencers' type and power, as well as the sellers' products as given, in order to focus on the sellers' hiring of influencers and influencers' impact on consumption. In Section 5, we allow the sellers to endogenize the products for sale. Our main findings are robust to having product decisions following seller-influencer matching. But in practice, firms decide on their business operations before exploring marketing channels, which is what our setup

[5] Advertising through conventional technology, e.g., through TV/newspaper, are often extremely costly (high c). We are cognizant that such a cost is used to indicate the existence of a price premium that can assure contractual performance in competitive equilibrium (Klein and Leffler, 1981). However, in an influencer economy with digital platforms and the proliferation of social-commercial network apps, c is relatively low and would not serve such a function for disclosing the presence of a large sunk "selling" costs.

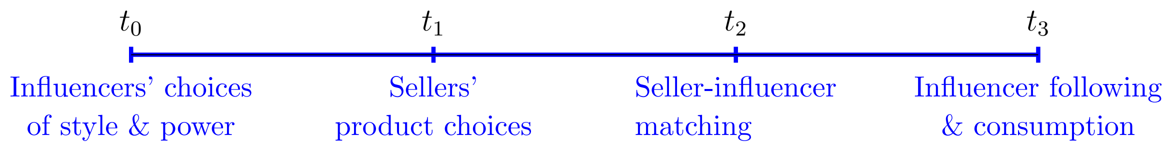

Figure 1: Timeline

captures. Finally, in Section 6, we endogenize influencers' power (which can be interpreted as skill training over the intermediate term) and type (which can be interpreted as culture, talent, or interest cultivated over the long run, perhaps through childhood education).

Matching and bargaining protocols. We use a general (bilateral) Nash bargaining protocol for the negotiation once influencers are hired by sellers.[6] Specifically, denote by γ and $(1-\gamma)$ the bargaining power assigned to the seller and the influencer respectively. Once sellers and influencers are matched, they have exogenous options outside the match, e.g., from revisiting the influencer market, which we normalize to zero. Anticipating such bargaining processes, sellers and influencers endogenously match. Our baseline setup focuses on one-to-one match, which can be interpreted as that in practice, the seller-influencer contracts either feature mutual exclusivity clauses or they are all allowed to have multiple relationships so that the matching is balanced. This negotiation-based approach is realistic, and is also common occurrence in negotiations for advertising price in the media industry (Dukes and Gal-Or, 2003; Gal-Or, 1997).

The joint matching and bargaining problems are non-trivial. In specifying the protocols, we strive to balance tractability, transparency, convention in the literature, coherence with our non-repeated game set-up, and realism. In fact, many key results are independent on how the surplus is divided between matched sellers and influencers, as long as they care about group surplus. One can alternatively specify equilibria that gives all net surplus to the sellers or the influencers. We discuss unbalanced matching and the welfare implications of exclusivity in Section 7 where the sellers have exclusivity and non-compete clauses imposed on the influencers, which are also commonly observed in the influencer markets, especially in its nascent stage.

3 Influencer-Induced Consumption and Technology

We start with a monopolist seller offering a homogenous unit-consumption product, which is marketed by influencer(s) and sold to consumers. The abstraction from seller competition and seller-influencer matching allows us to highlight the impact of general purpose technolo-

[6] Gal-Or (1999) and Dukes and Gal-Or (2003) discuss the advantages of this modeling approach as illustrated in the commercial media and healthcare industries.

gies on the influencer economy.

Fix an index set $\mathcal{J} \in \{1, \cdots, J\}$, we denote by $\Pi_{\mathcal{J}}$ the profit when all influencers in $\mathcal{J}$ are hired before advertising costs are deducted. For instance, $\Pi_{\{i\}}$ is the monopoly profit when only influencer $i \in \{1, 2\}$ is hired, and $\Pi_{\{1,2\}}$ the profit when both influencers 1 and 2 are hired. For ease of reference, denote by w_j the wage for influencer $j \in \mathcal{J}$ hired, and by U_k the kth seller's profit for all $k \in \{1, \cdots, K\}$.

3.1 Single-Influencer Benchmark and Bargaining Power

Before analyzing the general industrial organization of the influencer economy, let us consider the special case of one influencer to understand the formation of consumer base.

Lemma 1. *Given $J = K = 1$ and the values of (I, y, c), the potential consumer base is given by all $x \in \mathbf{S}^1$ such that $\|x - \theta\| \leq \min\left\{\frac{I}{c}, \pi\right\}$. There are two cases.*

(i) When $\frac{I}{c} \leq 2\pi$, the seller sets a monopoly price $p^* = \frac{y}{2}$, and the total revenue is $\Pi = \frac{yI}{2c}$. Only consumers with $\|x - \theta\| \leq \frac{I}{2c}$ are served.

(ii) When $\frac{I}{c} > 2\pi$, the seller set a price $p^* = y(1 - c\pi/I)$ and $\Pi = 2\pi p^* = 2\pi y(1 - c\pi/I)$.

Furthermore, the monopolist seller hires the influencer only when $\Pi \geq \varepsilon$. The seller's payoff and the influencer's wage are given by $U_1 = \gamma(\Pi - \varepsilon)$ and $w_1 = (1 - \gamma)(\Pi - \varepsilon)$.

Proof. See Appendix A.1. □

Lemma 1 illustrates how the seller targets a specific demographic by tapping into the influencer. Depending on the cost of background technology and influencers' ability, a monopolist seller may choose to target a subpopulation or the whole demographic of consumers. The seller only enters the market when the revenue is high enough. Figure 2 illustrates the consumer base in Lemma 1 when the background technology cost is big. Figure 3 illustrates the monopolist pricing in Lemma 1. Note that the vertical axis in Figure 3 corresponds to consumer utility. The monopolist price p^* is marked in red dashed line, and consumers served corresponds to the thick blue line.

3.2 Influencers's Competition and Technological Advances

Next, we examine the case with two representative influencers to understand how general purpose technologies such as digital social platforms and the Internet impact influencers' labor market. Without any loss of generality, we assume two heterogeneous influencers ($J = 2$) separated in type by $\|\theta_1 - \theta_2\| = \pi$, and that $I_1 \geq I_2$ (Influencer 1 is strong and Influencer 2 is weak). Furthermore, we impose Assumption 1 to ensure that at least one influencer is hired under equilibrium.

Assumption 1 (Sufficiently small technology cost). $c < \frac{yI_1}{2\varepsilon}$.

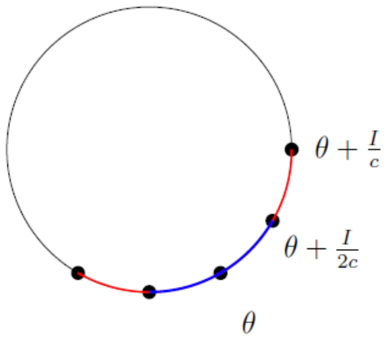

Figure 2: The circular market

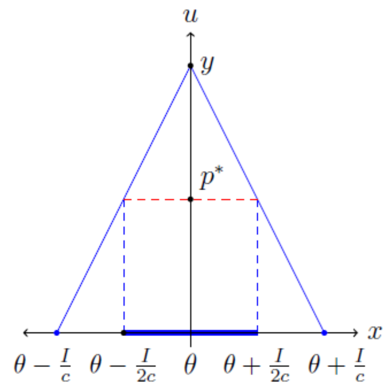

Figure 3: The monopolist pricing

Since $\Pi_{\{j\}}$ is fully characterized in Lemma 1, we present the joint profit $\Pi_{\{1,2\}}$ next.

Lemma 2 (Joint revenue function $\Pi_{\{1,2\}}$). *For $c \leq \frac{yI_1}{2\varepsilon}$, the joint revenue function $\Pi_{\{1,2\}}$ is:*

$$\Pi_{\{1,2\}} = \begin{cases} \frac{y(I_1+I_2)}{2c}, & \text{if } \frac{I_1+I_2}{2\pi} < c \\ 2\pi y\left(1 - \frac{c\pi}{I_1+I_2}\right), & \text{if } c < \frac{I_1+I_2}{2\pi} \end{cases} \quad \text{whenever} \quad \frac{I_2}{I_1} \leq \frac{\pi y - \varepsilon}{\varepsilon};$$

otherwise, $\Pi_{\{1,2\}} = 2\pi y\left(1 - \frac{c\pi}{I_1+I_2}\right)$.

Proof. See Appendix A.2. □

For ease of reference, define $\overline{c} = \frac{I_1}{I_2} * \frac{(I_1+I_2)\varepsilon}{2\pi^2 y}$.

Lemma 3 (Hiring for Joint Profit Maximization). *Assume a sufficiently small fixed hiring cost such that $\frac{\varepsilon}{y}(1 + \frac{I_1}{I_2}) < \pi$. To maximize the seller-influencers joint profit,*

(i) only the strong influencer is hired when $c \in \left(\frac{yI_2}{2\varepsilon}, \frac{yI_1}{2\varepsilon}\right) \cup (0, \overline{c})$.

(ii) both influencers are hired when $c \in \left[\overline{c}, \frac{yI_2}{2\varepsilon}\right]$.

Proof. See Appendix A.3. □

Given the suboptimality of hiring the weak influencer alone, whether to hire both influencers boils down to whether $\Pi_{\{1,2\}} - 2\varepsilon \geq \Pi_{\{1\}} - \varepsilon$. Lemma 3 shows that the joint-profit-maximizing market structure is non-monotonic in the technology cost. When the cost is too high (traditional advertising is very costly), joint profit maximization requires the seller to choose a small number of high ability influencers. For example, a seller may end up picking one influencer because hiring a second-best influencer(s) cannot break even. In this case, only the best influencer gets picked and a sub-population gets the product. One caveat is that this region vanishes if $I_2 = I_1$. As the background technology cost goes down, more people have local influence, and a seller should pick more influencers. Thus, the best influencer does not dominate, and others get hired too. As the cost continues to decrease, at some point, all these influencers are in competition and again only the best becomes dominant.

Next, we investigate the seller's incentive to hire influencers, and show how the incentive misalignment ubiquitously drives over-employment, as illustrated by Proposition 4 below. Define $\underline{c} = \frac{I_2}{I_1} * \frac{(I_1+I_2)\varepsilon}{2\pi^2 y}$, and $\underline{\gamma} := \inf_{c<\underline{c}} \frac{\Pi_{\{1,2\}}(c)-2\varepsilon}{\Pi_{\{1\}}(c)-\varepsilon}$.[7]

Lemma 4 (Inefficient over-employment & Bargaining power building)**.** *Whenever $\gamma < 1$, there exists inefficient over-employment when $c \in (\bar{c} - \delta, \bar{c})$ for $\delta > 0$ sufficiently small. Furthermore, whenever $\gamma \leq \underline{\gamma}$, there exists over-employment for all $c \leq \underline{c}$.*

Proof. See Appendix A.4. □

The inefficient over-employment is best illustrated when the competition between influencers is intense such that the profit gap between hiring both influencers and only hiring a strong influencer is so small that hiring the weak influencer cannot offset the additional fixed cost. However, even if it generates a negative welfare, the monopolist seller does have an incentive to hire the weak influencer, that is, by hiring the weak influencer, it places the seller in an advantageous position when bargaining with the strong seller. The seller, after hiring the weak influencer, is willing to share the net profit increment between that generated by hiring both influencers and that by hiring the weak influencer (i.e., $(\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{2\}} - \varepsilon)$). In contrast, if she hires only the strong influencer, the seller needs to give up a big portion of the profit (i.e., $(1-\gamma)(\Pi_{\{1\}} - \varepsilon)$). The smaller the seller's exogenous bargaining power, the stronger the incentive for over-employment. In particular, when the seller's bargaining power $\gamma \leq \underline{\gamma}$, both influencers are hired even when one influencer can serve the entire market.

In contrast, when the seller enjoys a sufficiently big bargaining power, the incentive for pursuing bargaining superiority through over-employment is alleviated and thus more aligned with the seller-influencers' joint profit. A direct ramification is that the observed concentration of influencers' market can be non-monotonic when γ gets close to 1.

Proposition 1 (Non-monotonicity in market concentration and influencers' payoffs)**.** *Assume that the fixed hiring cost is sufficiently small (i.e., $\frac{I_1}{I_2} * \frac{\varepsilon}{\pi y}\left(1 + \frac{I_1}{I_2}\right) \leq 1$). Then, as the technology cost c decreases, we have:*

(i) The concentration of influencers' market is non-monotonic when γ is sufficiently large;

(ii) The seller's payoff always increases;

(iii) Total payoffs for influencers first increase and then decrease for sufficiently small γ;

(iv) For sufficiently small γ, the distributional inequality between influencers' payoffs is increasing in the technology cost c for small c, and decreasing for large c.

Proof. See Appendix A.5. □

[7]Note that $\underline{\gamma} > \frac{\Pi_{\{1\}}(\underline{c})-2\varepsilon}{\Pi_{\{1\}}(\underline{c})-\varepsilon}$, and thus $\underline{\gamma} \to 1$ when $\varepsilon \to 0$.

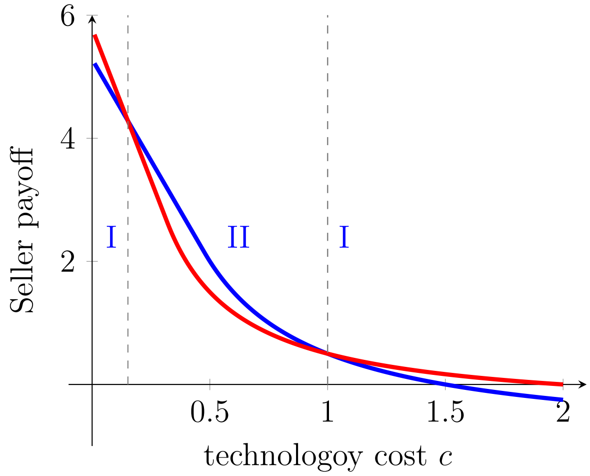

Figure 4: Non-monotonic market structure

The proposition highlights that there is a strong interaction between bargaining power and the technology costs and the impact of technological advances is more nuanced. The interesting non-monotonicity results in Proposition 1 is driven by two forces. First, the optimal hiring decision to maximize the joint profit for the seller and influencer(s) is non-monotonic in the general technology parameter c, as illustrated by Lemma 3, mainly because influencers are perfect substitutes when the general technology is sufficiently cheap. Second, the seller's incentive for building bargaining power distorts the hiring decision, causing the seller to hire the weak influencer even though it reduces their joint profit.

Let us discuss the intuition behind the non-monotonicity result. First, claim (i) says that when the seller enjoys a big bargaining power, his incentive is almost perfectly aligned with the group's joint profit and thus a non-monotonic market structure is observed. Figure 4 illustrates this when $y = 1, \varepsilon = 0.5, I_1 = 2, I_2 = 1$, and $\gamma = 1$. The blue line corresponds to the seller's profit when both influencers are hired, while the red one is the seller's profit when only one influencer (i.e., the strong influencer) is hired. Hence, in Region I, when the general technology is very cheap or very expensive (i.e., $c > 1$ or $c < 0.15$), only one is hired; and in Region II (i.e., $c \in (0.15, 1)$, both influencers are hired.

Second, claim (ii) holds because the seller's payoff is just the upper envelope of two payoff functions U_1 and $\hat{U}_1$, which corresponds to the two cases when the seller hires both influencers and when she only hires the strong influencer, and note that they are both continuous and strictly decreasing in the general technology parameter c. In Figure 4, it corresponds to the upper envelope of the blue curve and the red curve, and the seller's payoff obviously increases when the general technology is becoming cheaper.

Third, claim (iii) says that when the seller's bargaining power is not large enough, she has a strong incentive for bargaining power building by hiring both influencers. However, the over-employment can actually drive down influencers' wages, and in the extreme case when

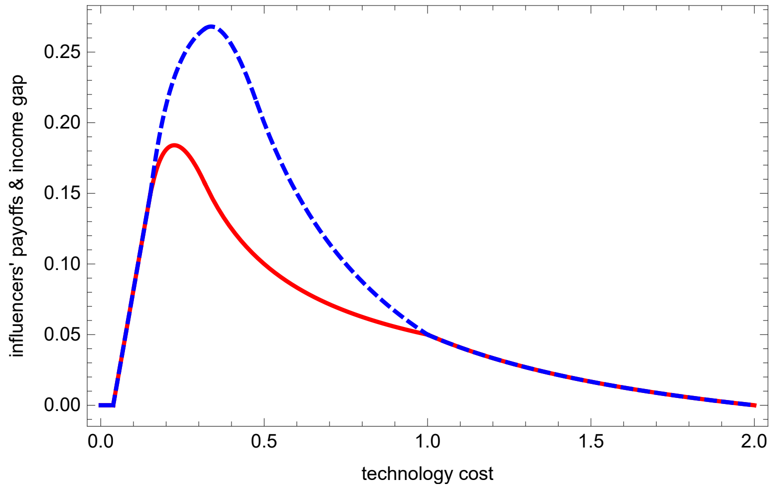

Figure 5: Non-monotonicity in influencers' payoffs and distributional inequality

turning on influence is very cheap, both influencers are strong and are perfect substitute. Thus, influencers are paid very little even when the general technology is sufficiently cheap because influencers are then perfect substitutes for each other.

Figure 5 illustrates the dynamics of total payoffs for both influencers and income gap when the background technology cost decreases. The parameters are specified as follows: $y = 1$, $I_1 = 2$, $I_2 = 1$, $\varepsilon = \frac{1}{2}$ and $\gamma = 0.8$. The blue line corresponds to the dynamics of total payoffs for both influencers. Initially, when the background technology is very costly, only influencer 1 is hired and the total revenue is very low, which bounds influencer 1's compensation. When the technology cost is reduced, more influencers are hired, and the total revenue increases. If the bargaining power is exogenously fixed, this transforms into a big increase in influencers' payoffs. Finally, when the background technology is very cheap, both the strong and the weak influencers can produce a big revenue if anyone is hired. This also implies that influencers are perfect substitutes, and by hiring both influencers, the seller can use fierce competition between influencers for bargaining power building and thus offer a minimal wage to influencers. Hence, total payoffs for both influencers first increase and then decrease when the general technology cost decreases.

The red line corresponds to the distributional inequality between influencers' payoffs, which is defined as the wage gap $w_1 - w_2$ between two influencers. When the technology cost is big, both influencers are paid very little because of the low joint profit. In contrast, when the technology cost is sufficiently small, both influencers are offered minimal wages, and thus the income gap is zero. Hence, we only see a big distributional inequality for intermediate technology cost range. Note that the non-monotonicity in concentration is related to the long-term sustainability and the unequal income emphasized in CBInsights (2021).

3.3 The Seller's Preferred Influencer Difference

Section 3.2 takes an exogenous style difference between the influencers. We now investigate the seller's preference over the style difference between influencers. For example, the seller might prefer working with influencers who overlap in consumer base, even though this results in a decreased overall total market coverage. This would basically be in line with some of the intuitions from Lemma 4.

For ease of reference, denote by $\beta := \|\theta_1 - \theta_2\|$ the style difference between the two influencers, where θ_j is the style location of influencer $j \in \{1, 2\}$. To simplify the reasoning, we remove the fixed hiring cost (i.e., $\varepsilon = 0$), which only plays a crucial role in the discussion of technological advances. This restriction implies that it is always optimal to hire both influencers. Given the bilateral Nash bargaining protocol, the payoffs for the seller and the two influencers are

$$w_1 = (1-\gamma)(\Pi_{\{1,2\}} - \Pi_{\{1\}}), \quad w_2 = (1-\gamma)(\Pi_{\{1,2\}} - \Pi_{\{2\}}),$$

and

$$U_1 = \Pi_{\{1,2\}} - w_1 - w_2 = (2\gamma - 1)\Pi_{\{1,2\}} - (1-\gamma)(\Pi_{\{1\}} + \Pi_{\{2\}}) \tag{2}$$

Note that only $\Pi_{\{1,2\}}$ depends on β, the style difference angle between the two influencers. Denote by β^S the style difference angle preferred by the seller.

Corollary 1 (Seller-preferred Influencers' Style Difference)**.** *Assume that* $\frac{I_1}{c} = \frac{I_2}{c} \in (0, \pi)$.[8] *Then,*

$$\beta^S = \begin{cases} \in [I, \pi], & \text{if} \quad \gamma > \frac{1}{2} \\ \in [0, \pi], & \text{if} \quad \gamma = \frac{1}{2} \\ 0, & \text{if} \quad \gamma < \frac{1}{2} \end{cases}$$

Proof. See Appendix A.6. □

Corollary 1 states that the seller prefers a zero style difference angle (i.e., $\theta_1 = \theta_2$) when her bargaining power parameter γ is small, and prefers a large style difference angle when γ is big, and is indifferent for any style difference angle for a knife-edge intermediate bargaining parameter.

The intuition behind Corollary 1 is as follows. Note that a larger style difference angle implies a bigger market coverage and a smaller over-lapping in consumer base for different influencers. When $\gamma > 0$ is small, a larger β can generate a bigger revenue but only a very

[8]This condition is imposed to recycle Lemma 9, and can be relaxed.

limited fraction of total revenue, determined by the bargaining power, flows to the seller. Furthermore, given a small γ, a smaller β decreases the market coverage and generates less revenue, but it also implies intense competition and perfect substitution between the two influencers and leads to a better bargaining position for the seller under the bilateral Nash bargaining protocol. In the limit where $\beta = 0$, the seller gets the total revenue of working with a sole influencer in the product market. In other words, a small style difference angle betters the cash show to the seller although it decreases the market coverage. In contrast, for a big $\gamma > 0$, there is no tension between bargaining and revenue generating, because the effect on revenue from the market coverage change always dominates that from influencers' wage changes. Hence, the seller's payoff increases with the total revenue from working with both influencers, and thus a large style difference angle is more desirable.

3.4 General Characterizations with Multiple Influencers

Next, consider a setting with many influencers with the background technology fixed, which we normalize to one. Then, we study how influencer competition affects the monopolist seller's payoff, influencers' wages, and total welfare.

Residual Multilateral Bargaining Protocol. We propose a residual bargaining protocol to handle the multilateral bargaining problem. Specifically, given the bargaining parameter $\gamma \in [0,1]$, when a group of influencers are hired $\mathcal{J} \in \{1, \cdots, J\}$, the seller only bargains with influencer $j \in \mathcal{J}$ for the "residual profit gap" $\Pi_{\mathcal{J}} - \Pi_{\mathcal{J}/\{j\}}$, which leads to

$$w_j = (1-\gamma)(\Pi_{\mathcal{J}} - \Pi_{\mathcal{J}/\{j\}} - \varepsilon)_+, \quad \text{and} \quad U_1 = \Pi_{\mathcal{J}} - \sum_{j \in \mathcal{J}} w_j. \tag{3}$$

where $\Pi_{\mathcal{J}}$ is the monopolist profit when the seller hires the influencer group $\mathcal{J}$, excluding all fixed hiring cost.[9]

Here, a_+ is defined as $a_+ := \max\{a, 0\}$. Note that, the term

$$\Pi_{\mathcal{J}} - \Pi_{\mathcal{J}/\{j\}} = \big(\Pi_{\mathcal{J}} - |\mathcal{J}|\varepsilon\big) - \big(\Pi_{\mathcal{J}/j} - |\mathcal{J}/j|\varepsilon\big)$$

measures the incremental change in residual profit gap from hiring influencer j.

The residual bargaining protocol has several desirable properties: (i) the allocation is unique and feasible because $\sum_{j\in\mathcal{J}} w_j + U_1 \leq \Pi_{\mathcal{J}}$; (ii) it is efficient because $\sum_{j\in\mathcal{J}} w_j + U_1 = \Pi_{\mathcal{J}}$; (iii) it cannot be blocked by a coalition of $S \subseteq \mathcal{J}$ for any $\gamma \in [0,1]$ because $\sum_{j\in S} w_j + U_1 \geq \Pi_S$ and $\sum_{j\in S} w_j \geq 0$; and (iv) it is consistent with the bilateral Nash

[9] The seller only shares the residual profit because of a credible threat that if influencer j rejects the offer, the seller switch to hiring the remaining group $\mathcal{J}/\{j\}$ and divide the profit $\Pi_{\mathcal{J}/\{j\}}$.

bargaining in the baseline model.[10] In essence, residual bargaining protocol is an equilibrium refinement. Note that property (iii) is applicable to all reasonable refinements because otherwise the allocation is blocked by excluding influencer j from the group $\mathcal{J}$, which further implies that $w_j \leq \Pi_{\mathcal{J}} - \Pi_{\mathcal{J}/\{j\}}$. In this sense, the residual bargaining protocol is the natural candidate satisfying property (iii) and property (iv).

To rule out the case that no influencer is hired, we assume that $\frac{yI}{2} \geq \varepsilon$. For simplicity, all influencers are also assumed to have identical power. First, note that a non-negligible hiring cost implies that only a finite number of influencers, rather than all influencers, are hired. Indeed, the number of influencers hired is capped at $\left[\frac{2\pi y}{\varepsilon}\right]$.[11] Define $\bar{\Pi}_J = \max_{|\mathcal{J}|=J} \Pi_{\mathcal{J}}$. We can show that $\bar{\Pi}_J$ is maximized when all $J \geq 1$ influencers are equally distanced, that is, the distance between any two neighboring influencers equals $2\pi/J$.

Lemma 5 (Equal-distanced Influencers)**.** *If the seller is restricted to hiring J influencers and can freely choose style locations for all influencers, then $\bar{\Pi}_J$ is achieved when all neighboring influencers hired are equally distanced (i.e., $\|\theta_j, \theta_{j+1}\| = \frac{2\pi}{J}$ for all $j \in \{1, \cdots, J\}$).*

Proof. See Appendix A.7. $\square$

With the aid of Lemma 5,

$$\bar{\Pi}_J = \begin{cases} \frac{yIJ}{2}, & \text{if} \quad J \leq \left[\frac{2\pi}{I}\right] \\ 2\pi y \left(1 - \frac{\pi}{IJ}\right), & \text{if} \quad J > \left[\frac{2\pi}{I}\right] \end{cases} \tag{4}$$

Eq. (4) follows from that all influencers are equally distanced and that the cutoff consumer indifferent between two neighboring influencers always receives a zero utility. Denote $\bar{J} = \operatorname{argmax}_{J\geq 0} \left(\bar{\Pi}_J - J\varepsilon\right)$. Furthermore, by Eq. (4), we can explicitly solve $\bar{J} = \pi\sqrt{\frac{2y}{I\varepsilon}}$.[12] In light of Lemma 5, we now consider the asymptotics when the number of influencers increases, given that all influencers are always equally distanced.

Lemma 6 (Asymptotics with many influencers)**.** *i) $1 \leq J^* \leq \left[\frac{2\pi y}{\varepsilon}\right]$; ii) For γ sufficiently large, $J^* = \bar{J}$ if $J = \bar{J} * m$ where $m \in \mathbb{N}$ or $J \to \infty$; iii) As $\varepsilon \to 0$, $J^* \to \infty$; iv) Given the residual bargaining protocol, the seller and influencers' payoffs are given by*

$$w_j^* = (1-\gamma)(\bar{\Pi}_{J^*} - \Pi_{\{1,\cdots,J^*\}/\{1\}} - \varepsilon), \quad \textit{and} \quad U_1^* = \bar{\Pi}_{J^*} - J^*\varepsilon - J^* w_1^*.$$

Proof. See Appendix A.8. $\square$

Lemma 6 yields the following insights. First, when the hiring cost is non-negligible, the number of influencers hired is finite. Second, even with endogenous bargaining power

[10] It can also be generated from coalitional Nash bargaining (Compte and Jehiel, 2010)

[11] The floor function $[x]$ returns the greatest integer less than or equal to $x \in \mathbb{R}$.

[12] When $\bar{J}$ is not an integer, it should be understood such that $\bar{J} \in \left\{\left[\pi\sqrt{2y/I\varepsilon}\right], \left[\pi\sqrt{2y/I\varepsilon}\right] + 1\right\}$.

building, the fixed cost guarantees all influencers can receive a positive wage. Third, when the seller enjoys a large bargaining power, the hiring plan always maximizes the joint profit for the seller and influencers in a large market. Fourth, when the hiring cost vanishes, the seller can afford hiring a large number of influencers so that each influencer is perfectly substitutable and thus she gets all the revenue.

4 Price Competition and Influencer Hiring

We next add seller competition to the analysis. In our influencer economy, sellers compete in both influencers' labor market and the product market. For tractability and transparency of the main mechanism, we focus on $J = K = 2$ for the remainder of the paper. We normalize the technology cost parameter $c = 1$ and take $\varepsilon = 0$ for simplicity.[13] We consider "balanced matching" in which the two sellers each can only hire one influencer. A seller-influencer group is then characterized by the 3-tuple (y_m, θ_m, I_m) for $m = 1, 2$. We discuss "unbalanced matching" where one seller can hire multiple influencers in Section 7.

4.1 Seller-Influencer Group Heterogeneity

We examine how product quality, influencer style, and influence power each affect the product competition in the influencer economy. For simplicity, we first consider one-dimension heterogeneity in endogenously formed seller-influencer groups. Note that when either the sellers or the influencers are homogeneous, the endogenous matching becomes trivial, allowing us to isolate the impact of the heterogeneity on influencer consumer-base acquisition and product market competition.

Heterogeneous product quality. To understand how product quality affects the competition, we set $I_1 = I_2 = I$ and $\theta_1 = \theta_2 = \theta$, but w.l.o.g., assume that $y_1 \geq y_2$. Proposition 2 fully characterizes the equilibrium. Obviously, when $y_1 = y_2$, the competition is most intense, and Bertrand competition leads to a zero profit equilibrium and both influencer-seller

[13] This assumption invites further explanation. The fixed hiring cost serves as an entry cost, and may lead to an endogenous entry problem. Recall that in a standard Bertrand price competition with endogenous entry, the seller offering a lower price always gets the whole market and the resulting payoff does not depend on the losing side seller's price. In contrast, unlike the previous Bertrand price competition, sellers' payoffs depend on the proposed prices by all sellers in our model setup. Depending on the equilibrium competition outcome, there are three cases: i) both sellers hire influencers with probability one, which occurs when equilibrium profits are sufficiently large; ii) one strong seller hires an influencer with probability one, the other seller hires an influencer with a probability less than one, which occurs when one seller-influencer group is much stronger; and iii) both sellers hire influencers with probability less than one, which occurs when competition between two groups are very intense. To circumvent the technical complexity, we either assume the equilibrium profits are sufficiently large, which reduces to heterogeneity among different seller-influencer groups, to cover the hiring cost, or simply discard the fixed hiring cost. We take the second approach because the fixed hiring cost is only crucial to the non-monotonicities result in Section 3.2 and do not generate extra insights in remaining sections.

groups set their own price to zero. However, as long as $y_1 > y_2$, there exists an equilibrium in which both groups obtain a positive profit. The intuition is clear. When $y_1 > y_2$, even Group 2 sets $p_2^* = 0$, the product by Group 1 is still more attractive for consumers with his style type close to the influencer, i.e., $\|x - \theta\| \to 0$. Thus, Group 1 has an incentive to set a positive price. This, in turn, implies that Group 2 can get a positive profit if Group 1 decides not to set $p_1^* = 0$ by attracting consumers not targeted by Group 1.

Denote by $k(j)$ the matched seller's identity for influencer $j \in \{1, 2\}$ under equilibrium.

Proposition 2. *Assume that $I_1 = I_2 = I$, $\theta_1 = \theta_2 = \theta$ and $y_1 \geq y_2$. There exists an equilibrium in which $k(j) = j$ for $j = 1, 2$. After matching, the two seller-influencer groups choose their prices such that $(p_1^C, p_2^C) = \left(\frac{2y_1(y_1-y_2)}{4y_1-y_2}, \frac{y_2(y_1-y_2)}{4y_1-y_2}\right)$. Group 1 targets consumers with type $\|x - \theta\| \leq \frac{2Iy_1}{4y_1-y_2}$ and group 2 targets type $\frac{2Iy_1}{4y_1-y_2} < \|x - \theta\| \leq \frac{3Iy_1}{4y_1-y_2}$. Correspondingly, their profits are given by*

$$\Pi_1^C = \frac{8Iy_1^2(y_1 - y_2)}{(4y_1 - y_2)^2}, \quad \textit{and} \quad \Pi_2^C = \frac{2Iy_1y_2(y_1 - y_2)}{(4y_1 - y_2)^2}. \tag{5}$$

and payoffs for sellers and influencers are given by

$$U_1 = \gamma\Pi_1^C, \; U_2 = \gamma\Pi_2^C, \; w_1 = (1 - \gamma)\Pi_1^C, \; \textit{and } w_2 = (1 - \gamma)\Pi_2^c$$

Proof. See Appendix A.9. □

Figure 6 illustrates the equilibrium described in Proposition 2. Specifically, the seller with a high-quality product targets those high value consumer demographic sufficiently close to the influencer's style, as illustrated with blue lines. In contrast, the group with a low-quality product targets those consumers relatively far from the influencer's style, and we illustrate it with red lines.

Furthermore, from Proposition 2, when $\frac{y_1}{y_2} \to 1$, it converges to the Bertrand competition, which features $(p_1^C, p_2^C) = (0, 0)$. In contrast, when $y_1 \gg y_2$, it converges to an equilibrium with $(p_1^C, p_2^C) = (y_1/2, y_2/4)$, which means that the product with a high quality is priced at its monopoly price, while the product of a low quality is priced at a monopoly price in the residual market after removing the market share taken by the strong seller.

Heterogenous influencer power. We now move on to examine the impact of influencer power on product prices. We let $y_1 = y_2$ and $\theta_1 = \theta_2$. Obviously, when $I_1 = I_2$, the only equilibrium outcome sustained is the Bertrand price competition. However, when $I_1 > I_2$, there exists an equilibrium in which both groups obtain a positive profit. The reason is as follows. Note that $p_1 \geq p_2$, otherwise the second influencer-seller group will be priced out

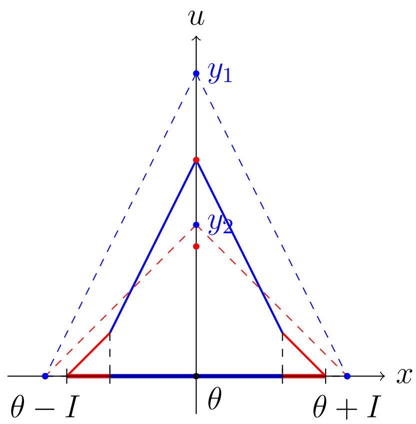

Figure 6: Heterogenous product quality

of the market. Moreover, the first group never wants to set price $p_1 = 0$ because consumers with type $x \in \mathbf{S}^1$ such that $I_2 < \|x - \theta\| \leq I_1$ always prefers Group 1 when $p_1 = 0$. Thus, group 1 can always ensure a positive profit by slightly increase the price. This in turn implies that the second group can get a positive profit by attracting consumers close to θ.

Proposition 3. *Assume that $y_1 = y_2 = y$, $\theta_1 = \theta_2 = \theta$ and that $I_1 \geq I_2$. There exists an equilibrium in which $k(j) = j$ for $j \in \{1, 2\}$. After matching, the two seller-influencer groups set $(p_1^C, p_2^C) = \left(\frac{2y(I_1 - I_2)}{4I_1 - I_2}, \frac{y(I_1 - I_2)}{4I_1 - I2}\right)$. Furthermore, group 1 targets consumers with types $\frac{I_1 I_2}{4I_1 - I_2} < \|x - \theta\| \leq \frac{I_1(2I_1 + I_2)}{4I_1 - I_2}$ and group 2 targets consumers with types $\|x - \theta\| \leq \frac{I_1 I_2}{4I_1 - I_2}$. Correspondingly, their profits are given by*

$$\Pi_1^C = \frac{4I_1^2(I_1 - I_2)y}{(4I_1 - I_2)^2}, \quad \textit{and} \quad \Pi_2^C = \frac{I_1 I_2(I_1 - I_2)y}{(4I_1 - I_2)^2}$$

and payoffs for sellers and influencers are given by

$$U_1 = \gamma \Pi_1^C, \; U_2 = \gamma \Pi_2^C, \; w_1 = (1 - \gamma)\Pi_1^C, \; \textit{and} \; w_2 = (1 - \gamma)\Pi_2^C.$$

Proof. See Appendix A.10. □

Figure 7 illustrates the equilibrium described in Proposition 3. Specifically, the group with a more powerful influencer targets the consumer demographic sufficiently far away from the influencer in style, as illustrated with blue lines. This is because they want to avoid tough price competition with the other group. Indeed, they can afford this because of the strong influencer power. In contrast, the group with a relatively weak influencer targets those consumers sufficiently close to the influencer's style, and we depict it with red lines.

The following corollary characterizes the two limit cases that $\frac{I_1}{I_2} \to \infty$ and that $\frac{I_1}{I_2} \to 1$.

Corollary 1. *When $\frac{I_1}{I_2} \to \infty$, the pricing strategies are given by $(p_1^C, p_2^C) = (\frac{y}{2}, \frac{y}{4})$, and when $\frac{I_1}{I_2} \to 1$, $(p_1^C, p_2^C) = (0, 0)$, the Bertrand price competition outcome.*

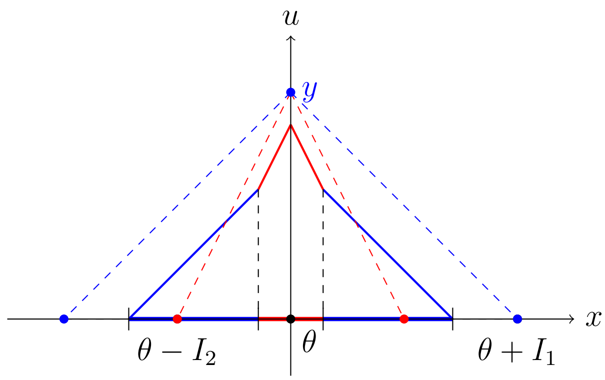

Figure 7: Heterogenous influence power

Note the difference in competition mode as it depends on how the power difference arises. When an influencer-seller group is more powerful because of the consumption value of the product, the stronger group focuses on attracting consumers with a taste similar to that of the influencer. In contrast, when the group's power comes from how easily the influencer attracts followers, the stronger group focuses on those consumers not reachable by the weaker group and sacrifice the loyal followers in the sense of taste proximity.

Heterogenous influencer style. Now, we consider how heterogeneity in influencers' style type affects market power and competition. Specifically, we study the equilibrium when there only exists heterogeneity in influencers' style, and we fix the other two dimensions (i.e., $y_1 = y_2$ and $I_1 = I_2$) to remove confounding effects.

Recall that $\beta = \|\theta_1 - \theta_2\|$. Define $\beta_0 := \frac{2}{67}(-7 + 5\sqrt{10})I \approx 0.263I$.

Proposition 4. *When $\beta \geq \beta_0$, there exists a pure strategy equilibrium such that $k(j) = j$. After matching, the two seller-influencer groups set prices such that*

$$p_1^C = p_2^C = \begin{cases} \frac{y}{5I}(2I + \beta), & \text{if } \beta_0 \leq \beta \leq \frac{6}{7}I \\ y * \left(1 - \frac{\beta}{2I}\right), & \text{if } \frac{6}{7}I < \beta \leq I \end{cases}$$

The two groups' profits are given by

$$\Pi_1^C = \Pi_2^C = \begin{cases} \frac{3y}{50I} * (2I + \beta)^2, & \text{if } \beta_0 \leq \beta \leq \frac{6}{7}I \\ y * \beta \left(1 - \frac{\beta}{2I}\right), & \text{if } \frac{6}{7}I < \beta \leq I \end{cases}$$

Moreover, for $\beta < \beta_0$, there exists no pure strategy equilibrium.

Whenever an equilibrium exists,

$$U_1 = \gamma \Pi_1^C, \; U_2 = \gamma \Pi_2 \; w_1 = (1-\gamma)\Pi_1^c, \; \text{and } w_2 = (1-\gamma)\Pi_2^c$$

Proof. See Appendix A.11. □

4.2 Multi-dimensional Heterogeneity

To focus on pure strategy equilibria that exists, we only extend the analysis to two cases of multi-dimensional heterogeneity. We investigate in the first case local monopoly when influencers' styles are sufficiently distinct. We then examine in a second case where one seller-influencer group dominates the entire market. Specifically, assume $y_1 \geq y_2$ and $I_1 \geq I_2$. We will later show that such a configuration can follow from the assortative matching and endogenous power acquisition between sellers and influencers.

Proposition 5 (Two Local Monopoly Sellers)**.** *If $\|\theta_1 - \theta_2\| \geq \frac{I_1+I_2}{2}$, the equilibrium matching is assortative, that is, $k(j) = j$ for $j \in \{1, 2\}$. Furthermore, the two influencer-seller groups are serving distinct subsets of consumers and their products are priced at their own monopoly prices, i.e., $p_k^* = \frac{y_k}{2}$. Profits are given by $\Pi_1 = \frac{y_1 I_1}{2}$ and $\Pi_2 = \frac{y_2 I_2}{2}$. Finally, $U_k = \gamma \Pi_k$ and $w_k = (1-\gamma)\Pi_k$ for $k \in \{1, 2\}$.*

Proof. See Appendix A.12. □

Proposition 5 reveals two key messages. First, when influencers' styles are sufficiently distinct, the competition is minimal and sellers enjoy local monopoly power, regardless of their product quality. Second, we establish assortative matching under regulated matching, which means that a seller with high quality products is matched with a more powerful seller. More discussions can be found in Section 6. Moreover, since influencers only help expose products and attract consumers and does not increase the common value of consumption y, the product with a better quality is always priced higher.

Next, we establish a market dominance result, i.e., a single seller-influencer group takes the entire product market. Note that when $\|\theta_1 - \theta_2\| < \frac{I_1+I_2}{2}$, none of the influencer-seller groups can get monopoly profits when the influence power and product quality do not differ between the two groups. However, when both the influence power and product quality of one group are sufficiently large relative to those of the other group, the market can be dominated by a monopolist. Assume that $y_1 > y_2$ and $I_1 > I_2$. Recall that $\beta = \|\theta_1 - \theta_2\|$.

Proposition 6 (Market Dominance)**.** *If $\frac{\beta}{I_1} \leq \min\{\frac{1}{2} - \frac{y_2}{y_1}, \frac{1}{2} - \frac{I_2}{I_1}\}$, then the second influencer-seller group has no market share and quits. Specifically, Seller 1 hires Influencer 1 and set the price at $p_1^* = \frac{y_1}{2}$ and Group 1's profit is $\frac{y_1 I_1}{2}$. Under equilibrium, payoffs for Seller 1 and Influencer 1 are given by $U_1 = \frac{\gamma y_1 I_1}{2}$ and $w_1 = \frac{(1-\gamma) y_2 I_2}{2}$. Additionaly, $U_2 = w_2 = 0$.*

Proof. See Appendix A.13. □

To gain market dominance, the influencers' style difference needs to be sufficiently small, and their power gap and product quality gap must be sufficiently large. Furthermore, note that it is not sufficient to just have much greater influence power or much better product to force out the rival group in the price competition. Suppose group 1 has way superior product, group 2 can still compete to gain some market share because the influencer he works with creates what is similar to a sufficiently large product differentiation ($\theta_1 \neq \theta_2$). We elaborate on this substitution by style difference for product differentiation in Section 5.

5 Style Heterogeneity and Product Differentiations

Having understood how sellers hire influencers and price products, we now endogenize sellers' production stage. Specifically, we investigate how influencer heterogeneity affects horizontal and vertical product differentiations, compared to traditional economies.

5.1 Horizontal Product Differentiation

Our first observation is that influencers' style difference and product specialization are intuitively substitutes. To illustrate the insight, we compare two economies with and without influencers. The former is what the baseline model describes. Both influencers have identical influence power $I_1 = I_2 = I$, but they are heterogeneous in their style, i.e., $\theta_1 \neq \theta_2$. In particular, θ_j are drawn from the unit circle $\mathbf{S}^1$ with a known density $f(\theta)$. Hence, both influencer's type and influence power are exogenously given, although there exists uncertainty.

The classical setup in Salop (1979) adequately describes an economy without influencers: Suppose that without influencers, the seller can enter the market selling their products by paying a fixed cost $F_H > 0$, and once they are in the market, they can costly change their style locations. In particular, consumers uniformly distributed on the unit circle $\mathbf{S}^1$ demands one unit and receives a utility of $u_i = y - p - t\|x_i - \alpha\|$, where $\|x - \alpha\|$ is the distance from consumer i to the seller, who selects a location α. Here, denote by y and p the product quality and the price charged. To simplify the analysis, we consider the case with two identical sellers competing through product specialization. In light of Eq. (1), the transportation cost setting in Salop (1979) is equivalent to an influencer economy setting with the exogenous influence power given by $\frac{y}{t}$.

Now, we turn to show the equilibrium construction in the influencer economy. Specifically, sellers can either choose to hire an influencer and accept his style type, or they can pay a big fixed cost and chooses any arbitrary location on the unit circle $\mathbf{S}^1$. As in Salop (1979), each seller can only select one style for her product, which means that she can only hire one influencer, or pay the fixed cost and select a location only once. Furthermore, to mostly recycle existing results, we can focus on the knife-edge case in which $I = \frac{y}{t} \in (0, \pi)$, although main insights still go through and are best understood in a more general sense. After realizations of influencers' style types/locations, sellers face a trade-off between incurring

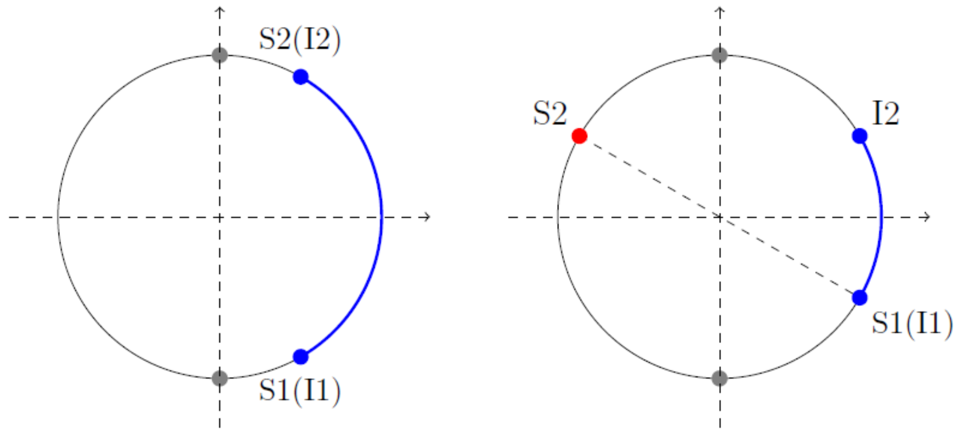

Figure 8: Influencer style difference and product differentiation

the fixed cost and accepting the potentially undesirable style locations by influencers.

Given the realizations of θ_j, recall that $\beta := \|\theta_1 - \theta_2\|$ and $\beta_0 = \frac{2}{67}(-7 + 5\sqrt{10})I$. By Proposition 4, if both sellers accept influencers' locations, the price competition outcome yields profits such that $\Pi_1(\beta) = \Pi_2(\beta)$, that is,

$$\Pi_1(\beta) = \begin{cases} \frac{3y}{50I} * (2I + \beta)^2, & \text{if } \beta \in [\beta_0, \frac{6}{7}I] \\ y * \left(\beta - \frac{\beta^2}{2I}\right), & \text{if } \beta \in (\frac{6}{7}I, I) \\ \frac{1}{2}yI, & \text{if } \beta \geq I \end{cases} \tag{6}$$

Moreover, for $\beta < \beta_0$, there exists no pure strategy equilibrium. To avoid the non-existence issue of equilibrium, we assume that $\beta \geq \beta_0$ and that

$$0 < F_H < \Pi_1(I) - \Pi_1(\beta_0), \tag{7}$$

which implies that there exists a unique β^* such that $\Pi_1(\beta^*) = \Pi_1(I) - F_H$ holds.

Proposition 7 (Horizontal Product Differentiation)**.** *Assume that the fixed cost F_H is relatively small (i.e., Eq. (7) holds), then there exists an equilibrium such that:*
(i) When $\beta \geq \beta^*$, each seller hires one influencer and accepts his style location.
(ii) When $\beta < \beta^*$, one seller hires an influencer and accepts his style location, and the other seller pays the fixed cost F_H and select a location such that $\|\alpha_1 - \alpha_2\| \geq I$.

Proof. See Appendix A.14. □

Figure 8 illustrates the equilibrium in Proposition 7. The left sub-figure entails a case with large style difference. The style locations for Influencers 1 and 2 are marked in blue nodes, while the two gray nodes illustrates the optimal style difference under the maximum differentiation principle (i.e., $\|\alpha_1 - \alpha_2\| = I \to \pi$). The blue arc corresponds to influencers style difference β. In this case, both sellers hire influencers to save the fixed cost in product differentiation and adapt to their corresponding influencer's style, and maximum differentiation principle fails. The right figure illustrates the equilibrium when influencers' style difference is small. In this case, one seller, say Seller 1, hires an influencer, say influencer 1. Given this fact, Seller 2 chooses product differentiation and incur the fixed cost F_H to avoid the toughness of price competition with Seller 1. In the figure, Seller 2's location is marked in red. Hence, maximum differentiation principle is restored.

Corollary 2 (Failure of Maximum Differentiation Principle)**.** *Whenever $\beta \in (\beta^*, I)$, the maximum differentiation principle fails, and thus influencers' style difference partly substitutes product differentiation. Furthermore, when influencers' style differentiation is small (i.e., $\beta_0 \leq \beta < \beta^*$), sellers engage in product differentiation. On the opposite, there exists no product differentiation when influencers' style difference is large (i.e., $\beta > \beta^*$).*

The main message in Corollary 2 is that the well-established principle of maximum differentiation found in the literature (e.g., d'Aspremont et al., 1979; De Frutos et al., 1999) would no longer hold in an influencer economy. More differentiated markets yield lower substitutability among products, therefore giving firms less elastic demand for the products. When influencers' style difference is small, sellers differentiate products to reduce competition. When influencers' style difference is large, sellers hires influencers and have no incentive to differentiate products.

5.2 Vertical Product Differentiation

The relationship between influencers' difference and vertical product differentiation (i.e., product quality differentiation) is more complex. When influencers are homogeneous and the influencer-seller matching is fixed, sellers have an incentive to differentiate product quality to avoid price competition (Shaked and Sutton, 1982). Consequently, we might observe heterogeneous product quality between sellers. Yet, when we allow for seller competition, heterogeneous product quality also arises, regardless of influencers' heterogeneity. This is mainly due to the incentive to grab the whole market and beat the competitor, rather than to avoid Bertrand style competition. Furthermore, vertical product differentiation is non-monotonic in the influencers' style difference.

Consider an endogenous balanced matching. Two influencers have identical influence powers and they might differ in their style locations (i.e., $I_1 = I_2$ and $\theta_1 \neq \theta_2$). Denote $\beta = \|\theta_1 - \theta_2\|$. The two sellers offer products with identical quality $y_1 = y_2 = \underline{y}$ ex ante. To reduce notation, we assume $I_1 = I_2 = 1$ and $\underline{y} = 1$. Influencers can pay a fixed cost F_V to invest in R&D to increase the product quality to $y > 1$. We use "NI" and "I" to denote "No Investing in high quality" and "Investing in high quality". Recall that $\beta_0 =$

$\frac{2}{67}(-7+5\sqrt{10})I \approx 0.263$. From Proposition 4, for $\beta \in [\beta_0, \frac{5}{6}I]$,[14]

$$\Pi_1^C = \frac{3y}{50I} * (2I+\beta)^2 = yA(\beta), \quad \text{where} \quad A(\beta) = \frac{3}{50}(2+\beta)^2.$$

For ease of reference, denote

$$V_1(\beta, y) = \Pi_{I,NI}^1 - \Pi_{NI,NI} = \Pi_{I,NI}^1 - A(\beta)$$
$$V_2(\beta, y) = \Pi_{I,I} - \Pi_{I,NI}^2 = yA(\beta) - \Pi_{I,NI}^2,$$

where $\Pi_{I,NI}^1 = \frac{y(1+2y)\left(2+8y+4y^2+\beta(4+3y)\right)^2}{(1+y)(8+19y+8y^2)^2}$, and $\Pi_{I,NI}^2 = \frac{(2+y)\left(4+8y+2y^2+\beta y(3+4y)\right)^2}{(1+y)(8+19y+8y^2)^2}$

Lemma 7 (Non-monotonic Vertical Differentiation)**.** *Assume that: i) $\beta_0 \leq \beta \leq \frac{5}{6}$; ii) $\overline{y}(1-\beta_0) \leq \underline{y}$; and iii) $V_1(\beta_0, y) < F_V < V_2(\frac{5}{6}, y)$. Then, there exists $\overline{\beta}$ and $\underline{\beta}$ such that:*

(i) $\beta \geq \overline{\beta}$, there exists one Nash Equilibrium (I, I);

(ii) $\underline{\beta} \leq \beta < \overline{\beta}$, there are two Nash Equilibrium: (I, NI) and (NI, I);

(iii) $\beta_0 \leq \beta < \underline{\beta}$, there exists one Nash Equilibrium (NI, NI).

Proof. See Appendix A.15. □

The regularity conditions in Lemma 7 have three. One, condition i) ensures the existence of pure strategy equilibrium before investment and reduce unnecessary complications in profits calculation. Two, condition ii) ensures that under asymmetric investment (i.e., (I, NI)), the seller-influencer group with higher product quality does not dominate and force the other group quit the market. Three, condition iii) focuses on the most interesting cost range in which non-monotonicity arises.[15]

Lemma 7 characterizes a non-monotonic relationship between influencers' style difference and vertical product differentiation. Figure 9 illustrates the equilibrium configuration in Lemma 7 with $y_1 = 5/4$ and $y_2 = 1$. Specifically, the horizontal axis β corresponds to the difference in influencers' style locations. The two functions, $V_1(\beta, y)$ and $V_2(\beta, y)$, corresponding to the blue and red solid line in the figure, measures the profit gap between "Investing in high quality" and "No Investing", given the other group chooses "No Investing" and "Investing in high quality". Fix the cost of investment $F_V > 0$ (i.e., the purple dashed line). There are three regions, "I", "II" and "III", divided by two cutoffs $\overline{\beta}$ and $\underline{\beta}$. In region III, $V_2(\beta, y) > F_V$, and thus both groups choose to invest in high quality. In region

[14]We restrict $\beta \in \left[\beta_0, \frac{5}{6}I\right]$, instead of $\beta \in \left[\beta_0, \frac{6}{7}I\right]$, to make profits formula simple (see Lemma 7).

[15]For $F_V \leq V_1(\beta_0, y)$, (I, I) is the unique Nash equilibrium. Similarly, for $F_V > V_1(\frac{5}{6}, y)$, (NI, NI) is the unique Nash equilibrium.

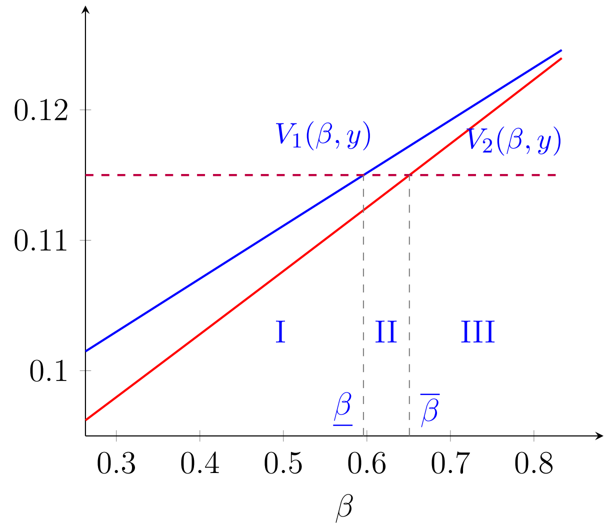

Figure 9: Vertical product differentiation

II, $V_2(\beta, y) < F_V$ and $V_1(\beta, y) > F_V$, which means only one groups chooses to invest in high quality. In region I, both groups chooses not to invest in high quality.

Lemma 7 implies that, when influencers' difference is small, as the style difference β increases, we first see more vertical differentiation (thus influencers' difference and vertical product quality differentiation are complements). In contrast, when influencers' difference is large, as the style difference increases, we see less vertical differentiation (thus these two are substitutes). This result is intuitive: For small β (i.e, influencers have similar style locations), the competition is very intense, which greatly limits the return from investing in high quality. Thus, both groups choose low quality and the vertical differentiation is minimal. For intermediate β, the competition is less intense which improves the investment return. However, the investment profit is only big enough to support one group investing, and if both groups invest, then one group cannot break even, leading to the observed vertical differentiation. Last, for large β, the competition is very minimal and even when both groups invest, they can break even. We only need to note that the investment profit is strictly increasing in the underlying influencer difference. Hence, we observe no vertical product differentiation again.

The following result follows directly:

Proposition 8. *When influencers' style difference is relatively small, it is a complement with vertical product differentiation; When influencers' style difference is large, it becomes a substitute for vertical differentiation.*

6 Endogenous Influence and Welfare Implications

We now move to t_0 to allow the influences to endogenize their influence either in power or type. First, we allow endogenous power acquisition and show that socially inefficient under-investment and over-investment in influence can arise due to externality and endogenous bargaining power issue. Second, we show that maximum style differentiation and assortative seller-influencer matching hold in the long run.

6.1 Inefficient Power Acquisition

This section discusses influencers' incentives to endogeneize their power selection. First, we illustrate this with a toy example. Second, we show that inefficient power acquisition can arise because of externality and endogenous bargaining power building.

Indeed, influencers have a strong incentive to compete through power/style differentiation to secure a more favorable outside option, as illustrated by Example 1 below.

Example 1. *Assume $\gamma < 1$. Consider the perfectly symmetric case with endogenous hiring (i.e., $y_1 = y_2 =: y$, $\theta_1 = \theta_2 =: \theta$, and $I_1 = I_2 =: I$). Influencer 2 can get a wage of*

$$w_2 = \frac{(1-\gamma)I_1 I_2 (I_1 - I_2) y}{(4I_1 - I_2)^2} \tag{8}$$

Initially, $I_1 = I_2 = I$ and thus $w_2 = 0$. However, if influencer 2 can commit to influence power reduction by choosing a small I_2, we can optimize over Equation (8) to get

$$I_2^* = \frac{4}{7} I, \text{ and } w_2 = \frac{yI}{48}.$$

In other words, Influencer 2 reduces influence power to avoid price competition.[16]

A related question is the welfare implications of endogenous influencers' ability. For instance, inefficiency can arise because of the potential arms race among influencers, which depends on how much additional utility is purely due to style preference. Note that the utility from consumption is bounded above, which means that many influencers spend effort to acquire power and too many endogenously become influencers.

Indeed, we can show that there might exist socially sub-optimal investment in power acquisition. To simplify the analysis, we focus on a very specific example with a monopolist seller and two identical influencers. Initially, $I_1 = I_2 = \pi$, $y_1 = y_2 = y$. Influencers can pay a fixed cost $C_T > 0$ to increase influence power to 2π before they are hired by sellers. The question is when arms race is welfare optimal? We use "I" and "NI" to denote the influencer

[16] Given that Influencer 2's voluntary power reduction, influencer 1 has no incentive to reduce power.

strategies entailing investing in power acquisition and not investing (keeping the power at π) respectively.

Proposition 9 (Power Acquisition & Inefficient Arms Race)**.** *Assume that $I_1 = I_2 = \pi$ and $\|\theta_1 - \theta_2\| = \pi$.*

The Nash Equilibrium for influencers to invest in power acquisition are given by:

(i) (NI, NI) is a Nash Equilibrium when $C_T > \frac{1}{3}(1-\gamma)y\pi$;

(ii) (I, NI) and (NI, I) are Nash Equilibrium when $C_T \in (\frac{1}{6}(1-\gamma)y\pi, \frac{1}{3}(1-\gamma)y\pi]$;

(iii) (I, I) is a Nash Equilibrium when $C_T \leq \frac{1}{6}(1-\gamma)y\pi$.

The optimal decision rule to maximize total welfare is given by

(i) when $C_T > \frac{1}{6}y\pi$, (NI, NI) is optimal, i.e., no influencer should invest;

(ii) when $\frac{1}{12}y\pi < C_T \leq \frac{1}{6}y\pi$, (I, NI) (or (NI, I)) is optimal, i.e., only one influencer should invest;

(iii) when $C_T \leq \frac{1}{12}y\pi$, (I, I) is optimal, i.e., both influencers should invest.

Proof. See Appendix A.16. □

Proposition 9 shows that power acquisition can exhibit over-investment, efficient investment, or under-investment relative to a socially efficient benchmark. To see it, we can check the incentive misalignment between power acquisition and total welfare. For instance, both influencers invest in power acquisition when $C_T \leq \frac{1}{6}(1-\gamma)y\pi$. In contrast, total welfare maximization requires $C_T \leq \frac{1}{12}y\pi$ for both influencers to invest. This implies that: (i) when $\gamma = \frac{1}{2}$, these two conditions coincide, which implies that power acquisition is welfare optimal; ii) when $\gamma > \frac{1}{2}$, there exists insufficient power acquisition for $C_T \in \left(\frac{1}{6}(1-\gamma)y\pi, \frac{1}{12}y\pi\right]$; and iii) when $\gamma < \frac{1}{2}$, there exists over-investment in power acquisition for $C_T \in \left(\frac{1}{12}y\pi, \frac{1}{6}(1-\gamma)y\pi\right]$.

There are two forces driving this result. One, the incentive misalignment between power acquisition and total welfare. Power acquisition does not consider externality on consumer welfare, as well as on other influencers. When the influencer market is not crowded, power acquisition can improve welfare by increasing consumer utility and it can exhibit under-investment when the positive externality on consumer welfare is not internalized. In contrast, when the influencer market is congested and all influencers compete for better wages, the arm race of potential influencers leads to wasting effort because the actual consumers they influence is very limited. Two, the bargaining power division among the seller and influencers. The bargaining power parameter assigned to influencers, $1-\gamma$, can also distort the incentive for power acquisition. A large γ reduces the incentive for power acquisition, while a small

γ (i.e., $1-\gamma \uparrow$) empowers the incentive for power acquisition. Hence, depending on the congestion of influencer market and the bargaining power, we might observe over-investment, efficient investment and under-investment in power acquisition.

6.2 Style Selection and Seller-influencer Matching

In this section, we allow influencers to choose influence style and study seller-influencer matching with endogenous styles. To this end, we first consider an example with costly style differentiation as illustrated in Example 2. Then, we formally recover the maximum differentiation principle in the limit case with costless style selection.

Example 2. *Consider $y_1 = y_2 =: y$, $I_1 = I_2 =: I$, and $\theta_1 \neq \theta_2$. To ensure the existence of a pure strategy equilibrium, we also assume that $\beta := \|\theta_1 - \theta_2\| \geq \frac{2}{67}(-7 + 5\sqrt{10})I$. Now, influencer 2 can pay a cost, $C(b)$, to select his own style location θ_2^*, where $b := \|\theta_1 - \theta_2^*\|$, and we assume $C(\beta) = 0, C'(\beta) = 0$ and $C''(b) > 0$ for $b \geq \beta$. Then, the optimal style type satisfies $d^* \in (\beta, I)$, that is, influencer 2 always invests in style differentiation as long as $\beta < I$.*

A detailed proof of the assertion in Example 2 can be found in Appendix A.17. Note that maximum style differentiation fails even when style selection is costly.

Now, we consider the other case when style selection is costless. First, we present an assortative matching result when the maximal style differentiation principle holds. Specifically, rank sellers and influencers by their product quality and influence power so that $y_1 \geq y_2$ and $I_1 \geq I_2$. Denote by $k(j)$ the matched seller identity $k(j)$ for influencer $j = 1, 2$.

Lemma 8. *Assume that $\|\theta_1 - \theta_2\| \geq \frac{I_1+I_2}{2}$. Then $k(j) = j$ for $j = 1, 2$, that is, the strong (weak) seller is matched with the strong (weak) influencer.*

Proof. See Appendix A.18. □

Lemma 8 shows the emergence of assortative matching when influencers' style locations are exogenous given such that the maximal style differentiation principle holds. The seller with a more valuable good can offer to hire a more powerful influencer by proposing a higher wage because the seller-influencer group's total profit is supermodular in influencer power and product quality parameter.[17]

Next, we turn to the problem of endogenous style location selection. We prove a special case in which $\frac{I_1+I_2}{2} \leq \pi$.

Proposition 10 (Maximal style differentiation and assortative matching)**.** *Assume that $\frac{I_1+I_2}{2} \leq \pi$. When style location selected is costless, the maximum style differentiation holds,*

[17] A twice-differentiable function $f : X \times Y \to \mathbb{R}$ is supermodular iff $\frac{\partial^2 f}{\partial x \partial y} \geq 0$ for all $(x, y) \in X \times Y$.

that is, $\|\theta_1^* - \theta_2^*\| \geq \frac{I_1+I_2}{2}$*, and there exists no overlapping in consumers served. Furthermore, assortative matching applies under endogenous style location selection.*

Proof. See Appendix A.19. □

Proposition 10 states that influencers follow the maximal style differentiation principle whenever possible under both seller competition and regulated matching, because it minimizes the competition among influencer-seller groups. Furthermore, assortative matching ensues under endogenous style location selection.

The result is also in stark contrast with Gal-Or and Dukes (2003) that discovers a minimum differentiation in commercial media markets. Overthere, product differentiation is taken as exogenous and thus the substitutability between style differentiation and product differentiation is absent.

7 Unbalanced Matching, Exclusivity, and Regulation

In practice, a seller often hire multiple influencers and require the influencers not to advertise rival sellers' products (e.g., Zietek, 2016). For example, a large survey of influencers by Mavrck (Katz, 2019) shows that the majority of influencers (61%) are receiving exclusivity requests from brands. In fact, exclusivity contracts have been prevalent in industries such as healthcare and infurance and have led to many antitrust cases (Gal-Or, 1999). However, policies are being introduced to better protect the influencers and to reduce market concentration through encouraging competition. The industry has also grown in awareness that exclusivity should be mutual.[18] This means that either both sides can contract with multiple counterparties or both sides have to exclusively collaborate—exactly our setting of balanced matching aims to capture.

Nevertheless, to better understand the welfare implication of exclusivity contracts in this emerging industry and to guide regulatory policies geared towards balancing the power of influencers and sellers, we extend the analysis to "unbalanced matching." In such a setting, a seller can hire multiple influencers, but not the other way round, which is consistent with the contracting landscape in the early stages of influencer industry (e.g., Zietek, 2016). We then compare settings with and without balanced matching to derive two key results:

[18]Influencers increasingly value long-term partnerships With brands rather than one-off exclusivity requests; they also expect to be compensated more when exclusivity is required. As early as 2008, the entertainment industry began to see the value behind full-time creators building multi-platform brands, and influencers started getting Hollywood agents to help negotiate bilateral exclusive contracts (Collectively, 2020). New York State Regulators, China's State Administration for Market Regulation, and the British Federal Trade Commissions are all increasing regulations regarding influencer contracting. See, e.g., https://www.ftc.gov/news-events/press-releases/2019/11/ftc-releases-advertising-disclosures-guidance-online-influencers; http://www.zhonglun.com/Content/2020/11-11/1619074334.html; https://wwd.com/business-news/legal/wwd-law-review-noncompetes-influencers-model-protection-11038135/.

First, balanced matching (mutual exclusivity contracting) is optimal under one-dimensional heterogeneity even when we allow sellers to compete for multiple influencers. Second, unbalanced matching (uni-directional exclusivity) can be optimal when influencers' style locations are sufficiently unique in uncongested influencer markets.

We begin analysis with useful lemma about the joint profit when a seller hires both influencers. Note that the joint profit in Lemma 2 is established under $\beta = \pi$ and that influencer power can be large. Recall that $\beta = \|\theta_1 - \theta_2\|$.

Lemma 9 (Joint Profit Function $\Pi_{\{1,2\}}$). *The joint profit function is given by*

$$\Pi_{\{1,2\}} = \begin{cases} yI, & \text{if } \beta \geq I \\ \frac{(2I+\beta)^2}{8I} y, & \text{if } \beta \leq \frac{2I}{3} \\ \frac{(2I\beta-\beta^2)}{I} y, & \text{if } \beta \in \left(\frac{2I}{3}, I\right) \end{cases} \tag{9}$$

and the pricing strategy is given by

$$p_1^* = p_2^* = \begin{cases} \frac{y}{2}, & \text{if } \beta \geq I \\ y * \left(\frac{\beta}{4I} + \frac{1}{2}\right), & \text{if } \beta \leq \frac{2I}{3} \\ y * \left(1 - \frac{\beta}{2I}\right), & \text{if } \beta \in \left(\frac{2I}{3}, I\right) \end{cases}$$

Proof. See Appendix A.20. □

Lemma 9 helps us establish that the equilibrium outcome coincides with that in regulated matching when heterogeneity is single-dimensional. Suppose that $y_1 \geq y_2$ and $I_1 \geq I_2$, we have:

Lemma 10 (Equilibrium under unbalanced matching). *Allowing uni-directional exclusivity contracts:*

(i) When the influencers or the sellers differ in a single dimension, the equilibrium coincides with that in Proposition 2, 3 and 4.

(ii) When influencers are sufficiently unique (i.e., $\beta \geq \frac{I_1+I_2}{2}$), seller 1 hires both influencers and offers prices at $p_1^* = p_2^* = \frac{y_1}{2}$. Payoffs for sellers and influencers satisfy:

$$U_1 = \frac{\gamma y_1 (I_1 + I_2)}{2}, U_2 = 0, \ w_1 = \frac{(1-\gamma) y_1 I_1}{2} \text{ and } w_2 = \frac{(1-\gamma) y_1 I_2}{2}.$$

Proof. See Appendix A.21. □

How do different forms of exclusivity affect welfare? Intuitively, compared to balanced matching, unbalanced matching features a monopolist seller with the relatively higher product quality. On the one hand, it increases welfare by replacing the seller with a low quality product, whose magnitude depends on the quality gap between sellers. On the other hand market concentration and monopolist pricing decreases surplus for consumers attracted, and prices out a large fraction of potential consumers. Note that when the quality gap between two products decreases, the former effect vanishes. Thus, unbalanced matching hurts consumers and social welfare when products have similar quality.

Proposition 11 compares the efficiency between balanced and unbalanced matchings.

Proposition 11 (Exclusivity Contracting and Welfare)**.**

(i) (Congested influencer market or homogeneous product market). Unbalanced matching lowers total welfare under one dimension heterogeneity, including heterogeneous product quality, heterogeneous influencer power and heterogeneous influencers' style locations.

(ii) (Uncongested influencer market). When $\beta > \frac{I_1+I_2}{2}$ *and* $y_1 > y_2$*, unbalanced matching dominates regulated matching in total welfare.*

Proof. See Appendix A.22. □

The key messages in Proposition 11 is intuitive. First, both product quality gap and influencer style difference affect intensity of seller competition. When the influencer market is not crowded and influencers' styles are distinct, regulation on mutual exclusivity contracting does not help encourage competition because of the inevitable local market power derived from influencer heterogeneity; given the economy features monopoly pricing anyway, uni-directional exclusivity is welfare-improving because it allows the better product to dominate.

In contrast, when products are close to being homogeneous or influencers are too similar in style, requiring mutual exclusivity and balanced matching can improve consumer welfare. Note that unbalanced matching always features joint profit maximization and a high quality product domination, while balanced matching features greater price competition. Simply put, the quality improvement channel is shut down in a homogeneous product market, while regulation can improve competition when the influencer market is crowded.

8 Conclusion

We build a model of the influencer economy in which (i) sellers produce goods and compete for consumers through influencers, (ii) sellers and influencers are matched in influencers' labor market and engage in Nash bargaining, and (iii) influencers acquire influence to attract consumers who identify with their style in addition to value the products they promote. We derive five key insights:

First, as technologies governing marketing outreach improve, the equilibrium features non-monotonicities in influencer market concentration, payoffs, and distributional inequality. Second, influencer heterogeneity and horizontal product differentiation are substitutes. At the same time, small style differences complement vertical product differentiation while large differences substitute. Third, assortative matching between sellers and influencers occurs under endogenous influence, with the maximum horizontal differentiation principle recovered in the limit of costless style selection. Fourth, the sellers' bargaining power counteracts the influencers' tendency to over-invest in influence power and they jointly determine the direction and magnitude of the sub-optimal acquisition. Fifth, regulations for balanced seller-influencer matching can encourage seller competition under single dimensional seller-influencer heterogeneity. But uni-directional exclusivity contracts are welfare-improving for sufficiently differentiated products and uncongested influencers' markets.

For tractability and to focus on the industrial organization of the influencer economy, we have largely abstracted away from the inner working of platforms and MCNs. In this regard, our findings constitute initial benchmark results rather than foregone conclusions. In our setting, platforms can be viewed as powerful influencing segments with large bargaining power, which leaves much to be desired. Profit sharing and contracting between influencers and platforms remain a crucial topic in understanding the digital economy. The organization of MCNs such as Douyin and Weibo, and their heterogeneity also constitute interesting future research.

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Online Appendix

A Relevant Proofs and Calculations

A.1 Proof of Lemma 1

Proof. First, consider the case that $\frac{I}{c} < \pi$. In this case, $D(p) \leq 2\pi$ for all $p \geq 0$. Thus, given the price $p \geq 0$, the total demand is given by

$$D(p) = 2(1 - p/y) * I/c$$

which further implies that $\Pi(p) = D(p) * p$ and thus $p^* = \frac{y}{2}$ and $\Pi(p^*) = \frac{yI}{2c}$.

Second, consider the case that $\frac{I}{c} \geq \pi$. In this case,

$$\Pi(p) = \begin{cases} 2\pi * p, & \text{if} \quad p \leq y(1 - c\pi/I) \\ 2p(1 - p/y) * I/c, & \text{if} \quad p > y(1 - c\pi/I) \end{cases}$$

Note that $\Pi(p)$ is continuous at $p = y(1 - c\pi/I)$. Again, depending on the value of $c \in \mathbb{R}_+$,

- $\frac{I}{c} \in [\pi, 2\pi]$. In this case, $y(1 - c\pi/I) \leq \frac{y}{2}$. The quadratic term implies that $\Pi(p)$ is strictly increasing for all $p \in [y(1 - c\pi/I), y/2]$ and strictly decreasing for $p > y/2$. Hence, $\Pi(p)$ is maximized at $p^* = \frac{y}{2}$, which yields $\Pi(p^*) = \frac{yI}{2c}$.

- $\frac{I}{c} \in (0, \pi)$. In this case, $y(1 - c\pi/I) > \frac{y}{2}$. This implies that $\Pi(p)$ is strictly increasing for $p \leq y(1 - c\pi/I)$ and strictly decreasing for $p > y(1 - c\pi/I)$ because the quadratic term is strictly decreasing for all $p > \frac{y}{2}$. Hence, $\Pi(p)$ is maximized at $p = y(1 - c\pi/I)$ and $\Pi(p^*) = 2\pi y(1 - c\pi/I)$.

Note that we can combine the two cases that $\frac{I}{c} < \pi$ and $\frac{I}{c} \in [\pi, 2\pi]$ to simplify the formula. The proof concludes. □

A.2 Proof of Lemma 2

Proof. Recall that $\Pi_{\{1,2\}}$ is the revenue when both influencers are hired by the monopolist seller, ignoring the fixed searching cost. When $c > \frac{I_1 + I_2}{2\pi}$, the seller sets prices $p_1^* = p_2^* = \frac{y}{2}$ and influencer j attracts consumers with $x \in \mathbf{S}^1$ such that $\|x - \theta_j\| \leq \frac{I_j}{2c}$ for $j = 1, 2$. Note that there exists no overlapping in consumers served since $\frac{I_1}{2c} + \frac{I_2}{2c} \leq \pi$. Thus, the monopoly pricing is feasible and thus $\Pi_{\{1,2\}} = \Pi_{\{1\}} + \Pi_{\{2\}} = \frac{y(I_1+I_2)}{2c}$.

Now, consider the case that $c < \frac{I_1+I_2}{2\pi}$. First, note that both influencers are active and serves a non-zero size of consumers. Otherwise suppose w.l.o.g. that influencer 2 is serving no consumer. Then, for any positive price p_1, we can always charge a slightly higher price for all consumers sufficiently close to θ_2. Now, suppose influencer 1 serves consumers $x \in \mathbf{S}^1$ such that $\|x - \theta_1\| \leq s_1$ with $s_1 \in [0, \pi)$. Prices p_1 and p_2 are set such that the marginal consumer (i.e., $x \in \mathbf{S}^1$ & $\|x - \theta_1\| = s_1$) gets zero surplus, otherwise it is profitable to increase the product price at least for one influencer. Hence,

$$p_1 = y\left(1 - \frac{cs_1}{I_1}\right), \text{ and } p_2 = y\left(1 - \frac{c * (\pi - s_1)}{I_2}\right)$$

Then, we can write down the total revenue as a function of s_1, that is,

$$\Pi_{\{1,2\}} = 2p_1 * s_1 + 2p_2 * (\pi - s_1).$$

Maximizing the joint revenue function, it yields

$$p_1^* = p_2^* = y * \left(1 - \frac{c\pi}{I_1 + I_2}\right), \tag{A.1}$$

and

$$s_1 = \frac{I_1 \pi}{I_1 + I_2}, \quad \text{and} \quad s_2 = \frac{I_2 \pi}{I_1 + I_2}. \tag{A.2}$$

which implies $\Pi_{\{1,2\}} = 2\pi y \left(1 - \frac{c\pi}{I_1 + I_2}\right)$ when $c < \frac{I_1 + I_2}{2\pi}$.

Finally, note that the condition $\frac{I_2}{I_1} \leq \frac{\pi y - \varepsilon}{\epsilon}$ holds iff $\frac{I_1 + I_2}{2\pi} < c \leq \frac{y I_1}{2\varepsilon}$. The proof concludes. $\square$

A.3 Proof of Lemma 3

Proof. First, note that it is always sub-optimal to hire influencer 2 alone, which is dominated by hiring influencer 1 alone. Furthermore, when both influencers are hired, the total profit is given by $\Pi_{\{1,2\}} - 2\varepsilon$. When only the strong influencer 1 is hired, the total profit is $\Pi_{\{1\}} - \varepsilon$. Hence, the joint maximal profit for the seller and two influencers is given by

$$W = \max\{\Pi_{\{1,2\}} - 2\varepsilon, \Pi_{\{1\}} - \varepsilon\}$$

Second, note that it is optimal to hire influencer 1 alone when $c \in \left(\frac{yI_2}{2\varepsilon}, \frac{yI_1}{2\varepsilon}\right)$. To see it, given that $\frac{\varepsilon}{y}(1 + \frac{I_1}{I_2}) < \pi$, the condition that $c > \frac{yI_2}{2\varepsilon}$ implies that $c > \frac{I_1 + I_2}{2\pi}$, and thus there exists no overlapping between consumers served if both influencers are served, that is, $\Pi_{\{1,2\}} = \Pi_{\{1\}} + \Pi_{\{2\}}$. This further implies that

$$\Pi_{\{1,2\}} - 2\varepsilon = \Pi_{\{1\}} - \varepsilon + \Pi_{\{2\}} - \varepsilon < \Pi_{\{1\}} - \varepsilon$$

and thus only the strong influencer 1 is hired.

Third, for $c \in [\bar{c}, \frac{yI_2}{2\varepsilon}]$, there are four cases.

- $c \in \left(\frac{I_1+I_2}{2\pi}, \frac{yI_2}{2\varepsilon}\right]$. We have: $\Pi_{\{1,2\}} = \frac{y(I_1+I_2)}{2c}, \Pi_{\{1\}} = \frac{yI_1}{2c}$, and $\Pi_{\{2\}} = \frac{yI_2}{2c}$.
- $c \in \left(\frac{I_1}{2\pi}, \frac{I_1+I_2}{2\pi}\right]$. We have: $\Pi_{\{1,2\}} = 2\pi y\left(1 - \frac{c\pi}{I_1+I_2}\right), \Pi_{\{1\}} = \frac{yI_1}{2c}$, and $\Pi_{\{2\}} = \frac{yI_2}{2c}$.
- $c \in \left(\frac{I_2}{2\pi}, \frac{I_1}{2\pi}\right]$.

$$\Pi_{\{1,2\}} = 2\pi y\left(1 - \frac{c\pi}{I_1+I_2}\right), \quad \Pi_{\{1\}} = 2\pi y\left(1 - \frac{c\pi}{I_1}\right), \quad \text{and} \quad \Pi_{\{2\}} = \frac{yI_2}{2c}$$

- $c \in \left(\bar{c}, \frac{I_2}{2\pi}\right]$.

$$\Pi_{\{1,2\}} = 2\pi y\left(1 - \frac{c\pi}{I_1+I_2}\right), \ \Pi_{\{1\}} = 2\pi y\left(1 - \frac{c\pi}{I_1}\right), \ \text{and } \Pi_{\{2\}} = 2\pi y\left(1 - \frac{c\pi}{I_2}\right)$$

We can directly verify that $\Pi_{\{1,2\}} - 2\varepsilon \geq \Pi_{\{1\}} - \varepsilon$.

Fourth, for $c \in (0, \bar{c})$, $\Pi_{\{1,2\}} = 2\pi y\left(1 - \frac{c\pi}{I_1+I_2}\right)$, and $\Pi_{\{1\}} = 2\pi y\left(1 - \frac{c\pi}{I_1}\right)$.

When $c = \bar{c}$, we have $\Pi_{\{1,2\}} - 2\varepsilon = \Pi_{\{1\}} - \varepsilon$ and it is easy to check that $\Pi_{\{1,2\}} - 2\varepsilon < \Pi_{\{1\}} - \varepsilon$ for all $c < \bar{c}$. The proof concludes. □

A.4 Proof of Lemma 4

Proof. The proof consists of two parts. First, we show that hiring both influencers are sub-optimal for all $c \leq \bar{c}$. Second, we prove there exists over-hiring for the two specific cases.

Optimal hiring for $c \leq \bar{c}$. Note that for $c \leq \bar{c}$, $\frac{I_1}{c} \geq 2\pi$ and the total welfare W_1 is given by

$$W_1 = 2\pi p_1 + 2 * \frac{\pi}{2}(y - p_1) - \varepsilon$$

Here, $p_1 = y\left(1 - \frac{c}{I_1}\pi\right)$. The first term $2\pi p_1$ is the seller-influencer group's joint profit, the second term $2 * \frac{\pi}{2}(y - p_1)$ is the consumer surplus, and the third term is the fixed hiring cost. This can be further simplified as

$$W_1 = \pi y + \pi y\left(1 - \frac{c}{I_1}\pi\right) - \varepsilon$$

Similarly, when both influencers are hired, the monopolist price and consumers served satisfy Eq (A.1) and (A.2). Actually, the monopolist pricing strategy coincides with that maximizing total welfare. The total welfare $\hat{W}_{1,2}$ is given by

$$\hat{W}_{1,2} = \big(2s_1p_1 + s_1(y - p_1)\big) + \big(2s_2p_2 + s_2(y - p_2)\big) - 2\varepsilon$$

Here, the first big term is the total welfare for consumers served by influencer 1, the second one by influencer 2, and the last term is the fixed hiring cost. By plugging the expressions from Eq (A.1) and (A.2), we can simplify it as

$$\hat{W}_{1,2} = \pi y + \pi y \left(1 - \frac{c\pi}{I_1 + I_2}\right) - 2\varepsilon$$

Note that when $c \leq 2\bar{c}$, $W_1 \geq \hat{W}_{1,2}$, and thus it is sub-optimal to hire both influencers when $c \leq \bar{c}$.

Case i). Note that $\Pi_{\{1,2\}} - 2\varepsilon = \Pi_{\{1\}} - \varepsilon$ and that social optimal hiring decision requires only influencer 1 is hired for all $c < \bar{c}$. First, if the monopolist seller only hire influencer 1, the Nash bargaining implies

$$U_1 = \gamma(\Pi_{\{1\}} - \varepsilon), \quad w_1 = (1-\gamma)(\Pi_{\{1\}} - \varepsilon), \quad \text{and} \quad w_2 = 0$$

In contrast, if the seller hires both influencer 1 and 2, the payoffs for the seller and influencers are given by

$$\begin{aligned} \hat{U}_1 &= \gamma(\Pi_{\{1,2\}} - 2\varepsilon) + (1-\gamma)(\Pi_{\{2\}} - \varepsilon), \quad w_2 = 0, \\ \text{and} \quad w_1 &= (1-\gamma)\big((\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{2\}} - \varepsilon)\big) \end{aligned} \tag{A.3}$$

By definition of $\bar{c}$, $\gamma(\Pi_{\{1\}} - \varepsilon) = \gamma(\Pi_{\{1,2\}} - 2\varepsilon)$, which implies that $U_1(\bar{c}) < \hat{U}_1(\bar{c})$. Furthermore, $U_1(c) < \hat{U}_1(c)$ for all $c \in (\bar{c} - \delta, c)$ for $\delta > 0$ sufficiently small. Thus, it sufficies to show the payoffs in Eq. (A.3).

Given two influencers are hired, the seller bargains with influencer 1 over the surplus $(\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{2\}} - \varepsilon)$, the marginal profit increment between hiring both influencers and only hiring influencer 2. If influencer 1 accepts this, Nash bargaining implies a payoff w_1 defined in Eq. (A.3). Indeed, on the off-equilibrium path (i.e, influencer 1 rejects the offer), the seller and influencer 2 shares the profit $\Pi_{\{2\}} - \varepsilon$. Similarly, the seller bargains with influencer 2 over the (negative) surplus $(\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{1\}} - \varepsilon) < 0$, the marginal profit gap between hiring both influencers and only hiring influencer 1. Due to individual rationality, influencer 2 only accepts $w_2 \geq 0$, and thus $w_2 = 0$. Indeed, on the off-equilibrium path, the seller and influencer 1 shares $\Pi_{\{1\}} - \varepsilon$.

Case ii). Define $\underline{c} = \frac{I_2}{I_1} * \frac{(I_1+I_2)\varepsilon}{2\pi^2 y}$, which satisfies that $\Pi_{\{1,2\}} - 2\varepsilon < \Pi_{\{1\}} - \varepsilon$ and that $\Pi_{\{1,2\}} - 2\varepsilon < \Pi_{\{2\}} - \varepsilon$. When the seller hires both influencer 1 and 2, the bargaining payoffs are given by:

$$\hat{U}_1 = \Pi_{\{1,2\}} - 2\varepsilon, \quad w_1 = 0, \quad \text{and} \quad w_2 = 0$$

When the seller only hires influencer 1, the Nash bargaining outcome is

$$U_1 = \gamma(\Pi_{\{1\}} - \varepsilon), \quad \text{and} \quad w_1 = (1-\gamma)(\Pi_{\{1\}} - \varepsilon)$$

Hence, it is optimal to hire both influencers when $\hat{U}_1 \geq U_1$, which implies that

$$\gamma \leq \frac{\Pi_{\{1,2\}} - 2\varepsilon}{\Pi_{\{1\}} - \varepsilon}$$

Thus, by definition of $\underline{\gamma}$, $\hat{U}_1(c) \geq U_1(c)$ for all $c < \underline{c}$ when $\gamma \leq \underline{\gamma}$.

In addition, note that

$$\frac{\Pi_{\{1,2\}} - 2\varepsilon}{\Pi_{\{1\}} - \varepsilon} > \frac{\Pi_{\{1\}} - 2\varepsilon}{\Pi_{\{1\}} - \varepsilon} \geq \frac{\Pi_{\{1\}}(\underline{c}) - 2\varepsilon}{\Pi_{\{1\}}(\underline{c}) - \varepsilon}$$

Here, we use the fact that $\Pi_{\{1,2\}} > \Pi_{\{1\}}$, and that the term $\frac{\Pi_{\{1\}} - 2\varepsilon}{\Pi_{\{1\}} - \varepsilon}$ is strictly decreasing in c. Hence, we have $\underline{\gamma} > \frac{\Pi_{\{1\}}(\underline{c}) - 2\varepsilon}{\Pi_{\{1\}}(\underline{c}) - \varepsilon}$. The proof concludes. □

A.5 Proof of Proposition 1

Proof. i) It follows directly from the observation that when $\gamma = 1$, the optimal hiring decision coincides with the social optimal market structure, which is monotone. Hence, by continuity, the argument goes through for sufficiently large $\gamma \to 1$.

- First, for $c > \frac{yI_2}{2\varepsilon}$, it is optimal to only hire influencer 1. This is because hiring influencer 2 generates a negative profit, which cannot help the seller bargain with influencer 1. Thus,

$$\hat{U}_1 = \gamma(\Pi_{\{1\}} - \varepsilon), \quad w_1 = (1-\gamma)(\Pi_{\{1\}} - \varepsilon), \quad \text{and} \quad w_2 = 0.$$

- Second, for $\bar{c} \leq c \leq \frac{yI_2}{2\varepsilon}$, it is optimal to hire both influencers. To see it, consider the case that the seller hires both influencers. In this case, the seller only shares with influencer 1 the surplus $(\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{2\}} - \varepsilon)$ by the same argument in the proof of Lemma 4 because influencer 2 is also hired. Similarly, the seller only shares with influencer 2 the surplus $(\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{1\}} - \varepsilon)$. Thus,

$$w_1 = (1-\gamma)\big((\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{2\}} - \varepsilon)\big), \text{ and } w_2 = (1-\gamma)\big((\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{1\}} - \varepsilon)\big)$$

and

$$\begin{aligned} U_1 &= \Pi_{\{1,2\}} - 2\varepsilon - w_1 - w_2 \\ &= \gamma(\Pi_{\{1,2\}} - 2\varepsilon) + (1-\gamma)(\Pi_{\{1\}} + \Pi_{\{2\}} - \Pi_{\{1,2\}}) \end{aligned}$$

In contrast, if the seller only hires influencer 1, she gets

$$\hat{U}_1 = \gamma(\Pi_{\{1\}} - \varepsilon)$$

By definition of $\bar{c}$, $\Pi_{\{1,2\}} - 2\varepsilon = \Pi_{\{1\}} - \varepsilon$ for all $\bar{c} \leq c \leq \frac{yI_2}{2\varepsilon}$. This further implies that $U_1 \geq \hat{U}_1$, and thus it is optimal for the seller to hire both influencers.

- Third, for $\underline{c} < c < \bar{c}$, we have both

$$\Pi_{\{1,2\}} - 2\varepsilon < \Pi_{\{1\}} - \varepsilon \quad \text{and} \quad \Pi_{\{1,2\}} - 2\varepsilon > \Pi_{\{2\}} - \varepsilon.$$

In this case, when the seller hires both influencers, the payoffs are given by

$$w_1 = (1-\gamma)\big((\Pi_{\{1,2\}} - 2\varepsilon) - (\Pi_{\{2\}} - \varepsilon)\big), \text{ and } w_2 = 0$$

and thus

$$U_1 = \gamma(\Pi_{\{1,2\}} - 2\varepsilon) + (1-\gamma)(\Pi_{\{2\}} - \varepsilon)$$

The payoff from hiring only influencer 1 is unchanged and given by $\hat{U}_1 = \gamma(\Pi_{\{1\}} - \varepsilon)$.

- Fourth, for $c \leq \underline{c}$, we have both

$$\Pi_{\{1,2\}} - 2\varepsilon < \Pi_{\{1\}} - \varepsilon \quad \text{and} \quad \Pi_{\{1,2\}} - 2\varepsilon \leq \Pi_{\{2\}} - \varepsilon.$$

By the residual surplus argument, it implies

$$U_1 = \Pi_{\{1,2\}} - 2\varepsilon, \quad w_1 = 0, \quad \text{and} \quad w_2 = 0.$$

To establish non-monotonicity, we just need to ensure the seller only hires influencer 1 for $\underline{c} < c < \bar{c}$, which requires that $U_1 < \hat{U}_1$, or equivalently

$$\Pi_{\{1,2\}} - 2\varepsilon < \gamma(\Pi_{\{1\}} - \varepsilon)$$

which reduces to

$$\gamma > \frac{\Pi_{\{1,2\}} - 2\varepsilon}{\Pi_{\{1\}} - \varepsilon}$$

Hence, if we take $\gamma \geq \sup_{c<\bar{c}} \left\{ \frac{\Pi_{\{1,2\}} - 2\varepsilon}{\Pi_{\{1\}} - \varepsilon} \right\} = \frac{\Pi_{\{1,2\}}(\bar{c}) - 2\varepsilon}{\Pi_{\{1\}}(\bar{c}) - \varepsilon} \in (0,1)$, then it is optimal to hire only influencer 1. The proof for part i) concludes. ■

ii) First, note that both U_1 and $\hat{U}_1$, the seller's payoff when both influencers are hired and only the strong influencer 1 is hired respectively, are strictly decreasing and continuous in c. Thus, the seller's payoff, $\max\{U_1, \hat{U}_1\}$, corresponds to the upper envelope of U_1 and $\hat{U}_1$, is also strictly decreasing in c and continuous. The proof for part ii) concludes. ■

iii) This directly follows from Lemma 4. Specifically, for $\gamma \leq \underline{\gamma}$, both influencers are hired for all $c \leq \underline{c}$. In this case, we have

$$w_1 = w_2 = 0, \quad \text{and} \quad w_1 - w_2 = 0.$$

Furthermore, for $\bar{c} < c < \frac{yI_2}{2\varepsilon}$, only the strong influencer 1 is hired. Thus,

$$w_1 = (1-\gamma)(\Pi_{\{1\}} - \varepsilon), \quad w_2 = 0, \quad \text{and} \quad w_1 - w_2 = (1-\gamma)(\Pi_{\{1\}} - \varepsilon).$$

Since $\Pi_{\{1\}}$ is strictly decreasing in c, we have both total wages for all influencers and the income gap between influencers are non-monotonic in the background technology cost parameter. The proof concludes for part iii) and iv). □

A.6 Proof of Corollary 1

Proof. Note that from Lemma 9, $\Pi_{\{1,2\}}$ is monotone in β. Thus, by Eq. (7), when $\gamma > \frac{1}{2}$, U_1 is strictly increasing in $\Pi_{\{1,2\}}$, and thus it achieves maximum for any $\beta \geq I$. The other two cases can be shown similarly. □

A.7 Proof of Lemma 5

Proof. Suppose not. Then, there exist three neighboring influencers hired such that $\|\theta_{j-1} - \theta_j\| \neq \|\theta_j - \theta_{j+1}\|$. For simplicity, assume that $\|\theta_{j-1} - \theta_j\| < \|\theta_j - \theta_{j+1}\|$. There are two cases to consider.

Case i). There exists some consumers not served along the arc between θ_{j-1} and θ_{j+1}. Without loss of generality, we assume there exists consumers not served between θ_j and θ_{j+1}. Then, we can keep (p_{j-1}, p_j, p_{j+1}) unchanged, and shift θ_j to $\theta_j + \delta$ for a sufficiently small $\delta > 0$. This weakly increases the total revenue Π_J, because it weakly increases the size of consumers served by influencer $j-1$.

Case ii). All consumers are served along the arc between θ_{j-1} and θ_{j+1}. For ease of notation, define $x_{i,i+1} \in \mathbf{S}^1$ the cutoff type indifferent between purchasing from influencer i and from influencer $(i+1)$.

Lemma 11. *The cutoff consumer type indifferent between following two neighboring influencers receive a zero utility.*

Proof. If not, suppose the cutoff type consumer $x_{j-1,j}$ receives a positive utility. Note that it cannot be the case that the consumer $x_{j,j+1}$ also receives a positive utility. Otherwise, we can increase p_j by a small amount without losing any consumers, which leads to a large total revenue. This implies that consumer $x_{j,j+1}$ receives a zero utility.

Now, we construct a hiring plan and a price scheme which generates more revenue. Consider the case that $p_{j-1} \geq p_j$ and the other case $p_{j-1} < p_j$ can be proved similarly. We shift θ_j to $\hat{\theta}_j = \theta_j + \delta$ and increase p_j to $\hat{p}_j = p_j + \frac{y}{I}\delta$ where $\delta > 0$ is small. Under the new hiring and pricing scheme, the cutoff consumer type $x_{j,j+1}$ remains unchanged, and the cutoff type $x_{j-1,j}$ shifts to $\hat{x}_{j-1,j} = x_{j-1,j} + \delta$. We can choose a sufficiently small $\delta > 0$ to ensure the consumer $\hat{x}_{j-1,j}$ still receives a positive utility. Now, all consumers between $x_{j-1,j}$ and $x_{j,j+1}$ either pay $p_{j-1} \geq p_j$ or $\hat{p}_j > p_j$. □

Lemma 11, combined with the condition that $\|\theta_{j-1} - \theta_j\| < \|\theta_j - \theta_{j+1}\|$, implies that $p_{j-1} > p_{j+1}$ because influencer $(j+1)$ needs to serve more consumers than influencer $(j-1)$ to ensure both consumers $x_{j-1,j}$ and $x_{j,j+1}$ receive a zero utility. Denote by a_{j-1} and a_{j+1} the size of consumers served by influencer $(j-1)$ and $(j+1)$, respectively. Since $p_{j-1} > p_{j+1}$, $a_{j-1} < a_{j+1}$ by Lemma 11.

However, the fact that $p_{j-1} > p_{j+1}$ implies there exists a price scheme more profitable. To see it, consider the new price scheme

$$(\hat{p}_{j-1}, \hat{p}_j, \hat{p}_{j+1}) = \left(p_{j-1} - \frac{y\delta}{I}, p_j, p_{j+1} + \frac{y\delta}{I}\right)$$

and

$$(\hat{\theta}_{j-1}, \hat{\theta}_j, \hat{\theta}_{j+1}) = \left(\theta_{j-1} + \frac{\delta}{2}, \theta_j + \delta, \theta_{j+1} - \frac{\delta}{2}\right)$$

Note that $\hat{a}_{j-1} = a_{j-1} + \delta$ and $\hat{a}_{j+1} = a_{j+1} - \delta$, and the size of consumers served other influencers remains unchanged, including influencer j. The total revenue change is given by

$$\begin{aligned}
\Delta &= \hat{p}_{j-1}\hat{a}_{j-1} + \hat{p}_{j+1}\hat{b}_{j+1} - p_{j-1}a_{j-1} - p_{j+1}b_{j+1} \\
&= \left(p_{j-1} - \frac{y\delta}{I}\right) * (a_{j-1} + \delta) + \left(p_{j+1} + \frac{y\delta}{I}\right) * (b_{j+1} - \delta) - p_{j-1}a_{j-1} - p_{j+1}b_{j+1} \\
&= \frac{y\delta}{I}(b_{j+1} - a_{j+1}) + (p_{j-1} - p_{j+1})\delta + O(\delta^2)
\end{aligned}$$

By the fact that $p_{j-1} > p_{j+1}$ and $a_{j-1} < a_{j+1}$, this generates a higher total revenue for a sufficiently small $\delta > 0$. This is a contradiction! Hence, it cannot be the case that all consumers are served but influencers are not equally distanced. The proof concludes. □

A.8 Proof of Lemma 6

Proof. **Claim i)**. The seller can only get a positive profit only when $1 \leq J^* \leq \left[\frac{2\pi y}{\varepsilon}\right]$.

Claim ii). Consider the first case that $J = \bar{J} * m$. Obviously, the seller can $\bar{J}$ equally-distance influencers and get the desired payoff $U_1 = \bar{\Pi}_{\bar{J}} - \bar{J} * w_j$. Indeed, this is optimal for the seller, given the residual bargaining protocol. Consider any alternative plan of hiring $n \in \mathbb{N}$ influencers ,

$$\bar{\Pi}_{\bar{J}} - \bar{J} * w_1^{\bar{J}} > \bar{\Pi}_n - n * w_1^n.$$

Note that when $\gamma = 1$, the above inequality trivially holds, by the definition of $\bar{J}$. By continuity, it also holds for all $\gamma \geq \gamma_n$ where $\gamma_n < 1$. Define $\bar{\gamma}(\varepsilon) = \sup_{1 \leq n \leq [2\pi/\varepsilon]} \gamma_n$. Thus, for all $\gamma \in [\bar{\gamma}(\varepsilon), 1]$, $J^* = \bar{J}$.

Now, consider the second case that $J \to \infty$. Fix $\delta > 0$. We can find J sufficiently large such that there exists a group of influencers $\theta_j (j \in \{1, \cdots, \bar{J}\})$ such that $\left| \|\theta_j - \theta_{j+1}\| - \frac{2\pi}{\bar{J}} \right| < \delta$. Then, we can use the same pricing scheme as in the discrete case that $J = m\bar{J}$. By the condition that $\left| \|\theta_j - \theta_{j+1}\| - \frac{2\pi}{\bar{J}} \right| < \delta$, the size of consumers served by each influencer and thus the revenue generated only deviate from those in the first case by an amount proportional to δ. Since $\delta > 0$ is arbitrary, the proof concludes.

Claim iii). When $\varepsilon = 0$, the joint revenue is weakly increasing in the number of influencers hired. Furthermore, if a sufficiently large number of equal-distanced influencers are hired, each influencer receives a wage close to zero because the incremental change in revenue is close to zero. In other words, $U_1^* = 2\pi y$ and $w_j = 0$ for $j \geq 1$.

Claim iv). It follows from the residual bargaining protocol. □

A.9 Proof of Proposition 2

Proof. First, note that the two influencers are symmetric for sellers, which means that sellers' incentives are trivial under regulated matching. Given this and the matched groups' profits, the influencer from the seller-influencer group with a bigger profit has no incentive to deviate when the bargaining power $(\gamma, 1-\gamma)$ is fixed. Thus, the other influencer also has no incentive to deviate, and we get $k(j) = j$ for $j \in \{1, 2\}$. Furthermore, given the group profit, payoffs for sellers and influencers just trivially follows from Nash bargaining.

Second, we construct an equilibrium in which $p_1^C \geq p_2^C \geq 0$ because the first influencer-seller group is stronger in the sense that it offers a product with a higher quality. This strategy implies that the first group is targeting the most valuable consumers and the consumer type boundary is pinned down by

$$y_1(1 - \|x^* - \theta\|/I) - p_1 = y_2(1 - \|x^* - \theta\|/I) - p_2$$

Obviously, since $y_1 \geq y_2$, all consumers with $\|x - \theta\| < \|x^* - \theta\|$ would purchase from Group 1. Furthermore, given this, the second group attracts those remaining consumers with $\|x - \theta\| \geq \|x^* - \theta\|$ and $\|x - \theta\| \leq \|x^{**} - \theta\|$ where

$$y_2(1 - \|x^{**} - \theta\|/I) - p_2 = 0$$

Thus, we can calculate the demand (i.e., $q_1 = 2\|x^* - \theta\|$ for Group 1 and $q_2 = 2(\|x^{**} - \theta\| - \|x^* - \theta\|)$ for Group 2)

$$q_1(p_1, p_2) = 2I\left(1 - \frac{p_1 - p_2}{y_1 - y_2}\right), \text{ and } q_2(p_1, p_2) = 2I\left(\frac{p_1 - p_2}{y_1 - y_2} - \frac{p_2}{y_2}\right)$$

and the profits as below.

$$\Pi_1 = 2Ip_1\left(1 - \frac{p_1 - p_2}{y_1 - y_2}\right), \quad \Pi_2 = 2Ip_2\left(\frac{p_1 - p_2}{y_1 - y_2} - \frac{p_2}{y_2}\right),$$

Taking derivatives over Π_m with respect to p_m for $m = 1, 2$,

$$\frac{2I(-2p_1^C + p_2^C + y_1 - y_2)}{y_1 - y_2} = 0, \text{ and } \frac{I(-4p_2^C y_1 + 2p_1^C y_2)}{(y_1 - y_2)y_2} = 0$$

Solving these two equations yields the desired solution. Moreover, the second order conditions are trivially satisfied and thus solutions given by the FOCs are optimal. By submitting the prices $(p_1^C, p_2^C) = \left(\frac{2y_1(y_1 - y_2)}{4y_1 - y_2}, \frac{y_2(y_1 - y_2)}{4y_1 - y_2}\right)$ into the demand functions and the profits, we obtain all the desired results after simple algebra manipulation. The proof concludes. □

A.10 Proof of Proposition 3

Proof. First, we focus on both groups' profits and skip the discussion on profit divisions between sellers and influencers and their incentives for matching, because the proof is almost the same as that in Proposition 2.

We construct an equilibrium which features $p_1^C \geq p_2^C \geq 0$. This is because, if $p_1 < p_2$, then the second influencer-seller group is priced out of the market because $\theta_1 = \theta_2$. Given that $p_1 \geq p_2$, consumers whose type is close to θ are attracted by the second group because the term $y\|x - \theta\|\left(\frac{1}{I_2} - \frac{1}{I_1}\right)$ only plays a secondary role compared the price gap $p_1 - p_2$. This yields the cutoff style type x^* such that all consumers with type x satisfying $\|x - \theta\| < \|x^* - \theta\|$ are served by the second group, i.e.,

$$y * (1 - \|x^* - \theta\|/I_1) - p_1 = y * (1 - \|x^* - \theta\|/I_2) - p_2$$

This leads to $\|x^* - \theta\| = \frac{I_1 I_2(p_1 - p_2)}{y(I_1 - I_2)}$.

However, consumers with $\|x - \theta\| > \|x^* - \theta\|$ are attracted by the first group whenever it generates a positive utility, which implies a second cutoff type $x^{**} \in \mathbf{S}^1$ such that

$$y\,(1 - \|x^{**} - \theta\|/I_1) - p_1 = 0$$

which leads to $\|x^{**} - \theta\| \leq I_1\,(1 - p_1/y)$. By construction, consumers with a type satisfying $\|x^* - \theta\| < \|x - \theta\| \leq \|x^{**} - \theta\|$ are served by the first group.

Now, we can compute the profits for two influencer-seller groups, i.e.,

$$\Pi_1 = p_1 * \left((1 - p_1/y) I_1 - \frac{I_1 I_2 (p_1 - p_2)}{y(I_1 - I_2)} \right)$$
$$\Pi_2 = p_2 * \frac{I_1 I_2 (p_1 - p_2)}{y(I_1 - I_2)}$$

Obviously, the profit function Π_m is concave in p_m for $m = 1, 2$ and thus the optimal solution is fully characterized by the first-order conditions as below.

$$\frac{I_1}{y(I_1 - I_2)} \big(I_2(p_2 - y) + I_1(y - 2p_1) \big) = 0$$
$$\frac{I_1 I_2}{y(I_1 - I_2)} (p_1 - 2p_2) = 0$$

Solving these two equations, we get the desired solution (p_1^C, p_2^C), and by simple algebra, we can obtain the profits for both groups. The proof concludes. □

A.11 Proof of Proposition 4

Proof. First, we focus on both groups' profits and skip the discussion on profit divisions between sellers and influencers and their incentives for matching, because the proof is almost the same as that in Proposition 2.

Second, let us fix p_2 and consider the profit function of the first group. If these two prices are sufficiently close, then we would expect the following condition

$$y(1 - \|x^* - \theta_1\|/I) - p_1 = y(1 - \|x^* - \theta_2\|/I) - p_2$$

or equivalently,

$$p_2 - p_1 = (\|x^* - \theta_1\| - \|x^* - \theta_2\|) * y/I$$

For ease of reference, denote $s_m = \|x - \theta_m\|$ for $m = 1, 2$. When p_1 and p_2 are sufficiently close and not too small (to be discussed shortly), x^* lies in between θ_1 and θ_2 in the short arc, then we can solve these by utilitizing the fact that $s_1 + s_2 = \|\theta_1 - \theta_2\|$ and get

$$s_1 = \frac{1}{2} \big(\|\theta_1 - \theta_2\| + (p_2 - p_1) * I/y \big),$$
$$s_2 = \frac{1}{2} \big(\|\theta_1 - \theta_2\| - (p_2 - p_1) * I/y \big)$$

Note that this holds only when there exists no consumer unserved in the short arc between θ_1 and θ_2, that is,

$$I * (1 - p_1/y) + I * (1 - p_2/y) \geq \|\theta_1 - \theta_2\|,$$

which further reduces to $p_1 + p_2 \leq (2 - \frac{\|\theta_1 - \theta_2\|}{I}) * y$.

Otherwise, $s_1 = I * (1 - p_1/y)$. Hence, to summarize,

$$s_1 = \begin{cases} \frac{1}{2}\big(\|\theta_1 - \theta_2\| + (p_2 - p_1) * I/y\big), & \text{if } p_1 + p_2 \leq (2 - \frac{\|\theta_1-\theta_2\|}{I}) * y \\ I * (1 - p_1/y), & \text{otherwise} \end{cases}$$

However, when p_1 is sufficiently low, then $s_1 > \|\theta_1 - \theta_2\|$ occurs, and in this case, $s_1 - s_2 = \|x - \theta_1\| - \|x - \theta_2\| = \|\theta_1 - \theta_2\|$. This implies that the first group grabs the whole market when

$$p_1 \leq p_2 - \|\theta_1 - \theta_2\| * y/I$$

and the first group loses all consumers when

$$p_1 \geq p_2 + \|\theta_1 - \theta_2\| * y/I$$

Henceforth, we can write down the profit function for the first group as below

$$\Pi_1^C = \begin{cases} 0, & \text{if } p_1 \geq p_2 + \|\theta_1 - \theta_2\| * y/I \\ 2p_1 * (1 - p_1/y)I, & \text{if } p_1 \leq p_2 - \|\theta_1 - \theta_2\| * y/I \\ p_1 * \left((1 - p_1/y) * I + s_1\right), & \text{if } p_1 + p_2 \leq (2 - \frac{\|\theta_1-\theta_2\|}{I}) * y \text{ and} \\ & p_1 \in \left(p_2 - \frac{\|\theta_1-\theta_2\| * y}{I}, p_2 + \frac{\|\theta_1-\theta_2\| * y}{I}\right) \\ 2p_1 * (1 - p_1/y)I, & \text{otherwise} \end{cases} \tag{A.4}$$

Note that there are two discontinuity points for the profit function above. Also note that the profit function Π_1^C is continuous at $p_1 = \left(2 - \frac{\|\theta_1-\theta_2\|}{I}\right) * y - p_2$. Similarly, we can write down the profit function for the second group by symmetry. Obviously, it is suboptimal to choose $p_1 \geq p_2 + \|\theta_1 - \theta_2\| * y/I$, which leads to a zero profit. Then, we start with the third case in Equation (A.4) and we can derive the first order conditions as follows.

$$\frac{y\|\theta_1 - \theta_2\| + I(p_2 - 6p_1 + 2y)}{2y} = 0,$$
$$\frac{y\|\theta_1 - \theta_2\| + I(p_1 - 6p_2 + 2y)}{2y} = 0$$

Solving these two equations, we can get $p_1^* = p_2^* = \frac{y(2I+\|\theta_1-\theta_2\|)}{5I}$, and we can further get the profit for the first group as below, i.e.,

$$\Pi_1 = \frac{3y(2I + \|\theta_1 - \theta_2\|)^2}{50I}$$

when $p_2 - \|\theta_1 - \theta_2\| * y/I < p_1 < p_2 + \|\theta_1 - \theta_2\| * y/I$.

However, we need to check that $p_1^* + p_2^* \leq \left(2 - \frac{\|\theta_1-\theta_2\|}{I}\right) * y$ indeed holds, which further reduces to

$$\|\theta_1 - \theta_2\| \leq \frac{6}{7}I.$$

For any $\|\theta_1 - \theta_2\| > \frac{6}{7}I$, define $\hat{p}_1^* = \hat{p}_2^* = \left(1 - \frac{\|\theta_1-\theta_2\|}{2A}\right) * y$. Fix $p_2 = \hat{p}_2^*$, the term $p_1 * ((1 - p_1/y) * I + s_1)$ is strictly increasing in p_1 for $p_1 \leq \frac{y(2I+\|\theta_1-\theta_2\|)}{5I}$. Thus, the first (influencer-seller) group has no incentive to deviate downward. Meanwhile, for $p_1 > \hat{p}_1^*$, the term $p_1 * (1 - p_1/y) * I$ is strictly decreasing for $p_1 > \hat{p}_1^*$ since $\hat{p}_1^* \geq \frac{y}{2}$. This implies that the first group has no incentive to deviate upward. Thus, $(\hat{p}_1^*, \hat{p}_2^*)$ constitutes an equilibrium when $\|\theta_1 - \theta_2\| \in (\frac{6}{7}I, I)$ whenever no group is priced out of the market.

Finally, to finish the equilibrium construction, we need to ensure that the construction in the third case is also globally optimal, which means that the first group has no incentive to deviate by a big price cut as illustrated in the second case in Equation (A.4).

Specifically, to undercut the second group, Group 1 only needs to set the price $p_1 = p_2^* - \|\theta_1 - \theta_2\| * y/I$, which leads to a profit as below

$$\hat{\Pi}_1 = 2I * (p_2^* - \|\theta_1 - \theta_2\| * y/I) * \left(1 - \frac{p_2^* - \|\theta_1 - \theta_2\| * y/I}{y}\right)$$

Note that this is the most profitable deviation since $\Pi_1 = 2I(1-p_1/y)p_1$ is strictly decreasing for all $p_1 \leq p_2^* - \|\theta_1 - \theta_2\| * y/I$. To support the equilibrium, it requires

$$\hat{\Pi}_1 \leq \Pi_1$$

which leads to the condition in the proposition. The proof concludes. □

A.12 Proof of Proposition 5

Proof. The assortative matching and profits derivation follows directly from Lemma 8 (see Appendix A.18). Payoffs for sellers and influencers follows from the exogenous Nash bargaining argument. The proof concludes. □

A.13 Proof of Proposition 6

Proof. Consider the following equilibrium conjecture.

- Assortative matching in the labor market. $k(j) = j$ for $j \in \{1, 2\}$.
- Product market. Group 1, the matching between seller 1 and influencer 1, prices the product at $p_1^* = \frac{y_1}{2}$, and earns a total profit of $\Pi_1 = \frac{y_1 I_1}{2}$. Group 2 prices the product at $p_2^* = 0$.

Now, we verify that this constitutes an equilibrium.

First, given $p_1^* = \frac{y_1}{2}$, a consumer with type $x \in \mathbf{S}^1$ such that $\|x - \theta_2\| \leq I_2$, always prefers the product from group 1 to that from group 2 even when group 2 sets the price at zero, as long as the following two conditions hold, that is, for type $x = \theta_2$,

$$y_1 * \big(1 - \beta/I_1\big) - p_1^* \geq y_2 \tag{A.5}$$

and for type $\|x - \theta_2\| = I_2$ & $\|x - \theta_1\| = \beta + I_2$,

$$y_1 * \big(1 - (\beta + I_2)/I_1\big) - p_1^* \geq 0 \tag{A.6}$$

Simplifying these two equations yields the condition that $\frac{\beta}{I_1} \leq \min\{\frac{1}{2} - \frac{y_2}{y_1}, \frac{1}{2} - \frac{I_2}{I_1}\}$.

Second, given consumers' equilibrium choice and group 2's pricing strategy, group 1 has no incentive to deviate from the monopolist profit. Meanwhile, given the assumed condition and other participants' equilibrium strategy, $p_2^* = 0$ because she cannot attract any consumer by setting $p_2^* > 0$.

Third, anticipating the equilibrium profits, influencer 1 chooses to match with seller 1 and gets a payoff of $w_1 = \frac{(1-\gamma)y_1 I_1}{2}$. Instead, if she deviates to seller 2, influencer 1 can at most get $\hat{w} - 1 = \frac{(1-\gamma)y_2 I_1}{2} < w_1$. Given influencer 1's equilibrium matching choice, influencer 2 can only match with seller 2. The proof concludes. □

A.14 Proof of Proposition 7

Proof. First, by the formula of $\Pi_1(\beta)$ (i.e., Eq. (6)), we can check that $\Pi_1(\beta)$ is strictly increasing for all $\beta \in \big(\beta_0, I\big]$, which implies that

$$G(\beta) := \Pi_1(\beta) - \big(\Pi_1(I) - F_H\big)$$

has at most one solution for all $\beta \in \big(\beta_0, I\big]$. Indeed, note that $G(I) = F_H$ and $G(\beta_0) < 0$ by the assumed condition Eq. (7). Hence, there exists a unique β^* well defined. Further, by monotonicity, we have $G(\beta) > 0$ for $\beta > \beta^*$ and $G(\beta) < 0$ for $\beta \in [\beta_0, \beta^*)$.

Second, consider $\beta > \beta^*$ under regulated matching (i.e., 1-1 matching). By symmetry, we consider the incentive for seller 1. Given that seller 2 hires an influencer, say influencer 2, seller 1 can choose to hire influencer 1 and get a payoff of $\gamma\Pi_1(\beta)$, or pay a fixed cost and select a location to achieve maximum differentiation (i.e., $\|\alpha_1 - \alpha_2\| \geq I$. Here, $\alpha_2 = \theta_2$ is influencer 2's style location. This yields a payoff of $\gamma(\Pi_1(I) - F_H)$. However, note that $G(\beta) > 0$ for $\beta > \beta^*$. Thus, it is optimal to hire influencer 1 because $\gamma\Pi_1(\beta) \geq \gamma(\Pi_1(I) - F_H)$.

Third, consider $\beta \leq \beta^*$ under regulated matching. We consider an asymmetric equilibrium in which seller 1 hires influencer 1, and seller 2 turns on the influence by paying the fixed cost F_H. First of all, for seller 2, given that seller 1 already hires influencer 1, she can hire influencer 2 and engage in price competition, which yields a payoff of $\gamma\Pi_1(\beta)$. On the other hand, she can pay the fixed cost F_H and select a location to avoid competition and thus gets $\gamma(\Pi_1(I) - F_H)$. By the fact $G(\beta) < 0$ for $\beta \leq \beta^*$, she has an incentive to pay the fixed cost F_H. Furthermore, given that seller 2 chooses maximum differentiation, seller 1 has the incentive to hire influencer 1 and gets a profit of $\gamma\Pi_1(I)$. The proof concludes. $\square$

A.15 Proof of Lemma 7

Proof. Before we get started, note that under regulated matching the Nash bargaining always gives the seller a fraction γ of the seller-influencer group's total profit. Hence, we can directly focus on total profits of groups in the proof.

i) Profits for (I, I) **and** (NI, NI)**.** First, under the assumed condition i) $\beta_0 \leq \beta \leq \frac{5}{6}$, Proposition 4 holds. Thus, when both groups choose to invest, each group gets a profit of

$$\Pi_{I,I} = yA(\beta).$$

and if they both choose not to invest, each group gets

$$\Pi_{NI,NI} = A(\beta).$$

ii) Profits for $(I, NI$ **(and** (NI, I)**).** Second, we compute profits for both seller-influencer groups when only one group, say, group 1, chooses to invest (and the other group choose not to invest). By the assumed condition ii), even if group 1 (seller 1 and influencer 1) chooses high quality y, group 2 can still attract some consumers even group 1 set a price at $p_1 = 0$. Now, denote by (p_1, p_2) the prices set by both groups.

For group 1, the size of consumers served is just $y_1(1 - \|x - \theta_1\|) - p_1 \geq 0$ or equivalently $(1 - p_1/y)$. For consumers between groups (along the short arc), the cutoff type $x^* \in \mathbf{S}^1$ satisfies

$$y(1 - \|x^* - \theta_1\|) - p_1 = 1 - (\beta - \|x^* - \theta_2\|) - p_2$$

Solving it yields

$$\|x^* - \theta_1\| = \frac{p_2 - p_1 + (y-1) + \beta}{y+1} =: s_1, \quad \text{and} \quad s_2 := \|x^* - \theta_2\| = \beta - s_1$$

We can further express profits for both groups as follows

$$\Pi^1_{I,NI} = (1 - p_1/y_1) * p_1 + p_1 * s_1, \quad \text{and} \quad \Pi^2_{I,NI} = (1 - p_2/y_2) * p_2 + p_2 * s_2$$

Taking first order conditions and solving them, we get

$$p_1^* = \frac{y_1(4y_1^2 + 8y_1y_2 + 3\beta y_1y_2 + 2y_2^2 + 4\beta y_2^2)}{8y_1^2 + 19y_1y_2 + 8y_2^2} = \frac{2(1+2\beta)y + (8+3\beta)y^2 + 4y^3}{8 + 19y + 8y^2}$$

and

$$p_2^* = \frac{y_2(4y_2^2 + 8y_1y_2 + 3\beta y_1y_2 + 2y_1^2 + 4\beta y_1^2)}{8y_1^2 + 19y_1y_2 + 8y_2^2} = \frac{4 + (8+3\beta)y + 2(1+2\beta)y^2}{8 + 19y + 8y^2}$$

We need to make sure that the cutoff type x^* gets a non-negative utility, that is,

$$y(1 - \|x^* - \theta\|) - p_1^* \geq 0,$$

which reduces to

$$\frac{y\big(2(5-6\beta)(1+y^2) + (22-25\beta)y\big)}{(1+y)(8+19y+8y^2)}$$

which trivially holds under the assumed condition i). Furthermore, we can directly verify that the second order conditions are satisfied.

Thus, profits for both groups can be further computed as

$$\Pi_{I,NI}^1 = \frac{y(1+2y)\big(2 + 8y + 4y^2 + \beta(4+3y)\big)^2}{(1+y)\left(8+19y+8y^2\right)^2}$$

and

$$\Pi_{I,NI}^2 = \frac{y(2+y)\big(4 + 8y + 2y^2 + \beta(3y+4y^2)\big)^2}{(1+y)\left(8+19y+8y^2\right)^2}$$

iii) Nash Equilibrium Construction. We first state some properties.

1. Fix y. $V_1(\beta, y) > V_2(\beta, y)$. Recall that $V_1(\beta, y) = \Pi_{I,NI}^1 - \Pi_{NI,NI}$ and $V_2(\beta, y) = \Pi_{I,I} - \Pi_{I,NI}^2$.

2. Fix y. Both $V_1(\beta, y)$ and $V_2(\beta, y)$ are strictly increasing in β.

Given these two properties, we can verify the equilibrium. First, by the assumed condition iii), $F_V \in (V_1(\beta_0, y), V_2(5/6, y))$ and property 1), we have

$$V_1(5/6, y) > V_2(5/6, y) > F_V > V_1(\beta_0, y) > V_2(\beta, y)$$

which, together with the strict monotonicity of $V_1(\beta, y)$ and $V_2(\beta, y)$, implies that there exist $\underline{\beta} \in (\beta_0, 5/6)$ and $\overline{\beta} \in (\beta_0, 5/6)$ such that

$$F_V = V_1(\underline{\beta}, y) = V_2(\overline{\beta}, y), \quad \text{and} \quad \overline{\beta} > \underline{\beta}.$$

To summarize,

a) For $\beta \geq \overline{\beta}$, $V_2(\beta, y) \geq F_V$, or equivalently, $\Pi_{I,I} - \Pi^2_{I,NI} \geq F_V$. Hence, given that influencer 1 chooses to invest, it is optimal for influencer 2 to invest. By symmetry, influencer 1 also chooses to invest, and thus (I, I) is a Nash Equilibrium.

b) For $\underline{\beta} \leq \beta < \overline{\beta}$, we have both $V_1(\beta, y) \geq F_V$ and $V_2(\beta, y) < F_V$, that is,

$$\Pi^1_{I,NI} - \Pi_{\{NI,NI\}} \geq F_V, \quad \text{and} \quad \Pi_{\{I,I\}} - \Pi^2_{\{I,NI\}} < F_V$$

These two conditions read as follows. One, given that influencer 2 chooses not to invest, it is optimal for influencer 1 to invest. Two, given that influencer 1 chooses to invest, it is optimal for influencer 2 not to invest. Thus, (I, NI) is a Nash Equilibrium, so is (NI, I).

c) For $\beta < \underline{\beta}$, $V_1(\beta, y) < F_V$, or equivalently, $\Pi^1_{I,NI} - \Pi_{\{NI,NI\}} < F_V$, which reads as, if influencer 2 does not invest, then it is optimal for influencer 1 not to invest. By symmetry, (NI, NI) is a Nash Equilibrium.

Now, it suffices to verify property 1) and 2) on $V_1(\beta, y)$ and $V_2(\beta, y)$. To this end, we write down the formulas and check them one by one.

- First, with y fixed, $V_1(\beta, y)$ is strictly increasing in β.

$$V_1(\beta, y) = \frac{(y-1)}{50(y+1)(8+19y+8y^2)} \times \left\{1600y^5 + 32M_1 * y^4 + 4M_2 * y^3 + 25M_3 * y^2 + 8M_4 * y + 192(2+\beta)^2\right\}$$

  where $M_1 = 251 + 51\beta - 6\beta^2$, $M_2 = 3704 + 1604\beta - 99\beta^2$, $M_3 = 500 + 340\beta + 3\beta^2$ and $M_4 = 623 + 548\beta + 62\beta^2$. With simple algebra, we can show that $M_j (j = 1, 2, 3, 4)$ are all positive for $\beta \in (\beta_0, 5/6)$.

- Second, with y fixed, $V_2(\beta, y)$ is strictly increasing in β.

$$V_2(\beta, y) = \frac{(y-1)}{50(y+1)(8+19y+8y^2)} \times \left\{1600 + 32M_1 * y + 4M_2 * y^2 + 25M_3 * y^3 + 8M_4 * y^4 + 192(2+\beta)^2 y^5\right\}$$

  Hence, fix y, then $V_2(\beta, y)$ is strictly increasing in β.

- Third, with y fixed, $V_1(\beta, y) > V_2(\beta, y)$ for all $\beta \in (\beta_0, 5/6)$.

$$V_1(\beta, y) - V_2(\beta, y) = \frac{(y-1)^2}{50(y+1)(8+19y+8y^2)} \times \left\{64 * M_5 + 40 * M_6 y + M_7 * y^2 + 40 * M_6 y^3 + 64 * M_5 y^4\right\}$$

where $M_5 = 13 - 12\beta - 3\beta^2$, $M_6 = 97 - 88\beta - 22\beta^2$, and $M_7 = 6196 - 5604\beta - 1351\beta^2$.

To see that $M_5 > 0$, note that there are two solutions $\beta_1 \approx -4.89$ and $\beta_2 \approx 0.89$. Hence, $M_5 > 0$ for all $\beta \in (-4.89, 0.89)$, and thus $M_5 > 0$ for all $\beta \in (\beta_0, 5/6)$. We can prove $M_6 > 0$ and $M_7 > 0$ in similar spirits.

All the proofs conclude. □

A.16 Proof of Proposition 9

Proof. The proof consists of three parts: i) initial equilibrium analysis; ii) Nash equilibrium construction; and iii) welfare analysis.

i) Initial Equilibrium Analysis. Initially, $I_1 = I_2 = \pi$. Because $\frac{I_1+I_2}{2} \leq \pi$, the seller, when hiring both influencers, set the price at $p_1^* = p_2^* = \frac{y}{2}$, which implies that before power acquisition,

$$\Pi^b_{\{1\}} = \frac{1}{2}y\pi, \quad \Pi^b_{\{2\}} = \frac{1}{2}y\pi, \quad \text{and} \quad \Pi^b_{\{1,2\}} = y\pi.$$

The Nash bargaining implies that

$$w_1^b = (1-\gamma)(\Pi_{\{1,2\}} - \Pi_{\{1\}}) = \frac{1}{2}(1-\gamma)y\pi.$$

Similarly, $w_2^b = \frac{1}{2}(1-\gamma)y\pi$.

ii) Nash Equilibrium Construction. First, we construct an equilibrium in which influencer 2 does not invest in power acquisition. Consider influencer 1's incentive to invest. If influencer 1 invests in power acquisition, then $I_1 + I_2 \geq 2\pi$ and thus by the proof of Lemma 2, the joint revenue function after power acquisition is given by[19]

$$\Pi^a_{\{1,2\}} = 2\pi y\left(1 - \frac{\pi}{I_1+I_2}\right) = 2\pi y(1 - 1/3) = \frac{4}{3}\pi y.$$

The optimal pricing strategy that maximizes the joint revenue function is given by

$$p_1^a = p_2^a = y\left(1 - \frac{\pi}{I_1+I_2}\right) = \frac{2y}{3}.$$

and consumers served by influencers satisfy $\|x - \theta_j\| \leq s_j$ where

$$s_1 = \frac{I_1\pi}{I_1+I_2} = \frac{2\pi}{3}, \quad \text{and} \quad s_2 = \frac{I_2\pi}{I_1+I_2} = \frac{\pi}{3}.$$

[19]Note that we assume that $c = 1$.

Similarly, if the seller only hires influencer j, profits are given by

$$\Pi^a_{\{1\}} = \frac{1}{2} y I_1 = y\pi, \quad \text{and} \quad \Pi^a_{\{2\}} = \frac{1}{2} y I_2 = \frac{1}{2} y\pi.$$

Hence, wages offered to influencers are given by

$$w^a_1 = (1-\gamma)(\Pi^a_{\{1,2\}} - \Pi^a_{\{2\}}) = (1-\gamma)\left(\frac{4}{3}y\pi - \frac{1}{2}y\pi\right) = \frac{5}{6}(1-\gamma)y\pi$$

and

$$w^a_2 = (1-\gamma)(\Pi^a_{\{1,2\}} - \Pi^a_{\{1\}}) = (1-\gamma)\left(\frac{4}{3}y\pi - y\pi\right) = \frac{1}{3}(1-\gamma)y\pi$$

Hence, given that influencer 2 does not invest, influencer 1 will invest in power acquisition when

$$w^a_1 - C_T \geq w^b_1 \implies C_T \leq \frac{1}{3}(1-\gamma)y\pi$$

Second, we come to check influencer 2's incentive.

(1) By symmetry, when $C_T > \frac{1}{3}(1-\gamma)y\pi$, influencer 2 will not invest in power acquisition, that is, (NI, NI) is a Nash Equilibrium.

Given that influencer 1 chooses to invest, if influencer 2 also invests,

$$\hat{\Pi}_{\{1,2\}} = 2\pi y\left(1 - \frac{\pi}{I_1 + I_2}\right) = 2\pi y(1 - 1/4) = \frac{3}{2}y\pi$$

and

$$\hat{\Pi}_{\{1\}} = \Pi^a_{\{1\}} = y\pi.$$

Nash bargaining implies that

$$\hat{w}_2 = (1-\gamma)(\hat{\Pi}_{\{1,2\}} - \hat{\Pi}_{\{1\}}) = \frac{1}{2}(1-\gamma)y\pi$$

Hence, influencer 2 will not invest if and only if

$$\hat{w}_2 - C_T < w^a_2 \implies C_T > \frac{1}{6}(1-\gamma)y\pi.$$

To summarize,

(2) When $\frac{1}{6}(1-\gamma)y\pi < C_T \leq \frac{1}{3}(1-\gamma)y\pi$, (I, NI) is a Nash Equilibrium.

(3) When $C_T \leq \frac{1}{6}(1-\gamma)y\pi$, (I, I) is a Nash Equilibrium.

iii) Welfare Analysis. Here, we show the optimal decision rule to maximize total welfare.

- First, when (NI, NI) is the outcome, the total welfare is achieved by letting influencer j serve consumers with $\|x-\theta_j\| \le \frac{\pi}{2}$ and $x \in \mathbf{S}^1$, that is,

$$\begin{aligned} SW = & \int_{y(1-\|x-\theta_1^*\|/I_1)-\frac{y}{2}\ge 0} y(1-\|x-\theta_1^*\|/I_1)dx \\ & + \int_{y(1-\|x-\theta_2^*\|/I_2)-\frac{y}{2}\ge 0} y(1-\|x-\theta_2^*\|/I_2)dx \\ & = \frac{3}{4}(yI_1 + yI_2) = \frac{3}{2}y\pi. \end{aligned}$$

- Second, when (I, NI) (or (NI, I)) is the outcome, total welfare is achieved by letting influencer j serve consumers with $\|x-\theta_j\| \le \frac{I_j}{I_1+I_2}\pi$ for $j = 1, 2$, because the cutoff type consumer $x \in \mathbf{S}^1$ satisfies

$$y\left(1 - \frac{\|x-\theta_1\|}{2\pi}\right) = y\left(1 - \frac{\|x-\theta_2\|}{\pi}\right)$$

  Then, total welfare is given by[20]

$$SW = \frac{5}{3}y\pi - C_T$$

- Third, wehn (I, I) is the outcome, total welfare is achieved by letting influencer j serve consumers with $\|x-\theta_j\| \le \frac{\pi}{2}$. Total welfare is given by

$$SW = \frac{7}{4}y\pi - 2C_T$$

Hence, we can calculate the optimal decision rule by comparing total welfare under different outcomes, that is,

1. when $C_T \le \frac{1}{12}y\pi$, (I, I) is optimal, i.e., both influencers should invest.
2. when $\frac{1}{12}y\pi < C_T \le \frac{1}{6}y\pi$, (I, NI) (or (NI, I)) is optimal, i.e., only one influencer should invest.
3. when $C_T > \frac{1}{6}y\pi$, (NI, NI) is optimal, i.e., no influencer should invest.

The proof concludes. □

[20] A detailed calculation is available upon request.

A.17 Proof of Example 2

Proof. By Proposition 4, influencer 2 is offered a wage of

$$w_2(b) = \begin{cases} \frac{3(1-\gamma)y}{50I} * (2I+b)^2, & \text{if } b \in [\beta_0, \frac{6}{7}I] \\ (1-\gamma)y * \left(b - \frac{b^2}{2I}\right), & \text{if } b \in (\frac{6}{7}I, I) \\ \frac{(1-\gamma)yI}{2}, & \text{if } b \geq I \end{cases}$$

Thus, if he decides to adjust his own style to $b > \beta$, the cost-benefit analysis reduces to

$$w_2^N(b) = \begin{cases} (1-\gamma)\left(\frac{3y(2I+b)^2}{50I} - C(b)\right), & \text{if } b \in [\frac{2}{67}(-7+5\sqrt{10})I, \frac{6}{7}I] \\ (1-\gamma)\left(y\left(b - \frac{b^2}{2I}\right) - C(b)\right), & \text{if } b \in (\frac{6}{7}I, I) \\ (1-\gamma)\left(\frac{yI}{2} - C(b)\right), & \text{if } b \in [I, \infty) \end{cases}$$

Note that,

$$\lim_{b \to \beta} \frac{dw_2^N}{db} = (1-\gamma)\left(\frac{3y(2I+b)}{25I} - C'(\beta)\right) > 0,$$
$$\lim_{b \to I-} \frac{dw_2^N}{db} = -(1-\gamma)C'(I-d_0) < 0$$

Moreover, note that $\frac{dw_2^N}{db}$ is strictly decreasing in b, and thus there exists a unique maximizer $b^* \in (\beta, I)$. The proof concludes. □

A.18 Proof of Lemma 8

Proof. First, since influencers weakly prefer being hired, there are two possible cases under regulated matching: $k(j) = j$ and $k(j) = 2-j$ where $j = 1, 2$. Second, given that $\|\theta_1 - \theta_2\| \geq \frac{I_1+I_2}{2}$, independent of the matching outcome, both seller-influencer groups can charge a monopolist price $p_1* = \frac{y_1}{2}$ and $p_2^* = \frac{y_2}{2}$. Thus, we can calculate payoffs for all sellers and influencers respectively, that is,

- $k(j) = j$. In this case,

$$U_1 = \frac{\gamma y_1 I_1}{2},\ U_2 = \frac{\gamma y_2 I_2}{2},\ w_1 = \frac{(1-\gamma)y_1 I_1}{2},\ \text{and } w_2 = \frac{(1-\gamma)y_2 I_2}{2}$$

- $k(j) = 2-j$. In this case,

$$\hat{U}_1 = \frac{\gamma y_1 I_2}{2},\ \hat{U}_2 = \frac{\gamma y_2 I_1}{2},\ \hat{w}_1 = \frac{(1-\gamma)y_2 I_1}{2},\ \text{and } \hat{w}_2 = \frac{(1-\gamma)y_1 I_2}{2}$$

Note that the matching $k(j) = 2 - j$ is not stable because both seller 1 and influencer 1 can fully anticipate the payoffs when they match, and thus both are willing to form a match. Indeed, $k(j) = j$ for $j = 1, 2$ is a stable matching. Given that seller 1 is matched with influencer 1, both of them have no incentive to deviate because $U_1 \geq \hat{U}_1$ and $w_1 \geq \hat{w}_1$. Given this, seller 2 and influencer 2 forms a match. The proof concludes. $\square$

A.19 Proof of Proposition 10

Proof. Consider the following equilibrium strategies conjectured as follows.

- On-equilibrium path, that is, $\|\theta_1 - \theta_2\| \geq \frac{I_1+I_2}{2}$.

  The equilibrium matching outcome and payoffs are specified as that in Lemma 8.

- Off-equilibrium path, that is, $\|\theta_1 - \theta_2\| < \frac{I_1+I_2}{2}$.

  Seller 1 is matched with influencer 1, and seller 2 is matched with influencer 2. Then, given the matching outcome, the two seller-influencer groups play a price competition game and if an equilibrium exists, payoff are specified according to the Nash bargaining. Whenever a pure strategy equilibrium does not exist, we assume that the payoffs for all sellers and influencers are bounded above by the worst equilibrium payoff, that is, the equilibrium with minimum total profits.

First, consider influencer 2. Under equilibrium, he gets a wage of $w_2 = \frac{(1-\gamma)y_2 I_2}{2}$. Instead, if he chooses θ_2 such that $\|\theta_1^* - \theta_2\| < \frac{I_1+I_2}{2}$. Then, given that seller 1 is matched with influencer 1, his wage $\tilde{w}_2$ cannot exceed w_2 because when he is matched with seller 2, their total profit is bounded above by $\frac{y_2 I_2}{2}$. More formally,

$$\Pi_2 \leq p_2(1 - p_2/y_2)I_2 \leq p_2^m(1 - p_2^m/y)I_2 = \Pi_2^m = \frac{y_2 I_2}{2}.$$

The first inequality says that influencer 2 can at most attract all consumers within his influence reach, and the second one says that it is weakly dominated by the monopoly price. Hence, he has an incentive to choose $\|\theta_1^* - \theta_2\| \geq \frac{I_1+I_2}{2}$. The same argument applies for influencer 1.

Second, given that $\|\theta_1^* - \theta_2^*\| \geq \frac{I_1+I_2}{2}$, by Lemma 8, both sellers have an incentive to accept the assortative matching outcome and the proposed equilibrium payoffs. The proof concludes. $\square$

A.20 Proof of Lemma 9

Proof. First, the case for $\|\theta_1 - \theta_2\| \geq I$ is easy, and the optimal pricing strategy is given by $p_1^* = p_2^* = \frac{y}{2}$ and each influencer serves consumers with types $\|x - \theta_j\| \leq \frac{I}{2}$ for $j = 1, 2$. Note that there is no overlapping in consumers served by the two influencers, and the monopoly profit is achieved, which yields

$$\Pi_{\{1,2\}} = 2\Pi_{\{1\}} = 2 * \frac{1}{2}yI = yI.$$

Second, we come to show that in the case that $\|\theta_1 - \theta_2\| < I$, we the following three properties hold: i) no gap for all consumers between influencer 1 and 2 on the short arc; ii) both influencers are actively serving consumers on the short arc; and iii) the pricing must be symmetric such that $p_1 = p_2$. We show these three properties one by one.

i) Suppose there is a gap a positive measure of consumers are not served between θ_1 and θ_2 on the short arc. Then, $\|x - \theta_j\| < \frac{I}{2}$ for at least some $j \in \{1, 2\}$ because $\|\theta_1 - \theta_2\| < I$. This implies $p_j > \frac{I}{2}$. Moreover, for such an influencer j, the profit is given by $\Pi_{\{j\}} = p_j(1 - p_j/y)I$, which is strictly decreasing for all $p_j \in (\frac{I}{2}, y]$. This means that it is profitable to choose a price $\hat{p}_j = p_j - \varepsilon$ with $\varepsilon > 0$ sufficiently small.

ii) Suppose influencer 2 is not active on the short arc and θ_2 is on the right of θ_1 on the short arc (i.e. θ_2 is ahead of θ_1 in the clockwise direction on the circle). Then, we have

$$y(1 - \|\theta_2 - \theta_1\|/I) - p_1 \geq y - p_2$$

This further implies that for all $x \in \mathbf{S}^1$ on the right of θ_2 (and thus $\|x - \theta_1\| = \|x - \theta_2\| + \|\theta_1 - \theta_2\|$)

$$y(1 - \|\theta_2 - \theta_1\|/I - \|x - \theta_2\|/I) - p_1 \geq y(1 - \|x - \theta_2\|/I) - p_2$$

Thus, influencer 2 is not actively serving any consumer $x \in \mathbf{S}^1$. However, this is suboptimal because if we only let influencer 1 be active on the market, the maximum profit cannot exceed $\Pi_{\{1\}}$ (i.e., the monopoly profit from hiring a single influencer) and it is dominated by the profit function proposed in the lemma.

iii) Otherwise suppose $p_1 \neq p_2$ and assume w.l.o.g. $p_1 < p_2$. Consider a new pair of prices $(\hat{p}_1, \hat{p}_2) = (p_1 + \varepsilon, p_2 - \varepsilon)$ with $\varepsilon > 0$ sufficiently small, and we show that $(\hat{p}_1, \hat{p}_2)$ dominates (p_1, p_2). By the "no gap" result in (i) and that both influencers are active, all consumers between θ_1 and θ_2 on the short arc are served and denote by x^* the cutoff consumer type, and define $s_j = \|x^* - \theta_j\|$ for $j \in \{1, 2\}$. By the indifference condition for consumer x^*, we have

$$y(1 - s_1/I) - p_1 = y(1 - s_2/I) - p_2$$

and

$$s_1 + s_2 = \|\theta_1 - \theta_2\|$$

These two conditions together yields

$$s_j = \frac{1}{2}\left(\|\theta_1 - \theta_2\| + \frac{I}{y}(p_{3-j} - p_j)\right)$$

Under (p_1, p_2), the profit is given by

$$\Pi = p_1 s_1 + p_2 s_2 + p_1(1 - p_1/y)I + p_2(1 - p_2/y)I$$

and under $(\hat{p}_1, \hat{p}_2)$, the profit is given by

$$\hat{\Pi} = \hat{p}_1 \hat{s}_1 + \hat{p}_2 \hat{s}_2 + \hat{p}_1(1 - \hat{p}_1/y)I + \hat{p}_2(1 - \hat{p}_2/y)I$$

Note that

$$\begin{aligned}
&\hat{p}_1(1 - \hat{p}_1/y)I + \hat{p}_2(1 - \hat{p}_2/y)I \\
&\quad = I * \left(p_1 + p_2 - (p_1 + \varepsilon)^2/y - (p_2 - \varepsilon)^2/y\right) \\
&\quad = p_1 + p_2 - p_1^2/y - p_2^2/y + 2(p_2 - p_1)\varepsilon I/y + O(\varepsilon^2) \\
&\quad > p_1(1 - p_1/y)I + p_2(1 - p_2/y)I
\end{aligned}$$

Similarly, we can show that

$$\begin{aligned}
\hat{p}_1 \hat{s}_1 - p_1 s_1 &= (p_1 + \varepsilon)\left(s_1 - \frac{I\varepsilon}{y}\right) - p_1 s_1 = \varepsilon s_1 - \frac{I\varepsilon}{y} p_1 \\
\hat{p}_2 \hat{s}_2 - p_2 s_2 &= (p_2 - \varepsilon)\left(s_2 + \frac{I\varepsilon}{y}\right) - p_2 s_2 = -\varepsilon s_2 + \frac{I\varepsilon}{y} p_2
\end{aligned}$$

which implies that

$$\hat{p}_1 \hat{s}_1 + \hat{p}_2 \hat{s}_2 - (p_1 s_1 + p_2 s_2) = (s_1 - s_2)\varepsilon + (p_2 - p_1)I\varepsilon/y > 0$$

Thus, (p_1, p_2) is strictly dominated, and the contradiction implies that $p_1 = p_2$.

Third, we find the optimal pricing strategy and the profit function $\Pi_{\{1,2\}}$. By property (iii), we denote $p_1 = p_2 = p$ and the profit function is

$$\Pi = p * \|\theta_1 - \theta_2\| + 2p(1 - p/y)I \tag{A.7}$$

as long as

$$y\left(1 - \left\|\frac{\theta_1 + \theta_2}{2} - \theta_1\right\| / I\right) - p \geq 0$$

The unconstrained optimizer to the profit function is given by

$$p^\star = \frac{y * \|\theta_1 - \theta_2\|}{4I} + \frac{y}{2}$$

By plugging this into the IR condition for the consumer with type $\frac{\theta_1+\theta_2}{2}$,

$$\|\theta_1 - \theta_2\| \leq \frac{2I}{3}.$$

and we get the profit

$$\Pi_{\{1,2\}} = \frac{(2I + \|\theta_1 - \theta_2\|)^2 y}{8I}.$$

Whenever $\|\theta_1 - \theta_2\| \in (\frac{2I}{3}, I)$, $p^\star$ violates the IR condition for the cutoff type $\frac{\theta_1+\theta_2}{2}$. However, note that Π in Equation (A.7) is strictly increasing for all $p \in (0, p^\star]$. Thus, the optimal price is given by the IR condition for the cutoff type, which yields

$$p^* = y * \left(1 - \frac{\|\theta_1 - \theta_2\|}{2I}\right)$$

and the profit is given by

$$\Pi_{\{1,2\}} = \frac{\|\theta_1 - \theta_2\| * (2I - \|\theta_1 - \theta_2\|) * y}{I}.$$

The proof concludes. □

A.21 Proof of Lemma 10

Proof. The proof has two parts.

Part i) Again, we check three cases one by one.

First, we consider heterogeneous product quality (i.e., $I_1 = I_2 = I, \theta_1 = \theta_2$ and $y_1 \geq y_2$). If the ex post matching is one-to-one, then it is characterized by Eq (5). In contrast, if optimal matching is such that both influencers are hired by seller 1, then

$$\hat{\Pi}_{\{1,2\}} = \frac{y_1 I}{2} = \hat{\Pi}_{\{1\}} = \hat{\Pi}_{\{2\}}$$

which implies that both influencers receive zero wages, that is, $\hat{w}_1 = \hat{w}_2 = 0$. This implies that influencer 2 will reject being hired by seller 1 together with influencer 1 since $w_2 = \frac{(1-\gamma)\Pi_2^C}{2} > \hat{w}_2$. Similarly, we can show that influencer 2 also rejects being hired together with influencer 1 by seller 2.

Second, we consider heterogeneous influence power (i.e., $\theta_1 = \theta_2$, $y_1 = y_2$ and $I_1 \geq I_2$). Suppose seller 1 hires both influencers, then the optimal pricing strategy is $p_1^* = p_2^* = \frac{y_1}{2}$.

Note that

$$\hat{\Pi}_{\{1,2\}} = \hat{\Pi}_{\{1\}} = \frac{y_1 I_1}{2}$$

which implies $\hat{w}_2 = 0$ by the Nash bargaining argument. Since $w_2 = \frac{(1-\gamma)\Pi_2^C}{2} > \hat{w}_2$ influencer 2 rejects being hired together with influencer 1 by seller 1. Similarly, influencer 2 rejects the offer that seller 2 hires both influencers.

We consider heterogeneous style difference (i.e., $y_1 = y_2 = y$, $I_1 = I_2 = I$ and $\theta_1 \neq \theta_2$). Here, we assume $\beta \geq \beta_0$ so that an equilibrium exists under regulated matching in Proposition 4. Since both sellers are identical, we compare regulated matching in Proposition 4 and the case when seller 1 hires both influencers.

If the ex post matching is regulated matching, then

$$w_1 = w_2 = (1-\gamma)\Pi_1^C$$

where

$$\Pi_1^C = \begin{cases} \frac{yI}{2}, & \text{if } \beta \geq I \\ y * \beta \left(1 - \frac{\beta}{2I}\right), & \text{if } \frac{6}{7}I < \beta < I \\ \frac{3y}{50I} * (2I+\beta)^2, & \text{if } \beta_0 \leq \beta \leq \frac{6}{7}I \end{cases}$$

If the ex post matching is unbalanced matching, then

$$\hat{w}_1 = \hat{w}_2 = (1-\gamma)(\Pi_{\{1,2\}} - \Pi_1) = (1-\gamma)\left(\Pi_{1,2} - \frac{yI}{2}\right)$$

where

$$\Pi_{\{1,2\}} = \begin{cases} yI, & \text{if } \ \beta \geq I \\ \frac{(2I+\beta)^2}{8I}y, & \text{if } \ \beta \leq \frac{2I}{3} \\ \frac{(2I\beta-\beta^2)}{I}y, & \text{if } \ \beta \in \left(\frac{2I}{3}, I\right) \end{cases}$$

Now, it reduces to check that

$$\Pi_1^C \geq \Pi_{\{1,2\}} - \frac{yI}{2}$$

Note that it is equivalent to check the inequality under the case $y = 1$ and $I = 1$. We can check it case by case. For instance, when $\beta \in [\beta_0, \frac{2I}{3}]$, it reduces to

$$\frac{3}{50}(2+\beta)^2 \geq \frac{1}{8}(2+\beta)^2 - \frac{1}{2}$$

Because $\frac{3}{50} < \frac{1}{8}$, the inequality is most restrictive when $\beta = \frac{2}{3}$. It is easy to check that it indeed holds for $\beta = 23$ for all $\beta \in [0, \frac{2}{3}]$, so it is true for all $\beta \in [\beta_0, \frac{2}{3}]$. Similarly, we can check it for when $\beta \in [2/3, 6/7]$, $\beta \in (6/7, 1]$ and $\beta > 1$.

To summarize, $\hat{w}_1 < w_1$, and thus influencer 1 rejects being hired together inlfuencer 2 by seller 1. By symmetry, seller 2 cannot hire both influencers under equilibrium.

Part ii) There are three possible matching outcomes: 1) regulated matching, which features assortative matching such that $k(j) = j$ (see Lemma 8); 2) unbalanced matching with seller 1 hiring both influencers; and 3) unbalanced matching with seller 2 hiring both influencers.

First, we consider regulated matching. In this case, $k(j) = j$ for $j = 1, 2$, and payoffs are given by:

$$U_1 = \frac{\gamma y_1 I_1}{2}, \quad U_2 = \frac{\gamma y_2 I_2}{2}, \quad w_1 = \frac{(1-\gamma) y_1 I_1}{2}, \quad \text{and } w_2 = \frac{(1-\gamma) y_2 I_2}{2}$$

Second, we consider the case that seller 1 hires both influencers. Note that

$$\Pi_{1,2} = \Pi_1 + \Pi_2, \quad \Pi_1 = \frac{y_1 I_1}{2} \quad \text{and} \quad \Pi_2 = \frac{y_1 I_2}{2}$$

and thus

$$U_1 = \frac{\gamma y_1 (I_1 + I_2)}{2}, \quad U_2 = 0, \quad w_1 = \frac{(1-\gamma) y_1 I_1}{2}, \quad and\ w_2 = \frac{(1-\gamma) y_1 I_2}{2}$$

Second, we consider the case that seller 2 hires both influencers. Note that

$$\Pi_{1,2} = \Pi_1 + \Pi_2, \quad \Pi_1 = \frac{y_2 I_1}{2} \quad \text{and} \quad \Pi_2 = \frac{y_2 I_2}{2}$$

and thus

$$U_1 = 0, \quad U_2 = \frac{\gamma y_2 (I_1 + I_2)}{2}, \quad w_1 = \frac{(1-\gamma) y_2 I_1}{2}, \quad and\ w_2 = \frac{(1-\gamma) y_2 I_2}{2}$$

Obviously, influencers would choose unbalanced matching with seller 1 hiring both influencers. The proof concludes. □

A.22 Proof of Proposition 11

Proof. The proof consists of two parts. Note that under unbalanced matching, the strong seller hiring both influencer always dominates the weak seller hiring both influencers because the product quality y enters the consumer utility in a product form, which implies total welfare and profits are proportional to the product quality.

- **Case i). Congested influencer market or homogeneous product market**

Denote by W_U the total welfare under (the best) unbalanced matching, and by W_R the total welfare under regulated matching. Now, it suffices to show that $W_U \leq W_R$ and we have three cases.

- Case 1). Heterogeneous influencer power (i.e., $y_1 = y_2 = y$, $\theta_1 = \theta_2 = \theta$ and $I_1 \geq I_2$). Total welfare only depends on the marginal consumer who is indifferent between consuming the product and zero consumption, which further depends on the equilibrium price.

Under unbalanced matching, $p_1^* = p_2^* = \frac{y}{2}$, and the total size of consumer served is just I_1 (i.e., $\{x \in \mathbf{S}^1 : y\left(1 - \frac{1}{I_1}\|x - \theta\|\right) - p_1^* \geq 0\}$).

In contrast, under regulated matching, the marginal consumer faces an equilibrium price given by $p_1^C = \frac{2y(I_1 - I_2)}{4I_1 - I_2}$ (see Proposition 3), which implies that the total size of consumer base is bigger than I_1 because

$$\frac{2y(I_1 - I_2)}{4I_1 - I_2} \leq \frac{y}{2}$$

Note that the total welfare only depends on the size of consumer base. Hence, unbalanced matching lowers total welfare in the heterogeneous influencer power case, that is, $W_U \leq W_R$.

- Case 2). Heterogeneous influencers' style type (i.e., $y_1 = y_2 = y$, $I_1 = I_2 = I$ and $\theta_1 \neq \theta_2$). Here, we only focus on the case in which a pure strategy price competition equilibrium exists (see Proposition 4), and thus the equilibrium price under regulated matching is given by

$$p_1^C = p_2^C = \begin{cases} \frac{y}{5I}(2I + \beta), & \text{if } \beta \in [\frac{2}{67}(-7 + 5\sqrt{10})I, \frac{6}{7}I] \\ y * \left(1 - \frac{\beta}{2I}\right), & \text{if } \beta \in (\frac{6}{7}I, I) \\ \frac{y}{2}, & \text{if } \beta \geq I \end{cases}$$

In contrast, under unbalanced matching (see Lemma 9),

$$p_1^* = p_2^* = \begin{cases} \frac{y}{2}, & \text{if } \beta \geq I \\ y * \left(\frac{\beta}{4I} + \frac{1}{2}\right), & \text{if } \beta \leq \frac{2I}{3} \\ y * \left(1 - \frac{\beta}{2I}\right), & \text{if } \beta \in \left(\frac{2I}{3}, I\right) \end{cases}$$

We can directly verify that $p_1^C = p_2^C \leq p_1^* = p_2^*$ for $\beta \geq \beta_0$. Thus, total welfare is higher under regulated matching, because a consumer who purchases the product under unmatched matching is also willing to buy it under regulated matching. Hence, $W_U \leq W_R$.

- Case 3). Heterogeneous product quality (i.e., $I_1 = I_2 = I$, $\theta_1 = \theta_2 = \theta$ and $y_1 \geq y_2$).

Under unbalanced matching, seller 1 hires both influencers and set a price at $p_1^* = p_2^* = \frac{y}{2}$. Influencer 2 is not active in the market. Hence, the total welfare is

$$W_U = \int_{\left\{x \in \mathbf{S}^1 : \|x - \theta\| \leq \frac{1}{2}I\right\}} y(1 - \|x - \theta\|/I)dx = \frac{3}{4}y_1 I.$$

In contrast, under regulated matching, the equilibrium outcome is given by Proposition 2, and thus the total welfare is given by

$$W_R = \int_{R_1} y(1 - \|x - \theta\|/I_1)dx + \int_{R_2} y(1 - \|x - \theta\|/I_2)dx$$

where R_1 are consumers served by the first influencer-seller group targets consumers, that is, $R_1 = \{x \in \mathbf{S}^1 : \|x - \theta\| \leq \frac{2Iy_1}{4y_1 - y_2}\}$. Similarly, R_2 are consumers served by the second influencer-seller targets, that is, $R_2 = \{x \in \mathbf{S}^1 : \frac{2Iy_1}{4y_1 - y_2} < \|x - \theta\| \leq \frac{3Iy_1}{4y_1 - y_2}\}$.

We can further simplify the expression of W_R as[21]

$$W_R = \frac{Iy_1(14y_1^2 - 4y_1y_2 - y_2^2)}{(4y_1 - y_2)^2} = Iy_1 * f(x)$$

where $f(x) = (14x^2 - x - 1)/(4x - 1)^2$ and $x := y_1/y_2$.

Note that $f'(x) = -\frac{12(x-1)}{(4x-1)^3} < 0$, and thus $f(x)$ is strictly decreasing for $x > 1$. Moreover, also note that

$$\lim_{x \to 1} f(x) = 1, \quad \text{and} \quad \lim_{x \to \infty} f(x) = \frac{7}{8}.$$

which further implies

$$W_R \geq \frac{7}{8}Iy_1 > \frac{3}{4}Iy_1 = W_U.$$

The proof concludes for the first part. ■

- **Case ii). Uncongested influencer market**

First, note that when $\beta \geq \frac{I_1+I_2}{2}$, each influencer charge a monopolist price $p_j^* = \frac{y}{2}$ when hired by a seller with product quality y, and serves a sub-population such that $x \in R_j$ and $R_j := \left\{x \in \mathbf{S}^1 : \|x - \theta_j\| \leq \frac{I}{2}\right\}$. Hence, there is zero over-lapping among consumers served by the two influencers.

Under unbalanced matching, both influencers are hired by the strong seller 1. Furthermore, total welfare can be calculated as

$$W_U = \frac{3}{4}y_1(I_1 + I_2).$$

Similarly, under regulated matching, the total welfare equals

$$W_R = \frac{3}{4}y_1I_1 + \frac{3}{4}y_2I_2$$

Obviously, $W_U > W_R$ as long as $y_1 > y_2$. ■

All the proofs conclude. □

[21] The skipped algebra is available upon request.