Equilibrium Equity and Variance Risk Premiums in A Cost-free Production Economy[0]

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Abstract

This paper provides a production-based equilibrium model with a recursive-preferences investor, which successfully explains the equity premium puzzle and the large negative variance risk premium with a low risk aversion setting, and theoretically generates the negative sign of the diffusive volatility risk premium. The empirical results show that the stochastic volatility with contemporaneous jumps (SVCJ) model built in our cost-free production economy can well capture the equity premium, the realized variance and the implied variance, so that the model can successfully explain both the large equity and variance risk premiums.

Keywords: Equity risk premium; variance risk premium; cost-free production economy; SVCJ model

JEL Classifications:: G12; G13.

1. Introduction

In this paper, we construct an equilibrium model in a cost-free production economy with a representative investor who has recursive preferences. To be simplified, we assume the process of the stock price follows a stochastic volatility with contemporaneous jumps (SVCJ) model.[1] After solving the equilibrium, we conclude that the SVCJ model built in our cost-free production economy can well capture the high equity risk premium (ERP) and the large negative variance risk premium (VRP). The explanation performance of our simpler production-based equilibrium model is similar to Drechsler (2013) who uses a more complicated consumption-based equilibrium model with a representative investor who model uncertainty that varies in intensity over time. To our knowledge, our model is the first production-based equilibrium model to successfully explain the equity premium puzzle and the large negative VRP.

To explain the high negative VRP, defined as the realized variance (RV) minus the implied variance (IV), is a very important topic.[2] In practically, the VRP is practitioners' cost to get a protection against high realized variance via buying variance swaps. How much they need to pay, of course, is the very curial issue that practitioners care about.[3] In empirically, Carr and Wu (2009) use the difference between the RV and this synthetic variance swap rate to quantify the VRP and find that there exists the large and negative mean of the VRP on five stock indexes and 35 individual stocks by using a large options data set. Todorov (2010); Bollerslev and Todorov (2011); Bollerslev et al. (2015) use the rare events to account for the large average VRP. Recently, González-Urteaga and Rubio (2016) discuss and test the volatility risk premium at the individual and portfolio level.[4] Barras and Malkhozov (2016) formally compare

[1]The SVCJ is one of most widely-used affine jump-diffusion (AJD) models (see Duffie et al. (2000)).

[2]The large negative VRP means the large average negative VRP. Our definition follows Carr and Wu (2009). Some papers propose the positive VRP as they defined the VRP as the implied variance minus the realized variance.

[3]Mixon and Onur (2015) report that gross vega notional outstanding for variance swaps, in 2014, is over USD 2 billion, with USD 1.5 billion in S&P 500 products and USD 3 billion in the CBOE VIX futures. The volatility market has become particularly popular over last decade.

[4]The difference between the variance risk premium and the volatility risk premium is whether we

the market VRP inferred from equity and option markets and find that the average difference between the two VRPs is essentially zero. Ait-Sahalia et al. (2015); Li and Zinna (2016) examine the term structures of the VRP by using variance swap rates data. In theoretically, Bollerslev et al. (2009); Drechsler and Yaron (2011); Bollerslev et al. (2012); Drechsler (2013); Jin (2015) adopt the long-run risks model (first proposed by Bansal and Yaron (2004)) to successfully explain the large average VRP. In addition, Buraschi et al. (2014) use a two-tree Lucas (1978) economy with two heterogeneous investors to well explain the volatility risk premium. All previous models in the literature are built in a consumption economy. In this paper, we construct a simple production-based equilibrium model and successfully explain the large ERP and VRP with a much lower the level of the relative risk aversion (RRA) of 1.1. However, the existing literature, Drechsler and Yaron (2011) choose a value of 9.5 to explain the VRP and Drechsler (2013) set it as 5.

The production-based equilibrium model adopted in this paper is developed from the neoclassical growth model in Constantinides (1990), which is first studied by Cox et al. (1985a) and followed by Bates (1991, 1996); Vasicek (2005); Zhang et al. (2012); Fu and Yang (2012); Ruan et al. (2016).[5] The main difference between our production-based equilibrium model and the classic consumption-based equilibrium model (e.g., Lucas (1978); Mehra and Prescott (1985); Bansal and Yaron (2004); Drechsler and Yaron (2011); Wachter (2013)) is that we construct our model starting from the index level (e.g., S&P 500), while the consumption-based model is based on the fundamentals (i.e., consumptions and dividends). This difference does affect a lot. For example, in terms of the equity premium puzzle, based on a consumption-based equilibrium model, Mehra and Prescott (1985) obtain that the equity premium (6.18%) is the product of the coefficient of the RRA and the variance of the growth rate of consumption ($3.57\%^2$),

take a square root of the variance.

[5]The cost-free production economy is taken from Constantinides (1990). However, Constantinides (1990) only solves a investment problem instead of an equilibrium problem. Thus, we adopt the equilibrium conditions in Cox et al. (1985a) and others to extend the investment model in Constantinides (1990) into a cost-free production-based equilibrium model.

which leads to the very large coefficient of RRA, 48.7, based on the sample of the U.S. economy from 1889 to 1978. On the other hand, with same sample, the production-based model (see the second term in Equation (3.1) in Zhang et al. (2012)) implies the very small coefficient of the RRA, only 2.2! This is because the ERP in Zhang et al. (2012) is the product of the coefficient of the RRA and the variance of the real return on S&P 500 ($16.54\%^2$). As the production-based equilibrium model works so well in explaining in the equity premium puzzle, in this paper, we extend it into explaining the VRP.

The reason why we call our model as the cost-free production-based equilibrium model, partially, is that it can be regarded as a special case of AK production model, that is developed from the neoclassical investment model (see Hayashi (1982)) and recently well extended by Bolton et al. (2011); DeMarzo et al. (2012); Wang et al. (2012); Bolton et al. (2013); Pindyck and Wang (2013) and others, in which there is no cost of installing capital (i.e., new investments) and the firm's productivity (i.e., "A") and the Tobin' q equals one, so that the value of the capital stock (i.e., stock price) is same to the capital stock (i.e., "K"). For example, if we assume the cost of installing capital is zero in Pindyck and Wang (2013), we will get the one-valued Tobin' q and solve that the stock price is the capital stock. In order to emphasize the important of our model, we name this specialized model as the cost-free production-based equilibrium model.

Should the diffusive volatility risk premium (DVRP), which is defined as the mean-reverting speed of the volatility in the risk-neutral measure minus that in the physical probability measure,[6] be theoretically positive or negative? As a detailed overview provided in Broadie et al. (2007), there exist the conflicting estimates of the DVRP for a long time in the literature. For example, the simple stochastic volatility model (SV), Jones (2003) estimate a positive DVRP, while, Pan (2002); Eraker et al. (2003) observe a negative DVRP. Thus, regarding the sign of the DVRP, Broadie et al. (2007)

[6]The first concept is proposed by Cox et al. (1985b).

argue that there is no theory providing a guidance. Actually, there are a few studies on that. Cox et al. (1985b); Bates (1991, 1996, 2000) show that the DVRP is negative as the volatility is negatively correlated to the S&P 500 index. Recently, Eraker and Wu (2014) propose a consumption-based equilibrium model and document that the DVRP is negative for any positive risk aversion coefficients. In order to qualify Broadie et al.'s (2007) argument, we use an equilibrium model to provide a guidance on the sign of the DVRP based on SVCJ model. Consistent to Cox et al. (1985b); Bates (1991, 1996, 2000); Eraker and Wu (2014), our production-based model documents again that the sign of the DVRP should be negative.

Finally, our model constructed in this paper involves the recursive preferences, which are well studied by Weil (1989); Epstein and Zin (1989, 1991). Later on, Duffie and Epstein (1992b,a) develop it into the continuous-time version. Now the recursive preferences are popularly used in asset pricing models (e.g., Bansal and Yaron (2004); Benzoni et al. (2011); Wachter (2013)). The main advantage of the recursive preferences is separating the RRA and the elasticity of intertemporal substitution (EIS). In spite of it increasing the complexity, we provide analytical expressions for all solutions, which are linked to the impact of the RRA and the EIS. Similar to the DVRP, there exist the conflicting estimates of the EIS in the literature. For example, Bansal and Yaron (2004) estimate EIS of 1.5 and Bansal et al. (2007) estimate the EIS of 2, while Hall (1988); Epstein and Zin (1991) and others estimate that EIS is below 1. Summarizing the estimates in the previous literature, we find that the large EIS (> 1) is accompanied by large RRA and the small EIS (< 1) is estimated with small RRA. In the paper, based on the equilibrium model, we discuss the two combinations, and find that the both are two possible candidates used to explain the large ERP and VRP. However, the former works better in our cost-free production-based equilibrium model. The reason is that the ERP is defined on the the variance of the real return on S&P 500 instead of the growth rate of consumption, as we mentioned above. We need the lower RRA to fit the ERP data.

There are at least two contributions in the paper: (i) We develop a cost-free production-based equilibrium model as the first production-based equilibrium model to successfully explain the equity premium puzzle and the large negative VRP. (ii) We provide a guidance on the sign of the DVRP based on SVCJ model.

The remainder of our article is organized as follows. Section 2 presents the definition of the ERP and VRP. Section 3 presents and solves the equilibrium model. Section 3 discusses the calibration and simulation. Section 4 concludes. Appendix A collects all proofs and Appendix B gives a comparison of estimates.

2. Equity and Variance Risk Premiums

2.1 Model-implied Equity and Variance Risk Premiums

The affine jump-diffusion models are well studied by Duffie et al. (2000). In this paper, we adopt the following SVCJ model (e.g., Eraker et al. (2003); Eraker (2004); Broadie et al. (2007)) to describe the joint dynamics of the stock price.[7] More general model is discussed in Duffie et al. (2000).

Under the physical probability measure $\mathbb{P}$, at time t, the stock price S_t follows,

$$\begin{cases} \frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dB_t^S + (e^x - 1)dN_t - \lambda m dt, \\ dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dB_t^V + y dN_t, \end{cases} \tag{1}$$

where B_t^S and B_t^V are a pair of correlated Brownian motions with correlation coefficient ρ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$; $N(t)$ is a Poisson process with the intensity λ; the jump size of the volatility has an exponential distribution $y \sim \exp(1/\mu_V)$ with mean μ_V and the jump size of the stock price is $x \sim N(\mu_S, \sigma_S^2)$; the growth rate $\mu = r + \phi_t$ where $ERP_t = 100 \times \phi_t = \eta_S V_t + \left(m\lambda - m^{\mathbb{Q}}\lambda^{\mathbb{Q}}\right)$ is the equity risk premium (ERP) contributed

[7]The SVCJ is the most popular AJD models used for option and other derivatives pricing (e.g., Eraker et al. (2003); Eraker (2004); Broadie et al. (2007); Zhu and Lian (2012); Zheng and Kwok (2014); Neumann et al. (2016) and others). They empirically document that the SVCJ works quite well to fit the S&P 500 Index.

by the diffusive risk premium $\eta_S V_t$ and the price jump risk premium $\left(m\lambda - m^{\mathbb{Q}}\lambda^{\mathbb{Q}}\right)$ scaled 100 (see Broadie et al. (2007));[8] κ, θ and σ_V are constant and $m = e^{\mu_S + \frac{1}{2}\sigma_S^2} - 1$.[9] The SVCJ model can be degenerated into two other model specifications often employed in the literature, namely the stochastic volatility model with jumps in prices (SVJ) ($\mu_V = 0$) and the simple stochastic volatility model (SV) ($\mu_V = \mu_S = \sigma_S = \lambda = 0$).

We specify the transition between the risk-neutral measure $\mathbb{Q}$ and the physical probability measure $\mathbb{P}$ by using the similar transformations to those applied in Eraker (2004); Broadie et al. (2007).[10] Then, the price process in a risk-neutral probability $\mathbb{Q}$ becomes

$$\begin{cases} \frac{dS_t}{S_t} = rdt + \sqrt{V_t}dB_t^S(\mathbb{Q}) + (e^x - 1)dN_t(\mathbb{Q}) - m^{\mathbb{Q}}\lambda^{\mathbb{Q}}dt, \\ dV_t = \left(\kappa\theta - \kappa^{\mathbb{Q}}V_t\right)dt + \sigma_V\sqrt{V_t}dB_t^V(\mathbb{Q}) + ydN_t(\mathbb{Q}), \end{cases} \tag{2}$$

where $m^{\mathbb{Q}} = e^{\mu_S^{\mathbb{Q}} + \frac{1}{2}\sigma_S^2} - 1$, $x \sim N(\mu_S^{\mathbb{Q}}, \sigma_S^2)$ and $y \sim \exp(1/\mu_V^{\mathbb{Q}})$.[11] In addition, ρ, $\kappa\theta$, σ_V, σ_S are the same across both measures (the detailed transformations are shown in Section 3). Hence the variance risk premium can be defined as follows.

Definition 1 (Model-implied Equity and Variance Risk Premiums). Having estimated the $\mathbb{P}$ and $\mathbb{Q}$ parameters, we can define the ERP as

$$\frac{ERP_t}{100} = \frac{1}{\tau}E^{\mathbb{P}}\left[\int_t^{t+\tau} d\ln S_t\right] - \frac{1}{\tau}E^{\mathbb{Q}}\left[\int_t^{t+\tau} d\ln S_t\right] = \mu - r = \eta_S V_t + \left(m\lambda - m^{\mathbb{Q}}\lambda^{\mathbb{Q}}\right), \tag{3}$$

where η_S is a constant and can be determined by the equilibrium, and define the VRP

[8] The constant η_S can be estimated, for example, see Neumann et al. (2016).

[9] The independence of jump sizes is consistent with the results of previous studies. For example, Eraker et al. (2003); Eraker (2004) report statistically insignificant correlations between the two jump sizes. In addition, this correlation primarily affects the conditional skewness of returns, μ_V and the correlation between the two jump sizes play a very similar role. Broadie et al. (2007) show that it is difficult to estimate this parameter precisely. Following Broadie et al. (2007), we assume two jump size are independent.

[10] In Broadie et al. (2007), they assume $\lambda = \lambda^{\mathbb{Q}}$. We set $\lambda \neq \lambda^{\mathbb{Q}}$ which is corresponding to the equilibrium models in Bates (2000); Liu et al. (2005); Zhang et al. (2012); Ruan et al. (2016). In section 3, we will confirm our setting.

[11] Following Broadie et al. (2007), a correlation between jumps in prices and volatility would be difficult to identify under $\mathbb{Q}$ because $\mu_V^{\mathbb{Q}}$ plays the same role in the conditional distribution of returns. In order to clearly define the volatility jump risk premium, it is necessary to assume the correlation between jumps should be zero.

as, based on SVCJ model,[12]

$$\begin{aligned}
\frac{VRP_t}{100^2} &= \underbrace{\frac{1}{\tau}E^{\mathbb{P}}\left[\int_t^{t+\tau}(d\ln S_t)^2\right]}_{RV_t} - \underbrace{\frac{1}{\tau}E^{\mathbb{Q}}\left[\int_t^{t+\tau}(d\ln S_t)^2\right]}_{IV_t} \\
&= \frac{1}{\tau}E^{\mathbb{P}}\left[\int_t^{t+\tau}V_s ds\right] - \frac{1}{\tau}E^{\mathbb{Q}}\left[\int_t^{t+\tau}V_s ds\right] + \lambda E^{\mathbb{P}}\left[x^2\right] - \lambda^{\mathbb{Q}}E^{\mathbb{Q}}\left[x^2\right] \\
&= \left(A - A^{\mathbb{Q}}\right)\cdot V_t + \left(B - B^{\mathbb{Q}}\right)
\end{aligned} \tag{4}$$

where

$$A = \frac{1-e^{-\kappa\tau}}{\kappa\tau}, \quad B = \left(\theta + \frac{\lambda\mu_V}{\kappa}\right)(1-A) + \lambda\left(\sigma_S^2 + \mu_S^2\right), \tag{5}$$

and

$$A^{\mathbb{Q}} = \frac{1-e^{-\kappa^{\mathbb{Q}}\tau}}{\kappa^{\mathbb{Q}}\tau}, \quad B^{\mathbb{Q}} = \left(\frac{\kappa\theta}{\kappa^{\mathbb{Q}}} + \frac{\lambda^{\mathbb{Q}}\mu_V^{\mathbb{Q}}}{\kappa^{\mathbb{Q}}}\right)(1-A^{\mathbb{Q}}) + \lambda^{\mathbb{Q}}\left(\sigma_S^2 + \mu_S^{\mathbb{Q}2}\right). \tag{6}$$

As we are interested in one month (21 business days) horizon variance risk premium, we set $\tau = 21$ days.

Based on the above definition, the model-implied equity risk premium can be decomposed into two components: the ERP contributed by the stochastic volatility (ERP^{SV}), the ERP contributed by the volatility jump (ERP^{PJ}),

$$ERP_t = ERP_t^{SV} + ERP_t^{PJ}, \tag{7}$$

where

$$\frac{ERP_t^{SV}}{100} = \eta_S V_t, \quad \frac{ERP_t^{PJ}}{100} = m\lambda - m^{\mathbb{Q}}\lambda^{\mathbb{Q}}. \tag{8}$$

Similarly, the model-implied variance risk premium can be decomposed into three components: the VRP contributed by the stochastic volatility (VRP^{SV}), the VRP mainly

[12] In SVCJ mdoel, according to Duan and Yeh (2010), $VIX^2 = \frac{1}{\tau}E^{\mathbb{Q}}\left[\int_t^{t+\tau}V_s ds\right] + 2\lambda^{\mathbb{Q}}E^{\mathbb{Q}}\left[e^x - x - 1\right]$, while, $IV_t = \frac{1}{\tau}E^{\mathbb{Q}}\left[\int_t^{t+\tau}V_s ds\right] + \lambda^{\mathbb{Q}}E^{\mathbb{Q}}\left[x^2\right]$. As $e^x = 1 + x + \frac{1}{2}x^2 + \mathcal{O}(x^3)$, we have $VIX^2 = IV_t + \mathcal{O}(x^3)$. So, we find that the difference between IV_t and VIX^2 is very small and then VIX^2 can be regarded as a good proxy of the IV_t.

contributed by the volatility jump (VRP^{VJ}) and the VRP contributed by the price jump (VRP^{PJ}).

$$VRP_t = VRP_t^{SV} + VRP_t^{VJ} + VRP_t^{PJ}, \tag{9}$$

where

$$\frac{VRP_t^{SV}}{100^2} = \left(A - A^{\mathbb{Q}}\right) \cdot V_t + \theta(1-A) - \left(\frac{\kappa\theta}{\kappa^{\mathbb{Q}}}\right)(1 - A^{\mathbb{Q}}), \tag{10}$$

$$\frac{VRP_t^{VJ}}{100^2} = \frac{\lambda\mu_V}{\kappa}(1-A) - \frac{\lambda^{\mathbb{Q}}\mu_V^{\mathbb{Q}}}{\kappa^{\mathbb{Q}}}(1 - A^{\mathbb{Q}}), \tag{11}$$

$$\frac{VRP_t^{PJ}}{100^2} = \lambda\left(\sigma_S^2 + \mu_S^2\right) - \lambda^{\mathbb{Q}}\left(\sigma_S^2 + \mu_S^{\mathbb{Q}2}\right). \tag{12}$$

From the expression (10), the VRP^{SV} is mainly from the contribution of the DVRP (i.e., $\kappa^{\mathbb{Q}} - \kappa$). The jump intensity risk premium, $\lambda - \lambda^{\mathbb{Q}}$, influences both VRP^{VJ} and VRP^{PJ} in (11)-(12). In Broadie et al. (2007), they conclude that the DVRP is insignificant in all SV, SVJ and SVCJ models because of the flat implied volatility term structure. In addition, they argue that even for the more efficient estimation procedure, they still can not confront the fact that the term structure is flat, which still implies the problem that DVRP is insignificant. In Section 3, we will give a economic explanation that the DVRP is large and negative in the reality, that suggests researchers need to choose more efficient data (e.g., Zhu and Lian (2012)), in order to get the significant and negative DVRP.

2.2 Equity and Variance Risk Premiums

According to Bollerslev et al. (2009); Drechsler and Yaron (2011); Bollerslev et al. (2012); Drechsler (2013); Jin (2015), we purposely rely on the readily available squared VIX index as our measure for the risk-neutral expected variance.[13] In addition, following Buraschi et al. (2014); González-Urteaga and Rubio (2016), we use daily returns of

[13]We download VIX from Yahoo Finance, see http://finance.yahoo.com/quote/%5EVIX/history.

S&P 500 to calculate the realized variance over 21-day windows.[14] The daily ERP is simply defined as the daily log returns of S&P 500 subtract the three-month Treasury bill which is obtained from Thomson Reuters Datastream (see Figure 1).[15]

[Insert Figure 1]

Definition 2 (Equity and Variance Risk Premium). We defined the (annualized) ERP as[16]

$$ERP_t = \underbrace{(R_t - r_t/252)}_{\text{Daily ERP}} \times 252, \tag{13}$$

where $R_t = (\ln S_t - \ln S_{t-1}) \times 100$ is the daily percentage returns of S&P 500 and r_t is the daily three-month Treasury bill rate. In addition, we defined the VRP as[17]

$$VRP_t = RV_t - VIX_t^2. \tag{14}$$

where RV_t calculated by the daily percentage returns of S&P 500 over 21-day windows at day t; VIX_t^2 is daily squared VIX index divided by 12 as one-month horizon at time t.

[Insert Table 1]

Table 1 shows that the mean of the daily equity risk premium is 0.01501 (or the mean of the annualized ERP is 3.7825) and the mean of variance risk premium in the sample period is -10.8647, which is significant negative. However, some days have the

[14] We download S&P 500 Index from Yahoo Finance, see http://finance.yahoo.com/quote/%5EGSPC/history.

[15] The three-month Treasury bill data is percentage annual risk-free rate.

[16] Throughout the paper, without added "daily", the ERP means annualized ERP.

[17] As $VIX_t^2 \neq E^{\mathbb{Q}}[RV_t] = IV_t$ based on Footnote 9, if there exist jumps in the return, we can rewrite the definition $VRP_t = RV_t - SWV_t + SWV_t - VIX^2$ where swap variance SWV_t is the sum of the difference between simple return and log return over 21-day windows and $VIX_t^2 = E^{\mathbb{Q}}[SWV_t]$. More details see Jiang and Yao (2013). We compute the average $RV_t - SWV_t$ is 0.01373 and 0.02228 during the period 02 January 1990 to 20 May 2016 and the subperiod 24 November 2010 to 20 May 2016, respectively, which are very small compared with the average VRP. Thus, we use VIX^2 as a good proxy of IV_t here.

positive VRP (e.g., maximum is 470.7357), which indicates the financial disasters. For example, in Figure 2, there are 41 days from August to November in 2008 whose VRP is larger than 100. In addition, the maximum exists on 25 September 2008. More cashes (e.g., 1997 Asian financial crisis, 1998 Russian financial crisis, stock market downturn of 2002, August 2011 stock markets fall and 2015–16 Chinese stock market crash) can be found in the positive-VRP periods. It suggests that the positive VRP can be an indicator of financial cashes.[18] Furthermore, unsurprisingly, the movement of daily 21-day RV is consistent to VIX^2 in Figure 3.

[Insert Figure 2]

[Insert Figure 3]

We use the estimates in Broadie et al. (2007) (see Table 2) to calculate the model-implied VRP and use the returns of S&P 500 Index to calculate the real variance risk premium based on Definition 2. In Table 3, we find that SVJ and SVCJ model with their estimates can well fit the real VRP (the mean in 1990-2003 is -14.2412). This gives us a motivation that the simply SVJ or SVCJ model built in our cost-free production economy may explain the high negative VRP.

[Insert Table 2]

[Insert Table 3]

[18]For example, Chen et al. (2016) use the sentiment factor to explain why the variance risk premium is positive in a financial crisis period.

3. A Cost-free Production-based Equilibrium Model

3.1 Model Setup

Our cost-free production economy follows Constantinides (1990).[19] There exists only one production good, which is also the consumption. There are two technologies for the good to consume or invest. There is no any cost for investments, so that we can call the two production technologies as the cost-free technologies and the economy as the cost-free production economy. The risky technology has stochastic return over the period $[t, t+dt]$, $\mu dt+\sqrt{V_t}dB_t^S+(e^x-1)dN_t-\lambda m dt$ where $dV_t=\kappa(\theta-V_t)dt+\sigma_V\sqrt{V_t}dB_t^V+ydN_t$. The riskless technology has constant returns rdt.

We suppose that there is a representative investor whose portfolio is $(u_t, 1-u_t)$, which represents the fraction of wealth invested in the risky and riskless technology, respectively. The consumption rate of the investor is c_t. Then the investor's capital process W_t with the initial capital $W_0>0$ satisfies a stochastic differential equation as follows,

$$\begin{cases} \frac{dW_t}{W_t}=[r+(\mu-r-\lambda m)u_t-\frac{c_t}{W_t}]dt+u_t\sqrt{V_t}dB_t^S+u_t(e^x-1)dN_t, \\ dV_t=\kappa(\theta-V_t)dt+\sigma_V\sqrt{V_t}dB_t^V+ydN_t. \end{cases} \tag{15}$$

In addition, the representative consumer has recursive preferences (see Duffie and Epstein (1992b,a)) given by

$$J_t=E_t\left[\int_t^\infty f(c_s,J_s)ds\right], \tag{16}$$

with

$$f(c,J)=\frac{\beta(1-\gamma)J}{1-\psi^{-1}}\left[\frac{c^{1-\psi^{-1}}}{((1-\gamma)J)^\omega}-1\right]=\frac{\beta}{1-\psi^{-1}}\frac{c^{1-\psi^{-1}}-((1-\gamma)J)^\omega}{((1-\gamma)J)^{\omega-1}}, \tag{17}$$

[19]Similar assumptions for the representative-consumer production economy can be found in Cox et al. (1985a); Bates (1991, 1996); Vasicek (2005); Zhang et al. (2012); Fu and Yang (2012); Ruan et al. (2016).

where the subjective time discount factor is denoted by β; ψ is the EIS and γ is the coefficient of RRA. In addition, we denote $\omega = (1 - \psi^{-1})/(1 - \gamma)$. Recursive utility allows to separate the effects of RRA and EIS. The special case of power utility results by setting the EIS equal to the inverse of the RRA coefficient. In this case, only innovations to consumption are priced. In the general case $\gamma \neq 1/\psi$, state variables carry a risk premium, too. We assume throughout that $\beta > 0$ and $\gamma > 0$. Most of the discussion focuses on that case $\gamma > 1$.

By choosing the investment u_t in the stock and the consumption rate c_t, the representative investor maximizes his/her expected objective function (16). Based on Cox et al. (1985a); Bates (1991, 1996); Vasicek (2005); Zhang et al. (2012); Fu and Yang (2012); Ruan et al. (2016), we define the market equilibrium in a production economy as follows.

Definition 3 (Market equilibrium). Equilibrium in our cost-free production economy is defined in a standard way: equilibrium consumption-portfolio pairs $(u_t,\ c_t)$ are such that the representative investor maximizes his/her expected objective function in (16), and markets clear, $u_t = 1$.

3.2 Solutions

After solving the equilibrium defined in Definition 3, we get the following proposition.

Proposition 1. The value function is

$$J(W, V) = e^{aV+b} \frac{W^{1-\gamma}}{1-\gamma}, \tag{18}$$

and the optimal consumption rate is

$$c^* = \left(e^{-\psi\omega(aV+b)}\beta^{\psi}\right) W, \tag{19}$$

where constant a and b satisfy,[20]

$$\begin{cases} 0 = -\frac{\beta(1-\gamma)}{1-\psi^{-1}} + \left[\left(\frac{1}{1-\psi^{-1}} - 1\right)(1-\gamma)\right]\left(\left(1+\psi\omega a\overline{V}\right)e^{-\psi\omega(a\overline{V}+b)}\beta^{\psi}\right) \\ \quad +r\left(1-\gamma\right) + \kappa\theta a + \lambda E\left[e^{ay+(1-\gamma)x} - 1\right] - (1-\gamma)\lambda E\left[e^{ay-\gamma x}(e^{x}-1)\right], \\ 0 = -ae^{-\psi\omega(a\overline{V}+b)}\beta^{\psi} + \gamma\left(1-\gamma\right)\frac{1}{2} - \kappa a + \frac{1}{2}\sigma_V^2 a^2, \end{cases} \tag{20}$$

with $\overline{V} = \frac{\kappa\theta+\lambda\mu_V}{\kappa}$.

The equity risk premium is solved as

$$\frac{ERP_t}{100} = \left(\gamma - \sigma_V \rho a\right)V + \lambda E\left[(1-e^{-\gamma x+ay})(e^{x}-1)\right] = \left(\gamma - \sigma_V\rho a\right)V + \lambda m - \lambda^{\mathbb{Q}} m^{\mathbb{Q}}. \tag{21}$$

The state-price density is given by

$$\frac{d\pi_t}{\pi_t} = -rdt - \gamma\sqrt{V_t}dB_t^S + a\sigma_V\sqrt{V_t}dB_t^V + \left(e^{-\gamma x+ay} - 1\right)dN_t - \lambda E\left(e^{-\gamma x+ay} - 1\right)dt. \tag{22}$$

The explicit transition between the risk-neutral measure $\mathbb{Q}$ and the physical probability measure $\mathbb{P}$ is given by

$$\kappa^{\mathbb{Q}} = \kappa + \left(\rho\gamma - a\sigma_V\right)\sigma_V, \quad \mu_S^{\mathbb{Q}} = \mu_S - \gamma\sigma_S^2, \quad \mu_V^{\mathbb{Q}} = \frac{1}{1-a\mu_V}\mu_V, \quad \lambda^{\mathbb{Q}} = \frac{\lambda e^{\frac{1}{2}\sigma_S^2\gamma^2 - \mu_S\gamma}}{1-a\mu_V}. \tag{23}$$

Following Constantinides (1990), if a firm has capital K_t at time t and it can be freely invested in two production technologies, we assume the firm invest $\delta_1 K_t$ capital in the risky technology and $(1-\delta_1)K_t$ in riskless one where δ_1 is constant. In addition, the firm is financed with the equity (stock) S_t and risk-free bond M_t where $dM_t/M_t = rdt$. We assume the leverage $\delta_2 = S_t/(S_t + M_t)$ which is a constant. Then we have the equality between the investment in two production technologies and the value of the

[20]Note here we choose $0 < a < 1/\mu_V$ and $\gamma > 1$.

firm.

$$
\begin{cases} dS_t + M_t r dt = \delta_1 K_t(\mu dt + \sqrt{V_t} dB_t^S + (e^x - 1)dN_t - \lambda m dt) + (1-\delta_1)K_t r dt, \\ dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dB_t^V + y dN_t, \end{cases} \tag{24}
$$

which can be rewritten as

$$
\begin{cases} \frac{dS_t}{S_t} = \frac{\delta_1}{\delta_2}\left(\mu - rdt + \sqrt{V_t} dB_t^S + (e^x - 1)dN_t\right) - \lambda m dt + rdt, \\ dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dB_t^V + y dN_t, \end{cases} \tag{25}
$$

In order to be consistent with our stock price process in Equation (1), following Constantinides (1990), we set $\frac{\delta_1}{\delta_2} = 1$. Thus, the process of the stock price (with dividends included) in (25) is same to (1).

Remark 1. Our production-based model can be regarded as an extension of the model in Cox et al. (1985a); Bates (1991, 1996, 2000); Vasicek (2005); Zhang et al. (2012); Fu and Yang (2012); Ruan et al. (2016), which only consider that the representative consumer has a constant relative risk aversion (CRRA) utility function. The production-based equilibrium model studied in previous literature is widely used in derivative pricing, especially in option pricing. It is reasonable to employ the production-based model to explain the VRP, because VIX index can be regarded as the variance swaps rates.

Remark 2. In Broadie et al. (2007), they suggest that $\frac{ERP_t}{100} = \eta_S V_t + \left(m\lambda - m^{\mathbb{Q}}\lambda^{\mathbb{Q}}\right)$ which is contributed by the diffusive risk premium $\eta_S V_t$ and the price jump risk premium $\left(m\lambda - m^{\mathbb{Q}}\lambda^{\mathbb{Q}}\right)$. The solution of the ERP in (21) supports their assumption. In addition, η_S can be solved as $\gamma - \sigma_V \rho a$, which is constant, and $\lambda m - \lambda^{\mathbb{Q}} m^{\mathbb{Q}} = \lambda E\left[(1 - e^{-\gamma x + ay})(e^x - 1)\right]$, which can be verified by using the transition in (23). If the volatility is a constant, σ, the ERP will become $\frac{ERP}{100} = \gamma\sigma^2 + \lambda E\left[(1 - e^{-\gamma x})(e^x - 1)\right]$ which has been studied in Zhang et al. (2012).

Remark 3. In the equilibrium model, we find out a stochastic density factor (i.e., state-price density) in (22), which provides a transition for parameters between the risk-neutral measure $\mathbb{Q}$ and the physical probability measure $\mathbb{P}$. The stochastic density factor in (22) captures all risks, that are two Brownian motion risks (B_t^S and B_t^V) and one jump risk (N_t). In the transition (23), it shows that the jump intensity in two measures are not equal, i.e., $\lambda \neq \lambda^{\mathbb{Q}}$, which is consistent to Bates (1991, 1996, 2000); Liu et al. (2005); Zhang et al. (2012); Ruan et al. (2016). It suggests that we have to estimate $\lambda^{\mathbb{Q}}$ as an independent risk-neutral parameter. In addition, as $\mu_S < 0, \gamma > 1$ and $0 < a < 1/\mu_V$, we have $\lambda < \lambda^{\mathbb{Q}}$. In other words, the jump intensity should be larger in risk-neutral probability measure than in the physical probability measure.

Remark 4. The transition in (23) documents that $\kappa^{\mathbb{Q}} - \kappa = (\rho\gamma - a\sigma_V)\,\sigma_V < 0$ for $\rho < 0, \gamma > 1, 0 < a < 1/\mu_V$ and $\sigma_V > 0$. In other words, the equilibrium model can generate the negative DVRP. It is not surprising. Actually, in the same production-based framework, Cox et al. (1985b); Bates (1991, 1996, 2000) argue that DVRP is negative because of the negative correlation between the volatility and the S&P 500 index. Recently, Eraker and Wu (2014) propose a consumption-based equilibrium model and get the similar result that $\kappa^{\mathbb{Q}} - \kappa < 0$ for any $\gamma > 0$.

Remark 5. For any $a > 0$ and $\gamma > 0$, we have $\mu_S^{\mathbb{Q}} < \mu_S < 0$ and $\mu_V^{\mathbb{Q}} > \mu_V > 0$. It is corresponding to the empirical results, e.g., Eraker (2004); Broadie et al. (2007); Neumann et al. (2016). It means that the jumps in the price and the volatility are larger in risk-neutral probability measure than in the physical probability measure.

3.3 Equilibrium Model-implied ERP and VRP

Plugging the explicit transition between the risk-neutral measure $\mathbb{Q}$ and the physical probability measure $\mathbb{P}$ given in (23) into Definition 1, we will get the model-implied VRP based on our cost-free production-based equilibrium model.

Definition 4 (Equilibrium model-implied Equity and Variance Risk Premiums). Having estimated the $\mathbb{P}$ parameters and the recursive preferences parameters of the representative investor, based on SVCJ model, we define the equilibrium model-implied ERP as

$$ERP_t = \left[(\gamma - \sigma_V \rho a) V + \lambda E\left((1 - e^{-\gamma x + ay})(e^x - 1)\right)\right] \times 100, \tag{26}$$

and the equilibrium model-implied VRP is defined as,

$$VRP_t = (A_\Delta \cdot V_t + B_\Delta) \times 100^2, \tag{27}$$

where

$$A_\Delta = \frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{1 - e^{-(\kappa + (\rho\gamma - a\sigma_V)\sigma_V)\tau}}{(\kappa + (\rho\gamma - a\sigma_V)\sigma_V)\tau},$$

and

$$\begin{aligned} B_\Delta = & \left(\theta + \frac{\lambda\mu_V}{\kappa}\right)(1 - \frac{1 - e^{-\kappa\tau}}{\kappa\tau}) + \lambda\left(\sigma_S^2 + \mu_S^2\right) \\ & - \frac{1}{\kappa + (\rho\gamma - a\sigma_V)\sigma_V}\left(\kappa\theta - \frac{\lambda\mu_V e^{\frac{1}{2}\sigma_S^2\gamma^2 - \mu_S\gamma}}{(1 - a\mu_V)^2}\right)\left[1 - \frac{1 - e^{-(\kappa + (\rho\gamma - a\sigma_V)\sigma_V)\tau}}{(\kappa + (\rho\gamma - a\sigma_V)\sigma_V)\tau}\right] \\ & + \frac{\lambda e^{\frac{1}{2}\sigma_S^2\gamma^2 - \mu_S\gamma}}{1 - a\mu_V}\left(\sigma_S^2 + \left(\mu_S - \gamma\sigma_S^2\right)^2\right). \end{aligned}$$

Equation (27) shows that VRP is a function of the physical measure parameters (κ, θ, σ_V, ρ, λ, μ_S, σ_S and μ_V) and the preferences parameters (β, γ and ψ).[21]

We set the baseline model parameters as $\kappa = 15, \theta = 0.02, \sigma_V = 0.5, \rho = -0.9, \lambda = 2, \mu_S = -0.01, \sigma_S = 0.02, \mu_V = 0.06, r = 0.008, \beta = 0.001, \gamma = 2.5, \psi = 2$ and $E[V] = 0.02$.[22] Then, the baseline average $ERP = 5.2023$, $VRP = -15.3317$ and $a = 6.9101$. we will analyse how those preferences parameters affect the ERP and VRP.

[Insert Figure 4]

[21]The parameter a is determined by β, γ and ψ in Equation (20).

[22]The value of parameters is according to our estimates in Table 9. The assumption, $RRA = 2.5 > 2$ and $EIS = 2 > 1$, is based on the parameter settings in Drechsler and Yaron (2011); Drechsler (2013).

[Insert Figure 5]

[Insert Figure 6]

From Figure 4, we find that, except θ, the higher κ, σ_V and the correlation ρ lead to the higher equity risk premiums. In terms of the VRP, we find that the larger σ_V and ρ produce the larger negative VRP but ρ less affects on the VRP. In addition, on the one hand, the VRP increases with $\kappa \leq 20$ and decreases with $\kappa > 20$. On the another hand, the VRP decreases with $\theta \leq 0.1$ and increases with $\theta > 0.1$. Empirically, the estimates of κ are less than 20 and the estimates of θ are less than 0.1 (e.g., Bakshi et al. (1997); Bates (2000); Eraker et al. (2003); Bates (2012); Neumann et al. (2016)). Thus, normally, we can say that the VRP increases with κ and decreases with θ.

In Figure 5, the larger value of jump parameters, i.e., λ, μ_S, σ_S and μ_V, leads to the larger ERP and VRP. It is consistent to (i) that the jump is the key factor to explain the equity premium puzzle (e.g., Rietz (1988); Barro (2006); Wachter (2013); Branger et al. (2016)); and (ii) the conclusion in Bakshi and Madan (2006) that the volatility spreads are positive (the VRP is negative) when the distribution of the index is negatively skewed and leptokurtic physical, which is denominated by the jump parameters. Furthermore, they have different sensitivities to the ERP and VRP. For example, with λ increasing from 2 to 20, the ERP goes up from 5.2023 to 13.9994 and the VRP goes up from -8.4328 to -12.6136 in Figure 5-(a), while, with μ_S decreasing from -0.01 to -0.1, the ERP jumps to 61.5961 and the VRP jumps to -31.3595 in Figure 5-(b).

Figure 6-(a) shows the ERP and VRP are a linear function of β, while, the ERP and VRP are a convex function of the RRA (e.g., γ) in 6-(b). In our numerical results, $ERP = 10.1944$ and $VRP = -21.7746$ when $\gamma = 2.2$ and then it sharply decreases, for example, $ERP = 3.5658$ and $VRP = -5.0358$ when $\gamma = 3$. The ERP reaches the minimal (2.7331) around $\gamma = 5$ and VRP reaches the minimal (-3.16) around $\gamma = 6$. After that, the ERP and VRP increase with the RRA. For example, $ERP = 39.0374$ and $VRP = -36.2261$ at $\gamma = 50$. The effects of the EIS on the

ERP and VRP are very different. Actually, when $\psi < 1$ (see Figure 6-(c)), it leads to the very large a closed to $1/\mu_V$, which will generate very large ERP and VRP (e.g., $ERP = 213.3938$ and $VRP = -4871.7$ when $\psi = 0.6$). It is not reasonable. Thus, our numerical results support that $EIS > 1$ is a good choice when $\gamma > 2$ (e.g., Bansal and Yaron (2004); Drechsler and Yaron (2011); Drechsler (2013)). In Figure 6-(d), we have $VRP = -0.8928, -8.7178$ and -0.9502 when $\psi = 1.01, 3$ and 100, respectively, which are starting, maximal and ending points. The evolution of ERP is similar to VRP. From the combination of Figure 6-(b) and (d), we can get a suggestion that if we assume $EIS > 1$, it is better to set a larger RRA (e.g., $RRA = 7.5$ in Bansal and Yaron (2004), $RRA = 9.5$ in Drechsler and Yaron (2011), $RRA = 10$ in Benzoni et al. (2011)).

[Insert Figure 7]

[Insert Figure 8]

[Insert Figure 9]

However, Hall (1988); Epstein and Zin (1991) and others estimate EIS is below 1. For example, Epstein and Zin (1991) estimate RRA is around 1 (i.e., $0.6 < RRA < 1.4$) and EIS is around 0.25 (i.e., $0.2 < EIS < 0.9$). Thus, we set $\gamma = 1.1$ and $\psi = 0.3$ and keep the rest of parameters. The results are given in Figure 7- 9. Except γ and ψ, the effects of other model parameters on the ERP and VRP do not change a lot. In Figure 9-(b) and (c), it shows that the parameter setting with a small RRA (which is around 1 but larger 1, e.g., $RRA = 1.1$) and a small EIS (e.g., $0.2 < EIS < 0.6$) is a very reasonable parameter setting. Actually, considering that the ERP and VRP should be a strictly increasing function with respect to RRA, $\{0 < EIS < 1, 1 < RRA < 2\}$ is a reasonable choice.

Based on the above numerical analysis, we get a conclusion related to parameter setting: (i) $\{EIS > 1, RRA > 2\}$ (e.g., $RRA = 5, 7.5$ or 10 and $EIS = 1.5$ or 2), and

(ii) $\{0 < EIS < 1, 1 < RRA < 2\}$ (e.g., $RRA = 1.1$ or 1.2 and $EIS = 0.3, 0.4, 0.5$ or 0.6) are two possible combinations to successfully capture the high VRP and ERP. However, based on our cost-free production-based equilibrium model, in the empirical analysis, the former is better to fit the data of the ERP and VRP. The reason is that the ERP is defined on the the variance of the real return on S&P 500 instead of the growth rate of consumption so that we need the lower RRA to fit the ERP data. Thus, all the following empirical results are obtained by using the former parameter setting.[23]

3.4 Equilibrium Model-implied ERP and VRP in Broadie et al. (2007)

We set the equilibrium model parameters as $\beta = -\ln(0.999)$ and $\gamma = 1.1$. The risk-free rate $r = 4.8512(\%)$ is the average three-month Treasury bill over the same period, 1990 to 1999. The physical model parameters are given in Broadie et al. (2007) (i.e., Table 2). The mean of the annualized log S&P500 returns, 14.2040, is from 1990 to 1999 which is the overlapping period between Eraker et al. (2003) (1980-1999) and Broadie et al. (2007) (1987-2003) after VIX lunched. Thus, the mean of the ERP is 9.3528, We report simulated statistics in Table 4 and 5 based on 10,000 simulations, with each statistic calculated using a sample size equal to its the ERP and VRP data counterpart.

[Insert Table 4]

[Insert Table 5]

We adjust ψ to fit the mean of the ERP in Table 4 and then we use the same parameters to calculate the VRP in Table 5. From Table 4 and 5, we have a conclusion that the SVCJ model built in our cost-free economy can explain not only the high ERP but

[23] Please note using the data in Mehra and Prescott (1985), the production-based model in Zhang et al. (2012) implies the very small coefficient of the RRA, only 2.2. If we add more economic uncertainty (e.g., stochastic volatility and jump risks), the model-implied RRA will be lower. A comparison can be found in Footnote 27.

also the large VRP (explanation rate is 56.91%). With the sensitivity analysis of model parameter above, we can choose the high EIS to produce the more negative VRP but the ERP will be slightly higher at same time. As a similar analysis will be studied in Section 4.3, we do not show the details here.

3.5 VRP Return Predictability

We calculate the conditional equity premium as

$$\begin{aligned} E_t\left[\frac{ERP_{t+1}}{100}\right] &= (\gamma - \sigma_V\rho a)\, E_t[V_{t+1}] + \lambda E\left[(1-e^{-\gamma x+ay})(e^x-1)\right] \\ &= (\gamma - \sigma_V\rho a)\left[e^{-\kappa}V_t + \left(\theta + \frac{\lambda\mu_V}{\kappa}\right)(1-e^{-\kappa})\right] + \lambda E\left[(1-e^{-\gamma x+ay})(e^x-1)\right] \\ &= (\gamma - \sigma_V\rho a)\, e^{-\kappa}V_t + (\gamma - \sigma_V\rho a)\left(\theta + \frac{\lambda\mu_V}{\kappa}\right)(1-e^{-\kappa}) \\ &\quad + \lambda E\left[(1-e^{-\gamma x+ay})(e^x-1)\right]. \end{aligned} \tag{28}$$

Equation (27) implies that $V_t = \frac{\frac{VRP_t}{100^2} - B_\Delta}{A_\Delta}$. Combining with Equation (28), we can get the predictive regression for the equity premium (excess market return),

$$\frac{ERP_{t+1}}{100} = \beta_{pred}\frac{VRP_t}{100^2} + \alpha_{pred} + \epsilon_{t+1}, \tag{29}$$

where

$$\beta_{pred} = \frac{\gamma - \sigma_V\rho a}{A_\Delta}e^{-\kappa}, \tag{30}$$

and

$$\alpha_{pred} = (\gamma - \sigma_V\rho a)\, e^{-\kappa}\frac{-B_\Delta}{A_\Delta} + (\gamma - \sigma_V\rho a)\left(\theta + \frac{\lambda\mu_V}{\kappa}\right)(1-e^{-\kappa}) + \lambda E\left[(1-e^{-\gamma x+ay})(e^x-1)\right]. \tag{31}$$

As $(\rho\gamma - a\sigma_V)\,\sigma_V < 0$ which leads to $A_\Delta < 0$, we have $\beta_{pred} < 0$. Therefore, the predictive coefficient is negative (with respect to the $\frac{VRP_t}{100^2}$ in Equation (4)). It is corresponding to empirical results in Bollerslev et al. (2009); Drechsler and Yaron

(2011); Jin (2015) and others. We report our empirical results from the daily predictive regressions of monthly equity premiums on variance risk premiums in Table 6. We will find β_{pred} is significantly negative with very large t-statistics. The predictive power from the fact that the volatility V_t determines the expected excess return in Equation (28) and the VRP in Equation (27). Thus, through the information of the volatility, the variance risk premium is able to predict the excess market return.

[Insert Table 6]

4. Empirical Analysis

Even though, we have shown that the SVCJ model can well explain the ERP and VRP based on the parameters in Broadie et al. (2007), in this section, we will use more recent data to robustly examine whether the SVCJ model built in a cost-free economy can explain the large ERP and VRP.

4.1 Estimates of Physical Measure Parameters

The first step is using S&P 500 returns data to estimate the physical measure parameters by using a Markov chain Monte Carlo (MCMC) sampler. Eraker et al. (2003); Eraker (2004); Amengual (2009); Zhu and Lian (2012); Kaeck and Alexander (2012) show that (i) MCMC yields very accurate estimates for jump-diffusion models; (ii) MCMC provides estimates of the latent volatility; (iii) MCMC outperforms the generalized method of moments (GMM), the quantile maximum likelihood estimation (QMLE) and the efficient method of moments (EMM); (iv) MCMC can utilize priors to disentangle jumps from diffusions in an intuitive manner. Hence, we use MCMC approach to estimate our affine jump-diffusion models. We present a time-discretization of Equation (1) by using the discrete scheme,

$$\begin{cases} R_t = \alpha + \sqrt{V_t}\epsilon_t^S + J_t^S q_t, \\ V_t = V_{t-1} + \kappa(\theta - V_{t-1}) + \sigma_V \sqrt{V_{t-1}}\epsilon_t^V + J_t^V q_t, \end{cases} \tag{32}$$

where $\alpha = r - \frac{1}{2}V_t + \eta_S V_t - \lambda m$; ϵ_t^S and ϵ_t^V are samples from two dependent standard normal distributions with correlation ρ; $R_t = (\ln S_t - \ln S_{t-1}) \times 100$, $y \sim \exp(1/\mu_V)$, $x \sim N(\mu_S, \sigma_S^2)$ and $q_t \sim Ber(\lambda)$. The parameters to be estimated are α, κ, θ, σ_V, ρ, λ, μ_S, σ_S, μ_V.

[Insert Table 7]

Following Eraker et al. (2003), we run the MCMC algorithm for 110,000 iterations, discarding the first 10,000 as a burn-in-period to achieve the convergence of the chain. For each parameter to be estimated, we use the same priors as in Eraker et al. (2003) and the sample period is from 24 November 2010 to 20 May 2016 (see Table 7). Estimates are shown in Table 8.

[Insert Table 8]

These reported parameters are quite informative. Table 8 shows that the values of the daily variance θ are 0.9298, 0.928 and 0.6983, respectively, for the SV, SVJ and SVCJ models, which are a little lower than the unconditionally sampled standard deviation of S&P500 return data, 0.9725^2. In the annualized view, the sampled standard deviation of data is 15.4380,[24] while, estimated $\sqrt{252\theta}$ are 15.3017, 15.3924 and 13.2654, respectively, for the SV, SVJ and SVCJ models. If we calculate the mean of the annualized standard deviation of the SVCJ model, $\sqrt{252(\kappa\theta + \lambda\mu_V)/\kappa}$, is 15.4564 which is almost same to the sampled value, 15.4380. Corresponding to S&P500 parameter estimates in Table III in Eraker et al. (2003), adding more jumps in the model leads to the lower estimated values of θ, σ_V, but higher value of ρ. During our sample

[24] $15.4380 = 0.9725\sqrt{252}$.

period (2010-2016), we have higher correlation ρ. It is not supervising. For example, in Duan and Yeh (2010), they report that ρ is -0.5268 in 1990-1995 and -0.7389 in 2001-2007. It seems that the current data has much high ρ.

4.2 Model-implied Risk-neutral Parameters and ERP

In order to get the risk-neutral parameters implied in our equilibrium model, we convert the daily percentage estimates into annualized values in Table 9.[25] Then set $\beta = -\ln(0.999)$ which is same to Drechsler and Yaron (2011); Drechsler (2013). Here we set $\gamma = 1.1$ which is much lower than existing literature (e.g., 9.5 in Drechsler and Yaron (2011); 5 in Drechsler (2013)). The risk-free rate $r = 0.0730(\%)$ which is the average three-month Treasury bill over the same period, 24 November 2010 to 20 May 2016. In order to fit the mean of ERP in Table 10, we set $\psi = 0.42, 0.42$ and 0.37 for SV, SVJ and SVCJ models, respectively, and then we get the annualized risk-neutral parameters implied in our equilibrium model in Table 11 and convert them into daily percentage value in Table 12.

[Insert Table 9]

[Insert Table 10]

[Insert Table 11]

[Insert Table 12]

In order to consider the estimate error for the MCMC spot variance, we also present the results by using the 10,0000 simulations in Table 10.[26] Comparing Panel A and B in Table 10, we find that there is very little distinguish between the two sets. This demonstrates that our main findings are robust to the MCMC spot variance. After

[25] A detailed guide for the conversion of parameters can be found in Appendix C of Branger and Hansis (2015).

[26] Neumann et al. (2016) do a comparison analysis for the MCMC spot variance and the spot variance estimated by the options data. They find that there is very little difference.

perfectly capturing the ERP data in Table 10, we obtain the optimal values of EIS and get the model-implied risk-neutral parameters in Table 12.

Here we discuss the parameters transition of the SVCJ model between the risk-neutral measure $\mathbb{Q}$ and the physical probability measure $\mathbb{P}$ in details. In physical measure, $\kappa = 0.06019$, $\lambda = 0.006985$, $\mu_S = -1.394$ and $\mu_V = 2.213$ given in Table 8, while, in risk-neutral measure, $\kappa^{\mathbb{Q}} = 0.0500$, $\lambda^{Q} = 0.0131$, $\mu_S^{\mathbb{Q}} = -1.4357$ and $\mu_V^{\mathbb{Q}} = 4.0925$ shown in Table 12. Based on that, our production-based equilibrium reveals that the DVRP, $\kappa^{\mathbb{Q}} - \kappa = -0.01019 < 0$, should be negative. From (23), we know that $\kappa^{\mathbb{Q}} - \kappa = (\rho\gamma - a\sigma_V)\,\sigma_V < 0$ for $\gamma > 1$ and $0 < a < 1/\mu_V$. Figure 10 shows that a increases with γ and always keeps positive. In addition, in risk-neutral probability measure, we have more negative mean of jump size in the price and larger mean of jump size in the volatility. It is consistent to the empirical estimates (e.g., Eraker (2004); Broadie et al. (2007)). Furthermore, the transition suggests that the estimate of the jump intensity in risk-neutral probability measure should be larger than in the physical probability measure. It will contribute part of the jump risk premium in the variance risk premium and the equality premium. Thus, here we suggest again that the risk-neutral parameter $\lambda^{\mathbb{Q}}$ should be estimated.

[Insert Figure 10]

4.3 VRP Implied in Equilibrium

We substitute the physical parameters in Table 8 and the risk-neutral parameters implied by the equilibrium model in Table 12 into the formula (4) and then compute the model-implied variance risk premium in Table 13.[27] Corresponding to Broadie

[27] If we use parameter setting, $\{EIS > 1, RRA > 2\}$, for example, the best combination $EIS = 1.1$ and $RRA = 3.5$, to get model-implied ERP, 10.0492, which is closed to the market ERP, 10.0150, using the MCMC spot variance, based on SVCJ model, then the mean of the VRP is -1.7409 whose absolute magnitude is lower than -4.2709 shown in the last row in in Table 13, Panel B. Thus, the comparison documents that the parameter setting, $\{EIS > 1, RRA > 2\}$, works worse than the parameter setting $\{0 < EIS < 1, 1 < RRA < 2\}$, based on our cost-free production-based equilibrium model.

et al. (2007) (See Table 3), the SVCJ model can explain more than 50% VRP. Table 14 provides statistics in details for the realized variance, the implied variance and the variance risk premium, all at the monthly horizon. It demonstrates that the model well fits the data of the realized variance and document that our estimates are very accurate. So far, we will find the SVCJ model can well explain the large VRP built in our cost-free production economy, after perfectly fitting the ERP data.

[Insert Table 13]

[Insert Table 14]

If we allow the model to generate a little bit higher ERP, the model-implied VRP will be higher. For example, we follow the fitting level of ERP in Drechsler (2013), i.e., Table 15, and then we get the corresponding results in Table 16. Comparing Table 15 and 16, our model can explain the VRP of 74%, which is same to the Drechsler's (2013) (in Table 16). In other words, our simpler production-based equilibrium has same power of explaining the equity and variance risk premiums, compared to the more complex consumption model with the model uncertainty in Drechsler (2013). Panel B in Table 16 examines the fraction of the VRP explained by the three components. We find that the jump VRP (especially, the volatility jump VRP) explains around 80% of the total VRP. It is consistent to Li and Zinna (2016) who calculate the fraction at around 79%.

[Insert Table 15]

[Insert Table 16]

In addition, according to numerical analysis in Table 7, we can continue to rise the value of EIS in order to fully capture the VRP data. In Table 17, with $EIS = 0.61$, the SVCJ model can perfectly fit the mean of VRP. In the Panel A of Table 17, the model generates slightly higher mean of the ERP which is around 14, compared to the real mean of the ERP that is around 10. But it is still acceptable. We give the statistics in details for the realized variance, the implied variance and the variance risk premium in Table 18. It shows that, with $EIS = 0.61$, our model well fits the data of the realized variance, implied variance and the variance risk premiums.

[Insert Table 17]

[Insert Table 18]

To summarize, the SVCJ model built in a cost-free production-based equilibrium model can well explain the equity premium puzzle and the high negative VRP with much low RRA and EIS coefficients.

5. Conclusions

The paper discusses the equity and variance risk premiums in a cost-free production-based economy with one representative investor who has a recursive preference. After solving the equilibrium, we provide an explicit transition for model parameters between the risk-neutral measure and the physical probability measure. This transition documents that the diffusive volatility risk premium should be negative. In addition, using the model-implied risk-neutral parameters, we calculate and analyse the model-implied equity and variance risk premiums. From the numerical analysis, we suggest that lower RRA and EIS setting is a good choice. Compared to the market data, we find that the SVCJ model built in our cost-free production economy can well explain the equity premium puzzle and the large negative VRP.

In contrast to Bollerslev et al. (2009); Drechsler and Yaron (2011); Drechsler (2013) who use a long-run risks model, and Buraschi et al. (2014) who use a two-tree Lucas (1978) economy with two heterogeneous investors, we employ a simpler cost-free production-based equilibrium model and successfully explain the large equity and variance risk premiums. Our analysis in this paper can be extended to study the skewness risk premium (e.g., Neuberger (2012); Kozhan et al. (2013)).

Appendix A: Proof of Proposition 1

The value function satisfies the following HJB equation,

$$0 = \max_{c,u} \left\{ f(c, J) + [r + (\phi - \lambda m)u - \frac{c}{W}]WJ_W + \frac{1}{2}u^2W^2VJ_{WW} + \kappa(\theta - V)J_V \right.$$
$$\left. + \frac{1}{2}\sigma_V^2 VJ_{VV} + \sigma_V\rho uWVJ_{WV} + \lambda E\left[J(W + uW(e^x - 1), V + y) - J\right] \right\}. \qquad (33)$$

This leads to the two first-order conditions (FOCs),

$$f_c(c, J) - J_W = 0, \qquad (34)$$
$$(\phi - \lambda m)WJ_W + uW^2VJ_{WW} + \sigma_V\rho WVJ_{WV},$$
$$+ \lambda E\left[J_W(W(1 + (e^x - 1)u), V + y)W(e^x - 1)\right] = 0. \qquad (35)$$

Applying the market clearing condition $u = 1$ to (35), we can solve the equity premium as,

$$\phi = \lambda m - \frac{1}{J_W}\left(WVJ_{WW} + \sigma_V\rho VJ_{WV} + \lambda E\left[J_W(We^x, V + y)(e^x - 1)\right]\right). \qquad (36)$$

In addition, from (34), we can solve the optimal consumption rate

$$c_t^* = \left(J_W\left[(1 - \gamma)J\right]^{\omega - 1}\beta^{-1}\right)^{-\psi}. \qquad (37)$$

Plugging (36) into (33) and using $u = 1$, we get the following partial differential equation (PDE),

$$0 = \frac{\beta(1 - \gamma)J}{1 - \psi^{-1}}\left[\frac{{c^*}^{1-\psi^{-1}}}{((1 - \gamma)J)^{\omega}} - 1\right] - c^*J_W + rWJ_W - \frac{1}{2}W^2VJ_{WW} + \kappa(\theta - V)J_V$$
$$+ \frac{1}{2}\sigma_V^2VJ_{VV} + \lambda E\left[J(We^x, V + y) - J\right] - \lambda E\left[WJ_W(We^x, V + y)(e^x - 1)\right]. \qquad (38)$$

We conjecture the value function has the following form,

$$J(W,V) = e^{aV+b}\frac{W^{1-\gamma}}{1-\gamma}, \tag{39}$$

where $a > 0$ and $\gamma > 1$ (similar assumptions see Wachter (2013)).
Substituting the conjectured value function in (39) into (37) and the PDE (38), the optimal consumption rate (37) can be rewritten as

$$c_t^* = \left(e^{-\psi\omega(aV+b)}\beta^{\psi}\right)W, \tag{40}$$

and the PDE (38) becomes

$$\begin{aligned}0 = &\frac{\beta(1-\gamma)}{1-\psi^{-1}}\left[e^{-\psi\omega(aV+b)}\beta^{\psi-1}-1\right] - (1-\gamma)\left(e^{-\psi\omega(aV+b)}\beta^{\psi}\right) + r\,(1-\gamma) + \gamma\,(1-\gamma)\,\frac{1}{2}V\\ &+ \kappa(\theta-V)a + \frac{1}{2}\sigma_V^2 V a^2 + \lambda E\left[e^{ay+(1-\gamma)x}-1\right] - (1-\gamma)\lambda E\left[e^{ay-\gamma x}(e^x-1)\right].\end{aligned}$$

Using the affine approximation method (e.g., see Benzoni et al. (2011)), we expand the exponential term in V near their long term mean level $\overline{V} = \frac{\kappa\theta+\lambda E[y]}{\kappa}$, $e^{-\psi\omega(aV+b)} \approx e^{-\psi\omega(a\overline{V}+b)} - \psi\omega a e^{-\psi\omega(a\overline{V}+b)}(V-\overline{V}) = \left(1+\psi\omega a\overline{V}\right)e^{-\psi\omega(a\overline{V}+b)} - \psi\omega a e^{-\psi\omega(a\overline{V}+b)}V$ and collect the terms with the same power of V. Then a and b can be solved in the following equation.

$$\begin{cases} 0 = -\frac{\beta(1-\gamma)}{1-\psi^{-1}} + \left[\left(\frac{1}{1-\psi^{-1}}-1\right)(1-\gamma)\right]\left(\left(1+\psi\omega a\overline{V}\right)e^{-\psi\omega(a\overline{V}+b)}\beta^{\psi}\right) \\ \quad +r\,(1-\gamma) + \kappa\theta a + \lambda E\left[e^{ay+(1-\gamma)x}-1\right] - (1-\gamma)\lambda E\left[e^{ay-\gamma x}(e^x-1)\right], \\ 0 = -a e^{-\psi\omega(a\overline{V}+b)}\beta^{\psi} + \gamma\,(1-\gamma)\,\frac{1}{2} - \kappa a + \frac{1}{2}\sigma_V^2 a^2. \end{cases} \tag{41}$$

Plugging (39) into (36), we get the equity premium as

$$\phi_t = (\gamma - \sigma_V \rho a)\,V_t + \lambda E\left[(1-e^{-\gamma x+ay})(e^x-1)\right], \tag{42}$$

and substituting (40) into (15), we obtain the optimal wealth process as follows,

$$\begin{cases} \frac{dW_t}{W_t} = [(\mu - \lambda m) - \left(e^{-\psi\omega(aV+b)}\beta^{\psi}\right)]dt + \sqrt{V_t}dB_t^S + (e^x - 1)dN_t, \\ dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dB_t^V + ydN_t. \end{cases} \tag{43}$$

Finally, we define the SDF as

$$\pi_t = \exp\left\{\int_0^t f_J(c, J)ds\right\} f_c(c, J), \tag{44}$$

or

$$\frac{d\pi_t}{\pi_t} = f_J(c, J)dt + \frac{df_c(c, J)}{f_c(c, J)}, \tag{45}$$

where

$$f_c(c, J) = J_W = e^{aV+b}W^{-\gamma}, \tag{46}$$

and

$$f_J(c, J) = \frac{\beta(1-\gamma)}{1-\psi^{-1}}\left[\frac{c^{1-\psi^{-1}}(1-\omega)}{((1-\gamma)J)^{\omega}} - 1\right] = \frac{\beta(1-\gamma)}{1-\psi^{-1}}\left[e^{-\psi\omega(aV+b)}\beta^{\psi-1}(1-\omega) - 1\right]. \tag{47}$$

Implying Itô's Lemma to (46), we have,

$$\frac{df_c(c, J)}{f_c(c, J)} = -\Gamma_t dt - \gamma\sqrt{V_t}dB_t^S + a\sigma_V\sqrt{V_t}dB_t^V + \left(e^{-\gamma x + ay} - 1\right)dN_t - \lambda E\left(e^{-\gamma x + ay} - 1\right)dt, \tag{48}$$

where

$$\begin{aligned}\Gamma_t =& \gamma\left(\mu - \lambda m - e^{-\psi\omega(aV+b)}\beta^{\psi}\right) - \frac{1}{2}\gamma(\gamma+1)V - a\kappa(\theta - V) \\ &- \frac{1}{2}\sigma_V^2 a^2 V + \gamma a\rho\sigma_V V - \lambda E\left(e^{-\gamma x + ay} - 1\right).\end{aligned}$$

Finally, we plug (47) and (48) into (45) and then we get

$$\frac{d\pi_t}{\pi_t} = -r_t dt - \gamma\sqrt{V_t}dB_t^S + a\sigma_V\sqrt{V_t}dB_t^V + \left(e^{-\gamma x+ay} - 1\right)dN_t - \lambda E\left(e^{-\gamma x+ay} - 1\right)dt, \tag{49}$$

where[28]

$$r_t = \Gamma_t - \frac{\beta(1-\gamma)}{1-\psi^{-1}}\left[e^{-\psi\omega(aV+b)}\beta^{\psi-1}(1-\omega) - 1\right]. \tag{50}$$

As B_t^S and B_t^V are a pair of correlated Brownian motions with correlation coefficient ρ, according to Girsanovs theorem (see, e.g., Theorem 1.32 and Theorem 1.34, Øksendal and Sulem (2007)), we present the transition between the risk-neutral measure $\mathbb{Q}$ and the physical probability measure $\mathbb{P}$,

$$\begin{aligned}
dB_t^S(\mathbb{Q}) =& dB_t^S + (\gamma - a\rho\sigma_V)\sqrt{V_t}, \\
dB_t^V(\mathbb{Q}) =& dB_t^V + \rho(\gamma - a\rho\sigma_V)\sqrt{V_t} - (1-\rho^2)a\sigma_V\sqrt{V_t} = dB_t^V + (\rho\gamma - a\sigma_V)\sqrt{V_t}, \\
\mu_S^{\mathbb{Q}} =& \mu_S - \gamma\sigma_S^2, \\
\mu_V^{\mathbb{Q}} =& \frac{1}{1-a\mu_V}\mu_V, \\
\lambda^Q =& \lambda E[e^{-\gamma x+ay}] = \lambda e^{\frac{1}{2}\sigma_S^2\gamma^2 - \mu_S\gamma}\frac{1}{1-a\mu_V}.
\end{aligned}$$

As Equation (41) can yield multiple solutions to a and b, we select a with the restriction, $a < 1/\mu_V$, to get make sure $\lambda^{\mathbb{Q}} > 0$. According to the empirical results in Eraker (2004); Broadie et al. (2007); Neumann et al. (2016), $\mu_V^{\mathbb{Q}} > \mu_V$, it suggests the restriction $a > 0$ as $\mu_V > 0$.

[28] Even though, the risk-free rate is determined by the equilibrium, we still assume it is exogenously given. Based on that, the equity premium and the growth rate of the stock are both endogenous.

Appendix B: A Comparison of Our Codes

In order to verify the accuracy of our MCMC codes, we present the replicated estimates by using the same sample in Eraker et al. (2003) and Yun (2011) with our MCMC codes based on OpenBUGS,[29] which are used in Section 4. We discard the first 10,000 runs as "burn-in"period and use the last 100,000 iterations in MCMC simulations to estimate model parameters. Specifically, we take the mean of the posterior distribution as parameter estimate and the standard deviation of the posterior as standard error in parentheses.

As the jumps of the SVCJ model in Eraker et al. (2003) are correlated, we only present the estimates of the SV and SVJ models. As a complementary comparison, we compare our estimates of the SV, SVJ and SVCJ models with Yun's (2011) which are estimated by using WinBUGS. From Table 19 and 20, we can confirm that our MCMC codes based on OpenBUGS can estimate similar model parameters with very small difference to Eraker et al. (2003) and Yun (2011).

[Insert Table 19]

[Insert Table 20]

[29]The reason we choose OpenBUGS instead of WinBUGS is that OpenBUGS running is faster and more functional. Changes between WinBUGS and OpenBUGS, see http://www.openbugs.net/w/OpenVsWin.

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Figure 1: S&P 500 Index and three-month Treasury bill (risk-free interest rate) from 02 January 1990 to 20 May 2016.

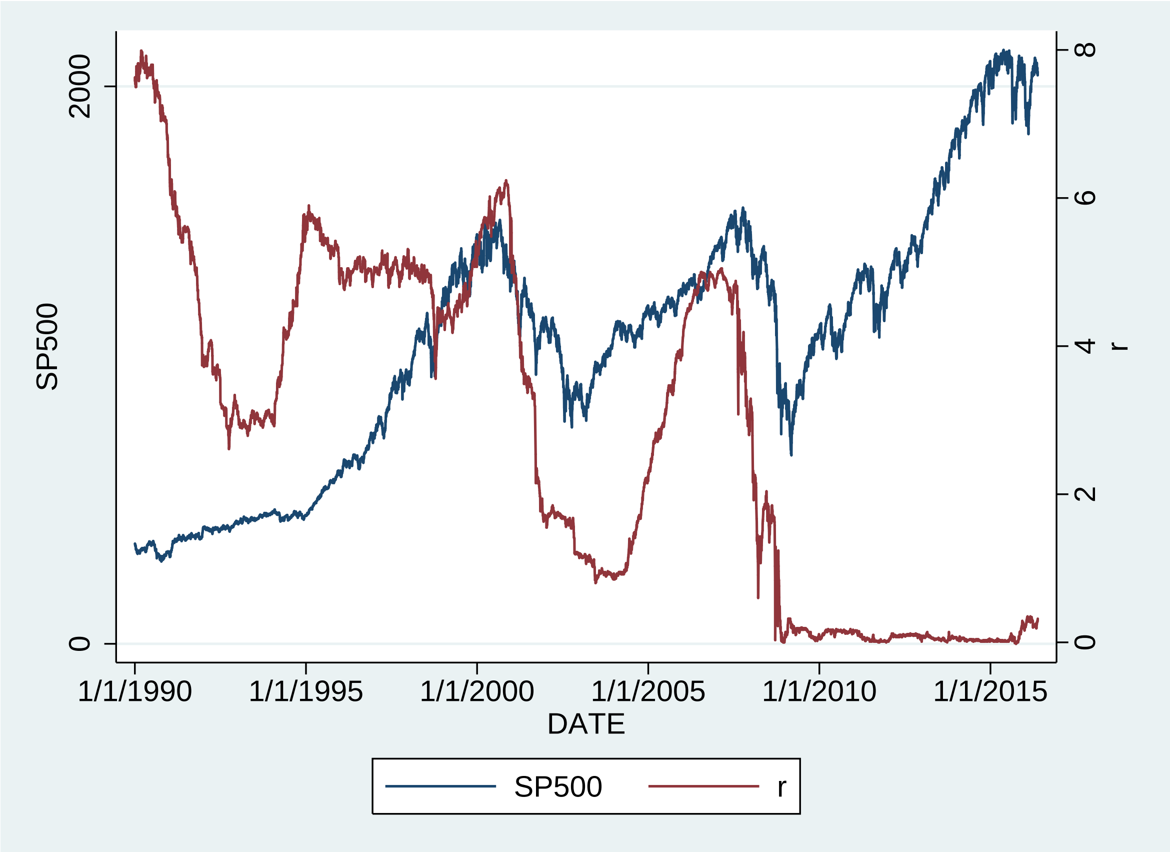


Table 1: Descriptive Statistics of RV, VIX^2 and VRP from 02 January 1990 to 20 May 2016. We compute the daily equity risk premium with $ERP_t = R_t - r_t/252$ where R_t is the daily percentage returns of S&P 500 and r_t is the three-month Treasury bill rate. The variance risk premium is $VRP_t = RV_t - VIX_t^2$ where RV_t calculated by the daily percentage returns of S&P 500 over 21-day windows at day t; VIX_t^2 is daily squared VIX index divided by 12 as one-month horizon at time t.

	R_t	r_t	Daily ERP_t	RV_t	VIX_t^2	VRP_t
Mean	0.02645	2.8822	0.01501	27.0496	37.8532	-10.8647
Median	0.05307	3.0600	0.04220	14.5692	26.9101	-10.8003
Minimum	-9.4695	-0.0200	-9.4704	1.8478	7.2230	-272.4370
Maximum	10.9572	7.990	10.95624	590.7901	544.8616	470.7357
Std	1.1342	2.2922	1.1342	47.9894	38.7397	34.4880
Skewness	-0.2377	0.1076	-0.23128	6.9159	4.9105	6.0733
Kurtosis	11.5721	1.7034	11.5688	64.0032	40.9442	78.7593

Figure 2: VRP **from 02 January 1990 to 20 May 2016.** We defined the variance risk premium as $VRP_t = RV_t - VIX_t^2$ where RV_t calculated by the daily percentage returns of S&P 500 over 21-day windows at day t; VIX_t^2 is daily squared VIX index divided by 12 as one-month horizon at time t.

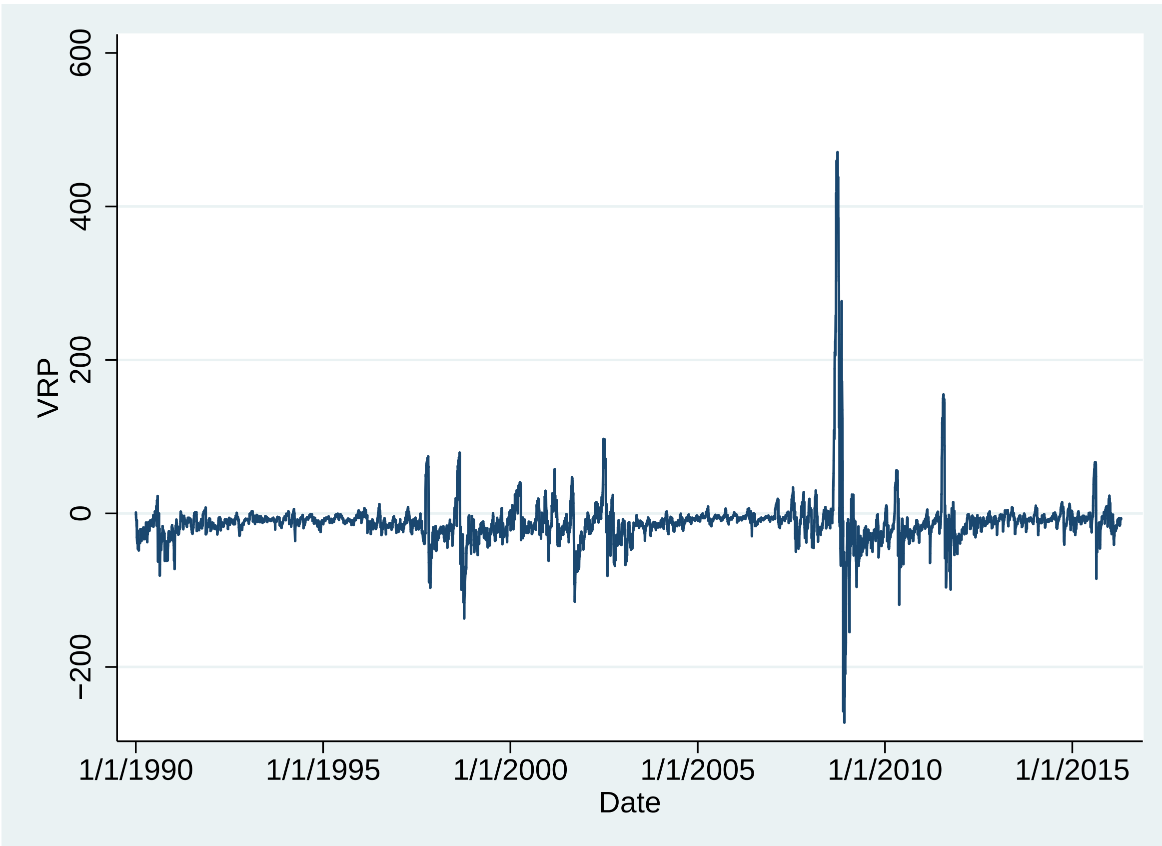


Figure 3: RV and VIX^2 from 02 January 1990 to 20 May 2016. We calculate RV_t by using the daily percentage returns of S&P 500 over 21-day windows at day t; VIX_t^2 is daily squared VIX index divided by 12 as one-month horizon at time t.

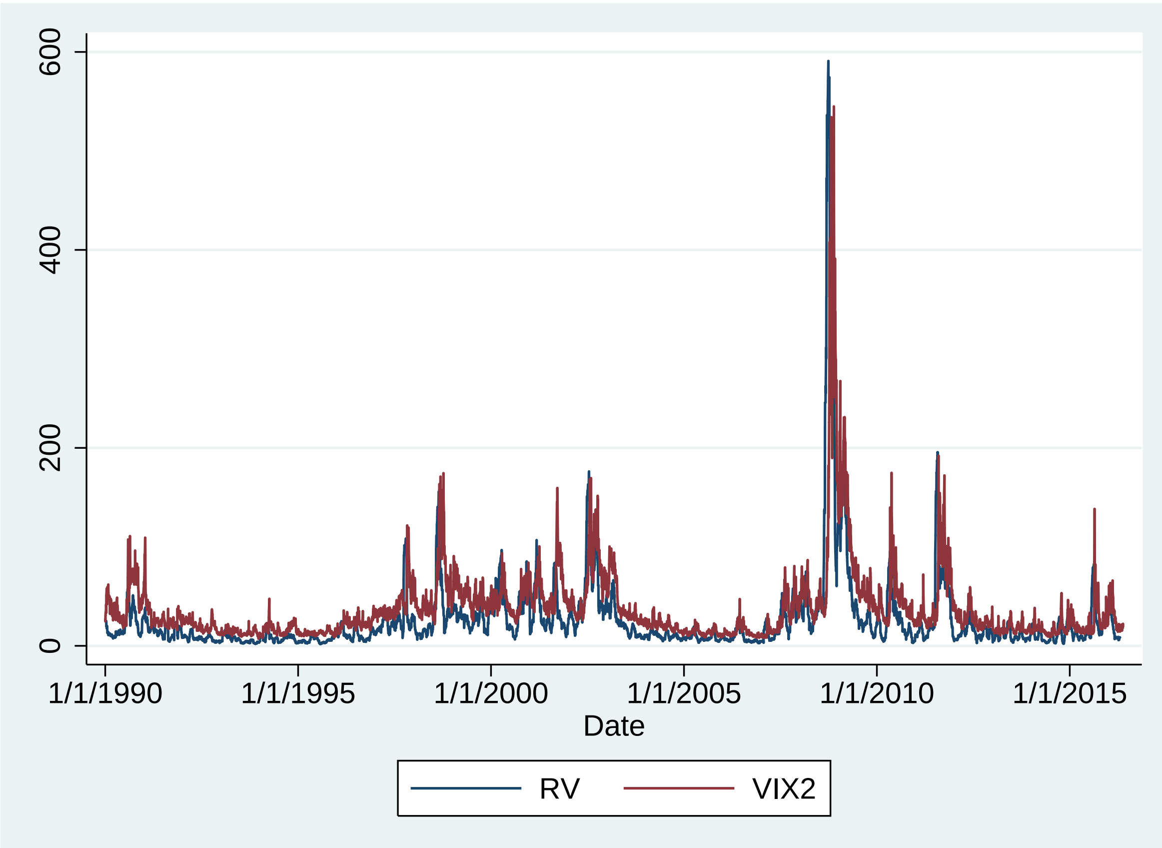


Table 2: Estimates in Broadie et al. (2007). The physical measure parameters estimated by Eraker et al. (2003) with sample (1980 to 1999). The risk-neutral parameters estimated by options on S&P 500 futures with sample (1987 to 2003). The parameter values correspond to daily percentage returns.

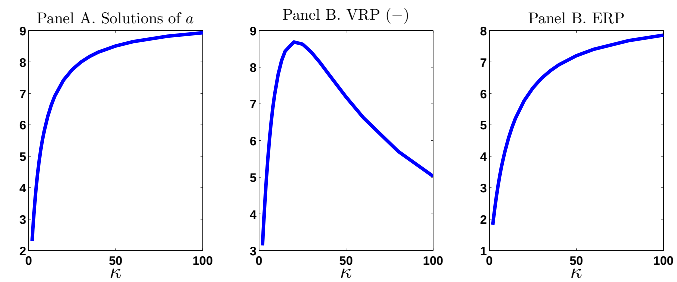
Model	κ	θ	σ_V	ρ	λ	μ_S	σ_S	μ_V	$\kappa^{\mathbb{Q}}$	$\mu_S^{\mathbb{Q}}$	$\mu_V^{\mathbb{Q}}$
SV	0.023	0.90	0.14	-0.40					0.028		
SVJ	0.013	0.81	0.10	-0.47	0.006	-2.59	4.07		0.023	-9.97	
SVCJ	0.026	0.54	0.08	-0.48	0.006	-2.63	2.89	1.48	0.056	-6.58	10.81

Table 3: Model-implied Variance Risk Premium in Broadie et al. (2007). We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its VRP data counterpart. We report the mean of the VRP from 1990 to 1999 which is the overlapping period between Eraker et al. (2003) and Broadie et al. (2007) after VIX lunched.

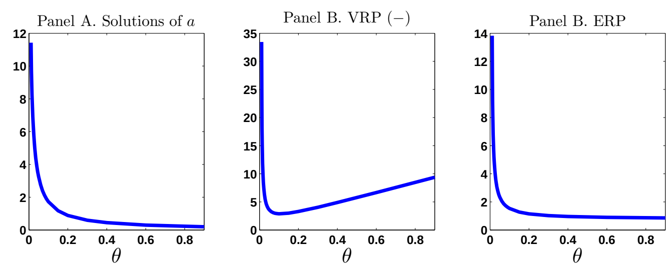
Model	$E[VRP^{SV}]$	VRP^{PJ}	VRP^{VJ}	Model-based $E[VRP]$	$E[VRP]$	Explanation rate
SV	0.8241	0	0	0.8241	-14.5495	5.66%
SVJ	1.5341	-11.6793	0	-10.1452	-14.5495	69.83%
SVCJ	3.8162	-4.5838	-8.3754	-9.1430	-14.5495	62.84%

Figure 4: The effects of the volatility parameters (I). We set the baseline model parameters as $\kappa = 15, \theta = 0.02, \sigma_V = 0.5, \rho = -0.9, \lambda = 2, \mu_S = -0.01, \sigma_S = 0.02, \mu_V = 0.06, r = 0.008, \beta = 0.001, \gamma = 2.5, \psi = 2$ and $E[V] = 0.02$. The baseline average $ERP = 5.2023$, $VRP = -15.3317$ and. The effects of a parameter on the solutions of a and the VRP are only against the corresponding parameter and other parameters keep the baseline value. $(-)$ means the real value is negative but we present it's absolute value.

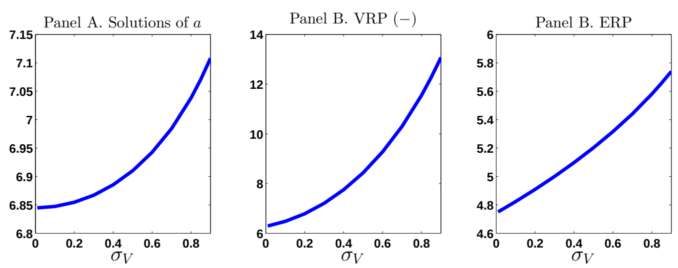
(a) The effects of κ



(b) The effects of θ



(c) The effects of σ_V



(d) The effects of ρ

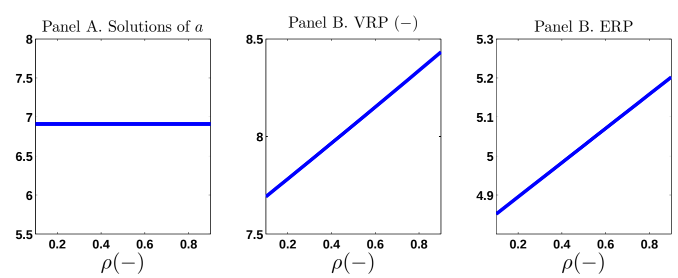
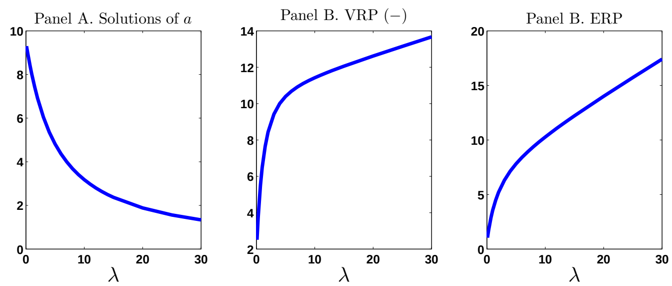
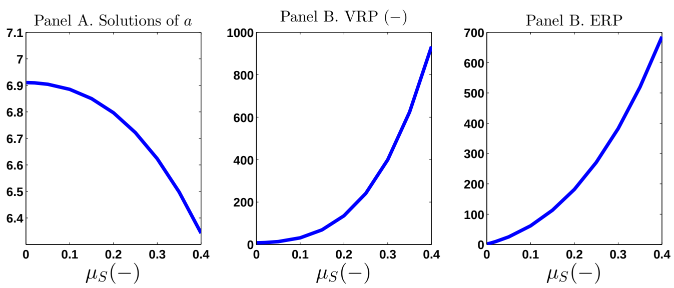


Figure 5: The effects of the jump parameters (I). We set the baseline model parameters as $\kappa = 15, \theta = 0.02, \sigma_V = 0.5, \rho = -0.9, \lambda = 2, \mu_S = -0.01, \sigma_S = 0.02, \mu_V = 0.06, r = 0.008, \beta = 0.001, \gamma = 2.5, \psi = 2$ and $E[V] = 0.02$. The baseline average $ERP = 5.2023$, $VRP = -8.4532$ and $a = 6.9193$. The effects of a parameter on the solutions of a and the VRP are only against the corresponding parameter and other parameters keep the baseline value. $(-)$ means the real value is negative but we present it's absolute value.

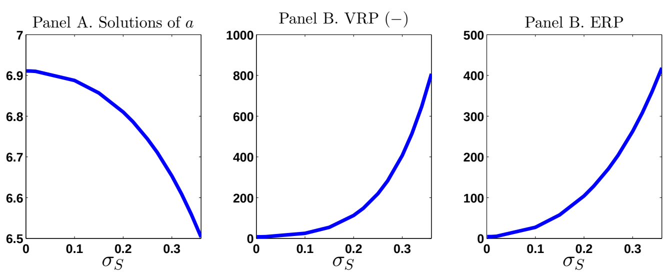
(a) The effects of λ



(b) The effects of μ_S



(c) The effects of σ_S



(d) The effects of μ_V

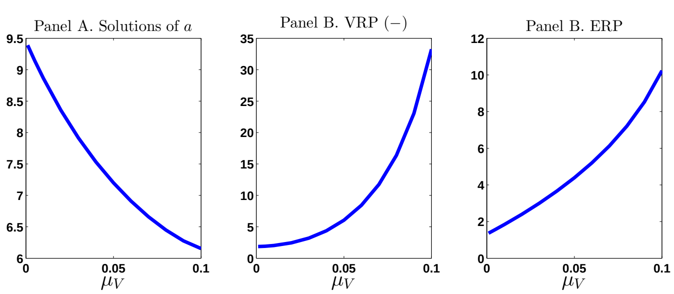
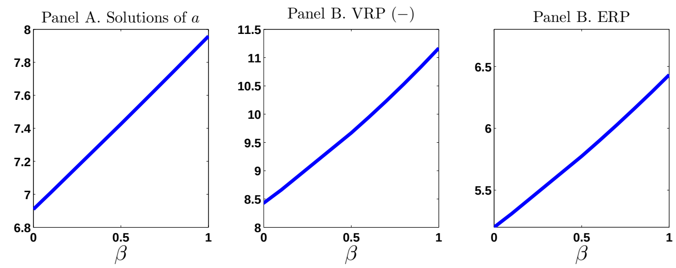
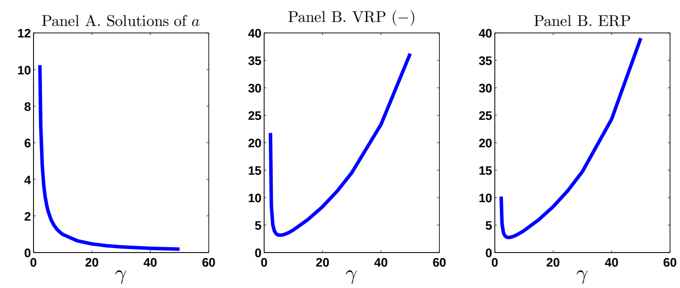


Figure 6: The effects of the preferences parameters (I). We set the baseline model parameters as $\kappa = 15, \theta = 0.02, \sigma_V = 0.5, \rho = -0.9, \lambda = 2, \mu_S = -0.01, \sigma_S = 0.02, \mu_V = 0.06, r = 0.008, \beta = 0.001, \gamma = 2.5, \psi = 2$ and $E[V] = 0.02$. The baseline average $ERP = 5.2023$, $VRP = -8.4532$ and $a = 6.9193$. The effects of a parameter on the solutions of a and the VRP are only against the corresponding parameter and other parameters keep the baseline value. $(-)$ means the real value is negative but we present it's absolute value.

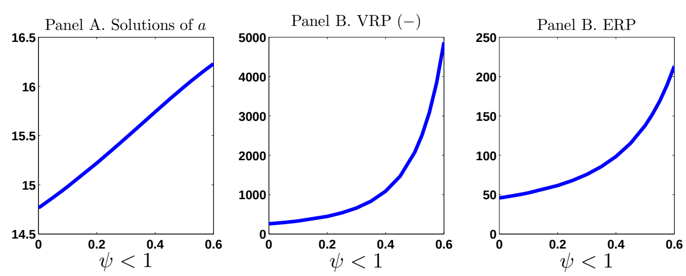
(a) The effects of β



(b) The effects of $\gamma > 2$



(c) The effects of $\psi < 1$



(d) The effects of $\psi > 1$

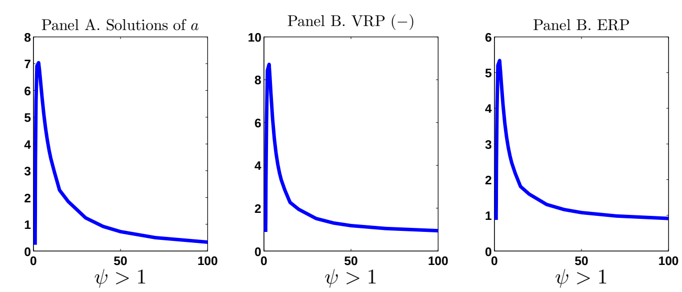
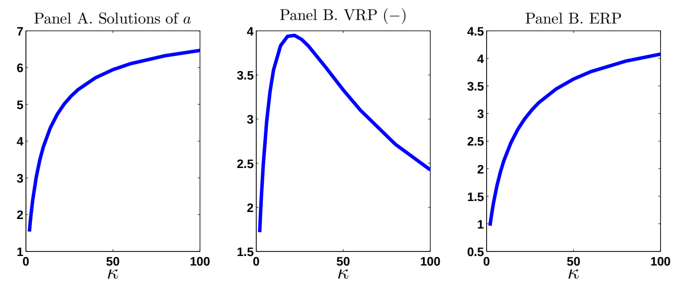
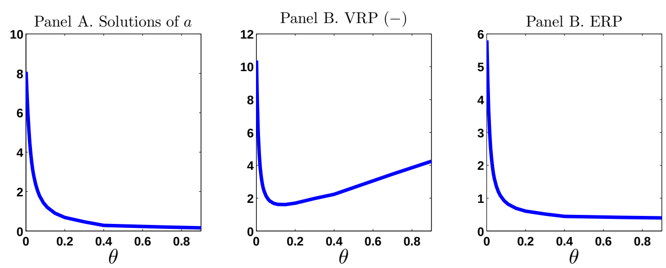


Figure 7: The effects of the volatility parameters (II). We set the baseline model parameters as $\kappa = 15, \theta = 0.02, \sigma_V = 0.5, \rho = -0.9, \lambda = 2, \mu_S = -0.01, \sigma_S = 0.02, \mu_V = 0.06, r = 0.008, \beta = 0.001, \gamma = 1.1, \psi = 0.3$ and $E[V] = 0.02$. The baseline average $ERP = 2.5359$, $VRP = -3.8701$ and $a = 4.4609$. The effects of a parameter on the solutions of a and the VRP are only against the corresponding parameter and other parameters keep the baseline value. $(-)$ means the real value is negative but we present it's absolute value.

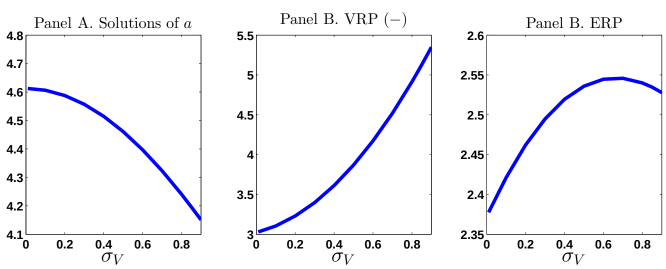
(a) The effects of κ



(b) The effects of θ



(c) The effects of σ_V



(d) The effects of ρ

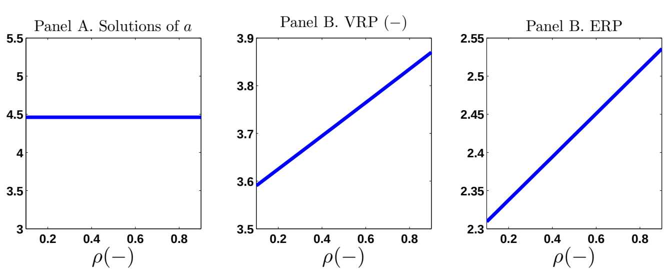
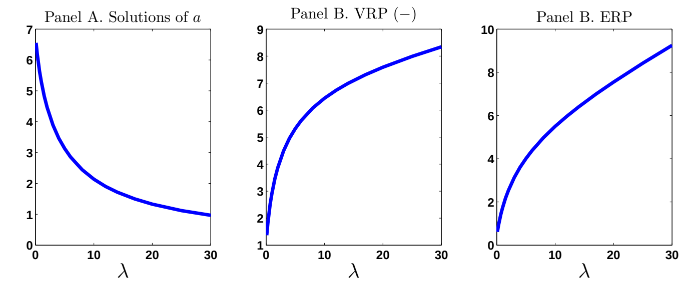
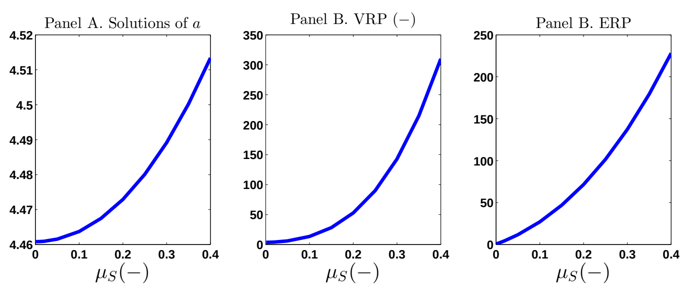


Figure 8: The effects of the jump parameters (II). We set the baseline model parameters as $\kappa = 15, \theta = 0.02, \sigma_V = 0.5, \rho = -0.9, \lambda = 2, \mu_S = -0.01, \sigma_S = 0.02, \mu_V = 0.06, r = 0.008, \beta = 0.001, \gamma = 1.1, \psi = 0.3$ and $E[V] = 0.02$. The baseline average $ERP = 2.5359$, $VRP = -3.8701$ and $a = 4.4609$. The effects of a parameter on the solutions of a and the VRP are only against the corresponding parameter and other parameters keep the baseline value. $(-)$ means the real value is negative but we present it's absolute value.

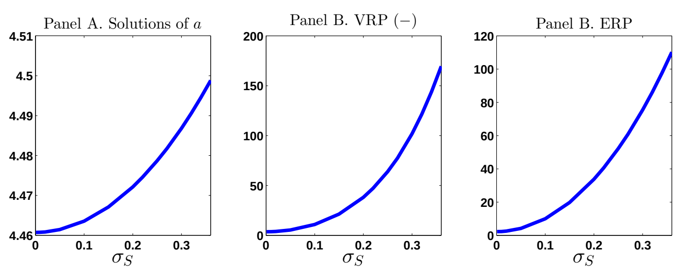
(a) The effects of λ



(b) The effects of μ_S



(c) The effects of σ_S



(d) The effects of μ_V

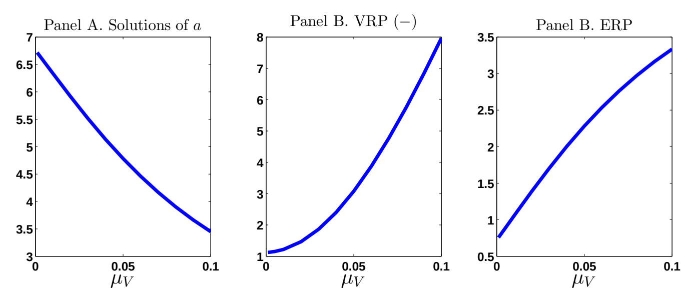
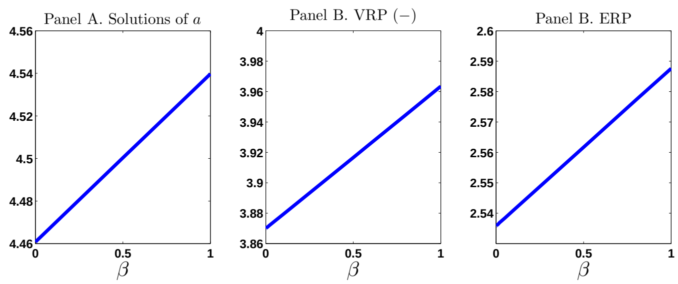
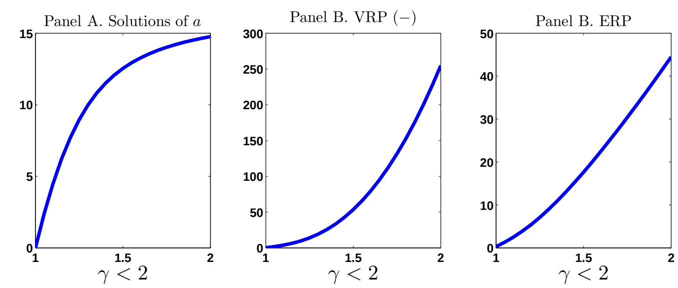


Figure 9: The effects of the preferences parameters (II). We set the baseline model parameters as $\kappa = 15, \theta = 0.02, \sigma_V = 0.5, \rho = -0.9, \lambda = 2, \mu_S = -0.01, \sigma_S = 0.02, \mu_V = 0.06, r = 0.008, \beta = 0.001, \gamma = 1.1, \psi = 0.3$ and $E[V] = 0.02$. The baseline average $ERP = 2.5359$, $VRP = -3.8701$ and $a = 4.4609$. The effects of a parameter on the solutions of a and the VRP are only against the corresponding parameter and other parameters keep the baseline value. $(-)$ means the real value is negative but we present it's absolute value.

(a) The effects of β



(b) The effects of $\gamma < 2$



(c) The effects of $\psi < 1$

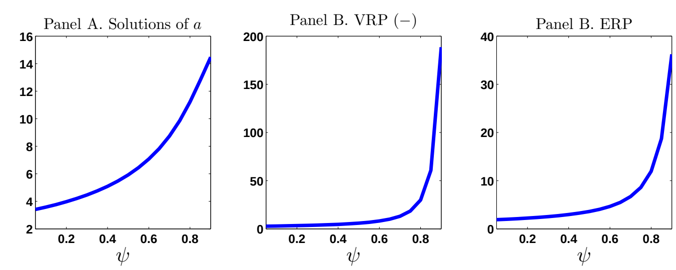


Table 4: Equilibrium Model-implied Equity Premium based on Broadie et al. (2007). We set the equilibrium model parameters as $\beta = -\ln(0.999)$, $r = 4.8512(\%)$. We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its market equity premium data counterpart. The mean of the annualized log S&P500 returns, 14.2040, is from 1990 to 1999 which is the overlapping period between Eraker et al. (2003) and Broadie et al. (2007) after VIX lunched. The risk-free rate $r = 4.8512$ is the average three-month Treasury bill over the same period, 1990 to 1999. The three-month Treasury bill data is from Thomson Reuters Datastream.

$E[ERP^{SV}]$	ERP^{PJ}	Model-implied $E[ERP]$	$E[ERP]$
	SV Model ($\gamma = 1.1, \psi = 0.84$)		
9.6275	0	9.6275	9.3528
	SVJ Model($\gamma = 1.1, \psi = 0.84$)		
9.0845	0.3885	9.4731	9.3528
	SVCJ Model ($\gamma = 1.1, \psi = 0.76$)		
5.4048	4.5802	9.9850	9.3528

Table 5: Equilibrium Model-implied Variance Risk Premium based on Broadie et al. (2007). We set the equilibrium model parameters as $\beta = -\ln(0.999)$. We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its VRP data counterpart. We report the mean of the VRP from 1990 to 1999 which is the overlapping period between Eraker et al. (2003) and Broadie et al. (2007) after VIX lunched. The risk-free rate $r = 4.8512$ is the average three-month Treasury bill over the same period, 1990 to 1999. The three-month Treasury bill data is from Thomson Reuters Datastream.

VRP_t^{SV}	VRP_t^{PJ}	VRP_t^{VJ}	Model-based VRP_t	VRP_t	Explanation rate
SV Model ($\gamma = 1.1, \psi = 0.84$)					
-2.1308	0	0	-2.1308	-14.5495	14.65%
SVJ Model ($\gamma = 1.1, \psi = 0.84$)					
-1.3123	-0.2146	0	-1.5269	-14.5495	10.49%
SVCJ Model ($\gamma = 1.1, \psi = 0.76$)					
-0.4107	-2.2695	-5.6005	-8.2807	-14.5495	56.91%

Table 6: Return predictability. This table reports empirical results from the daily predictive regressions of the equity premiums (S&P 500 log returns subtract the risk-free rate) on variance risk premiums (i.e., regression (29)). We defined the monthly ERP as $(R_t - r_t/252) \times 21$ where R_t is the daily percentage returns of S&P 500 and r_t is the three-month Treasury bill rate. The variance risk premium is $VRP_t = RV_t - VIX_t^2$ where RV_t calculated by the daily percentage returns of S&P 500 over 21-day windows at day t; VIX_t^2 is daily squared VIX index divided by 12 as one-month horizon at time t.

Periods	02 January 1990 - 20 May 2016			24 November 2010 - 20 May 2016		
	β_{pred}	t-statistic	adj. R^2 (%)	β_{pred}	t-statistic	adj. R^2 (%)
	-13.5463	-16.26	3.82	-20.8929	-8.79	5.31

Table 7: Descriptive Statistics of R_t, RV, VIX^2 and VRP from 24 November 2010 to 20 May 2016. We compute the daily equity risk premium with $ERP_t = R_t - r_t/252$ where R_t is the daily percentage returns of S&P 500 and r_t is the daily three-month Treasury bill rate. We defined the variance risk premium as $VRP_t = RV_t - VIX_t^2$ where RV_t calculated by the daily percentage returns of S&P 500 over 21-day windows at day t; VIX_t^2 is daily squared VIX index divided by 12 as one-month horizon at time t.

	R_t	r_t	Daily ERP	RV_t	VIX_t^2	VRP_t
Obs.	1381	1381	1381	1360	1360	1360
Mean	0.0400	0.07301	0.03974	19.9787	28.3325	-8.3538
Median	0.0549	0.0500	0.0546	12.4189	21.2934	-8.9812
Minimum	-6.8958	-0.0200	-6.8960	2.1505	8.8752	-99.1070
Maximum	4.6317	0.3500	4.6316	195.5504	192	155.0615
Std	0.9725	.07027	0.9725	25.2038	22.6837	22.7515
Skewness	-0.4732	1.8761	-0.4731	3.9569	3.0577	3.3699
Kurtosis	7.5650	6.4181	7.5649	22.4983	14.1150	24.6868

Table 8: S&P 500 Parameter Estimates. Parameter estimates for the S&P 500 index data, 24 November 2010 to 20 May 2016. The estimates correspond to percentage changes in the index value. We discard the first 10,000 runs as "burn-in" period and use the last 100,000 iterations in MCMC simulations to estimate model parameters. For each parameter to be estimated, we use the same priors as in Eraker et al. (2003). Specifically, we take the mean of the posterior distribution as parameter estimate and the standard deviation of the posterior as standard error in parentheses.

Parameters	κ	θ	σ_V	ρ	λ	μ_S	σ_S	μ_V
SV	0.04416	0.9298	0.2357	-0.8686				
	(0.006846)	(0.116)	(0.01692)	(0.03013)				
SVJ	0.04371	0.928	0.2312	-0.8714	0.004833	-0.3886	2.018	
	(0.006993)	(0.1187)	(0.01549)	(0.03114)	(0.004345)	(2.665)	(0.5229)	
SVCJ	0.06019	0.6983	0.2169	-0.9128	0.006985	-1.394	1.948	2.213
	(0.008618)	(0.08918)	(0.01673)	(0.02666)	(0.003284)	(1.222)	(0.4666)	(0.4543)

Table 9: Annualized S&P 500 Parameter Estimates. Annual decimals are converted by scaling: 252κ, $252\theta/10000$, $252\sigma_V/100$, 252λ, $\mu_S/100$, $\sigma_S/100$ and $252\mu_V/10000$ except for ρ.

Parameters	κ	θ	σ_V	ρ	λ	μ_S	σ_S	μ_V
SV	11.1283	0.02343	0.5940	-0.8686				
SVJ	11.0149	0.02339	0.5826	-0.8714	1.2179	-0.003886	0.02018	
SVCJ	15.1679	0.01760	0.5469	-0.9128	1.7602	-0.01394	0.01948	0.05577

Table 10: Equilibrium Model-implied Equity Premium. We set the equilibrium model parameters as $\beta = -\ln(0.999)$, $r = 0.0730(\%)$. We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its market equity premium data counterpart. The mean of the annualized log S&P500 returns, 10.0880, is from 24 November 2010 to 20 May 2016. The risk-free rate $r = 0.0730$ is the average three-month Treasury bill over the same period, 24 November 2010 to 20 May 2016. The three-month Treasury bill data is from Thomson Reuters Datastream.

	Panel A: Simulated results.		
$E[ERP^{SV}]$	ERP^{PJ}	Model-implied $E[ERP]$	$E[ERP]$
	SV Model ($\gamma = 1.1, \psi = 0.42$)		
0	0	10.6216	10.0150
	SVJ Model ($\gamma = 1.1, \psi = 0.42$)		
10.5046	0.0566	10.5612	10.0150
	SVCJ Model ($\gamma = 1.1, \psi = 0.37$)		
9.4221	1.2624	10.6845	10.0150
	Panel B: MCMC results		
$E[ERP^{SV}]$	ERP^{PJ}	Model-implied $E[ERP]$	$E[ERP]$
	SV Model ($\gamma = 1.1, \psi = 0.42$)		
0	0	9.7402	10.0150
	SVJ Model ($\gamma = 1.1, \psi = 0.42$)		
9.5668	0.0566	9.6234	10.0150
	SVCJ Model ($\gamma = 1.1, \psi = 0.37$)		
8.8124	1.2624	10.0749	10.0150

Table 11: Annualized model-implied risk-neutral parameters. We set the equilibrium model parameters as $\beta = -\ln(0.999)$ and $r = 0.0730(\%)$ which is the average three-month Treasury bill over the same period, 24 November 2010 to 20 May 2016. The three-month Treasury bill data is from Thomson Reuters Datastream.

Model	β	γ	ψ	a	$\kappa^{\mathbb{Q}}$	λ^{Q}	$\mu_S^{\mathbb{Q}}$	$\mu_V^{\mathbb{Q}}$
SV	-ln(0.999)	1.1	0.42	6.6251	8.2160			
SVJ	-ln(0.999)	1.1	0.42	6.6542	8.1977	1.2234	-0.0043	
SVCJ	-ln(0.999)	1.1	0.37	13.3679	11.9858	2.6061	-0.014	0.0813

Table 12: Model-implied risk-neutral parameters. The estimates correspond to daily percentage changes in the index value.

Model	$\kappa^{\mathbb{Q}}$	λ^{Q}	$\mu_S^{\mathbb{Q}}$	$\mu_V^{\mathbb{Q}}$
SV	0.0326			
SVJ	0.0325	0.0049	-0.4334	
SVCJ	0.0500	0.0131	-1.4357	4.0925

Figure 10: The solution of a against γ based on SVCJ. This figure shows the solution of a against γ. The SVCJ model parameters are given in Table 9 and 11.

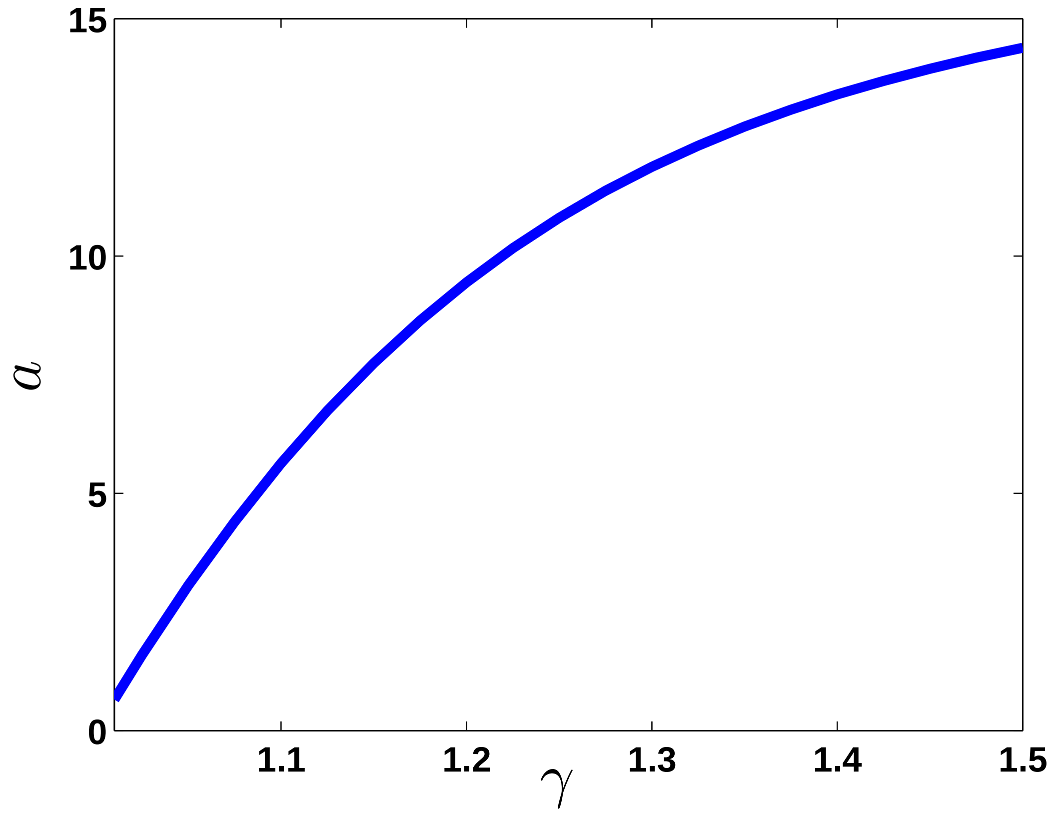


Table 13: Model-implied Variance Risk Premium. We set the equilibrium model parameters as $\beta = -\ln(0.999)$, $r = 0.0730(\%)$. We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its market equity premium data counterpart. The MCMC results use the latent variance V_t from the MCMC estimation of the physical parameters. The mean of the annualized log S&P500 returns, 10.0880, is from 24 November 2010 to 20 May 2016. The risk-free rate $r = 0.0730$ is the average three-month Treasury bill over the same period, 24 November 2010 to 20 May 2016. The three-month Treasury bill data is from Thomson Reuters Datastream.

		Panel A: Simulated results.			
$E[VRP^{SV}]$	VRP^{PJ}	VRP^{VJ}	Model-based $E[VRP]$	$E[VRP]$	Explanation rate
		SV Model ($\gamma = 1.1, \psi = 0.42$)			
-1.9146	0	0	-1.9146	-8.3538	22.92%
		SVJ Model ($\gamma = 1.1, \psi = 0.42$)			
-1.8489	-0.0057	0	-1.8546	-8.3538	20.20%
		SVCJ Model ($\gamma = 1.1, \psi = 0.37$)			
-0.9757	-0.4301	-2.9110	-4.3169	-8.3538	51.68%
		Panel B: MCMC results.			
$E[VRP^{SV}]$	VRP^{PJ}	VRP^{VJ}	Model-based $E[VRP]$	$E[VRP]$	Explanation rate
		SV Model ($\gamma = 1.1, \psi = 0.42$)			
-1.7970	0	0	-1.7970	-8.3538	21.51%
		SVJ Model ($\gamma = 1.1, \psi = 0.42$)			
-1.7264	-0.0057	0	-1.7321	-8.3538	20.73%
		SVCJ Model ($\gamma = 1.1, \psi = 0.37$)			
-0.9297	-0.4301	-2.9110	-4.2709	-8.3538	51.13%

Table 14: Variance risk premium based on SVCJ model. We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its full-sample data counterpart. The MCMC results use the latent variance V_t from the MCMC estimation of the physical parameters.

	Data	Model	Model
Type of V_t		MCMC	Simulations
$mean(VRP)$	-8.3538	-4.2709	-4.3158
$std(VRP)$	22.7515	1.4304	0.6941
$skew(VRP)$	3.3699	-3.5938	-2.6488
$kurt(VRP)$	24.6868	20.2760	14.3162
$mean(RV)$	19.9787	20.1776	20.8975
$std(RV)$	25.2038	12.5286	11.1274
$skew(RV)$	3.9569	3.5938	2.6438
$kurt(RV)$	22.4983	20.2760	14.2903
$mean(IV)$	28.3325	24.4485	25.2132
$std(IV)$	22.6837	13.3097	11.8276
$skew(IV)$	3.0577	3.5938	2.6488
$kurt(IV)$	14.1150	20.2760	14.3162

Table 15: Model-implied Equity and Variance Risk Premiums in Drechsler (2013). Data estimates are reported for samples starting in 1930 and ending in 2009.

Model-implied $E[ERP]$	Data $E[ERP]$	Explanation rate
6.29	4.93	127.59%
Model-based $E[VRP]$	Data $E[VRP]$	Explanation rate
-8.17	-10.55	77.44%

Table 16: Model-implied Equity and Variance Risk Premiums based on SVCJ model ($\gamma = 1.1, \psi = 0.52$). We set the equilibrium model parameters as $\beta = -\ln(0.999)$, $r = 0.0730(\%)$. We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its market equity premium data counterpart. The MCMC results use the latent variance V_t from the MCMC estimation of the physical parameters. The mean of the annualized log S&P500 returns, 10.0880%, is from 24 November 2010 to 20 May 2016. The risk-free rate $r = 0.0730\%$ is the average three-month Treasury bill over the same period, 24 November 2010 to 20 May 2016. The three-month Treasury bill data is from Thomson Reuters Datastream.

		Panel A: ERP with SVCJ Model ($\gamma = 1.1, \psi = 0.52$).		
$E[ERP^{SV}]$	ERP^{PJ}	Model-implied $E[ERP]$	$E[ERP]$	Explanation rate
		(a) Simulated results.		
11.0897	1.7310	12.8207	10.0150	128.01%
		(b): MCMC results		
10.3804	1.7310	12.1114	10.0150	120.93%

		Panel B: VRP with SVCJ Model ($\gamma = 1.1, \psi = 0.52$).			
$E[VRP^{SV}]$	VRP^{PJ}	VRP^{VJ}	Model-based $E[VRP]$	$E[VRP]$	Explanation rate
		(a) Simulated results.			
-1.2126	-0.5927	-4.4013	-6.2066	-8.3538	74.30%
		(b) MCMC results.			
-1.1561	-0.5927	-4.4013	-6.1501	-8.3538	73.62%

Table 17: Model-implied Equity and Variance Risk Premiums based on SVCJ model ($\gamma = 1.1, \psi = 0.61$). We set the equilibrium model parameters as $\beta = -\ln(0.999)$, $r = 0.0730(\%)$. We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its market equity premium data counterpart. The MCMC results use the latent variance V_t from the MCMC estimation of the physical parameters. The mean of the annualized log S&P500 returns, 10.0880%, is from 24 November 2010 to 20 May 2016. The risk-free rate $r = 0.0730\%$ is the average three-month Treasury bill over the same period, 24 November 2010 to 20 May 2016. The three-month Treasury bill data is from Thomson Reuters Datastream.

		Panel A: ERP with SVCJ Model ($\gamma = 1.1, \psi = 0.61$).		
$E[ERP^{SV}]$		ERP^{PJ}	Model-implied $E[ERP]$	$E[ERP]$
		(a) Simulated results.		
12.5428		2.2474	14.7902	10.0150
		(b): MCMC results		
11.7424		2.2474	13.9898	10.0150
		Panel B: VRP with SVCJ Model ($\gamma = 1.1, \psi = 0.61$).		
$E[VRP^{SV}]$	VRP^{PJ}	VRP^{VJ}	Model-based $E[VRP]$	$E[VRP]$
		(a) Simulated results.		
-1.4225	-0.7718	-6.2600	-8.4543	-8.3538
		(b) MCMC results.		
-1.3563	-0.7718	-6.2600	-8.3881	-8.3538

Table 18: Variance risk premium based on SVCJ model ($\gamma = 1.1, \psi = 0.61$). We report simulated statistics based on 10,000 simulations, with each statistic calculated using a sample size equal to its full-sample data counterpart. The MCMC results use the latent variance V_t from the MCMC estimation of the physical parameters.

	Data	Model	Model
Type of V_t		MCMC	Simulations
$mean(VRP)$	-8.3538	-8.3881	-8.4543
$std(VRP)$	22.7515	1.1411	1.0154
$skew(VRP)$	3.3699	-3.5938	-2.6496
$kurt(VRP)$	24.6868	20.2760	14.3113
$mean(RV)$	19.9787	20.1776	20.9044
$std(RV)$	25.2038	12.5286	11.1483
$skew(RV)$	3.9569	3.5938	2.6496
$kurt(RV)$	22.4983	20.2760	14.3113
$mean(IV)$	28.3325	28.5656	29.3587
$std(IV)$	22.6837	13.6698	12.1637
$skew(IV)$	3.0577	3.5938	2.6496
$kurt(IV)$	14.1150	20.2760	14.3113

Table 19: A comparison to Eraker et al. (2003). We present the replicated estimates by using same MCMC methodology and the sample in Eraker et al. (2003) with our MCMC codes based on OpenBUGS, which will be used in Section 4. We discard the first 10,000 runs as "burn-in"period and use the last 100,000 iterations in MCMC simulations to estimate model parameters. For each parameter to be estimated, we use the same priors as in Eraker et al. (2003). Specifically, we take the mean of the posterior distribution as parameter estimate and the standard deviation of the posterior as standard error in parentheses.

Model		κ	θ	σ_V	ρ	λ	μ_S	σ_S
SV	Eraker et al. (2003)	0.0231	0.9052	0.1434	-0.3974			
		(0.0068)	(0.1077)	(0.0128)	(0.0516)			
	Our estimates	0.02889	0.9002	0.1544	-0.407			
		(0.00524)	(0.07645)	(0.01185)	(0.0502)			
SVJ	Eraker et al. (2003)	0.0128	0.8136	0.0954	-0.4668	0.0060	-2.5862	4.0720
		(0.0039)	(0.1244)	(0.0104)	(0.057)	(0.0021)	(1.3034)	(1.7210)
	Our estimates	0.01979	0.8534	0.1184	-0.479	0.005394	-2.978	4.046
		(0.004218)	(0.08437)	(0.0106)	(0.05199)	(0.00201)	(1.304)	(0.7853)

Table 20: A comparison to Yun (2011). We present the replicated estimates by using same MCMC methodology and the sample in Yun (2011) with our MCMC codes based on OpenBUGS, which will be used in Section 4. We discard the first 10,000 runs as "burn-in"period and use the last 100,000 iterations in MCMC simulations to estimate model parameters. For each parameter to be estimated, we use the same priors as in Eraker et al. (2003). Specifically, we take the mean of the posterior distribution as parameter estimate and the standard deviation of the posterior as standard error in parentheses.

Model		κ	θ	σ_V	ρ	λ	μ_S	σ_S	μ_V
SV	Yun (2011)	0.0288	1.0210	0.1813	-0.4960				
		(0.0052)	(0.1133)	(0.0128)	(0.0528)				
	Our estimates	0.02796	1.051	0.1811	-0.4904				
		(0.005127)	(0.1168)	(0.01303)	(0.05238)				
SVJ	Yun (2011)	0.0204	0.9856	0.1450	-0.5912	0.0066	-3.3490	3.7040	
		(0.0048)	(0.1340)	(0.0132)	(0.0511)	(0.0025)	(1.4190)	(0.7329)	
	Our estimates	0.0184	1.055	0.1423	-0.5838	0.006439	-3.389	3.976	
		(0.0046644)	(0.1548)	(0.01364)	(0.05226)	(0.002371)	(1.461)	(0.8397)	
SVCJ	Yun (2011)	0.0362	0.7026	0.1459	-0.5712	0.0049	-4.6460	2.7100	2.1040
		(0.0060)	(0.0711)	(0.0131)	(0.0500)	(0.0017)	(1.1250)	(0.8013)	(0.4025)
	Our estimates	0.03495	0.7216	0.1451	-0.5611	0.005264	-4.149	2.514	2.044
		(0.005897)	(0.07657)	(0.01199)	(0.0582)	(0.001824)	(1.18)	(0.6005)	(0.3855)