A Systemic Change of Measure from Central Clearing[†]

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Abstract

The default of a Nasdaq Clearing Commodities member in 2018 and the ongoing COVID-19 pandemic underscore the importance of a holistic risk management practices by central counterparties to forestall the loss spread throughout the entire system. This paper investigates the systemic impact of central clearing in the presence of counterparty credit risk by utilizing a statistical model of a financial network in which edge weights represent the sensitivities of one participant's failure to its counterparties' default likelihood. The reduced-form model specifies the mechanism of systemic risk concentration under central clearing in that the introduction of a central counterparty into a market redistributes the probability mass of the systemic failure from the center of the distribution into its tail. Numerical illustrations with a novel importance sampling technique shed light on a policy-oriented implication towards regulating the adverse dependence between risk concentration under central clearing and the resiliency of the financial system via proper margin policies at a collective level.

Keywords: Central Clearing; Systemic Risk; Measure Change; Tail Risk Concentration; Monte Carlo Simulation; Margin Policy; Goal ProgrammingJEL Codes: C15, C46, C61, G17, G23

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1. Introduction

The origins of central counterparty clearing (CCC) can be traced back to the beginning of the modern banking system in the 18th century to exchange checks and coins. Such a method of clearing was, of course, prone to error and abuse. However, around 1770, bank clerks in London began to meet at Five Bells, a tavern on Lombard Street, to exchange all their checks and settle their balances in cash (Nevin & Davis, 1970). In the 19th century, CCC arrangements became popular in produce and stock exchanges in the U.S. and Europe (Emery, 1896). In today's financial markets, CCC arrangements are used to settle transactions for various derivatives, securities and money markets. They operate by inserting a central counterparty (CCP) between every pair of market participant.

Systemic risk often refers to the likelihood of observing a cascade of failures in the financial system. Central clearing has been introduced to mitigate such a risk by isolating financial institutions from their counterparty credit risk. As a result, recent financial regulations have pushed for the introduction of CCPs into various markets to reduce complexity and enhance transparency and stability. Nevertheless, by becoming a nexus of netting and the sole absorbent of default impacts, the default of the CCP may cause uncontrollable credit risk propagation through the centralized network. In this regard, the default of a Nasdaq Clearing Commodities member in 2018 and the ongoing COVID-19 market turbulence underscore the importance of a holistic risk management practices by CCP's to forestall the loss spread at the systemic level.

This paper investigates the trade-off between the system-wide cost and benefit of central clearing when considering counterparty default risk. While CCPs are intended to reduce systemic risk in the financial system, the centralized approach inherent in CCC arrangements can also lead to a concentration of risk for financial markets. In other words, CCPs are systemically important interconnectors in the financial system as their operations transform systemic risk. It is noteworthy that the introduction of a CCP into a market redistributes the probability mass of system-wide defaults from the center of the distribution into its tail. In other words, the centrality of a CCP leads to concentration of risk accompanied by contagion effects based on mutually strengthening interactions with the source of systematic risk. We use a probability measure change technique to drive the distributional changes of aggregate default rate induced by the introduction of the CCP and initial margin schemes. Trade-offs often appear between a CCP's benefit from margin collection, isolating clearing members (CMs) from other CMs' defaults and the cost of joining the central clearing. For example, reducing the first moment of the total number of defaults under a central clearing arrangement often has an adverse effect on its tail risk. We provide meaningful implications by showing how the design of margin policy can mitigate CCP-driven systemic risk concentration.

Duffie & Zhu (2011) pioneered the research on central clearing and its netting efficiency. They show that aggregate expected exposure may arise from central clearing as a trade-off result in a model in which multilateral netting through the CCP deprives netting opportunities across non-centrally cleared contracts. Research from Cont & Kokholm (2014) relax the strong assumptions in Duffie & Zhu (2011) and derive more plausible conditions of efficient multilateral netting. Another study by Menkveld (2017) employs the framework of Duffie & Zhu (2011), and turns the focus onto the total exposure of the CCP against CMs to investigate how the concentration of positions can cause distress to the CCP. Additionally, Garratt & Zimmerman (2020) look into the mean-variance of the expected exposure under financial network structures, revealing that there is a strict subset relation among networks.

This strand of literature recognizes the vital role of CCPs in mitigating and managing systemic risk; however, there has been no serious investigation on the likelihood of systemic default clustering under central clearing. Our study attempts to fill this gap by specifying a stochastic default intensity model. Our proposed model framework sheds light on the impact of introducing central clearing into the financial market of participants in the presence of counterparty default risk. Specifically, we posit a network in which edge weights represent the exposure that is the total potential losses incurred from counterparties' defaults without netting. Under a bilateral arrangement, we assume that those exposures determine the sensitivities of one counterparty's failure to another participant's default. In turn, the (potentially dynamic) interdependency drives the contagion in this network with defaults jointly correlated via an intensity-based model. It is worthwhile to mention that the general description of the bilateral model provides sufficient details so that a meaningful central clearing entity (i.e., CCC) can be introduced in the system. Accordingly, we apply a well-defined intensity-based default model under central clearing. The procedure entails a change of measure under which one can appropriately address systemic contribution of CCP in the system as a whole, and the probability measure change is driven by the likelihood ratio of the modified measure under CCC divided by the original probability measure under the bilateral arrangement. We highlight that the bilateral and CCC models are consistent in a way that direct comparisons can be analyzed in the context of risk concentration.

The central quantity of our investigation is the distribution of the aggregate default rate, defined as the number of defaults divided by the total number of participants in the system. The statistical properties of the total number of defaults for a fixed time horizon can be deduced via Monte Carlo (MC) simulation in the setting of bilateral and centrally cleared markets with the application of a novel importance sampling technique for tail probability estimation. Our findings indicate that introducing a CCP to a bilateral counterparty arrangement has non-trivial effects on the tail distribution of the aggregate default rate.

Moreover, this tail distribution shows non-trivial dependence on the margin policy along with the protocol for CCP default proceedings. Our intensity-based network model allows the framework under which trade-offs arise before and after the introduction of central clearing. In this context, we further investigate the optimal risk management of the CCP in totalizing and minimizing systemic risk in order to provide meaningful implication by showing how we can mitigate the CCP-driven systemic risk concentration.

Our main findings show that systemic trade-offs may appear under the central clearing scheme as reducing the first moment of the aggregate default rate often has an adverse effect on the tail risk and vice versa. Central clearing is more likely to decrease the mean of the aggregate default rate if the CCP protects survivors against counterparties' defaults. However, the delayed default feedback may cause the clustering of failures once the CCP defaults. As a price for lowering the average default rate in regular time, a systemic risk concentration arises. Given a reasonable choice of preference parameter sets governing the policymaker's priorities over different goals to achieve, constant and/or countercyclical margin schemes generally dominate a procyclical margin scheme. Moreover, as the survivor probability of the CCP is backed better by the constant and countercyclical margin schemes, there remain less reasons to adhere to procyclical margin schemes.

2. Preliminaries

Consider a measure space (Ω, \mathcal{F}) equipped with a right continuous and complete information filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$. We suppose that there are $n \in N$ participants in a financial market. Let τ_i represents the time of default of the *i*th participant. We further assume $\{\tau_i\}_{i=1}^n$ are almost surely distinct and totally inaccessible \mathbb{F} -stopping times.¹ Each participant *i* is associated with a default indicator process N^i defined by

$$N_t^i = \mathbf{1}_{\{\tau_i \le t\}},\tag{1}$$

where $t \geq 0$ and $\mathbf{1}_{\mathcal{A}}$ is the indicator of $\mathcal{A} \in \mathcal{F}$.

We take the financial regulators' point of view in that the primary consideration in our analysis is to quantify and manage systemic risk. Specifically, we search for the likelihood of an event that a certain fraction of the total population in our consideration defaults. Giesecke & Kim (2011) propose measures of systemic risk based on the number of failures of firms in the financial system.

¹Stopping time τ is totally inaccessible if $\mathbf{P}(\tau = \tau^* < \infty) = 0$ for all predictable stopping times τ^* .

They define the systemic risk as the conditional probability that a sufficiently large fraction of the total population in the financial system fails to meet their obligations. This definition of systemic risk is based on the actual failures of participants in the financial system, enabling the analyses of the impact on surviving participants' risk stemming from the defaulters' spillover effect. Accordingly, we define the system-wide default counting process given by

$$C_t = \sum_{i=1}^n N_t^i.$$
 (2)

For a given risk horizon T > 0, the central quantity of interest is the distribution of the aggregate default rate in the system given by

$$D_T = \frac{C_T}{n} \in [0, 1],$$
 (3)

where the D_T represents the fraction of the defaulting members in the entire system by time T. Notably, the right tail distribution of D_T quantifies the systemic risk, as the conditional probability that a certain fraction of participants defaults (e.g., widely accepted tail risk measures include Value-at-Risk (VaR) and Expected Shortfall (ES)). In the following sections, we derive the distributions of D_T under various financial networks and compare them to find the systemic optimal choice of the clearing scheme.

2.1. Networked market, exposures and defaults

Total inaccessibility of the default times presumes that the credit events come as a *surprise* to the observer who sees only the information in the market filtration \mathbb{F} . The defaults, however, must be positively correlated with the default sensitivities, as feedback exposures, of the market participants to one another. We will suppose the gross exposures, the total potential losses incurred from counterparties' defaults without netting, is modeled by a càdlàg nonnegative process

$$\delta_t^{ij} \ge 0 \quad \text{for} \quad i, j = 1, \cdots, n \quad t \ge 0.$$

The underlying network on n vertices is then characterized by a directed graph with edge weights specified by $\left\{\delta_t^{ij}\right\}_{i,j=1}^n$ at each time t. In particular, $\delta_t^{ij} = 0$ implies that at time t no directed edge exists from vertex j to i, equivalently ihas no exposure against j. Further, at time t the respective netted exposure of i to j is max $\left(\delta_t^{ij} - \delta_t^{ji}, 0\right)$.

Totally inaccessible stopping times admit an intensity under mild conditions. In particular, we suppose each N_t^i admits an intensity λ_t^i , i.e.,

$$N_t^i - \int_0^t \lambda_s^i ds \tag{5}$$

is a **P**-martingale for a nonnegative process, where **P** is a probability measure associated with the financial network $\left\{\delta_t^{ij}\right\}_{i,j=1}^n$.² We assume λ_t^i to be càdlàg for the time being. This **P**-intensity model does not assign a direct economic interpretation to its variables. Instead, we regard it more as a statistical model with parameters fit to market data.

In this financial market, the default of one participant is transmitted through a network of exposures. In this regard, we take the $\{\lambda^i\}_{i=1}^n$ to be of the form

$$\lambda_t^i = \left(X_t^i + \sum_{j=1}^n f\left(\delta_t^{ij}, N_t^j\right)\right) \left(1 - N_t^i\right) \quad i = 1, \cdots, n, \tag{6}$$

where each X^i is a nonnegative càdlàg process and $f\left(\delta_t^{ij}, N_t^j\right)$ is the feedback of j's default to the survivor *i*. For example, letting $f\left(\delta_t^{ij}, N_t^j\right) = \delta_t^{ij} N_t^j$ implies a default of participant *j* will impact the intensity of participant *i* by $\delta_t^{ij} > 0$ for $t \ge \tau_j$. To reflect the one-time nature of the default events, we multiply $1 - N_t^i$ to make the defaulter's intensity zero after the default. Typically, in credit risk applications, $X_t^i = w^i Y_t + Z_t^i$ for some $w^i \ge 0$, a combination of a common factor Y_t and an idiosyncratic factor Z_t . This specification is sufficiently general to accommodate virtually any model used in practice. For instance, the time

 $^{^2 \}rm Notice$ that the financial network structure determines the default propagation mechanism and the likelihood of the defaults of participants as well.

dependence of δ_t^{ij} allows us to model mean-reverting and non-mean-reverting jumps. We place no assumptions, aside from path regularity, on the $\{X_t^i\}_{i=1}^n$ nor the $\{\delta_t^{ij}\}_{i,j=1}^n$.

2.2. Bilateral arrangement and central clearing

Under a bilateral arrangement, we posit a network of exposure with no restriction on connectivity between any pair of participants. That is, any pair of *i* and *j* in the network do not need an intermediary to have exposure against each other. We denote the probability measure under such a network by **B**. Under a bilateral arrangement, we suppose each N_t^i admits an **B**-intensity b_t^i , i.e.,

$$N_t^i - \int_0^t b_s^i ds \tag{7}$$

is a **B**-martingale. Then, we introduce a special market participant, a nonoperating CCP, indexed by i = 0 which under **B** will be held separate from the other market participant.³ Its default intensity may be taken as

$$b_t^0 = X_t^0 \left(1 - N_t^0 \right), \tag{8}$$

for some càdlàg, nonnegative process X_t^0 . Note that under **B**, the CCP has no network relationship with any of the participants.

With the inauguration of central clearing, we let all bilateral clearing participants become a clearing member and novate their contracts with the CCP, resulting in each δ_t^{ij} breaking into δ_t^{i0} and δ_t^{0j} . With default protocols of central clearing, the change of clearing scheme amounts to a transformation of default likelihoods of participants and associated probability measures. Under the central clearing associated with a new probability measure **C**, we posit N_t^i admits a **C**-intensity c_t^i .

 $^{^{3}}$ The existence of a non-operating CCP and its default likelihood are assumed for measure change. An event that cannot occur in a probability space cannot occur in an equivalent probability space.

2.3. Measure change

Adopting Theorem 3.1 of Giesecke & Shkolnik (2020), we define $\mathcal{L}_T \left(= \frac{d\mathbf{C}}{d\mathbf{B}}\right)$ as the Radon-Nikodym derivative (or likelihood ratio) which drives the change of probability measure from **B** to **C** given by

$$\mathcal{L}_{T} = \exp\left(\sum_{i=0}^{n} \int_{0}^{T} \left(b_{s}^{i} - c_{s}^{i}\right) ds - N_{T}^{i} \log\left(b_{\tau_{i}-}^{i}/c_{\tau_{i}-}^{i}\right)\right).$$
(9)

The likelihood ratio \mathcal{L}_T satisfies $\mathbb{E}^{\mathbf{B}}(\mathcal{L}_T) = 1$ and $\mathbb{E}^{\mathbf{B}}(\mathcal{L}_T|\mathcal{A}) = \frac{\mathbf{C}(\mathcal{A})}{\mathbf{B}(\mathcal{A})}$ for $\mathcal{A} \in \mathcal{F}$ under mild conditions; see Giesecke & Shkolnik (2020) for technical details.

One of the primary intentions of central clearing is to control the propagation of defaults, lowering the expected default rate for a given horizon. However, the risk does not vanish itself but only shift toward some other occasions. If a central clearing redistributes the probability mass of D_T from the center into its (right) tail, a systemic risk concentration occurs. Specifically, the risk concentration from central clearing can be defined as the maximal monotonic increase in the right tail of the D_T distribution caused by moving from bilateral clearing into central clearing. It is noteworthy that \mathcal{L}_T shapes the distributional change of D_T induced by the introduction of CCP.

3. A system-level cost-benefit analysis

As the CCP insulates CMs from their counterparties defaults under its tenure, a default feedback is less prone to propagate through the financial network of exposure and the expected aggregate default rate can be lowered by central clearing to some extent. However, any form of risk does not vanish away by itself but only transfer to others in a re-packaged form. A possible scenario is that central clearing redistributes the probability mass of the aggregate default rate from the center to its right tail by delaying the impact of CMs' defaults until it is resumed to bilateral clearing upon the CCP's default. Such a systemic risk concentration can be mitigated by well-designed collateral schemes



Figure 1: An illustration of systemic risk concentration from central clearing

Note. This figure illustrates the mechanism of systemic risk concentration from central clearing. On the left panel, which depicts the probability distributions of D_T under each clearing scheme, the increase in probability on the left-side of the distribution is noticeable. It appears the right tail distribution under bilateral clearing has higher probabilities than that under central clearing. The right panel, in contrast, highlights that the introduction of a central counterparty into a market redistributes the probability mass of the systemic failure from the center of the distribution into its right tail.

that carefully redistributes likelihoods of the aggregate default rate from the right tail.

3.1. Modeling the impact of margin policy

The standard risk management tools against counterparties defaults are precautionary capital reserves such as margin requirements and loss mutualization funds. Initial margin is known to be the most effective and in fact, most widely used. Of course, the initial margin scheme has its benefit and cost at the same time. Initial margin reduces the counterparty credit risk exposure as the defaulter pays a part of exposure up-front while it comes as a cost by imposing additional capital burden which increases the default likelihood of the margin payer. We model these two aspects of initial margin on default likelihoods in our intensity model. When modeling and searching for the optimal initial margin policy, one should bear in his mind about the procyclicality of initial margin. As most risk-based initial margin schemes impose less in a calm market but more in a distressed market, those margin schemes act as amplifiers of margin payers' risk and become the potential triggers of systemic events. In this paper, we specify initial margin schemes cyclical to systemic risk cycle and look into their impact on the aggregate default rate distribution, especially by looking at its mean and tail probabilities.

We define three types of initial margin policies based on the cyclicality of the scheme to the systemic risk cycle. The margin rate $\mu \in [0, 1]$ specifies the percentage of the sensitivity of λ_t^i to j's default that is reduced by posted initial margins. The notional amount of initial margin is the margin rate times the size of exposure of counterparty to the margin payer. The constant margin rate is defined by

$$\eta_t = \mu, \tag{10}$$

the procyclical margin rate is defined by

$$\eta_t = \mu D_t \tag{11}$$

and the countercyclical margin rate is defined by

$$\eta_t = \mu \left(1 - D_t \right) \tag{12}$$

for some constant $\mu > 0$.

3.2. Default intensities under bilateral arrangement

We make virtually no modeling assumptions on the bilateral market (under **B**) except the default transmission channel in Eq. (6). When passing to central clearing (under **C**), at least some modeling is inevitable. First, we posit a static network of exposure $\left\{\delta_t^{ij}\right\}_{i,j=1}^n$ drawn from i.i.d. uniform distribution over [0, 0.5]. This simplifies the default feedback model and scales back the mag-

nitude of impact on the survivors' intensity. The intensity of market participant $i = 1, \cdots, n$ is defined as

$$b_{t}^{i}(\eta) = \left(X_{t}^{i} + \sum_{j=1}^{n} e^{-\kappa(t-\tau_{j})} (1-\eta_{t}) \,\delta_{t}^{ij} N_{t}^{j} + \sum_{j=1}^{n} e^{-\kappa t} \eta_{t} \delta_{t}^{ji} \left(1-N_{t}^{j}\right) \right) \left(1-N_{t}^{i}\right),$$
(13)

where X_t^i is the baseline intensity and $e^{-\kappa(t-\tau_j)}$ captures an exponential decay impact of the default of j on $b_t^i(\eta)$ for $\tau_j \leq t$ with a half-life period parameter $\kappa > 0$. We let X_t^i follow a square-root diffusion process to maintain intensities positive. Note the initial margin mainly reduces the magnitude of feedback from defaulted CMs by $1 - \eta_t$ while it imposes additional burden toward surviving counterparties. The intensity of a non-operating CCP is defined in Eq. (8).

3.3. Default intensities under central clearing

Before characterizing intensities under central clearing, we need to clarify the default proceedings under central clearing. We assume that under CCP's tenure, all payments of defaulted contracts are guaranteed by the CCP; thereby the default impact is absorbed by the CCP and the surviving counterparty is insulated. Meanwhile, bilateral clearing resumes once the CCP defaults, and the resulting withheld defaulted contracts are also returned to the original obligators. Under such a model, the CCP no longer insulates its members from each other's defaults and may impose initial margin policies that lead to postponed shocks.

Default intensity of the CCP under central clearing is given by

$$c_t^0(\eta) = \left(X_t^0 + \sum_{i=1}^n e^{-\kappa(t-\tau_i)} \left(1 - \eta_t\right) \delta_t^{0i} N_t^i\right) \left(1 - N_t^0\right),\tag{14}$$

where $\delta_t^{0i} = \sum_{j=1}^n \delta_t^{ji}$ by the definition of central clearing. The intensity of

clearing member i is defined by

$$c_{t}^{i}(\eta) = \left(X_{t}^{i} + N_{t}^{0}e^{-\kappa(t-\tau_{j})}\sum_{j=1}^{n} (1-\eta_{t}) \,\delta_{t}^{ij} N_{t}^{j} + \sum_{j=1}^{n} \zeta \,\delta_{t}^{ij} \left(1-N_{t}^{j}\right) + \sum_{j=1}^{n} e^{-\kappa t} \eta_{t} \delta_{t}^{ji}\right) \left(1-N_{t}^{i}\right),$$
(15)

where $\zeta \in [0, 1]$ is the attrition rate of central clearing. ζ represents the fraction of surviving CMs' exposure suffering the losses when resuming the bilateral arrangement. The rest $1 - \zeta$ of each contract resumes bilateral clearing without suffering the losses.

It is noteworthy that the additional capital burden materializes in CMs' intensities and the delayed default feedback decreased by the margin rate as well. The selection of initial margin policy shapes the probability measure under central clearing. Interestingly, such shocks implied by well-designed margin policies could mitigate the propagation of systemic risk at the macro level, which will be shown in the numerical section.

3.4. Multi-objectives in a downside risk framework

A policymaker may be concerned about the multifaceted consequence of introducing central clearing. By selecting a central clearing margin policy, denoted by η , a policymaker seeks to minimize a set of M nonnegative objective functions $\mathbf{f}_1(\eta), \dots, \mathbf{f}_M(\eta)$. We consider the following marginal objectives.

It is desirable to minimize the mean of D_T given by

$$\sum_{k=0}^{n} \frac{k}{n} \mathbf{P} \left(D_T = \frac{k}{n} \middle| \eta \right).$$
(16)

From a macro-prudential perspective, the policymaker should be more concerned about the failure of an abnormally large portion of the total population in the system. With the predetermined critical level $c \in [0, 1]$, the probability of D_T exceeding the threshold is given by

$$\sum_{k\geq cn}^{n} \mathbf{P}\left(\left.D_{T}=\frac{k}{n}\right|\eta\right).$$
(17)

The conditional mean of D_T exceeding the threshold $c \in [0, 1]$ is given by

$$\mathbb{E}^{\mathbf{P}}\left(D_T | D_T \ge c\right). \tag{18}$$

The policymaker could also consider quantile-based tail-risk measures to summarize the information in the right tail of the distribution of D_T . The widely accepted VaR at level $\alpha \in (0, 1)$ is defined with a discrete distribution by

$$\operatorname{VaR}_{\alpha}(\eta) = \min\left\{ x \ge 0 : \sum_{k=x}^{n} \mathbf{P}\left(\left. D_{T} = \frac{k}{n} \right| \eta \right) \le 1 - \alpha \right\}.$$
(19)

Subsequently, the definition of ES is given by

$$\mathrm{ES}_{\alpha}(\eta) = \frac{1}{1-\alpha} \int_{\alpha}^{1} \mathrm{VaR}_{\beta}(\eta) d\beta, \qquad (20)$$

which can be computed with a discrete distribution as

$$\mathrm{ES}_{\alpha}(\eta) = (1 - \omega_{\alpha}) \sum_{k > \mathrm{VaR}_{\alpha}(\eta)} \frac{k}{n} \mathbf{P} \left(D_{T} = \frac{k}{n} \middle| \eta \right) + \omega_{\alpha} \mathrm{VaR}_{\alpha}(\eta), \qquad (21)$$

where ω_{α} is given by

$$\omega_{\alpha} = \frac{1}{1 - \alpha} \left(\sum_{k \le \text{VaR}_{\alpha}(\eta)} \mathbf{P} \left(\left. D_T = \frac{k}{n} \right| \eta \right) - \alpha \right).$$
(22)

The failure of the CCP is another concern of the policy maker in that its failure has a direct impact on the CM's default risk. The probability of a default of the CCP is given by

$$\mathbf{C}\left(\tau_0 \le T | \eta\right). \tag{23}$$

Note that the CCP may default under a bilateral arrangement in our model, but no meaningful interpretation can be drawn from the event.

3.5. A min-max goal programming formulation

In general, the multi-objective optimization problem does not have a solution satisfying all goals simultaneously. In this context, goal programming (GP) can provide a reasonable compromise by balancing the trade-off between the multiple objectives. Our GP formulation proceeds in two steps. The policymaker first finds the stand-alone optimal mth goal given by

$$G_m^{\star} = \min\left\{f_m(\eta) : \eta \in \mathcal{H}\right\},\tag{24}$$

where \mathcal{H} is a set of various central clearing margin policies under consideration. Then, the policymaker obtains the unwanted squared relative deviations

$$d_m(\eta) = \left(\frac{f_m(\eta) - G_m^{\star}}{G'_m - G_m^{\star}}\right)^2 \in [0, 1],$$
(25)

where G'_k is the worst *m*th goal given by

$$G'_m = \max\left\{f_m(\eta) : \eta \in \mathcal{H}\right\}.$$
(26)

The policymaker finally considers the optimization problem given by

$$\min_{\eta \in \mathcal{H}} \mathbb{C}(\eta; \gamma), \tag{27}$$

where the preference-weighted systemic cost function $\mathbb{C}(\eta; \gamma)$ is given by

$$\mathbb{C}(\eta;\gamma) = \sum_{m=1}^{M} \gamma_m d_m(\eta)$$
(28)

for a nonnegative preference parameter set $\gamma = (\gamma_1, \dots, \gamma_M)$, which governs the policymaker's priorities over different goals to achieve. The systemic cost function $\mathbb{C}(\eta; \gamma)$ is the weighted sum of squared deviations from stand-alone goals. The min-max GP formulation was introduced by Flavell (1976). Note that we assume no individual objective has priority order over one another but has a preference weight over one another.

4. Numerical results

This section illustrates our numerical analyses in the GP formulation as described in Section (3.5). After the description of our setup of MC simulation with conditional importance sampling, we discuss the trade-off impact of the initial margin policy by redistributing risk among participants in the system. Subsequently, alternative tail risk specification confirms and substantiates the interpretation of our numerical results.

4.1. Simulation setup

We consider n = 20 market participants, all potentially central CMs, which approximates the size of the primary dealers in the U.S. repo market.⁴ The risk horizon is T = 1 year. The parameters of Y_t and Z_t^i satisfy the Feller condition to ensure that $X_t^i = w^i Y_t + Z_t^i > 0$ holds almost surely. The selected parameters model a high credit quality system with $\mathbf{B}(D_T = 0) = 0.1743735$. We further assume the attrition rate of central clearing, $\zeta = 0.5$.

We perform Monte Carlo (MC) simulation to estimate the distribution of D_T . Since the events that significant portions of the total population default are extremely rare, a high computational cost is required to obtain reliable estimates with naive MC simulation. For this reason, we apply a conditional importance sampling technique to reduce the simulation variance significantly for tail probability estimation; see Kim & Shkolnik (2020) for details. Our estimation results are based on the conditional importance sampling scheme with 10⁷ replications.

 $^{^4}$ As of July 2017, ICE CDS Clearing maintains 30 clearing members in the U.S. market, whereas 22 members interface between the CCP and clients for ICE Clear Europe's CDS clearing operation.

We formulate the multiple objectives balancing problem with equal preference weight on each marginal objective. The choice of marginal objectives is mean, $\text{ES}_{95\%}$, $\text{ES}_{99.9\%}$ and the default probability of the CCP. ESs at two different significance levels are used to consider tail and extreme tail risk of D_T distribution simultaneously. As the default probability of the CCP may cause a rippling cascade, this probability should be considered together with other marginal risk management objectives.

The policymaker may set a threshold level of the default rate as a criterion towards preemptive actions. For instance, the conditional mean of the aggregate default rate on it exceeding a certain threshold level and the triggering probability can be an alternative tail risk measure. In effect, the probability that the aggregate default rate exceeds 1/3 of the total population is much less than 1% in our simulation setting. Figure 4 illustrates each of the risk management goals alongside various margin rates under consideration.

4.2. Main findings and implications

Figure 2 shows the effect of central clearing by graphically representing the conditional expectations of likelihood ratios on initial margin scheme and aggregate default rate. The domain where the conditional expectation is higher than 1 indicates the increase of the probability mass of D_T , vice versa. We verify that the margin rates lower than 0.1 cause systemic risk concentration in all initial margin policy specifications by lowering the mean and increasing the right tail of D_T . However, as the margin rate increases, constant and countercyclical margin schemes lower the mean and the right tail probabilities at the same time. The cost of these risk reduction is imposed on the left tail distribution of D_T , which increases the chance of any defaults. Procyclical schemes also subdue the mean of D_T while it intensifies the cost to the right tail extremely as the margin rate increases.



Note. These figures illustrate the effect of central clearing by showing the conditional expectations of likelihood ratios on initial margin scheme and aggregate default rate. In each panel, the conditional expectations characterize the risk concentration. The domain where the conditional expectation is higher than 1 indicates increases of the probability mass of D_T , vice versa. Interestingly, as the margin rate increases, constant and countercyclical margin schemes lower the mean and the right tail probabilities at the same time. Procyclical scheme also subdue the mean of D_T while it intensifies the cost to the right tail extremely.

Figure 3: The transformations of the aggregate default rate distributions from central clearing



Note. These figures illustrate the risk concentration of the aggregate default rate distributions from central clearing under different initial margin schemes. The probability redistributions, thereby risk concentrations, are determined by the conditional expectation of likelihood ratios as illustrated in Figure 2.

	Margin Rate		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Clearing Scheme													
Bilateral Arrangement	Constant	Mean	0.0919	0.1062	0.1198	0.1328	0.1451	0.1569	0.1681	0.1788	0.1891	0.1989	0.2082
		Stdev	0.0695	0.0729	0.0755	0.0775	0.0789	0.0799	0.0805	0.0808	0.0809	0.0807	0.0804
		Skewness	0.7697	0.6653	0.5786	0.5044	0.4395	0.3814	0.3285	0.2795	0.2334	0.1894	0.1468
		Kurtosis	3.6660	3.5111	3.4180	3.3678	3.3495	3.3562	3.3839	3.4301	3.4934	3.5733	3.6699
	Procyclical	Mean	0.0919	0.0923	0.0928	0.0932	0.0936	0.0940	0.0944	0.0949	0.0953	0.0957	0.0961
		Stdev	0.0695	0.0699	0.0703	0.0707	0.0711	0.0715	0.0719	0.0722	0.0726	0.0730	0.0734
		Skewness	0.7697	0.7706	0.7714	0.7721	0.7728	0.7733	0.7737	0.7741	0.7744	0.7746	0.7747
		Kurtosis	3.6660	3.6613	3.6564	3.6512	3.6458	3.6401	3.6343	3.6282	3.6219	3.6154	3.6087
	Countercyclical	Mean	0.0919	0.1058	0.1188	0.1311	0.1428	0.1538	0.1642	0.1742	0.1836	0.1926	0.2012
		Stdev	0.0695	0.0725	0.0747	0.0764	0.0775	0.0782	0.0786	0.0788	0.0787	0.0785	0.0781
		Skewness	0.7697	0.6653	0.5802	0.5089	0.4478	0.3943	0.3466	0.3035	0.2639	0.2269	0.1920
		Kurtosis	3.6660	3.5178	3.4337	3.3932	3.3841	3.3992	3.4340	3.4858	3.5529	3.6346	3.7305
Central Clearing	Constant	Mean	0.0859	0.0874	0.0889	0.0905	0.0920	0.0935	0.0950	0.0965	0.0980	0.0994	0.1009
		Stdev	0.0634	0.0637	0.0641	0.0645	0.0648	0.0652	0.0655	0.0658	0.0661	0.0664	0.0667
		Skewness	0.7371	0.7214	0.7058	0.6904	0.6751	0.6600	0.6451	0.6304	0.6158	0.6014	0.5872
		Kurtosis	3.7594	3.7200	3.6816	3.6442	3.6078	3.5723	3.5379	3.5044	3.4718	3.4401	3.4093
	Procyclical	Mean	0.0859	0.0859	0.0860	0.0860	0.0861	0.0861	0.0862	0.0862	0.0863	0.0864	0.0864
		Stdev	0.0634	0.0634	0.0635	0.0635	0.0636	0.0636	0.0637	0.0637	0.0638	0.0638	0.0639
		Skewness	0.7371	0.7370	0.7370	0.7369	0.7368	0.7368	0.7367	0.7367	0.7366	0.7365	0.7365
		Kurtosis	3.7594	3.7567	3.7540	3.7513	3.7486	3.7460	3.7433	3.7407	3.7380	3.7354	3.7328
	Countercyclical	Mean	0.0859	0.0873	0.0888	0.0903	0.0917	0.0932	0.0946	0.0960	0.0975	0.0989	0.1003
		Stdev	0.0634	0.0637	0.0640	0.0643	0.0646	0.0649	0.0652	0.0654	0.0657	0.0659	0.0661
		Skewness	0.7371	0.7215	0.7060	0.6907	0.6756	0.6606	0.6458	0.6312	0.6168	0.6026	0.5885
		Kurtosis	3.7594	3.7227	3.6870	3.6522	3.6184	3.5854	3.5534	3.5223	3.4920	3.4625	3.4338

 Table 1: Summary statistics of the aggregate default rate distributions

Note. This table provides summary statistics of the aggregate default rate by varying the clearing schemes as combinations of network centrality and margin imposition schemes. The graphical representation of this table is provided in Figure 3.

Table 1 provides the summary statistics of the simulated distribution of the aggregate default rates across different clearing schemes. The clearing schemes are combinations of network centrality and margin schemes, which are further decomposed into margin cyclicality and their rates. Our simulation results imply that the aggregate default rate distributions transform into different shapes with the introduction of central clearing. For the fixed margin scheme, the first and second moments of the default rate decrease under the central clearing scheme in all cases. This observation indicates that the default rate has a lower mean and more centered distribution in the centrally cleared system. For the acceptable margin rates, the kurtosis of the default rate distribution increases in constant and countercyclical margin schemes, indicating that the decrease in the expected default rate is realized with the systemic cost of increasing the downside tail risk.

Figure 4 shows the extensive set of marginal objectives used in our analyses. A primary concern of the policymaker is the mean level of the aggregate default rate. The policymaker may monitor a clustering of defaults and take preemptive actions when pre-specified fractions of the total population have defaulted. The aggregate default probability and the conditional mean of the aggregate default rate exceeding a critical level in Panels B and C illustrate that risk. Panels D and E shows ES estimates at the 95% and 99.9% significance levels, respectively. The role of initial margins in shielding the CCP from the CMs' defaults is shown in Panel F.

Figure 5 shows the ratio of each marginal objective values under central clearings to that under bilateral clearings for fixed margin schemes. The ratio greater than 1 indicates that the central clearing scheme increases the risk measure compared to bilateral clearing, and vice versa. Most risk measures except the default probability of the CCP benefit from central clearing and such a benefit becomes more pronounced as the margin rate increases. Even the cases when tail risk arises as illustrated in Panel D or E, a little increase in margin rate resolves the risk concentration. Note that the introduction of central clearing doubles the default probability of the CCP while the relative magnitude is mechanical artifact coming from our CCP default model specification under



Figure 4: Marginal risk management objectives

Note. This figure illustrates the policy maker's marginal risk management alongside initial margin policies. Panel A depicts the mean level of the aggregate default rate, a primary concern of the policymaker. Panels B and C provide the probability and conditional mean of the aggregate default rate exceeding a critical level. In this example, we assume the policymaker monitors the systemic risk and may take preemptive action when 1/3 of the total population has defaulted. Panels D and E show ES estimates at the 95% and 99.9% significance levels, respectively. Panel F shows the role of margin schemes in shielding the CCP from the CMs' defaults.



Figure 5: The ratios of marginal risk management objectives

Note. This figure illustrates the relative magnitude of each marginal objectives under central clearings to bilateral clearings conditioned on fixed margin schemes. The ratio greater(smaller) than 1 states the introduction of central clearing increases(decreases) the risk measure.



Figure 6: Goal programming results and the optimal margin rates

Note. This figure depicts the mean-tail-extreme tail-SIFI balancing optimization result with equal preference weights on each objective. In this figure, tail and extreme tail risk are estimated by $ES_{95\%}$ and $ES_{99.9\%}$, respectively. SIFI indicates the default probability of the CCP. The mean-tail-extreme tail-SIFI balancing goal programming results the optimal margin rates (constant, procyclical, countercyclical) = (0.61, 1.00, 0.68).

bilateral clearing; the increase in the default probability of the CCP is caused from default feedback terms in Eq. (14).⁵ In short, we observe that a larger initial margin collected from CM can mitigate such a systemic risk under central clearing.

Figure 6 provides a graphical illustration of our GP results by showing the total systemic cost function values across various margin rates. To visualize the pure effect of the introduction of central clearing, we compute the marginal deviations based on the best and worst systemic costs in the pool of all central clearing schemes we consider. In general, both constant and countercyclical initial margin schemes are desired as they offer lower total systemic cost compared to procyclical scheme. The constant margin scheme maintains its dominance

 $^{^5 \}rm Recall$ that the default probability of the non-operating CCP under bilateral clearing has insignificant economic implications to the system.



Figure 7: Goal programming results and the optimal margin rates with alternative marginal objectives

Note. This figure depicts the mean-tail-extreme tail-SIFI balancing optimization result with equal preference weights on each objective. In this figure, $\text{ES}_{95\%}$ measures the tail risk and the extreme tail events are defined as the conditional mean default rate on the more or equal to 1/3 of the total population defaults. SIFI indicates the default probability of the CCP. The mean-tail-extreme tail-SIFI balancing goal programming results the optimal margin rates (constant, procyclical, countercyclical) = (0.55, 1.00, 0.61).

over the countercyclical scheme in the low margin rate domain while the relation reverts as the margin rate arises.

Figure 7 provides the graphical illustrations of the total systemic cost function with alternative marginal objective specifications. Specifically, we apply the extreme tail risk measure, $ES_{99.9\%}$, along with the conditional expectation of the default rate given that the aggregate default rate exceeds the threshold level of 1/3. Policymakers may establish a pre-specified criterion level of aggregate default rate and take preemptive actions to prevent systemically severe damage in the financial network. The general tendency remains the same under central clearing schemes, while the optimal margin level slightly decreases from the original specification. Still, the constant margin scheme provides the best outcome by minimizing the total systemic cost.

4.3. Supplementary analysis

Instead of considering multiple objectives separately and simultaneously, we alternatively consider a sole risk measure encompassing the entire distribution of D_T focusing on the right tail at the same time. Acerbi (2002) suggests spectral risk measures that assign more weights to the tail of risk distributions of interest when calculating statistics across the entire distribution. Kou & Peng (2016) discuss a generalized version of a spectral risk measure of a random variable X and it is defined by

$$\rho(X) = \int_{(0,1)} F_X^{-1}(u)\phi(u)d(u), \qquad (29)$$

where $\phi(\cdot)$ is an increasing and nonnegative function. Accordingly, Cherny & Madan (2009) propose a distort function $\phi(u) = 1 - (1 - x^{1/(1+\alpha)})^{1+\alpha}$ for some $\alpha > 0$. We employ this form of distort function to describe the relative significance in the lower survival rate region. In our spectral risk measure, the distort function is specified as

$$1 - (1 - D_T)^{1/m}$$
 for $m > 0.$ (30)

Figure 8 illustrates our distort function and the associate spectral risk measure alongside margin rates under various clearing schemes. It seems the tradeoff of increasing the margin rate in estimating the systemic cost is evident. The general tendency alongside the margin rate does not change from our multiple objectives balancing analyses while the countercyclical scheme dominates in all clearing scheme specifications with the spectral risk measure.

5. Conclusion

The failure of a member at Nasdaq Clearing in 2018 and the ongoing COVID-19 market turbulence underscore the importance of a holistic risk management practices by CCP's to forestall the loss spread at the systemic level. Despite the original intention of introducing the CCP into the system for the mitigation

Figure 8: A distort function and a spectral risk measure



Note. Panel A shows the distort function $1 - (1 - D_T)^{1/m}$ for an arbitrarily chosen m = 50, which is intended to assign more weights to the right tail distribution of D_T in calculating the weighted expectation. Panel B illustrates the implication of our proposed spectral risk measure by showing the weighted expectation of D_T based on our distortion function described in Panel A. Interestingly, the countercyclical margin scheme becomes the best margin policy in minimizing the total systemic cost across most of the margin rates we consider.

of systemic risk, this paper finds that systemic trade-offs may appear under the central clearing scheme as reducing the first moment of the aggregate default rate often has an adverse effect on the tail risk and vice versa. Our main findings indicate that the statistical properties of the full distribution exhibit a non-trivial dependence on the initial margin policy, which redistributes burdens among participants as well as the CCP in the system. Central clearing is more likely to decrease the mean of the aggregate default rate, if the CCP guards survivors against counterparties' defaults. However, the delayed default feedback may cause the clustering of failures once the CCP defaults. As a price for lowering the average default rate in regular time, a systemic risk concentration arises.

This paper formulates a multi-objective optimization problem based on a min-max GP approach involving a social cost minimization of preference-weighted marginal risk management objectives under consideration. The results of our numerical analyses indicate that the systemic risk concentration arises from central clearing. Although putting weights on the tail risk justifies bilateral clearing, constant and/or countercyclical margin schemes generally dominate a procyclical margin scheme based on a reasonable choice of preference parameter set governing the policymaker's priorities over different marginal goals to achieve.

Appendix: The sketch of the conditional importance sampling (cIS) algorithm

This appendix briefly summarizes the algorithm of conditional importance sampling (cIS) proposed by Kim & Shkolnik (2020) for the purpose of estimating tail probabilities efficiently. The objective is to estimate $\mathbf{P}\left(D_T = \frac{k}{n}|\eta\right)$ with acceptable accuracy for a margin policy η and a large $k \leq n$, where $\mathbf{P}\left(D_T \geq \frac{k}{n}|\eta\right)$ is sufficiently small.

Importance sampling is a variance reduction technique that is widely used in MC simulations by choosing an efficient simulation measure to obtain reliable estimates of tail probabilities. The conditional importance sampling approach is based on adaptive probability measure changes conditional on the tail event of interest without the need for calculating any optimal parameters prior to, or during, simulation; see Kim & Shkolnik (2020) for details.

With all notations defined in this paper, we adopt Corollary 1 in Kim & Shkolnik (2020) with the definition of the cIS simulation measure \mathbf{Q}_T^k as

$$\mathbf{Q}_{T}^{k}\left(\mathcal{A}\right) = \mathbb{E}^{\mathbf{B}}\left(\frac{\mathbf{1}_{\left\{D_{T}=\frac{k}{n}\right\}\cap\mathcal{A}}}{\mathcal{M}_{T}^{k}}\right) \quad \text{for all } \mathcal{A} \in \mathcal{F}_{T},$$
(31)

where

$$\mathcal{M}_T^k = \exp\left(-\int_0^T \bar{b}_s ds\right) \prod_{i=1}^k \bar{b}_{\tau_i -} \frac{T^k}{k!},\tag{32}$$

with $\overline{b}_t = \sum_{i=1}^n b_i^i$. Recall that our default intensity model under bilateral

clearing has the form of

$$b_t^i(\eta) = \left(X_t^i + \sum_{j=1}^n e^{-\kappa(t-\tau_j)} \left(1 - \eta_t\right) \delta_t^{ij} N_t^j + \sum_{j=1}^n e^{-\kappa t} \eta_t \delta_t^{ji} \left(1 - N_t^i\right)\right) \left(1 - N_t^i\right),$$
(33)

For $i = 0, 1, \dots, n$. We let $X_t^i = w^i Y_t + Z_t^i$ for some constant w^i and the squared root diffusion processes Y_t and (Z_t^0, \dots, Z_t^n) given by

$$dY_t = \kappa^Y \left(\theta^Y - Y_t\right) dt + \sigma^Y \sqrt{Y_t} dW_t^Y, \tag{34}$$

where W^Y is a **B**-Brownian motion with some $\kappa^Y > 0$, $\theta^Y > 0$ and $\sigma^Y > 0$, and

$$dZ_t^i = \kappa^i \left(\theta^i - Z_t^i\right) dt + \sigma^i \sqrt{Z_t^i} dW_t^i, \tag{35}$$

where (W^0, \dots, W^n) is a vector of mutually independent **B**-Brownian motions, $\kappa^Y > 0, \ \theta^Y > 0 \text{ and } \sigma^Y > 0.$

Our numerical analyses are based on the parameter set for the systematic factor (Y_t) given by $(\kappa^Y, \theta^Y, \sigma^Y) = (1.0, 0.0597911, 0.2)$ along with the initial value $Y_0 = 0.0597911u^Y$ where u^Y is drawn from [0.5, 1.25] uniformly. For $i = 0, \dots, n$, the systematic factor loadings w^i are drawn from $[0, 1], \kappa^i$ from [0.5, 1.5] and θ^i from [0.025, 0.125] uniformly. We set $\sigma^i = \max\left(\sqrt{2\kappa^i\theta^i}, \overline{\sigma}^i\right)$ where $\overline{\sigma}^i$ is drawn from [0.1, 0.4] uniformly. The conditions for parameters are intended for Y_t and Z_t^i to satisfy the Feller condition to maintain $X_t^i > 0$ almost surely under **B**. The exposures $\left\{\delta_t^{ij}\right\}_{i,j=1}^n$ are drawn from [0, 0.025] uniformly for $i \neq j$. $\delta_t^{ii} = 0$ for all $i = 0, \dots, n$ by definition.

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