Variance and Skew Risk Premiums for the Volatility Market: The VIX Evidence *

José Da Fonseca† Yahua Xu‡

July 8, 2016

Abstract

We extract variance and skew risk premiums from volatility derivatives in a model-free way and analyze their relationships along with volatility index and equity index returns. These risk premiums can be synthesized through option trading strategies. Using a time series of option prices on the VIX, the most liquid volatility derivative market, we find that variance swap excess return can be partially explained by volatility index and equity index excess returns while these latter variables carry little information for the skew swap excess return. The results sharply contrast with those obtained for the equity index option market underlining very specific characteristics of the volatility derivative market.

JEL Classification: G11; G12; G13

Keywords: VIX option market, variance swap, skew swap, risk premiums.

*We thank the conference participants of the First PKU-NUS Annual International Conference on Quantitative Finance and Economics in Shenzhen, China, 2016; the First Annual China Derivatives Markets Conference in Suzhou, China, 2016; 2016 Greater China Area Finance Conference in Xiamen, China, 2016; International Conference on Applied Financial Economics in Shanghai, China, 2016, for helpful comments. Parts of this research were conducted while the first author was visiting fellow at the University of Technology, Sydney, whose financial support is gratefully acknowledged. The usual caveat applies.

†Corresponding author: Department of Finance, Business School, Auckland University of Technology, Private Bag 92006, 1142 Auckland, New Zealand. Phone: +64 9 9219999 extn 5063. Email: jose.dafonseca@aut.ac.nz

‡Department of Finance, Business School, Auckland University of Technology, Private Bag 92006, 1142 Auckland, New Zealand. Phone: +64 9 9219999 extn 8944. Email:yaxu@aut.ac.nz

1 Introduction

The rapid growth of volatility products, among which the VIX index is by far the most well known, has turned the volatility an asset class in itself, see Whaley (1993) and Zhang et al. (2010). The availability of VIX options suggests to apply option-implied moment estimation methodologies, which were extensively used on equity (index) options, to that market as they allow the extraction of information on the VIX distribution without specifying any parametric model for it. They are often qualified as model-free approaches. These methodologies are favored over more traditional historical estimation strategies as options embed a more forward looking point of view of asset moments' distribution. Also, options contain risk neutral information and when combined with historical information enables the determination of risk premiums that are the key quantities for risk management.

These methodologies have been extensively applied to equity index options and/or individual stock options, and to foreign exchange options and are based on the analytical results proposed in Derman et al. (1997) and Carr and D.Madan. (1998). The literature is so important that we will restrict ourselves to quote the works of Bakshi et al. (2003) and Neuberger (2012) as entry points in this field. Among many possible applications let us mention the use of higher risk neutral moments for asset pricing models in Bakshi et al. (2003), the estimation of an investor's risk aversion from volatility spread in Duan and Zhang (2014) or volatility forecasting using implied moments in Neumann and Skiadopoulos (2013) and Byun and Kim (2013), and more recently the analysis of variance and skew swaps for the SP500 option market in Kozhan et al. (2013). Quite surprisingly, the use of these results for the VIX option market remains largely unexplored.

Following Kozhan et al. (2013), we use VIX options to compute variance swap and skew swap excess returns and analyze their relationships to VIX index excess return and SP500 index excess return. All the quantities involved are obtained from options in a model-free way and correspond to tradable strategies some of which, like the variance swap, are actively used on the market nowadays. A byproduct of these results is to draw some conclusions on variance and skew risk premiums for the VIX market that will certainly hold for other less developed volatility markets. The results also underline certain differences between the equity (index) option market and the volatility (index) option market that may not be a surprise as equity and volatility dynamics are profoundly different.

Our paper contributes to the literature by analyzing variance and skew risk premiums for the volatility market and finds that both quantities are negative. For this market, we show that variance swap excess return can be partially explained by volatility index excess return and equity index excess return but also by skew swap excess return. However, considering all these explanatory variables together does not fully capture the return of a variance swap trading strategy as the capital asset pricing model (CAPM) does not hold. To explain the skew swap excess return the most important variable is the variance swap excess return from which we deduce that higher order moments of the volatility distribution can hardly be hedged using equity index trading strategies. Overall, our results depict the volatility index market as being structurally different from the equity index market.

The paper is organized is as follows. We present the key ingredients to obtain the quantities from option prices in Section 2. A description of the empirical data used in our analysis is provided in Section 3. Regression tests and analysis are performed in Section 4 and Section 5 concludes the paper by providing some open questions.

2 Pricing formulas

The main purpose of this work is to analyze the variance and skew risk premiums for the volatility market and these keys quantities will be computed using VIX call and put options. To this end, we will denote by $C_{t,T}(K)$ and $P_{t,T}(K)$ the European call and put option prices at time t with maturity T and strike K on the VIX whose value at time t is VIX_t. It is often more convenient to use the forward value of the VIX, so we write $F_{t,T}$ for the forward value at time t with maturity T that is related to the spot value through the standard equality $F_{t,t} = \text{VIX}_t$. Lastly, we will also make use of $r_{t,T} = \ln F_{T,T} - \ln F_{t,T}$, the log return of a position on the forward contract. The availability of these derivative products allows us to compute the variance and skew risk premiums in a model-free way as shown in the literature with the important contribution provided by Kozhan et al. (2013) that we will closely follow (see also Neuberger (2012)).

Extracting distribution information from option prices, like higher moments, has a long history. Let us quote without pretending to be exhaustive the work of Carr and D.Madan. (1998), and has found many applications, see among many others the works of Bakshi et al. (2003) for individual options; Bakshi and Madan (2006) and Carr and Wu (2009) for a variance risk premium analysis of equity index options (options on SP500, SP100 and other major indexes as well as equity); Fleming (1998), Neumann and Skiadopoulos (2013), Byun and Kim (2013) and Konstantinidi and Skiadopoulos (2016) for forecasting aspects (using SP500 options); Ammann and Buesser (2013) for variance risk premium properties for the foreign exchange market; and investors' risk aversion analysis as in Kostakis et al. (2011) and Duan and Zhang (2014) (using SP500 options); variance risk premiums for the commodity markets in Prokopczuk and Wese Simen (2014).

To the best of our knowledge we are only aware of the work of Huang and Shaliastovich (2014) that exploits VIX options to extract volatility higher moments (in their case the second moment, that is to say, the volatility of the VIX or the volatility of volatility) and performs a joint analysis with the second moment extracted from SP500 options (i.e. the square of the VIX) along with high frequency quantities such as realized volatility and bi-power variation (these two latter quantities allow the authors to isolate the role of jumps). As our work follows Kozhan et al. (2013), it differs from Huang and Shaliastovich (2014) by focusing also on the skewness of the VIX distribution and this aspect is important as it controls the shape of the VIX option smile and is also related to the "inverse" leverage effect, or positive skew, for the volatility market.[1] What is more, it is really at the skewness level that the volatility index option market departs from the equity index option market.

The variance risk premium A variance swap contract, receiver of the floating leg and payer of the fixed leg, is a contract between two counterparties that involves receiving at maturity T of the contract the realized variance of a given asset while at initiation t of the contract there is

[1] Regarding parametric approaches to VIX option pricing the literature is substantial, let us mention the works of Grünbichler and Longstaff (1996), Detemple and Osakwe (2000), Sepp (2008), Lian and Zhu (2013) and Park (2016). For variance, skew and kurtosis swaps within the affine framework see Zhao et al. (2013).

a payment of a premium (i.e. the fixed leg). The premium is given by

$$Var_{t,T} = \frac{2}{B_{t,T}} \left(\int_0^{F_{t,T}} \frac{P_{t,T}(K)}{K^2} dK + \int_{F_{t,T}}^{+\infty} \frac{C_{t,T}(K)}{K^2} dK \right) \tag{1}$$

with $B_{t,T}$ being the zero-coupon at time t with maturity T. It was shown in the literature that a variance swap contract enables to hedge changes in the variance of the underlying asset and in the present case it will be the VIX. Eq.(1) involves a continuum of options and as in the market only a finite number of options is available this quantity is approximated by the following sum

$$Var_{t,T} = 2 \sum_{K_i \leq F_{t,T}} \frac{P_{t,T}(K_i)}{B_{t,T} K_i^2} \Delta I(K_i) + 2 \sum_{F_{t,T} \leq K_i} \frac{C_{t,T}(K_i)}{B_{t,T} K_i^2} \Delta I(K_i), \tag{2}$$

with the weight function $\Delta I(K_i)$ defined as

$$\Delta I(K_i) = \begin{cases} \frac{K_{i+1} - K_{i-1}}{2}, 0 \leq i \leq N (\text{with } K_{-1} = 2K_0 - K_1, K_{N+1} = 2K_N - K_{N-1}) \\ 0, \quad \text{otherwise.} \end{cases}$$

The floating leg of the swap is the realized variance of the underlying asset and is given by

$$rVar_{t,T} = \sum_{i=t}^{T-1} \left[2 \left(\exp\left(r_{i,i+1} \right) - 1 - r_{i,i+1} \right) \right], \tag{3}$$

and by combining Eq.(1) and Eq.(3) we deduce the value of a realization of the variance swap, $rVar_{t,T} - Var_{t,T}$, from which we obtain the variance risk premium after averaging under the historical probability measure. Also of interest is the excess return of an investment made on the variance swap, it is denoted by

$$xVar_{t,T} = \frac{rVar_{t,T}}{Var_{t,T}} - 1. \tag{4}$$

As the options have monthly maturities there will be monthly observations for the variance risk premium and variance swap excess return. As a result, in the previous equations t will run through the first days following the option maturity dates while T will be the first maturity date available posterior to a given t. Notice that $rVar_{t,T}$ is known at time T while $Var_{t,T}$ is known at time t. So actually, for one-month options, $rVar_{t,T}$ is determined one month later than $Var_{t,T}$.

The skew risk premium A skew swap, receiver of the floating leg and payer of the fixed leg, is a contract between two counterparties that involves receiving at maturity T of the contract the realized skewness of a given asset while at initiation t of the contract there is a payment of a premium (i.e. the fixed leg). The premium is given by

$$Skew_{t,T} = \frac{6}{B_{t,T}} \left(- \int_0^{F_{t,T}} \frac{F_{t,T} - K}{K^2 F_{t,T}} P_{t,T}(K) dK + \int_{F_{t,T}}^{+\infty} \frac{K - F_{t,T}}{K^2 F_{t,T}} C_{t,T}(K) dK \right) \tag{5}$$

and as explained in Kozhan et al. (2013) it depends on the skewness of the underlying asset distribution (i.e. the skewness of the log of VIX distribution) and is implied from the options. If we define

$$V^E_{t,T} = \frac{2}{B_{t,T}} \left(\int_0^{F_{t,T}} \frac{P_{t,T}(K)}{K F_{t,T}} dK + \int_{F_{t,T}}^{+\infty} \frac{C_{t,T}(K)}{K F_{t,T}} dK \right) \tag{6}$$

then $Skew_{t,T} = 3(V^E_{t,T} - Var_{t,T})$. As $Var_{t,T}$ can be approximated by Eq.(2), to determine $Skew_{t,T}$ from option market prices we just need to discretize Eq.(6) and it is done as

$$V^E_{t,T} = 2 \sum_{K_i \leq F_{t,T}} \frac{P_{t,T}(K_i)}{B_{t,T} K_i F_{t,T}} \Delta I(K_i) + 2 \sum_{F_{t,T} \leq K_i} \frac{C_{t,T}(K_i)}{B_{t,T} K_i F_{t,T}} \Delta I(K_i), \tag{7}$$

with $\Delta I(K_i)$ previously defined.

The floating leg of the skew swap is given by

$$rSkew_{t,T} = \sum_{i=t}^{T-1} \left[3 \Delta V^E_{i,T} \left(\exp\left(r_{i,i+1}\right) - 1 \right) + 6 \left(2 - 2 \exp\left(r_{i,i+1}\right) + r_{i,i+1} + r_{i,i+1} \exp\left(r_{i,i+1}\right) \right) \right], \tag{8}$$

with $\Delta V^E_{i,T} = V^E_{i+1,T} - V^E_{i,T}$ the daily change of $V^E_{t,T}$. Combining Eq.(2), Eq.(7) and Eq.(8) we deduce the value of a realization of the skew swap, $Skew_{t,T} - rSkew_{t,T}$, from which we obtain the skew risk premium after averaging under the historical probability measure. As for the variance risk premium, of interest is the excess return of an investment made on the skew swap, it is denoted by

$$xSkew_{t,T} = \frac{rSkew_{t,T}}{Skew_{t,T}} - 1. \tag{9}$$

Similarly to variance swap quantities there will be monthly observations for the skew risk premium and skew swap excess return. As a result, in the previous equations t will run through the first days following the option maturity dates while T will be the first maturity date available posterior to a given t. We might drop the dependency with respect to T in the different quantities related to premiums, either variance or skew, to lighten the notations although the reader should keep in mind that they are of monthly frequency.

Remark 2.1 *In Kozhan et al. (2013), the authors define the skew as* $Skew_{t,T}/(Var_{t,T})^{\frac{3}{2}}$ *and the realized skew as* $rSkew_{t,T}/(Var_{t,T})^{\frac{3}{2}}$*, thereby following the usual mathematical definition while in our case we do not normalize by the term* $(Var_{t,T})^{\frac{3}{2}}$*. It would be more accurate to term Eq.*(5) *and Eq.*(8) *as risk neutral and realized third moments but nevertheless we will use the word skew. Following Kozhan et al. (2013), most if not all of the regressions will involve the skew swap excess return given by Eq.*(9) *and whether the normalization by* $(Var_{t,T})^{\frac{3}{2}}$ *is performed or not is irrelevant for that quantity with the consequence that our results can be compared to those of Kozhan et al. (2013). Lastly, in Kozhan et al. (2013) Table 3 in that paper mentions the "normalized" implied skew and realized skew while the corresponding section (3.1 Time variation in risk premiums) refers only to third implied and realized moments (see Eq.(31) and Eq.(32)). As a result, we are uncertain whether these normalized quantities are ever used in the regressions presented in that paper.*

3 Data and descriptive statistics

To compute the variance and skew risk premiums we use European options, both calls and puts, written on the VIX that are traded on the Chicago Board Options Exchange (CBOE) and mature every month. The data are provided by Thomson Reuters Ticker History (TRTH) from SIRCA[2] and contain option information such as ticker, option type, maturity date, strike and bid-ask option quotes. The sample period starts from January 4, 2010 and ends in September 10, 2015. VIX options started to be traded in 2006 while for futures contracts on the VIX it was in March 2004 but as it can be seen on Figure 1 the trading activity on VIX futures contracts, that constitute the natural hedging tool for VIX options, only took off in 2010. This justifies, along with the fact that we wanted to avoid the Global Financial Crisis, to start the sample in 2010. The VIX futures contracts are obtained from Datastream and cover the same period and we use them as froward value for the VIX. For each option the corresponding futures contract value is used. Whenever a risk free rate is needed we use Libor rates provided by Bloomberg.

[2]http://www.sirca.org.au/

[Insert Figure 1 here]

In sharp contrast with options on the SP500, options on the VIX display a positive skew that is also related to the positive relationship between the VIX and its volatility, see Figure 2. Figure 2 also reports the smile for SP500 options with the usual negative slope related to the leverage effect. Thanks to the availability of the VVIX (VIX of VIX options) this fact can be intuited from the evolutions of the VIX and the VVIX shown in Figure 3. It mirrors the well known leverage effect of SP500 options that is negative and will explain some of the differences between variance and skew premiums for SP500 and VIX options.

[Insert Figure 2 here]

[Insert Figure 3 here]

The Figure 4 shows the evolution of risk-neutral variance (Var) and realized variance ($rVar$), while Figure 5 depicts the comovement between risk-neutral skew ($Skew$) and realized skew ($rSkew$). Both figures suggest positive correlations between the variables. In general, the curve of Var ($Skew$) is above that of $rVar$ ($rSkew$), except for some spikes where the curve of Var ($Skew$) is below that of $rVar$ ($rSkew$). As a result, the sample averages of the Var and the $Skew$ are higher than that of $rVar$ and $rSkew$, respectively, leading to negative risk premiums. Lastly, variance of risk neutral quantities are lower than their realized counterparts.

[Insert Figure 4 here]

[Insert Figure 5 here]

The variance and skew risk premiums, namely, the excess return from the variance swap ($xVar$) and the excess return from the skew swap ($xSkew$), are computed on a monthly basis using pricing formulas given in the previous section. The comovement of variance and skew risk premiums shown by Figure 6 demonstrates their positive correlation. Magnitudes of both $xVar$ and $xSkew$ increase dramatically when the market is volatile, as there are spikes for both $xVar$ and $xSkew$ during the 2010-2012 period. Moreover, the larger spikes given by $xSkew$ suggest that skew risk premium is more sensitive to market downturns. In other words, it is likely that

$xSkew$ captures more important uncertainty information in the market. More in-depth analysis will be given later.

[Insert Figure 6 here]

Following the methodology presented in the previous section the quantities are computed on a monthly basis. Taking into account our data sample it will produce monthly observations ranging from January 2010 (involving options with maturity February 2010) to July 2015 (involving options with maturity August 2015). Table I reports the descriptive statistics of the variables we defined in the above part, namely, Var, $rVar$, $Skew$, $rSkew$, $xVar$ and $xSkew$. Again, let us stress the fact that variables have monthly frequency. The sample average values of the risk-neutral variance and realized variance are 0.071 and 0.049, respectively. Holding options in fact provides insurance against bad states of the economy, which suggests that option-implied risk-neutral volatility is usually slightly higher than the actual historical realized volatility. Therefore, a negative variance swap excess return with value of -0.283 will be generated implying that an investor is willing to lose 28.3% of the premium in order to hedge variance risk. It is interesting to note that the value is close to the one obtained in Kozhan et al. (2013) for the variance swap excess return extracted from SP500 options. For the skewness, the average risk-neutral skew is 0.018 while the realized skew is only 0.031 leading to an excess return for the skew swap of -0.293, or -29.3%, a value close to variance swap excess return value. The financial interpretation is therefore similar, an investor entering in a skew swap paying a fixed leg (i.e. the premium) and receiving a floating leg will lose on average 29.3% of the premium to hedge skew risk.[3] The excess return for the skew swap and variance swap are close and it contrasts with the results for the SP500 for which excess return for the skew swap is roughly twice that for the variance swap. For the standard deviations, the salient fact is the value for the skew swap excess return that is twice the value for the variance swap contract and this result is in line with the values for SP500 options of Kozhan et al. (2013).

[Insert Table I here]

[3] Similar results are obtained using a parametric approach in Zhao et al. (2013)

The correlations between the variables are reported in Table II. Consistent with the positive slope observed on the VIX option smile the correlation between the risk neutral variance and skew is positive at 0.202, an increase of the VIX level is associated with an increase of its volatility leading to a right tail volatility distribution thicker than the left tail distribution, thus a larger (positive) value for the skewness. This result is in line with the implementation of parametric models such as Sepp (2008) or Lian and Zhu (2013) although the strength of the approach used by Kozhan et al. (2013) is to be model free. Thus, the results can be used to assess the relevance of the few existing parametric models implemented for the VIX option market.

Regarding realized quantities a much stronger dependency, at 0.888, occurs between the realized variance and skew. Consistently with the relationship between the variables $xVar$, Var and $rVar$ a negative but weak correlation of -0.098 is obtained between the risk neutral variance (Var) and the variance swap excess return ($xVar$) while there is a strong and positive correlation of 0.933 between the realized variance ($rVar$) and variance swap excess return ($xVar$). Similar results are obtained for skew related variables, the correlation between the risk neutral skew and skew swap excess return is negative and equal to -0.287 and smaller (in absolute value) than the correlation between this same excess return and the realized skew that is equal to 0.661 suggesting that high demand for hedging variance risk and skew risk will take place at same the time on the market.

Overall, the information content of the shape of the VIX option smile, and more precisely the positive slope, applies to the realized distributions as well as to the swap trading strategies. Although there are some similarities with the results found for SP500 options by Kozhan et al. (2013) some specifics to the volatility derivative option market appear that will have important consequences as we will see on the regression analysis.

[Insert Table II here]

4 Empirical results

In order to deepen our understanding of the relations between the variables, a thorough empirical analysis will be carried out in this section. The first aspect we shall focus on is whether the risk-neutral variance and skew, which are forward looking measures, contain any predictive information regarding their realized counterpart. The second aspect worth analyzing is whether the excess return from investing in a variance (skew) swap can be explained by the Capital Asset Pricing Model (CAPM). It is known from Ang et al. (2006) that market-wide volatility risk can explain cross-section of stock returns. Yang et al. (2013) further proved that higher-order moment risk, such as market-wide skew, is also a pricing factor for cross-sectional stock returns. More recently, Kozhan et al. (2013) showed that excess return from variance or skew swap from equity index options can be partially explained by market index excess returns. They further enhance their analysis by considering cross-moment effects by quantifying the impact, along with index market return, of variance swap excess return on skew swap excess return and vice-versa. Our objective is to perform similar analysis for the volatility market that during the past years has grown so much that it has become an asset class in itself and deserves a specific analysis.

Risk-neutral and realized variance (skew) It is of interest to assess whether risk-neutral quantities, either variance or skew, can explain their corresponding realized counterpart as it gives some insights into the market price of risks. Therefore, we perform the univariate regression of realized variance (skew) on risk-neutral variance (skew) given by the set of equations

$$rVar_t = a_0 + a_1 Var_t + \epsilon_t^a, \tag{10}$$

$$rSkew_t = b_0 + b_1 Skew_t + \epsilon_t^b, \tag{11}$$

where we used $Skew_t$ instead of $Skew_{t,T}$, $rSkew_t$ instead of $rSkew_{t,T}$ to simplify the notations. We report in Table III the results.

[Insert Table III here]

For Eq.(10) the coefficient is positive at 0.392, thus consistent with the correlation value reported in Table II, and significant. Notice, however, that the R^2 is relatively low as it is equal to 2.91%.[4] Regarding the skew variables, the risk neutral skew provides no information on the realized skew as the coefficient is not significant and the R^2 is close to 0%. These two results suggest that the driving factors for the risk neutral and realized quantities, either variance or skew, are different.

Factor models for variance swap excess return: In this part we focus on understanding the determinants of variance swap excess return, to which extent they are related to market returns and other risk factors. First, we consider regressions

$$xVar_t = a_0 + a_1 xm_t^{\text{VIX}} + \epsilon_t^a, \tag{12}$$

$$xVar_t = b_0 + b_1 xm_t^{\text{SP}} + \epsilon_t^b, \tag{13}$$

with the results reported in Table IV columns (1) and (2), respectively. These equations measure how variance swap excess return from the volatility market is spanned by volatility excess return, given by xm^{VIX}, and equity index excess return, given by xm^{SP}. Both of these returns can be achieved by trading on futures contracts available for these two indexes (for VIX futures contracts see Zhang and Zhu (2006) and Zhu and Zhang (2007)). Eq.(12) leads to a positive and significant coefficient for xm^{VIX} with a R^2 of 41.9%. The positive sign is consistent with the positive skew of the smile observed on the VIX option market (and reported in Figure 2). Indeed, an increase of the volatility leads to an increase of the volatility of volatility, thus a positive relationship exists between xm^{VIX} and $xVar$. Notice also that the constant term is significant and negative at -0.18 and larger than the mean value of -0.283 reported in Table I. As xm^{VIX} is the excess return that can be achieved by trading on the volatility market and as the VIX is closely related to the SP500, one of the largest indices, thus it can be considered as the market volatility excess return. Eq.(13) regresses variance swap excess return on SP500 excess return, it leads to a negative and significant parameter of -0.9261 with a R^2 of 24%. Here also, the sign is consistent with the well known leverage effect as a negative value for xm^{SP} (i.e. bear market condition) will lead to an increase of the VIX (i.e. xm^{VIX}) and the positive skew of the VIX option smile, illustrated in the previous regression, implies an increase of VIX's volatility, that is to say, an increase of $xVar$. Hence, the consistency of these regression results with the shapes of both the SP500 smile and VIX smile. The fact that the R^2 for Eq.(12) is larger than the R^2 for Eq.(13)

[4] In fact it is an adjusted R-square but will omit the term adjusted in the sequel.

is reasonable as $xVar$ is related to the volatility of the VIX, namely the volatility of volatility, while the VIX is related to volatility of the SP500. Lastly, as xm^{SP} can be identified with the market excess return and as the constant term in Eq.(13) is significant it suggests that the capital asset pricing model does not hold for the variance swap contract written on the volatility.

[Insert Table IV here]

Using xm^{VIX} and xm^{SP} as explanatory variables for $xVar$, it leads to estimate the following factor model

$$xVar_t = a_0 + a_1 xm_t^{\text{VIX}} + a_2 xm_t^{\text{SP}} + \epsilon_t, \tag{14}$$

with the results given in Table IV column (3). The coefficient on the second variable turns out to be not significant, although it has the correct sign, suggesting that regarding variance swap excess return the information content of SP500 excess return is already spanned by the VIX excess return. The constant term is significant and negative much in line with the values obtained either for Eq.(12) or Eq.(13).

The $xVar$ variable depends on the second moment of the VIX distribution (both risk neutral and historical) and so far we have considered as determinants the first moment of the VIX distribution, through the VIX excess return, and the first moment of the SP500 distribution also through the same quantity. The following regression

$$xVar_t = a_0 + a_1 xSkew_t + \epsilon_t, \tag{15}$$

with results reported in column (4) of Table IV, assesses the impact of the third moment distribution, through the skew swap excess return, on the second moment distribution given by $xVar$. The coefficient of $xSkew$ is positive and significant while the R^2 is high at 35.9%. The finding is in line with the high correlation between $xVar$ and $xSkew$ reported in Table II. It is interesting to note that the positive sign is also consistent with the positive slope of the VIX option smile. Indeed, as already explained an increase of the VIX implies an increase of the VIX's volatility, thus a thicker right tail distribution leading to a greater skew value. Notice also the constant term is negative and significant at -0.201, suggesting that skew swap excess return cannot fully explain variance swap excess return and, thus, of the existence of another source of risk for variance risk premium.

Combining the previous regressions, it is natural to consider

$$xVar_t = a_0 + a_1 xm_t^{\text{VIX}} + a_2 xSkew_t + \epsilon_t^a, \tag{16}$$

$$xVar_t = b_0 + b_1 xm_t^{\text{SP}} + b_2 xSkew_t + \epsilon_t^b, \tag{17}$$

as it allows us to understand whether skew swap excess return carries complementary information to the VIX and SP500 excess returns considered separately. The results are reported in columns (5) and (6) of Table IV for Eq.(16) and (17), respectively. The first regression leads to significant coefficients for both xm^{VIX} and $xSkew$ implying complementary information between these two variables. Notice also the sign values, they are consistent with intuition. Lastly, the R^2 culminates at 57.44% and the constant term is still significant. If the pair $(xm^{\text{SP}}, xSkew)$ of explanatory variables is considered instead, very similar conclusions are obtained. Namely, coefficients are both significant with correct signs and the R^2 at 52.99% is slightly lower than the previous case but remains high while the constant term is negative and significant.

Using all the variables as explanatory variables the following regression

$$xVar_t = a_0 + a_1 xm_t^{\text{SP}} + a_2 xm_t^{\text{VIX}} + a_3 xSkew_t + \epsilon_t \tag{18}$$

is performed with results given in column (7) of Table IV. In line with the regressions Eq.(14) and Eq.(16), the SP500 excess return is not significant while the others display coefficient signs consistent with the VIX smile. The R^2 is nearly equal to the one obtained when regressing only on the pair $(xm^{\text{VIX}}, xSkew)$ and the constant term is significant and negative suggesting that the CAPM does not hold as the SP500 excess return is often used as a proxy for the market excess return.

Factor models for skew swap excess return In this section we analyze the determinants of skew swap excess return by performing several regressions. We start by considering the two regressions

$$xSkew_t = a_0 + a_1 xm_t^{\text{VIX}} + \epsilon_t^a, \tag{19}$$

$$xSkew_t = b_0 + b_1 xm_t^{\text{SP}} + \epsilon_t^b, \tag{20}$$

with the results reported in Table V column (1) for Eq.(19) and column (2) for Eq.(20). The first regression leads to a positive and significant regression coefficient for the variable xm^{VIX} thereby suggesting that an increase of volatility excess return induces an increase of skew swap excess return. The more the market is volatile the more skewed is the volatility distribution and the more attractive is the skew swap for an investor. Again, this result is consistent with the positive slope of the VIX option smile as explained in the previous regressions. It can also be intuited from Table II as an increase of VIX level leads to a higher level of VIX's volatility, either risk neutral or realized, that is positively related to VIX skewness distribution, either risk neutral or realized, so a positive relationship between xm^{VIX} and $xSkew$ is natural. The R^2 is 11.49%, lower than the 41.89% found when VIX excess return is used to explain variance swap excess return. Therefore, the VIX excess return has more explanatory power for second moment related quantities than third moment related quantities, a result that seems very reasonable but nevertheless appealing to find. In contrast with results obtained for variance swap excess return regressions the constant term is not significant and won't be for any regression having the skew swap excess return as dependent variable. From Eq.(20) we deduce that SP500 excess return has no explanatory power for skew swap excess return (or third moment related quantities) as the coefficient is not significant although its sign is consistent with the shapes of the smiles (i.e. VIX and SP500 option smiles). The very weak relationship between these two variables is also confirmed by the R^2 of 0.66% that is close to zero.

[Insert Table V here]

Combining xm^{VIX} and xm^{SP} as explanatory variables for $xSkew$ leads to the regression

$$xSkew_t = a_0 + a_1 xm_t^{\text{VIX}} + a_2 xm_t^{\text{SP}} + \epsilon_t^a, \tag{21}$$

with the estimates given in column (3) of Table V. The results are consistent with those of Eq.(19) and Eq.(20) as only the coefficient of xm^{VIX} is significant and positive, as expected. The R^2 is 13.25%, marginally higher than its counterpart in regression Eq.(19) (i.e. 11.49%).

Let us now focus on explanatory power of the variance swap excess return $xVar$ for the skew swap excess return $xSkew$ by performing the set of regressions

$$xSkew_t = a_0 + a_1 xVar_t + \epsilon_t^a, \tag{22}$$

$$xSkew_t = b_0 + b_1 xm_t^{\mathrm{VIX}} + b_1 xVar_t + \epsilon_t^b, \tag{23}$$

$$xSkew_t = c_0 + c_1 xm_t^{\mathrm{SP}} + c_2 xVar_t + \epsilon_t^c, \tag{24}$$

with results reported in columns (4), (5) and (6) of Table V. For Eq.(22), we already know that the regression coefficient a_1 will be significant and positive, consistent with financial intuition, while the R^2 is equal to 35.9% from the regression Eq.(15). But in contrast with this latter regression results, the constant term will not be significant (as already mentioned). Eq.(23) and its estimated coefficients clearly show that VIX excess return does not provide any additional information than $xVar$ when it comes to explain skew swap excess return as the xm^{VIX} coefficient is not significant and the R^2 is around 35%, so identical to the one obtained when regressing on $xVar$ alone. Very similar conclusions can be achieved if the pair $(xm^{\mathrm{SP}}, xVar)$ of explanatory variables is considered (although the R^2 mildly increases from 35.9%, when $xVar$ is considered alone, to 38.3% for Eq.(24)).

Lastly, combining all the variables, the following factor model is estimated

$$xSkew_t = a_0 + a_1 xm_t^{\mathrm{VIX}} + a_2 xm_t^{\mathrm{SP}} + a_2 xVar_t + \epsilon_t^a, \tag{25}$$

with the results given in column (7), the last one, of Table V. The estimates are very much in line with those of the previous regressions. More precisely, we find that only the coefficient of variance swap excess return ($xVar$) is significant and with correct sign; the R^2 at 38% is close to its value when regressed on the pair $(xm^{\mathrm{SP}}, xVar)$ of Eq.(24); the constant term is not significant.

Considering jointly Table IV and Table V, we can conclude that to explain variance swap excess return, or quantities related to second moment of the volatility, third moment of the volatility, given by skew swap excess return, and quantities related to first moments, either volatility or stock, provide complementary information. However, to explain volatility higher moments (such as skewness) lower order moment quantities (first moments of volatility or stock) perform very poorly with the SP500 excess return being irrelevant. Though this latter finding appears to be reasonable it provides some confidence on the consistency of the results. Putting these results into a hedging portfolio point of view it suggests that higher moment related quantities (of the volatility) can only be hedged using contracts related to higher moments if a static hedge is performed and to some extent a separation between the volatility and equity markets.[5] Needless to say that a dynamic hedging strategy would allow us to solve that problem.

Fama-French factor models Following Kozhan et al. (2013) we build two linear models using Fama-French factors and take xm^{SP} as a reasonable proxy for the equity market excess return. Thus, the two variables SMB (the size factor) and HML (the book-to-market factor) are considered along with the variance excess returns $xVar$ and skew excess returns $xSkew$. We estimate the models

$$xVar_t = a_0 + a_1 xm_t^{\text{VIX}} + a_2 xSkew_t + a_3 SMB_t + a_4 HML_t + \epsilon_t^a, \tag{26}$$

$$xVar_t = b_0 + b_1 xm_t^{\text{VIX}} + b_2 xm_t^{\text{SP}} + b_3 xSkew_t + b_4 SMB_t + b_5 HML_t + \epsilon_t^b, \tag{27}$$

$$xSkew_t = c_0 + c_1 xm_t^{\text{VIX}} + c_2 xVar_t + c_3 SMB_t + c_4 HML_t + \epsilon_t^c, \tag{28}$$

$$xSkew_t = e_0 + e_1 xm_t^{\text{VIX}} + e_2 xm_t^{\text{SP}} + e_3 xVar_t + e_4 SMB_t + e_5 HML_t + \epsilon_t^e, \tag{29}$$

and the coefficient values, reported in Table VI, allow us to draw conclusions very similar to those obtained by Kozhan et al. (2013). Whether $xVar$ or $xSkew$ is considered as dependent variable the Fama-French factors are not significant. This slightly contrasts with Carr and Wu (2009) where it is shown that for both SP500 (SPX) and SP100 (OEX) the size factor is significant. To some extent there is an inconsistency between Kozhan et al. (2013) and Carr and Wu (2009) as they find contradicting conclusions for the size factor for the SP500 that may be explained by the samples that are different.

[Insert Table VI here]

The impacts of SMB and HML are insignificant here and this disappointing performance of the two Fama-French factors (i.e. SMB and HML) might be explained by the fact that they are built using using two equity portfolios. As mentioned earlier, volatility products can be treated as an asset class itself, with specific properties different from the equity market. Although these two markets are related, the well-known leverage effect being a good example, one needs specific

[5]Indeed, by construction the linear regression suggests a static hedging strategy.

or bespoke factors for this market. A similar problem appeared for the currency market and it is precisely the contribution of Lustig et al. (2011) to propose a common risk factor for that market that they name HML_{FX}. Its relevance was illustrated in Jurek and Xu (2014) on a problem related to carry trade. Our results suggest that similar factors, specific to the volatility market, should be built but this objective is beyond the scope of the present paper.

Differences between volatility index and equity index markets It is of interest to compare the results obtained here for the volatility index market, through the use of VIX options, and those obtained for the equity index market, through the use of SP500 options, analyzed in Kozhan et al. (2013) as these are two different asset classes (i.e. volatility versus equity). It would allow us to assess whether there are structural differences between these two markets. Notice that Table II, reporting variables' correlation, already displays important differences with those observed for the SP500. Also, the close relationship between the VIX and the SP500 suggests to consider them jointly. As the most important variables are the skew swap excess return, denoted $xSkew$ in this work while denoted xs in Kozhan et al. (2013), and the variance swap excess return, denoted $xVar$ in this work while denoted xv in Kozhan et al. (2013), we will restrict the comparison to these variables as well as to the regressions involving them.

The first important difference is the correlation between $xSkew$ and $xVar$ that is equal to 0.897 in Kozhan et al. (2013), a value much higher than the 0.607 obtained here, that leads these authors to question whether there are two separate risk premiums. In Kozhan et al. (2013), when regressing the variance swap excess return on equity index excess return the constant term is significant and similar conclusion is achieved for the skew swap excess return suggesting that the capital asset pricing model does not hold (see Table 4 of Kozhan et al. (2013)). For the VIX, this conclusion mainly holds for $xVar$, see Table IV column (2) along with Eq.(13), but for the skew, Table V column (2) along with Eq.(20) show that the constant term is not significant.[6]

For the SP500 option market when regressing $xVar$ on the equity index excess return xm^{SP} and the skew swap excess return ($xSkew$) the constant term is not significant and swapping $xVar$

[6]Notice that in this latter case the R^2 is close to zero, so to some extent the conclusion also holds for the skew but the argument is different.

and $xSkew$ leads to similar conclusion (see Table 5 of Kozhan et al. (2013)). Thus, these authors rightly conclude that $xVar$ is spanned by the pair $(xm^{\text{SP}}, xSkew)$ and $xSkew$ by $(xm^{\text{SP}}, xVar)$. For the VIX market the situation is slightly different as the constant term remains significant only when $xVar$ is regressed on the other two variables, see Table IV column (6) where the t-statistics is found to be -2.03, suggesting that an additional factor is needed to fully explain variance swap excess return beyond the information provided by xm^{SP} and $xSkew$.

Related to the high correlation level between $xVar$ and $xSkew$ in Kozhan et al. (2013) (i.e. 0.897), whenever one of the variables is used as explanatory variable while the other is used a dependent one, the R^2 is extremely high leaving little room for other factors (still these authors found xm^{SP} to be significant). In the VIX case, although there is a decent correlation between $xVar$ and $xSkew$, additional variables such as equity index excess return xm^{SP} or volatility excess return xm^{VIX} do improve the R^2 when $xVar$ is the dependent variable.

Lastly, the coefficient signs remain consistent with intuition for all factor models, and in particular adding a new explanatory variable to a given regression does not change the coefficient signs. From an hedging point of view, it means that one does not have to reverse a position on a given instrument, an aspect that is extremely appealing in practice. This sharply contrasts with Kozhan et al. (2013) as they find a negative coefficient sign for xm^{SP} (equity index excess return) when the skew swap excess return $xSkew$ is regressed on this variable alone (see Table 4 in that paper) while the sign turns out to be positive when the variance swap excess return is added as an explanatory variable. This is a direct consequence of the strong correlation between $xSkew$ and $xVar$ extracted from SP500 options. Also, for skew swap excess return the equity index excess return is irrelevant for the volatility option market while it is for the SP500 option market. It suggests that for quantities related to volatility higher moments quantities related to equity (index) moments convey little information, as a result there is an idiosyncratic volatility factor.

These are the salient differences between the equity index option market and the volatility index option market underlining very specific characteristics of the volatility market with the related consequences in terms of hedging strategies.

5 Conclusion

In this work we provide an analysis of variance and skew risk premiums for the volatility market using VIX options and the methodology proposed by Kozhan et al. (2013). We find that swap excess return can be partially explained by equity and volatility excess returns along with skew swap excess return. On the contrary, to explain skew swap excess return only the variance swap contract is relevant, investing on the equity index (i.e. SP500 in that case) is of no use and volatility index provides no additional information beyond the variance swap contract. Quite remarkably, all the results are consistent with the shape of the smiles observed on the SP500 and VIX option markets. Certain results are in sharp contrast with those obtained for the SP500 option market, they underline certain specifics of the volatility market.

Our work suggests several extensions. First, applying the methodology to other volatility option markets, such as crude oil for example, should confirm the results obtained here. Second, the availability of exotic volatility options, such as options on leveraged volatility exchange traded fund as presented in Bao et al. (2012), is an alternative to extract implied-moments for the volatility distribution and assessing whether all these products convey consistent information is an important question. We leave these open problems for future research.

References

M. Ammann and R. Buesser. Variance risk premiums in foreign exchange markets. *Journal of Empirical Finance*, 23:16–32, 2013. doi: 10.1016/j.jempfin.2013.04.006.

A. Ang, R. Hodrick, Y. Xing, and X. Zhang. The Cross-Section of Volatility and Expected Returns. *The Journal of Finance*, 61(1):259–299, 2006. doi: 10.1111/j.1540-6261.2006.00836.x.

G. Bakshi and D. Madan. A theory of volatility spreads. *Management Science*, 52(12):1945–1956, 2006. doi: 10.1287/mnsc.1060.0579.

G. Bakshi, N. Kapadia, and D. Madan. Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. *Review of Financial Studies*, 16(1):101–143, 2003. doi: 10.1093/rfs/16.1.0101.

Q. Bao, S. Li, and D. Gong. Pricing VXX Option with Default Risk and Positive Volatility Skew. *European Journal of Operational Research*, 223:246–255, 2012. doi: 10.1016/j.ejor.2012.06.006.

S. J. Byun and J. S. Kim. The information content of risk-neutral skewness for volatility forecasting. *Journal of Empirical Finance*, 23:142–161, 2013. doi: 10.1016/j.jempfin.2013.05.006.

P. Carr and D.Madan. Towards a Theory of Volatility Trading. *in R. Jarrow (ed.), Risk Book on Volatility*, pages 417–427, 1998.

P. Carr and L. Wu. Variance Risk Premiums. *Review of Financial Studies*, 22(3):1131–41, 2009. doi: 10.1093/rfs/hhn038.

E. Derman, I. Kani, and M. Kamal. Trading and Hedging Local Volatility. *Journal of Financial Engineering*, 6 (3):233–268, 1997.

J. Detemple and C. Osakwe. The Valuation of Volatility Options. *European Finance Review*, 4(1):21–50, 2000. doi: 10.1023/A:1009814324980.

J.-C. Duan and W. Zhang. Forward-looking market risk premium. *Management Science*, 60(2):521–538, 2014. doi: 10.1287/mnsc.2013.1758.

J. Fleming. The quality of market volatility forecasts implied by S&P 100 index option prices. *Journal of Empirical Finance*, 5:317–345, 1998. doi: 10.1016/S0927-5398(98)00002-4.

A. Grünbichler and F. Longstaff. Valuing Futures and Options on Volatility. *Journal of Banking and Finance*, 20:985–1001, 1996. doi: 10.1016/0378-4266(95)00034-8.

D. Huang and I. Shaliastovich. Volatility-of-volatility risk. *WorkingPaper SSRN-2497759*, 2014.

J. W. Jurek and Z. K. Xu. Option-Implied Currency Risk Premia. *WorkingPaper SSRN-2338585*, 2014. doi: 10.2139/ssrn.2338585.

E. Konstantinidi and G. Skiadopoulos. How does the market variance risk premium vary over time? Evidence from S&P 500 variance swap investment returns. *Journal of Banking & Finance*, 62:6275, 2016. doi: 10.1016/j.jbankfin.2015.10.006.

A. Kostakis, N. Panigirtzoglou, and G. Skiadopoulos. Market timing with option-implied distributions: A forward-looking approach. *Management Science*, 57(7):1231–1249, 2011. doi: 10.1287/mnsc.1110.1346.

R. Kozhan, A. Neuberger, and P. Schneider. The Skew Risk Premium in the Equity Index Market. *Review of Financial Studies*, 26(9):2174–2203, 2013. doi: 10.1093/rfs/hht039.

G.-H. Lian and S.-P. Zhu. Pricing vix options with stochastic volatility and random jumps. *Decisions in Economics and Finance*, 36(1):71–88, 2013. doi: 10.1007/s10203-011-0124-0.

H. Lustig, N. Roussanov, and A. Verdelhan. Common Risk Factors in Currency Markets. *The Review of Financial Studies*, 24(11):3731–3777, 2011. doi: 10.1093/rfs/hhr068.

A. Neuberger. Realized skewness. *Review of Financial Studies*, 25(11):3423–3455, 2012. doi: 10.1093/rfs/hhs101.

M. Neumann and G. Skiadopoulos. Predictable dynamics in higher order risk-neutral moments: Evidence from the S&P 500 options. *Journal of Financial and Quantitative Analysis*, 48(3):947–977, 2013. doi: 10.1017/S002210901300032X.

W. Newey and K. West. A simple positive semidefinite, heteroscedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3):703–708, 1987. doi: 10.2307/1913610.

Y. Park. The effects of asymmetric volatility and jumps on the pricing of VIX derivatives. *Journal of Econometrics*, 192(1):313–328, 2016. doi: 10.1016/j.jeconom.2016.01.001.

M. Prokopczuk and C. Wese Simen. Variance risk premia in commodity markets. *WorkingPaper SSRN-2195691*, 2014. doi: 10.2139/ssrn.2195691.

A. Sepp. VIX option pricing in a jump-diffusion model. *Risk Magazine*, 21(4):84–89, 2008.

R. E. Whaley. Derivatives on Market Volatility: Hedging Tools Long Overdue. *The Journal of Derivatives*, 1: 71–84, 1993.

B. Yang, P. Christoffersen, and K. Jacobs. Market Skewness Risk and the Cross Section of Stock Returns. *Journal of Financial Economics*, 107(1):46–68, 2013. doi: 10.1016/j.jfineco.2012.07.002.

J. E. Zhang and Y. Zhu. VIX futures. *Journal of Futures Markets*, 26(6):521–531, 2006. doi: 10.1002/fut.20209.

J. E. Zhang, J. Shu, and M. Brenner. The new market for volatility trading. *Journal of Futures Markets*, 30(9): 809–833, 2010. doi: 10.1002/fut.20448.

H. M. Zhao, J. E. Zhang, and E. C. Chang. The Relation between Physical and Risk-neutral Cumulants. *International Review of Finance*, 13(3):345–381, 2013. doi: 10.1111/irfi.12013.

Y. Zhu and J. E. Zhang. Variance term structure and VIX futures pricing. *International Journal of Theoretical and Applied Finance*, 10(1):111–127, 2007. doi: 10.1142/S0219024907004123.

A Tables

Table I: Descriptive statistics

	Mean	Std. dev.	Q1	Median	Q3
Var	0.071	0.022	0.057	0.065	0.081
$rVar$	0.049	0.041	0.023	0.032	0.068
$xVar$	-0.283	0.616	-0.668	-0.511	-0.118
$Skew$	0.031	0.012	0.024	0.033	0.040
$rSkew$	0.018	0.022	0.005	0.011	0.019
$xSkew$	-0.293	1.341	-0.835	-0.686	-0.329

Note: Descriptive statistics such as mean, standard deviation, the 25th percentile, median, and 75th percentile for the variables: the risk neutral variable Var given by Eq.(2), the realized variance $rVar$ given by (3), the variance swap excess return $xVar$ given by Eq.(4), the risk neutral skew $Skew$ given by Eq.(5), the realized skew given by Eq.(8) and the skew swap excess return $xSkew$ given by Eq.(9). Sample with monthly frequency ranging from January 2010 to July 2015.

Table II: Correlations

	Var	$rVar$	$xVar$	$Skew$	$rSkew$	$xSkew$
Var	1.000	0.209	-0.098	0.202	0.115	0.137
$rVar$		1.000	0.933	0.058	0.888	0.702
$xVar$			1.000	0.046	0.868	0.607
$Skew$				1.000	0.133	-0.287
$rSkew$					1.000	0.661
$xSkew$						1.000

Note: Correlation between the variables: the risk neutral variable Var given by Eq.(2), the realized variance $rVar$ given by (3), the variance swap excess return $xVar$ given by Eq.(4), the risk neutral skew $Skew$ given by Eq.(5), the realized skew $rSkew$ given by Eq.(8) and the skew swap excess return $xSkew$ given by Eq.(9). Sample with monthly frequency ranging from January 2010 to July 2015.

Table III: Realized moments versus implied moments

	Const.	Var	$Skew$	Adj. R^2(%)
$rVar$	0.022	0.392		2.91
	(1.63)	(2.24)		
$rSkew$	0.010		0.243	0.29
	(1.37)		(1.25)	

Note: Regressions of $rVar$, the realized variance, given by (3) on Var, the risk neutral variable, given by Eq.(2) and $rSkew$, the realized skew, given by Eq.(8) on $Skew$, the risk neutral skew, given by Eq.(5). The t-statistics are computed according to Newey and West (1987). Sample with monthly frequency ranging from January 2010 to July 2015.

Table IV: Factor models for variance swap excess return

				$xVar$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Const.	-0.186	-0.188	-0.182	-0.201	-0.151	-0.130	-0.137
	(-3.31)	(-2.36)	(-3.21)	(-2.57)	(-2.89)	(-2.03)	(-2.61)
xm^{VIX}	2.010		1.906		1.539		1.148
	(6.10)		(5.44)		(5.26)		(4.18)
xm^{SP}		-9.621	-0.874			-8.096	-3.093
		(-3.89)	(-0.43)			(-4.46)	(-1.92)
$xSkew$				0.279	0.197	0.250	0.207
				(2.47)	(2.65)	(3.41)	(2.97)
Adj. R^2(%)	41.89	24.18	41.09	35.9	57.44	52.99	57.99

Note: Regressions of $xVar$, the variance swap excess return given by Eq.(4), on explanatory variables based on $(xm^{\text{VIX}}, xm^{\text{SP}}, xSkew)$. xm^{VIX} is the excess return of the VIX index, xm^{SP} is the excess return of the SP500 and $xSkew$, given by Eq.(9), is the skew swap excess return. The t-statistics are computed according to Newey and West (1987). Sample with monthly frequency ranging from January 2010 to July 2015.

Table V: Factor models for skew swap excess return

				$xSkew$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Const.	-0.177	-0.233	-0.221	0.081	0.087	0.060	0.042
	(-0.87)	(-1.25)	(-1.31)	(0.32)	(0.33)	(0.25)	(0.18)
xm^{VIX}	2.395		3.668		-0.457		0.920
	(2.84)		(2.16)		(-0.52)		(1.40)
xm^{SP}		-6.093	10.743			8.870	12.004
		(-0.89)	(0.96)			(0.97)	(1.19)
$xVar$				1.322	1.419	1.555	1.442
				(3.60)	(2.78)	(2.68)	(2.73)
Adj. R^2(%)	11.49	0.66	13.25	35.9	35.18	38.4	38.13

Note: Regressions of $xSkew$, the skew swap excess return given by Eq.(9), on explanatory variables based on $(xm^{\text{VIX}}, xm^{\text{SP}}, xVar)$. xm^{VIX} is the excess return of the VIX index, xm^{SP} is the excess return of the SP500 and $xVar$ the variance swap excess return given by Eq.(4). The t-statistics are computed according to Newey and West (1987). Sample with monthly frequency ranging from January 2010 to July 2015.

Table VI: Fama-French risk factors for variance and skew swap excess returns

	Const.	xm^{VIX}	xm^{SP}	$xVar$	$xSkew$	SMB	HML	Adj. R^2(%)
$xVar$	-0.147	1.525			0.203	-0.018	0.003	56.53
	(-2.70)	(5.56)			(2.74)	(-0.78)	(0.12)	
$xVar$	-0.134	1.154	-2.948		0.212	-0.016	0.002	56.95
	(-2.51)	(4.38)	(-1.83)		(3.06)	(-0.70)	(0.065)	
$xSkew$	0.070	-0.435		1.406		0.110	-0.011	36.53
	(0.29)	(-0.53)		(2.96)		(1.62)	(-0.21)	
$xSkew$	0.032	0.802	10.785	1.428		0.096	-0.005	38.74
	(0.14)	(1.29)	(1.05)	(2.64)		(1.54)	(-0.098)	

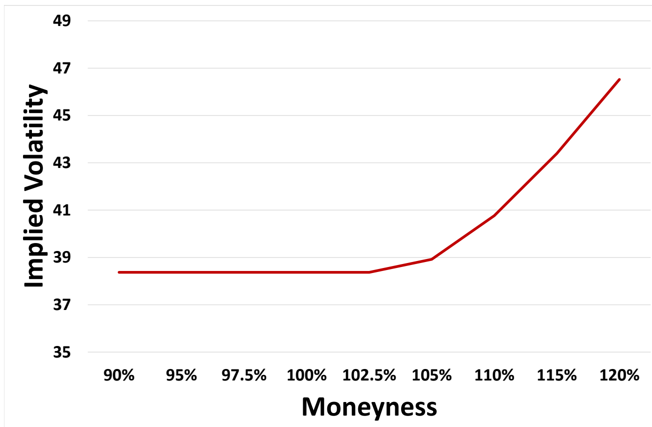
Note: Regressions of the skew swap excess return $xVar$, given by Eq.(4), and the skew swap excess return $xSkew$, given by Eq.(9), on explanatory variables based on $(xm^{\text{VIX}}, xm^{\text{SP}}, SMB, HML)$. xm^{VIX} is the excess return of the VIX index, xm^{SP} is the excess return of the SP500 and the two Fama-French factors SMB (the size factor) and HML (the book-to-market factor). The t-statistics are computed according to Newey and West (1987). Sample with monthly frequency ranging from January 2010 to July 2015.

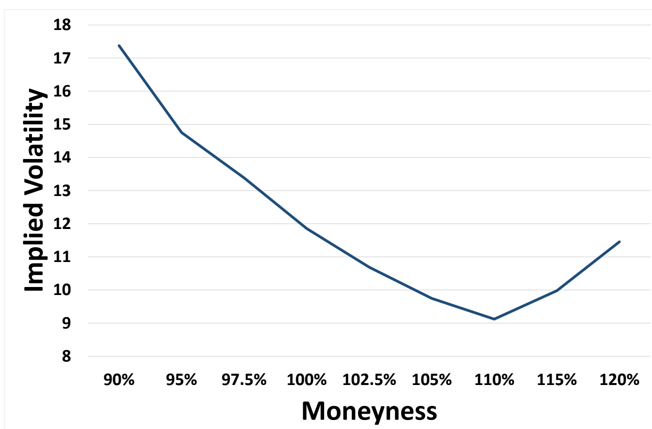
B Figures

Figure 1: VIX futures open interest

VIX Futures open interest from March 2004 October 2015. The trading activity on these products took off in 2010 and are the natural tools to hedge options written on the VIX.

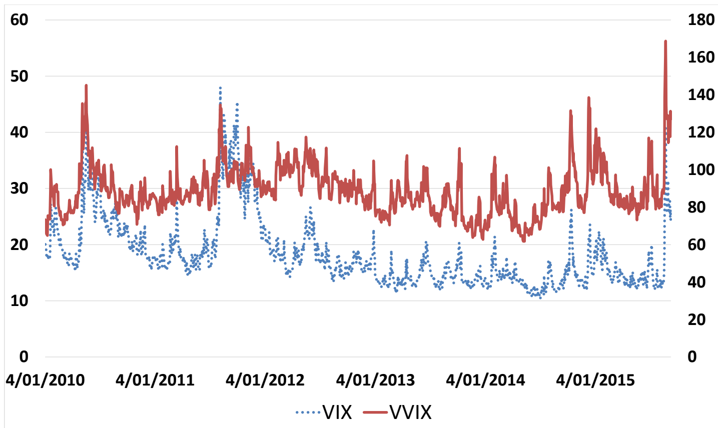
Figure 2: Smile of VIX and SP500 options





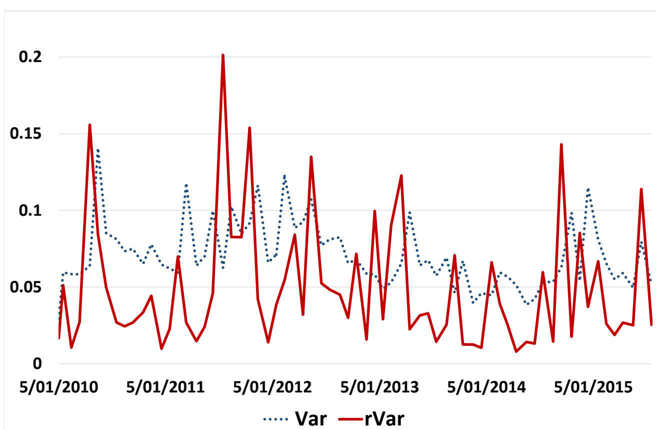
Implied volatility for VIX options (upper figure) and SP500 options (lower figure) on June 2, 2014 with 3-month time to maturity. The positive skew is one of the main feature on the VIX smile and contrast with the downward sloping curve observed for SP500 options.

Figure 3: Evolutions of VIX and VVIX



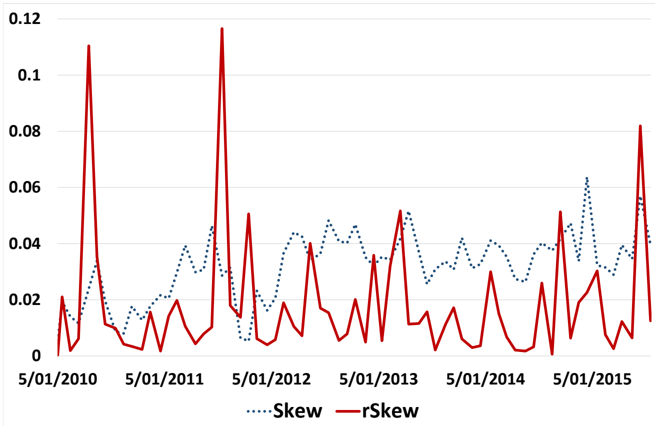
Evolutions of VIX and VVIX (VIX of VIX options) from January 4, 2010 to September 10, 2015 (round dot for the VIX and solid line for the Vvix). The positive correlation between these two indexes is apparent.

Figure 4: Risk-neutral and realized variances



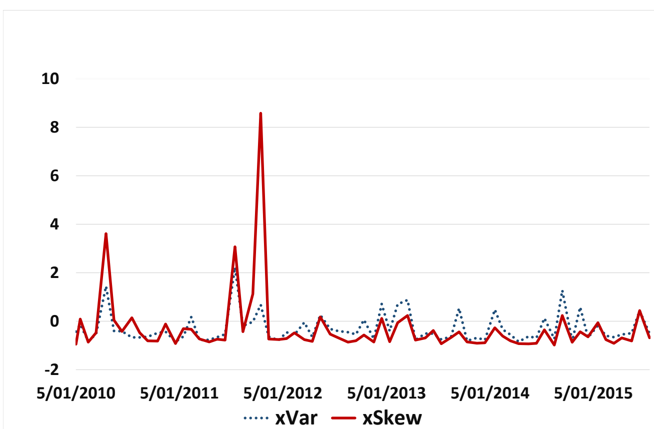
Evolutions of risk-neutral and realized variances from January 4, 2010 to September 10, 2015 (blue round dot for the risk-neutral variance and red solid line for the realized variance). In general, risk-neutral variance is greater than realized variance, while the volatility of the former is smaller than the latter.

Figure 5: Risk-neutral and realized skews



Evolutions of risk-neutral and realized skews from January 4, 2010 to September 10, 2015 (blue round dot for the risk-neutral skew and red solid line for the realized skew). In general, risk-neutral variance is greater than realized variance, while the volatility of the former is smaller than the latter.

Figure 6: Evolutions of VIX variance and skew risk premiums



Evolutions of VIX variance and skew risk premiums from January 4, 2010 to September 10, 2015 (blue round dot for the variance swap excess return and red solid line for the skew swap excess return). The positive correlation between these two indexes is apparent. Moreover, skew risk premium is more sensitive to market crashes as the magnitude of spikes are greater when a crisis occurs.