Optimal Contracting, Ownership Structure and Asset Pricing*

Hae Won (Henny) Jung
University of Melbourne
hae.jung@unimelb.edu.au

Qi Zeng
University of Melbourne
qzeng@unimelb.edu.au

Ajay Subramanian
Georgia State University
asubramanian@gsu.edu

December 10, 2014

Abstract

This paper proposes a dynamic equilibrium model of asset pricing and executive compensation. A representative, all-equity firm is owned by two types of risk-averse investors — a large shareholder (blockholder) and a continuum of small shareholders. The large shareholder holds a long-term view and trades the firm's shares infrequently. From her controlling equity stake in the firm, she plays a dominant role in hiring a risk-averse manager and implementing an optimal compensation scheme to incentivise the manager to exert effort. Small dispersed and competitive shareholders, however, trade the firm's shares continuously, from which its equilibrium stock price is determined. Our unified framework allows us to explore the endogenous determination of key variables in equilibrium — the firm's ownership dynamics, managerial contracts, and stock prices and excess dollar returns. We show that, in contrast to the benchmark owner-manager case, incorporating the optimal contracting problem results in a stochastically evolving stock price and nontrivial dynamics for the large shareholder's ownership. In addition, we predict the correlations among the equilibrium variables in response to variations in the model parameters, which can help to explain the mixed empirical evidence in the literature.
Keywords: Ownership Structure, Optimal Managerial Contracts, Equilibrium Asset Pricing
JEL Code: G12, G32, D81

*We thank Gil Aharoni, Lyndon Moore, Neal Galpin, Bruce Grundy, Spencer Martin, Filippo Massari and seminar participants at the 2014 SED Meetings, the 2013 FIRN Australian Finance Conference, and the University of Melbourne. Any remaining errors are ours.

1 Introduction

Research in financial economics reveals a fundamental dichotomy between its two cornerstones; asset pricing and corporate finance. At a broad level, corporate finance research focuses on the separation of ownership and control in modern corporations. Agency conflicts among a firms various stakeholders that arise from this separation endogenously influence firms cash flows, but the markets valuation of the cash flows through the underlying stochastic discount factor or pricing kernel is typically exogenously specified. In contrast, asset pricing models typically view corporate cash flows as exogenous and focus on identifying a stochastic discount factor or pricing kernel to price the assumed cash flows (Gorton et al., 2014). In reality, however, the cash flows of firms and their valuation by market participants are simultaneously and endogenously determined. A recent and nascent literature has started to pay attention to this issue by exploring potential implications of agency problems for influencing corporate cash flows and thereby the prices of corporate securities.

Even though it is clearly important to incorporating agency problems in studying asset prices, ignoring two central governance mechanisms that have been extensively addressed as typical remedies for those agency problems may overestimate or underestimate the impact of agency conflicts. However, it is, in general, challenging to link optimal contracting and asset pricing models. This paper aims to propose a unified framework that separates a representative, all-equity firm's strategic principal-agent problem from the competitive asset pricing process by distinguishing a large shareholder from small atomistic shareholders. The large shareholder holds a long-term view and trades the firm's shares infrequently. From her controlling equity stake in the firm, she acts strategically both in the security market and in major corporate decisions. Specifically, she plays a dominant role in hiring a risk-averse manager and implementing an optimal compensation scheme to incentivise the manager to exert effort. Small dispersed shareholders, however, trade the firm's shares continuously, from which its equilibrium stock price is competitively determined.

The asset pricing framework with both large and small shareholders mainly builds on Admati et al. (1994) and DeMarzo and Urošević (2006) that analyze the optimal trading and ownership policy of a large shareholder who can affect the firm's output and asset prices through a costly monitoring. These studies, however, abstract away from the optimal contracting problem between the manager and shareholders. DeMarzo and Urošević (2006) formulate a moral hazard problem as a reduced-form that directly takes the usual trade-off between incentives and risk sharing rather than deriving the trade-off through agency contracting. In the context of our general framework, their model corresponds to the benchmark scenario in which the firm is controlled by an owner-manager whose costly action directly affects the firm's output process.

Using our framework, we provide a full-fledged and dynamic equilibrium description of the interactions among a firms large ownership dynamics, managerial contracts, and stock prices and returns. We show that incorporating the standard managerial moral hazard problem and solving for the optimal agency contract significantly alters the implications of DeMarzo and Urošević (2006), which hereafter we denote by D-U, for equity prices and large ownership dynamics. In the benchmark owner-manager case, which refers to the scenario studied by D-U, one obtains the counterfactual implication that the large shareholder's trading decision and the firm's share price are time deterministic. In contrast, the incorporation of the separation between the large shareholder and the manager results in a stochastically evolving stock price and nontrivial dynamics for the large shareholder's ownership. Also, the owner-manager case involves a more volatile stock return, a higher expected stock return, and a higher Sharpe ratio overall, controlling for potentially different underlying parameters including the firm's intrinsic risk and productivity and risk preferences of the manager and shareholders. This observation partially explains the higher equity risk premium in emerging markets, in which owner-manager firms, that is, family-controlled public firms are more prevalent, than in developed markets. Further, in contrast to the owner-manager case, the large shareholder can optimally choose the manager's contract which has an impact on the firm's stock return volatility and thus the large shareholder's risk premium.

Consequently, the large shareholder tends to hold a higher equity stake in the firm when the firm's cash flows are more volatile or when the firm's productivity is lower, which is the opposite to the prediction from the owner-manager case. Lastly, the large shareholder may find it optimal to sell her stake entirely relatively quickly when the firm's underlying risk falls sufficiently far, when the relative bargaining power of the manager is higher, or when recontracting with more expensive contract terms needs to take place more frequently.

As a firm's large shareholder ownership, managerial incentives, and stock returns are all endogenously determined in our framework, our comparative statics analysis shows their equilibrium correlations as one of the underlying parameters changes. In particular, we can address how two central governance mechanisms – large shareholder ownership and managerial incentives — interact to affect the firm's expected stock returns and stock return volatility. First, we show that both large shareholder ownership and managerial incentives are negatively correlated with the firm's expected stock return except when the manager's or shareholders' risk preference varies. Consequently, an empirical investigation of the impact of each governance mechanism on a firm's expected stock return needs to be performed by taking into account the risk preferences of different players. Second, the negative relation between managerial incentives and the stock return volatility always holds with any change in parameter values. Third, the two governance mechanisms tend to be complements to each other (that is, a positive relation between managerial incentives and large shareholder ownership) rather than substitutes for each other except when the relative risk aversion between the large shareholder and the manager varies.

This paper is closely related to the recent literature that incorporates an agency problem into an asset pricing model — in particular, Ou-Yang (2005) and Gorton et al. (2014). In these studies, shareholders act competitively in the stock market, yet they strategically coordinate together to provide managerial incentives. In contrast, as noted earlier, we separate the optimal contracting problem between the large shareholder and the manager from the determination of asset prices by adopting the framework of D-U. This is consistent with what we observe in reality from typical large shareholders, such as venture capitalists and angel investors before a firm goes public, and institutional investors as well as other significant blockholders afterwards. Further, regarding the managerial optimal contract, Gorton et al. (2013) implement the manager's compensation by his share trading and Ou-Yang (2005) considers a lump-sum compensation payment in line with work by Holmstrom and Milgrom (1987). In contrast, we consider a more general contracting problem by adopting the continuous value approach of recent continuous-time contracting models (e.g., Sannikov, 2008; Williams, 2009; He, 2011) in which the agent's utility process is an additional state variable for the principal's problem and intermediate consumption by both the principal and agent are allowed.

Among studies in the vast literature on corporate governance, our study is also close to the ongoing literature initiated by Gompers, Ishii and Metrick (2003) that explores the effects of corporate governance mechanisms on equity price. Cremers and Nair (2005) show complementary interactions between internal (active shareholders) and external (market for corporate control) governance mechanisms in generating long-term abnormal returns. Parigi, Pelizzon, and von Thadden (2013) both theoretically and empirically show that the quality of corporate governance, which is endogenously chosen, correlates positively with CAPM beta and idiosyncratic volatility and negatively with returns on assets.

Lastly, in the asset pricing literature, there are also several studies that examine possible implications of large shareholders (Cvitanic, 1997; El Karoui, Peng and Quenez, 1997; Cuoco and Cvitanic, 1998). In this literature, however, the effects of their trading on asset prices are assumed to be exogenous, which distinguishes ours from the existing literature because we endogenously derive the effects of the large shareholder's optimal contracting decision on asset pricing.

The remainder of the paper is as follows. Section 2 lays out the general setup of our model. In Section 3, we analyze a benchmark owner-manager case using a CARA-normal framework in which the owner-manager makes a trading policy and effort choice in discrete time. Section 4 solves for the equilibrium in our main model with optimal contracting. Section 5 discusses the implications of our model, and Section 6 concludes. All detailed proofs are presented in the appendix.

2 General Setup

We study a continuous-time principal-agent problem in an asset pricing framework. There is a representative all-equity firm in the economy. As in Holmstrom and Tirole (1993), Admati et. al (1994) and DeMarzo and Urošević (2006), the firm's shareholders are comprised of two groups: "large" shareholders or blockholders who hold a block of shares in the firm and a continuum of dispersed small shareholders who collectively hold the remaining equity stake. In reality, major strategic corporate decisions require the approval of corporate boards that are significantly influenced by large shareholders (e.g., see Shleifer and Vishny (1997), Holderness (2003), Tirole (2006).) Cronqvist and Fahlenbrach (2009) document evidence for significant blockholder effects on a number of important corporate decisions such as investment, financial, and executive compensation policies. For simplicity, we ignore strategic behaviors among different blockholders, that is, they behave as a monolithic unit in their collective interest, so that we refer to them as a single representative blockholder. Because our focus is on asset pricing implications, we assume that both types of shareholders are risk averse.

The large shareholder plays a dominant role in hiring a risk-averse manager to operate the firm and determining the manager's optimal compensation contract. As in traditional principal-agent models with moral hazard (see Laffont and Martimort (2002)), the manager can affect the firm's output process by exerting costly effort. The large shareholder does not observe the manager's effort directly, but instead indirectly induces the manager to choose a recommended level of effort by offering him an optimal incentive contract that is contingent on the firm's observable output. The manager owns no shares in the firm, but her optimal incentive contract could potentially be implemented through (restricted) stock grants.

Based on prior research on firm ownership including the aforementioned studies, we consider different holding periods for the two groups of investors. As in DeMarzo and Urošević (2006), small shareholders trade shares of the firm continuously, whereas the large shareholder trades the firm's shares at a discrete set of finite dates with her shareholdings within each time interval being held constant. Our analysis allows for the time interval between successive trading dates to be arbitrary so that we can consider the continuous-time limit in which the large shareholder trades continuously. For simplicity, we assume that the large shareholder offers a new contract to the manager at each trading date and commits to the contract until the next trading date. In addition to the firm's shares, both types of investors can trade a risk-free bond (savings account) continuously. We now describe the various elements of the model in detail.

2.1 Firm's Output Process

We consider a continuous-time model with finite horizon $[0, T]$. Uncertainty is generated by a standard Brownian motion Z_t on a probability space $(\Omega, \mathcal{F}, P)$ with the complete and augmented information filtration $\mathbb{F} = \{\mathcal{F}_t, 0 \leq t \leq T\}$. The firm's cumulative output X_t evolves as follows:

$$dX_t = \mu(a_t, t)dt + \sigma dZ_t, \tag{1}$$

where a_t is the manager's effort level and the constant $\sigma > 0$ is the volatility of cash flows. As in the standard principal-agent model, managerial effort influences the drift but not the volatility of the firm's output. The output process is publicly observable, and all instantaneous cash flows net of the manager's compensation are paid out to shareholders.

2.2 Large and Small Shareholders

The firm has a representative large shareholder L and a continuum of small, dispersed shareholders S (uniformly indexed over the unit interval, that is, $S \in [0, 1]$). We normalize the total number of the firm's shares outstanding to one. Small shareholders trade continuously, while the large shareholder trades on a finite set of dates $\{t_i\}\,; i = 0, ..., N$ with $t_N = T$. We denote the shares held by a small shareholder, S, at date t as $\Theta_{S,t}$ and the shares held by L as Θ_t. $L's$ shareholding process is right continous with left limits. If L's shareholdings just prior to her trade at date t_i are $\Theta_{t_i}^-$, the difference $\Theta_{t_i} - \Theta_{t_i}^- > 0 (< 0)$ represents the number of shares that she optimally purchases (sells) at date t_i. Her equity stake remains constant within the interval between any subsequent dates: that is, $\Theta_t = \Theta_{t_i}$ for $t \in [t_i, t_{i+1})$. For the case $i = N$, $t_{N+1} = \infty$. Due to the market clearing condition for the firm's shares, the total number of shares collectively held by small shareholders is $\int_0^1 \theta_{S,t} dS = 1 - \Theta_t = 1 - \Theta_{t_i}$.

Investors also have access to a risk-free bond that is in perfectly elastic supply so that it pays a continuously compounded constant return $r > 0$. This assumption can be relaxed by alternatively assuming that the risk-free bond is in zero net supply, which would endogenously determine the risk-free rate over time. The investors are initially endowed with wealth $Y_{i,0} \geq 0,$ for $i = L, S$. Both groups of investors are risk averse and their preferences are described by the utility function $u^i(c_{i,t}),$ for $i = L, S$ with the following properties: $u_c^i \equiv \partial u^i / \partial c > 0$ and $u_{cc}^i \equiv \partial^2 u^i / \partial c^2 < 0$ where $c_{i,t}$ represents their instantaneous consumption at time t. The investors maximize their expected utilities by making optimal portfolio and consumption decisions.

2.3 Managerial Preferences and Contracting

L ires a risk-averse manager, M, and offers him a sequence of explicit contracts with each contract, Π_i, valid over the time interval, $[t_i, t_{i+1})$. The manager continuously chooses an effort level $a_t \in [\underline{a}, \overline{a}]$ that affects the firm's output X_t as shown in (1), and incurs the instantaneous effort cost, $\Psi(a_t, t)$ that is twice continuously differentiable, increasing and convex in the effort level, a_t. The manager's choice of effort is unobservable by shareholders. It is convenient to augment the definition of the manager's contract to include the recommended effort process as well as the compensation payments to the manager, that is, $\Pi_i = \{c_{M,t}, a_t;\ \text{for } t_i \leq t < t_{i+1}\}$. If $\mathcal{F}_t$ denotes the information filtration generated by the history of the firm's output process up to time t, both the manager's effort and compensation flows specified in the contract are $\mathcal{F}_t$-adapted stochastic processes. We assume that the manager is restricted to trade the firm's shares and the risk-free bond, so that the only source of income for his consumption is the compensation payment.[1]

The manager's utility function is $u^M(c_{M,t}, a_t)$ with the following properties: $u_c^M \equiv \partial u^M/\partial c > 0, u_{cc}^M \equiv \partial^2 u^M/\partial c^2 < 0, u_a^M \equiv \partial u^M/\partial a < 0$ and $u_{aa}^M \equiv \partial^2 u^M/\partial a^2 < 0$. Given the manager's contract Π_i that lasts for $[t_i, t_{i+1})$, the manager's continuation value at time t, that is, his expected utility from the contract, is given by

$$W_{M,t}(c_M, a) \equiv E_{M,t}\left[\int_t^{t_{i+1}} e^{-\delta_M(\tau-t)} u^M(c_{M,\tau}, a_\tau)d\tau\right]. \tag{2}$$

The manager's contracts must be incentive compatible for the manager. That is, given the stream of the contractually promised compensation payoff, it is optimal for the manager to choose the effort levels specified by the contract:

$$a = \arg\max_{a'} \; W_{M,t}(c_M, a'). \tag{3}$$

In addition, the manager's participation constraint requires that the contract deliver to the manager at least his reservation value $\underline{W}_{M,t_i}$. The participation constraint is imposed only on each recontracting date t_i so that both L and M commit to the contract until date t_{i+1}. Hence,

$$W_{M,t_i}(c_M, a) \geq \underline{W}_{M,t_i}. \tag{4}$$

In general, the reservation value $\underline{W}_{M,t_i}$ can be time deterministic or stochastic depending on the contractual environment such as the relative bargaining power of the large shareholder and the manager in the firm or labor-market circumstances.

[1] He (2011) shows that there is no loss of generality by restricting attention to incentive-compatible and no-savings contracts when the principal has the same saving technology (with the same interest rate) as the agent.

2.4 Equilibrium Characterization

As we show in the following sections, an equilibrium of the model is essentially determined by L's endogenous ownership dynamics Θ_{t_i}. As in DeMarzo and Urošević (2006) and Gorton, He, and Huang (2014), we focus on Markov-perfect equilibria in which L's optimal trading strategy at each date t_i only depends on the current state — the date t_i and her prior holdings $\Theta_{t_{i-}} = \Theta_{t_i-1}$ — rather than the whole history. L's ownership dynamics in a Markov-perfect equilibrium are time consistent.

The time line of events between large shareholder's any two trading dates $t \in [t_i, t_{i+1})$ is as follows. Given her prior shareholdings $\Theta_{t_{i-1}}$, the large shareholder makes a trading decision Θ_{t_i} and determines the manager's contract $\Pi_i = c_{M,t}, a_t$ that specifies the compensation payment to the manager and the recommended level of effort. Due to the incentive compatibility of the contract, it is optimal for the manager to exert the recommended effort level given the promised compensation payoff. By rationally anticipating the large shareholder's trading and contracting decisions, small investors competitively trade shares of the company based on their optimal consumption and portfolio choices $(c_{S,t}, \theta_{S,t})$, which determines the equilibrium stock price P_t through the market clearing condition. The next period starts with the large shareholder's next trading and recontracting decisions.

In order to solve for the equilibrium fully, we assume the following for the rest of the paper. First, all the players have constant absolute risk aversion (CARA) preferences over their consumption and their risk aversion parameters are γ_L, γ_M and γ_S, respectively. More specifically, their utility functions are given by

$$\begin{aligned} u^M(c_{M,t}, a_t) &= -\frac{1}{\gamma_M} e^{-\gamma_M\left(c_{M,t} - \Psi(a_t)\right)}, \\ u^i(c_{i,t}) &= -\frac{1}{\gamma_i} e^{-\gamma_i c_{i,t}} \text{ for } i = L, S. \end{aligned} \tag{5}$$

Second, we assume that the firm's mean cash flow and the cost of exerting effort are time-independent and given by

$$\mu(a_t, t) = \mu_0 + \mu_1 a_t;\ \Psi(a_t, t) = \frac{1}{2}\psi a_t^2; \tag{6}$$

where the constants μ_0, μ_1 and ψ are positive.

3 A Benchmark Model: Owner-Manager

We first analyze a benchmark model that corresponds to the model of DeMarzo and Urošević (2006) . In the benchmark model, L directly runs the company by exerting costly effort herself, that is, there is no agency problem between L and M. We then turn to the main analysis of the full model with the agency problem. Comparing our model to the benchmark model without the principal-agent problem will serve to illustrate key implications of optimal contracting for the large shareholder's ownership dynamics and asset returns. All the detailed proofs are provided in the Appendix.

We first consider L's problem. Suppose that L's ownership policy in the firm is given by $\Theta = \{\Theta_{t_i}; i = 0, 1, \ldots, N\}$. Given the ownership dynamics, L's problem is to choose her optimal effort and consumption decisions that maximize her expected utility

$$W_{L,t} = \max_{c_L, a} E_t \left[\int_t^\infty\ e^{-r(\tau - t)} u(c_{L,\tau}, a_\tau) d\tau \right]. \tag{7}$$

subject to the budget constraint

$$dB_{L,t} = (rB_{L,t} - c_{L,t})dt + \Theta_t dX_t - P_t d\Theta_t = \left[rB_{L,t} - c_{L,t} + \Theta_t \mu(a_t)\right] dt + \Theta_t \sigma dZ_t - P_t d\Theta_t. \tag{8}$$

The budget constraint (8) reflects L's risk-free account balance change in a time interval $(t, t + dt)$ due to her instantaneous consumption and dividend payment (the change in the firm's cumulative output in (1)). We solve the above problem using the Hamilton-Jacobi-Bellman (HJB) approach and summarize the results in the following lemma. To distinguish the equilibrium solution in the owner-manager case from the solution of our main model with optimal contracting, we indicate the equilibrium variables in this scenario by superscript o.

Lemma 1
Given the large shareholder's ownership dynamics Θ and the stock price process P, the large shareholder's value function at time t has the form of

$$W_{L,t}^{o}(B_{L,t},t) = -\frac{1}{\gamma_L r} e^{-\gamma_L \left[r(B_{L,t}+G^{o}(t|\Theta,P)) + \frac{\delta_L - r}{\gamma_L r} \right]}. \tag{9}$$

Her optimal effort and consumption policies are

$$\begin{aligned} a_t^{o} &= \frac{\mu_1}{\psi}\Theta_t, \\ c_{L,t}^{o} &= r(B_{L,t} + G^{o}(t|\Theta,P)) + \Psi(a_t^{o}) + \frac{\delta_L - r}{\gamma_L r}. \end{aligned} \tag{10}$$

In the above, the time dependent certainty-equivalent payoff, $G^{o}(t|\Theta,P)$, is

$$G^{o}(t|\Theta,P) = \int_t^{\infty} e^{-r(\tau-t)} \; \left[k_L^{o}(\Theta_\tau,\tau)d\tau - P_\tau d\Theta_\tau \right],$$

where $k_L^{o}(\Theta_t,t)$ is the net benefit flow to the large shareholder from holding Θ_t at time t

$$k_L^{o}(\Theta_t,t) = \Theta_t \mu(a_t^{o}) - \Psi(a_t^{o}) - \frac{1}{2}\gamma_L r \Theta_t^2 \sigma^2. \tag{11}$$

Note that if t is in a time interval $[t_i, t_{i+1})$, L's shareholdings are held constant ($\Theta_t = \Theta_{t_i}$), and her optimal choice of effort and net benefit flow from holding a block of shares in the firm are also constant. In this case, L's certainty-equivalent payoff satisfies the following recursive equation:

$$G^{o}(t|\Theta,P) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right) k_L^{o}(\Theta_t,t) + e^{-r(t_{i+1}-t)} G^{o}(t_{i+1}|\Theta,P). \tag{12}$$

L exerts greater effort when she holds a larger block of shares in the firm, when the productivity of effort is higher, or when the unit cost of effort is lower. L's net benefit flow from holding Θ_t represents the investor's optimal trade-off between expected payoff and risk from her shareholdings. Note that the expected payoff is reduced by the effort cost solely borne by L in the owner-manager case.

We now turn to the problem of small investors who, given an anticipated trading policy $\widehat{\Theta}_t$ and effort choice $\widehat{a}_t$ for L, make optimal portfolio and consumption decisions continuously as shown below:

$$W_{S,t} = \max_{c_S, \theta_S} E_t \left[\int_t^{\infty} e^{-r(\tau - t)} u(c_{S,\tau}) d\tau \right]. \tag{13}$$

These atomistic investors trade the shares of the firm competitively as price takers, but by forming expectations regarding L's optimal policies. The investor's risk-free bond account changes within a time interval $(t, t+dt)$ as

$$dB_{S,t} = (rB_{S,t} - c_{S,t})dt + \theta_{S,t} dX_t - P_t d\theta_{S,t}, \tag{14}$$

where $B_{S,t}$ and $\theta_{S,t}$ are the investor's holdings of the risk-free bond and the firm's stock at time t. The investor's wealth process $Y_{S,t}$ from the portfolio holdings is $Y_{S,t} = B_{S,t} + \theta_{S,t} P_t$, which thereby evolves according to

$$dY_{S,t} = (rY_{S,t} - c_{S,t})dt + \theta_{S,t}(dP_t + dX_t - rP_t dt). \tag{15}$$

From the above problem for small shareholders, we derive the equilibrium stock price P_t through the market clearing condition, which is described in the next lemma.

Lemma 2
The equilibrium stock price is given by

$$\begin{aligned} P_t^o &= \int_t^{\infty} e^{-r(\tau - t)} k_S^o(\Theta_\tau, \tau) d\tau \\ \text{where } k_S^o(\Theta_t, t) &= \mu(a_t^o) - (1 - \Theta_t)\gamma_S r \sigma^2. \end{aligned} \tag{16}$$

The expected dollar return of a stock and its volatility are

$$\mu_R^o = (1 - \Theta_t)\, \gamma_S r \sigma^2; \;\; \sigma_R^o = \sigma. \tag{17}$$

In the above, k_S represents the marginal benefit flow to the atomistic investors from holing an additional share of the firm, which can be interpreted as those investors' valuation of a share. One can easily check that the investors' valuation linearly increases with the L's shareholdings Θ_t. In addition, as the excess dollar return for holding a share of the firm's stock within the time interval $(t, t+dt)$ is defined as

$$dR_t = dP_t + dX_t - rP_t dt = (1 - \Theta_t)\gamma_S r\sigma^2 dt + \sigma dZ_t, \tag{18}$$

we can obtain the expected excess dollar return or risk premium of the stock and its volatility. Note that L's shareholdings affect the risk premium for the stock only through its effect on the supply of the shares, not the demand for the shares. As L holds a larger block of the company's stock, the stock's liquidity available to small shareholders is lower, thereby increasing the current stock price and, thus, lowering the expected stock return, which is shown by the term $(1 - \Theta_t)$ in (17). Note that the impact of L's effort choice on the firm's cash flow process is expected by small investors and thereby captured by the stock price, which is why the demand for the shares is not affected by L's ownership level.

We note here that the equilibrium stock price is a *deterministic* function of time, that is, the stock price is not dependent upon the realization of the output process. As we show in the next section, the equilibrium stock price is *stochastic* when we introduce the agency conflict between the large shareholder and the manager. If L were to hold the same equity stake in the firm over time, the resulting stock price would remain unchanged. As she makes a trading policy on each of the finite set of dates, the equilibrium stock price jumps due to the change in her ownership stake which is rationally anticipated by investors. Thus, we have the following recursive form of the stock price at L's trading date t_i :

$$P^o_{t_i} = P^o(\Theta_{t_i}, t_i) = \phi_i k^o_S(\Theta_{t_i}, t_i) + (1 - r\phi_i)P^o(\Theta_{t_{i+1}}, t_{i+1}). \tag{19}$$

where $\phi_i = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t_i)}\right)$. This form of the equilibrium stock price is useful for the characterization of L's optimal trading policy over time as we show below.

As noted earlier, we focus on a Markov-perfect equilibrium for L's optimal ownership dynamics that maximizes her certainty-equivalent payoff (11) and, therefore, her value function (9) given her prior holdings $\Theta_{t_{i-1}}$ and the stock price process P^o.

Lemma 3
The large shareholder's optimal trading policy in the owner-manager case is characterized as follows.

$$G^o(\Theta_{t_{i-1}}, t_i) = \max_{\Theta} \ \phi_i k_L^o(\Theta, t_i) - (\Theta - \Theta_{t_{i-1}})P^o(\Theta, t_i) + (1 - r\phi_i)G^o(\Theta, t_{i+1}), \tag{20}$$

whose first order condition for Θ *is given by*

$$k_{L1}^o(\Theta, t_i) - k_S^o(\Theta, t_i) - (\Theta - \Theta_{t_{i-1}})k_{S1}^o(\Theta) = 0. \tag{21}$$

Using the above, we obtain the analytical solution for the large shareholder's optimal holdings $\Theta_{t_i}(\Theta_{t_{i-1}})$ *as*

$$\Theta_{t_i}^o(\Theta_{t_{i-1}}) = \frac{\nu\Theta_{t_{i-1}} + \gamma_S r\sigma^2}{\nu + (\gamma_L + \gamma_S)r\sigma^2}. \tag{22}$$

where $\nu = \frac{\mu_1^2}{\varphi} + \gamma_S r\sigma^2 > 0$ *and* $\gamma = \gamma_L + \gamma_S$.

In the first-order condition (21), the first two terms represent L's and the small investors' marginal benefit flows from holding an additional share. The wedge between the value to L and the value to the small investors is necessary to induce a trade on date t_i. One can see that if the wedge is positive (negative), then L purchases additional shares (sells part of their shares), that is, $\Theta_{t_i} > \Theta_{t_{i-1}}$ ($\Theta_{t_i} < \Theta_{t_{i-1}}$). With our specific forms of the functions, L's time deterministic trading decision is analytically characterized as shown by (22). In addition, note that there is a unique solution $0 < \Theta_{t_i} < 1$ on each date t_i, which confirms the existence of the unique subgame perfect equilibrium in the owner-manager case. Further, it is straightforward to show that L will trade gradually on each date toward the following ownership level:

$$\overline{\Theta}^o = \frac{\gamma_S}{\gamma_L + \gamma_S}, \tag{23}$$

which is in effect the price-taking (or competitive) equilibrium that is determined by (21) without taking into account the impact of L's trading policy on the stock price. In other words, the level of the pricing-taking equilibrium is the one at which there is no wedge in marginal valuations of these groups of investors, thereby inducing no further trading. If L's initial holdings of the stock is higher (lower) than this level, she will divest (accumulate) her shares gradually toward the price-taking outcome. Lastly, as in DeMarzo and Urosevic (2006), the time interval $\Delta_i = t_{i+1} - t_i$ between L's trading dates, that is, the frequency of trade, does not affect L's ownership dynamics. The implication of this result is that, in the continuous-time limit as the time interval between successive trades by L tends to zero, L immediately trades to the price-taking level, $\overline{\Theta}^o$, and maintains her holdings at this level

4 The Model with Optimal Contracting

We now examine the main model that incorporates the agency conflict between L and M.

4.1 Optimal Contracting

Suppose that L's trades over time are given by $\Theta = \{\Theta_{t_i};\ i = 0, 1, \ldots, N\}$ and the stock price process is P. As described in Section 2.3, we consider a sequence of contracts $\{\Pi_i\}$ under the assumption that L recontracts with the manager on each of the finite set of dates when she changes her ownership stake in the firm. Each of the managerial contracts, which specifies the instantaneous compensation payoff and recommended effort level at each date over the period during which the contract lasts, is determined in a way that maximizes L's expected utility subject to the incentive compatibility and participation constraints for the manager as specified in (3) and (4).

Following the dynamic contracting literature (Williams, 2009; Sannikov, 2008; Cvitanić and Zhang, 2013), we take the manager's continuation value $W_{M,t}$ defined in (2) as another state variable whose value evolves according to the following stochastic process

$$dW_{M,t} = \left(\delta_M W_{M,t} - u^M(c_{M,t}, a_t)\right) dt + \chi_{M,t} \sigma dZ_t. \tag{24}$$

In the above, $\chi_{M,t}$ is a $\mathcal{F}_t$-adapted process that represents the sensitivity of the manager's continuation value to the exogenous shock in the firm's output and plays a key role in the provision of incentives, as will become clear later.

As the aforementioned studies show with a detailed proof (see, for example, Williams (2009)), the incentive compatibility constraint for the manager can be replaced by the following local incentive compatibility constraint under certain technical conditions:

$$\chi_{M,t} = -\frac{u_a^M(c_{M,t}, a_t)}{\mu'(a_t)} = \frac{\Psi'(a_t)}{\mu'(a_t)} H(c_{M,t}, a_t) > 0, \tag{25}$$

where $H(c_{M,t}, a_t) = e^{-\gamma_M (c_{M,t} - \Psi(a_t))}$. This constraint corresponds to the first-order approach in the standard static principal-agent problem. When facing the contract that promises to provide the manager with the continuation value $W_{M,t}$ satisfying the local incentive compatibility, the manager will find it optimal to choose the effort level recommended by the contract.

The large shareholder's optimal contracting problem given her ownership policy and stock price process is now defined as the problem with the following two state variables: (i) L's risk-free account balance $B_{L,t}$ in her budget constraint,

$$dB_{L,t} = (rB_{L,t} - c_{L,t})dt + \Theta_t(dX_t - c_{M,t}dt) - P_t d\Theta_t, \tag{26}$$

which now reflects the reduction in the instantaneous dividend payoff due to the manager's compensation payment, and (ii) the manager's continuation value $W_{M,t}$ in (24) after substituting (25) for $\chi_{M,t}$. We solve the problem using the HJB approach that is similar to the owner-manager case and summarize the results in the following proposition.

Proposition 1
Given the large shareholder's ownership dynamics Θ and the stock price process P, the large shareholder's value function at time t has the form of

$$W^*_{L,t}(B_{L,t}, W_{M,t}, t) = -\frac{1}{\gamma_L r} e^{-\gamma_L \left[r(B_{L,t} + G^*(t|\Theta, P)) + \frac{\Theta_t}{\gamma_M} \ln(-W_{M,t}) + \frac{\delta_L - r}{\gamma_L r} \right]}. \tag{27}$$

The optimal policies are

$$\begin{aligned} c^*_{L,t} &= r(B_{L,t} + G^*(t|\Theta, P)) + \frac{\Theta_t}{\gamma_M} \ln(-W_{M,t}) + \frac{\delta_L - r}{\gamma_L r} \\ a^*_t &= \alpha_t, \\ c^*_{M,t} &= -\frac{1}{\gamma_M} \ln(\beta_t) - \frac{1}{\gamma_M} \ln(-W^*_{M,t}) + \Psi(a^*_t). \end{aligned} \tag{28}$$

The manager's expected utility process from his optimal employment contracts follows

$$d\ln(-W_{M,t}^{*}) = \left(\mu_{W,t}^{*} - \frac{1}{2}(\sigma_{W,t}^{*})^2\right)dt - \sigma_{W,t}^{*}dZ_t, \tag{29}$$

where

$$\mu_{W,t}^{*} = \delta_M - \frac{\beta_t}{\gamma_M};\ \sigma_{W,t}^{*} = \frac{\psi}{\mu_1}\alpha_t\beta_t\sigma. \tag{30}$$

In the above, α_t and β_t are determined by

$$\beta_t = \gamma_M r\left[\gamma_M \alpha_t^{*}(\psi\alpha_t^{*} - \mu_1) + 1\right],$$
$$-\left(1 + \frac{\gamma_L}{\gamma_M}\Theta_t\right){\sigma_{W,t}^{*}}^2 + (\gamma_L r\Theta_t\sigma)\sigma_{W,t}^{*} + \left(r - \frac{\beta_t}{\gamma_M}\right) = 0. \tag{31}$$

The large shareholder's certainty-equivalent payoff, $G^{*}(t|\Theta, P)$, is

$$G^{*}(t|\Theta, P) = \int_t^{\infty} e^{-r(\tau - t)}\ \left[k_L^{*}(\Theta_\tau, \tau)d\tau - P_\tau d\Theta_\tau\right]$$

where the net benefit flow to the large shareholder from holding Θ_t at time t is given by

$$\begin{aligned} k_L^{*}(\Theta_t, t) &= \Theta_t\left[\mu(a_t^{*}) - \left(-\frac{1}{\gamma_M}\ln\beta_t - \frac{1}{\gamma_M r}\left(\mu_{W,t}^{*} - \frac{1}{2}(\sigma_{W,t}^{*})^2\right) + \Psi(a_t^{*})\right)\right] \\ &- \frac{1}{2}\gamma_L r\Theta_t^2\left(\sigma - \frac{\sigma_{W,t}^{*}}{\gamma_M r}\right)^2. \end{aligned} \tag{32}$$

In the above, both L's consumption and the manager's compensation are linear in the log of the manager's continuation value process $W_{M,t}^{*}$. The linear forms are consistent with other dynamic contracting models with a CARA-normal setting (e.g, Williams, 2009). Also, note that the manager's recommended effort level α_t and the fixed component of the manager's compensation payoff β_t are dependent upon L's shareholdings Θ_t in a non-linear manner.

Because the manager's expected utility is generated from his optimal contract, we can infer some implications for his compensation payoff by looking at how his continuation value evolves over time. Using the optimal policies in (28) we can specify the evolution of the state variable, the manager's expected utility (or continuation value) process, as a geometric Brownian motion with drift $\mu_{W,t}^{*}$ and volatility $\sigma_{W,t}^{*}$, which are dependent upon L's holdings Θ_t and other parameters, as shown in (30).

The manager's pay-performance sensitivity), denoted by PPS_t^1, is the dollar change in the CEO's pay for a dollar change in the firm's cumulative output X_t. By deriving the incremental increase in the manager's compensation payment for the interval $[t, t+dt]$ from (28) and (29),

$$dc^*_{M,t} = -\frac{1}{\gamma_M}\left(\mu^*_{W,t} - \frac{1}{2}(\sigma^*_{W,t})^2\right)dt + \frac{\sigma^*_{W,t}}{\gamma_M}dZ_t, \tag{33}$$

we obtain the incentive measure related to the firm's output as follows:

$$PPS^1_t = \frac{dc^*_{M,t}}{dX_t} = \frac{\sigma^*_{W,t}}{\gamma_M \sigma} = \frac{1}{\gamma_M}\left(\frac{\psi}{\mu_1}\alpha_t\beta_t\right), \tag{34}$$

where the last equality follows from the definition of $\sigma^*_{W,t}$ in (30). This incentive measure, as well as another measure that will be defined later, is positively associated with the volatility term σ^*_W of the manager's expected utility process. To put it differently, the manager's contracts provide the manager with effective incentives by optimally adjusting the volatilities of the manager's consumption and his continuation value. Importantly, as α_t and β_t are dependent upon Θ_t, the manager's PPS with respect to the firm's cash flows is also related to L's holdings of the stock.

4.2 Equilibrium Stock Price

By rationally anticipating L's ownership stake and her decision on the manager's optimal contract, small shareholders trade the shares of the firm competitively, that is, they take the stock price as given in making their portfolio choice. The following proposition characterizes the stock market equilibrium derived from the optimal portfolio and consumption problem of small shareholders and the stock market clearing condition.

Proposition 2
As small investors rationally anticipate the large shareholder's trading Θ and compensation Pi policies, the equilibrium stock price is derived as

$$\begin{aligned} P^*_t &= e^{rt}\Lambda(\Theta_{t_i}, t_i) + \frac{1}{\gamma_M r}\ln(-W_{M,t}) \\ &= \int_t^\infty e^{-r(\tau-t)}k^*_S(\Theta_\tau, \tau)d\tau + \frac{1}{\gamma_M r}\ln(-W_{M,t}) \end{aligned} \tag{35}$$

where

$$\begin{aligned} d\Lambda &= -e^{-rt}k_S^*(\Theta_t,t)dt \\ k_S^*(\Theta_t,t) &= \mu(a_t^*) - \left(-\frac{1}{\gamma_M}\ln\beta_t - \frac{1}{\gamma_M r}\left(\mu_{W,t} - \frac{1}{2}\sigma_{W,t}^2\right) + \Psi(a_t^*)\right) - (1-\Theta_t)\gamma_S r\left(\sigma - \frac{\sigma_{W,t}}{\gamma_M r}\right)^2 \end{aligned} \quad (36)$$

The the expected dollar return of a stock and its volatility are

$$\mu_{R,t}^* = (1-\Theta_t)\gamma_S r\left(\sigma - \frac{\sigma_{W,t}^*}{\gamma_M r}\right)^2;\ \sigma_{R,t}^* = \sigma - \frac{\sigma_{W,t}^*}{\gamma_M r}. \quad (37)$$

It is important to note that the equilibrium stock price in the presence of optimal contracting consists of both time deterministic and stochastic (as it is dependent upon the state variable $\ln(-W_{M,t}^*)$) components, which represents a key point of distinction from the equilibrium stock price in the owner-manager case (see Lemma 2). We can further derive the recursive form of the stock price on L's trading date t_i as

$$P_{t_i}^* = P^*(\Theta_{t_i},t_i) = \phi_i k_S^*(\Theta_{t_i},t_i) + (1-r\phi_i)\Lambda(\Theta_{t_{i+1}},t_{i+1}) + \frac{1}{\gamma_M r}\ln(-W_{M,t_i}). \quad (38)$$

As shown by Propositions 1 and 2, the manager's optimal compensation payment $c_{M,t}^*$ and the equilibrium stock price P_t^* are negatively and positively related to the log of the state variable, $\ln(-W_{M,t}^*)$, respectively, which suggests a negative relation between CEO pay and contemporaneous stock price. This negative relation appears because the current stock price reflects the costs of compensating the manager that are shared by all shareholders.

The excess dollar return for holding a share of the firm's stock within the time interval $(t, t+dt)$ is defined as

$$\begin{aligned} dR_t^* &= dP_t^* + dX_t - c_{M,t}^*dt - rP_t^*dt \\ &= (1-\Theta_t)\gamma_S r\left(\sigma - \frac{\sigma_{W,t}^*}{\gamma_M r}\right)^2 dt + \left(\sigma - \frac{\sigma_{W,t}^*}{\gamma_M r}\right)dZ_t. \end{aligned} \quad (39)$$

By combining the above definition of the excess dollar return on the stock and the equation for the incremental change in CEO pay, (33), we can define another pay-performance sensitivity measure as the dollar change in CEO pay for a dollar change in the stock return:

$$PPS_t^2 = \frac{dc_{M,t}^*}{dR_t^*} = \frac{r\sigma_{W,t}^*}{\gamma_M r\sigma - \sigma_{W,t}^*}. \tag{40}$$

Further, the drift and volatility terms in the second equation in (39) represent the expected stock return and stock return volatility in equilibrium. As one can easily see in (37), there is a positive relation between the stock market variables,

$$\mu_{R,t}^* = (1 - \Theta_t)\gamma_S r {\sigma_{R,t}^*}^2, \tag{41}$$

which arises because the collective demand for the stock by small shareholders with CARA preferences is competitively determined by the expected stock returns adjusted for the investor's risk premium, $\mu_{R,t}^*/(\gamma_S r {\sigma_{R,t}^*}^2)$, which must equal the supply of the shares, $1 - \Theta_t$, to ensure market clearing.

It is worth noting that, in contrast to the owner-manger case, L's trading policy Θ_t has both direct and indirect influence on the expected stock return $\mu_{R,t}^*$ in the case of agency contracting. On the one hand, her ownership stake reduces the stock's liquidity available to small shareholders, thereby increasing the current stock price and, thus, lowering the expected stock return, as shown by the term $(1 - \Theta_t)$. By Proposition 1, on the other hand, her ownership also affects the volatility $\sigma_{W,t}^*$ of the manager's expected utility process. As implied by the two measures of managerial incentives, (34) and (40), the volatility term is effectively chosen by L through the implementation of the manager's contract. That is, a higher sensitivity of CEO pay to the firm's output or stock return leads to more volatile compensation payments to the manager, thus rendering more volatile expected utility process for the manager. Since shareholders receive the dividend streams net of the manager's compensation payments, more volatile CEO pay implies less volatile residual cash flows and, thereby, lower stock return volatility.

4.3 Large Shareholder's Optimal Ownership Policy

We now look at L's decision for her optimal trading policy on date t_i that maximizes her certainty-equivalent continuation payoff (32), given her prior holdings $\Theta_{t_{i-1}}$ and the stock price process $P^*_{t_i}$.

Proposition 3
The large shareholder's optimal trading policy at date t_i is

$$G^*(\Theta_{t_{i-1}}, t_i) = \max_{\Theta} \ \phi_i k_L(\Theta, t_i) - (\Theta - \Theta_{t_{i-1}})P^*(\Theta, t_i) + (1 - r\phi_i)G^*(\Theta, t_{i+1}) + \frac{\Theta}{\gamma_M r} \ln(-W_{M,t}). \quad (42)$$

The first order optimality condition is given by

$$k_{L1}(\Theta^*_{t_i}, t_i) - k_S(\Theta^*_{t_i}, t_i) - (\Theta^*_{t_i} - \Theta_{t_{i-1}})k_{S1}(\Theta^*_{t_i}) + \frac{(1 - r\phi_i)}{\phi_i}\frac{1}{\gamma_M r} \ln(-\underline{W}_{M,t_i}) = 0. \quad (43)$$

By comparing the above first order condition to (21), one can notice a few distinctions in the optimal contracting case from the owner-manager case. First, the the share value to L, k^*_{L1} and the share value to the small investors, k^*_S, are now affected by the manager's optimal contract, which are reduced by the expected value of the manager's instantaneous compensation payoff and increased by the reduction in the stock return volatility due to optimal risk sharing with the manager through the contract. Since k_L and k_S are non-linear in Θ_t, it is difficult to find an analytical solution. We thus characterize L's ownership dynamics by solving the above first-order condition numerically. Second, in sharp contrast with the owner-manager case, L's optimal trading policy is affected by the manager's reservation value $\underline{W}_{M,t_i}$ on the date of recontracting as well as the time interval Δ_i between L's trading dates (as embodied in ϕ). Consequently, L's optimal trading policy does depend on the frequency of trading because the latter affects the duration of the manager's contracts. Because of the non-linearity of the first-order condition (43), an analytical characterization of the effects of optimal contracting for general parameter values cannot be obtained. Hence, we provide a more detailed discussion of the effects of optimal contracting after numerically exploring L's ownership dynamics. Lastly, we can still obtain the long-term or steady-state level, $\overline{\Theta}^*$, of L's holdings from the price-taking equilibrium, which also depends on the steady-state levels of the manager's reservation value and L's trading and recontracting frequency. As we will see later, the optimal contracting dynamics depending on the manager's contract environment influences both the long-term (steady-state) level of L's equity stake in the firm and the time-path of her ownership dynamics.

5 Model Implications

We numerically characterize the equilibrium solution in the full model and the benchmark "owner-manager" model. We first look at the case in which the large shareholder's trading takes place only once. The single-round case is intended to discuss the equilibrium correlations between the three key variables — the large shareholder's ownership, managerial incentives, and stock returns. We then discuss the large shareholder's ownership dynamics when she dynamically trade.

5.1 Baseline Parametrization

Table I reports the baseline parameters of the model. We take the risk-free rate $r = 2\%$ (Guvenen, 2009)) and the firm's output (cash flow) volatility $\sigma = 0.25$ (He, 2011). For simplicity we set each economic agent's time discount rate δ_i to 0.0513 so that the annual discount factor $e^{-\delta_i} = 0.95$ for $i = L, M, S$. The time interval between the large shareholder's successive trading dates is set to 1 year. We set the unit cost ψ of effort and the constant term μ_0 in the mean cash flow function $\mu(a_t)$ to 1, respectively.

The remaining parameters include the coefficients of absolute risk aversion for the manager and for shareholders, the productivity of effort, and the manager's reservation value that is assumed to be constant over time. We assume that both groups of shareholders are identical in terms of their risk aversions. These parameters are chosen to match the following empirical moments reported in Table II: (i) the median shares (37%) owned by large shareholders (or blockholders) in a randomly selected, CRSP- and Compustat-listed corporation as reported in Holderness (2009); (ii) the manager's pay-performance sensitivity (the dollar-to-dollar measure) of 0.3% (that is, a manager's wealth rise of about $3 for each increase of $1,000 in firm value) as documented in the empirical literature (Jensen and Murphy, 1990; Edmans et al., 2008); (iii) the average Sharpe ratio in the U.S. data that is reported in Guvenen (2009); (iv) the median market-to-book ratio (1.183) of Compustat-listed firms as documented in Riddick and Whited (2009).

Given that these moments documented in the literature are relatively stable over time, we match them with the long-term (or steady-state) equilibrium of the model with agency contracting where the large shareholder finds it optimal to hold the same level of shareholdings over time. In particular, to match the observed market-to-book ratio, we define the model-predicted ratio ($\overline{MB}^*$) in the steady-state using the fact that the stock price of a firm represents its total market value because there is one share of asset outstanding. The market value of equity ($\overline{MV}^*$) is thus the steady-state equilibrium stock price, which can be obtained from (35) as below by noting that small investors' marginal valuations of the dividend flow $k_S^*(\cdot)$ in (45) is constant over time at the steady-state level of the large shareholder's ownership $\overline{\Theta}^*$:

$$\overline{MV}^* = \frac{1}{r} k_S^*(\overline{\Theta}^*) + \frac{1}{\gamma_M r} \ln(-\underline{W}_M). \tag{44}$$

Our proxy for the book value of equity (BV^*) is the stock price of the hypothetical firm in the absence of the manager's human capital inputs (e.g., Jung and Subramanian, 2014; Ou-Yang, 2005).

$$\overline{BV}^* = \frac{1}{r} \tilde{k}_S^*(\overline{\Theta}^*) \text{ where } \tilde{k}_S^*(\overline{\Theta}^*) = \mu_0 - (1 - \overline{\Theta}^*)\gamma_S r \sigma^2. \tag{45}$$

By matching their ratios to the median book-to-market ration from data, we can pin down the parameters in the mean cash flow function, μ_0 and μ_1.

The baseline parameters in Table I suggest that, in order to match the empirical moments reasonably as shown in Table II, the manager is more risk averse than shareholders, which reflects the anecdotal and empirical evidence. Also, it is necessary to have the manager's reservation value that is different from minus one, which implies the importance of taking into account the agency contracting environment to generate the equilibrium outcome consistent with the data. It should be noted, however, that this calibration exercise is only intended to ensure that our subsequent quantitative analysis provides empirically relevant predictions rather than estimating the parameters (Strebulaev and Whilted, 2011).

Table II reports the steady-state equilibrium values of the key variables for the optimal contracting case and the owner-manager case. The large shareholder's long-term ownership in the presence of optimal contracting is lower than in the owner-manager case in which the large ownership is simply determined by the perfect risk-sharing between the large shareholder and small investors based only on their relative risk aversion. The large shareholder can induce the manager to put greater effort through optimal contracting than the effort level she would exert directly in the owner-manager case where the large shareholder needs to bear the full cost of effort. The sensitivities of the manager's optimal contract, which are captured by the manager's positive PPS measures, reduce the volatility of the firm's dividend process, thereby effectively lowering its stock return volatility and, therefore, the expected excess dollar return from holding its share. The resulting Sharpe ratio is slightly higher in the owner-manager case than in the contracting case.

5.2 Ownership Structure, Stock Returns, and Managerial Incentives

In this subsection, we explore the three key variables that are endogenously determined in the equilibrium of the model with the optimal agency contracting — the large shareholder's ownership, stock returns, and the managerial incentives. To focus on their equilibrium correlations, we restrict our attention in this subsection to the case in which the large shareholder's trading takes place only once on date zero. For simplicity, we assume that the large shareholder holds no equity stake in the firm and contemplates purchasing a certain number of shares of the firm.

One can easily check that the large shareholder's ownership decision on date zero is reduced to

$$\Theta^* = \arg\max_{\Theta} \ \Omega(\Theta){\sigma_R^*}^2(\Theta), \ \text{where } \Omega(\Theta) \equiv \left[\gamma_S \Theta(1-\Theta) - \frac{1}{2}\gamma_L \Theta^2\right] \tag{46}$$

where σ_R^* is given by (37). The large shareholder's objective function above consists of two parts. The first term is a quadratic function of Θ in the bracket that depends on the risk aversions of the large and small shareholders. The second one is, more importantly, ${\sigma^*}_R^2(\Theta)$, which, as we discussed earlier, reflects the effects the large shareholder's ownership has on the firm's stock return volatility through her control over the manager's compensation contract. If there exists an interior solution for the optimal large ownership $\Theta^* \in (0,1)$, then the solution satisfies

$$-\frac{{\sigma_R^*}'(\Theta^*)}{\sigma_R^*(\Theta^*)} \left(= \frac{{\sigma_W^*}'(\Theta^*)}{\gamma_M r \sigma - \sigma_W^*(\Theta^*)} \right) = \frac{\Omega'(\Theta^*)}{2\Omega(\Theta^*)}. \tag{47}$$

where σ_W^* is given by (30). In the above, the term on the left-hand side represents the rate of increase or decrease in the firm's stock return volatility with respect to an increase in the large shareholder's ownership stake that arises through its effect on the manager's optimal contract. In other words, this term captures the large shareholder's trade-off between incentive provision and risk-sharing with the manager in choosing an optimal contract for the manager. The right-hand side, however, represents the large shareholder's risk-sharing with small shareholders based on their relative risk aversions. Under a reasonable set of parameter values (including the baseline parameter values), the equilibrium stock return volatility declines with the large shareholder's ownership stake, that is, ${\sigma_R^*}'(\Theta) < 0$.

To provide further implications of the model, we now explore the effects of the model parameters on the equilibrium outcome with one round of the large shareholder's trading. Specifically, we vary each of the parameters — the firm's cash flow volatility σ, the productivity of effort, and each player's risk aversion γ_i for $i = L, M, S$,—about their baseline values, respectively, and obtain the equilibrium solution in each scenario. In addition to assessing the impacts of each parameter change on the three key equilibrium variables including the large shareholder ownership, managerial incentives, and stock returns that are endogenously determined by the same underlying factors, we can also derive their relationships in a general equilibrium setting that generate some important empirical implications.

Figure 1 shows the equilibrium correlations of the key variables in response to variations in the firm's cash flow volatility. As the firm's underlying cash flows become more volatile, the firm's dividend process and, therefore, its expected stock return and stock return volatility increase with the underlying risk. As the uncertainty in the firm's output increases, it is more costly for the large shareholder to induce the manager's effort effectively, thereby lowering the sensitivity of the manager's compensation payment to the firm's output. As the risk sharing effect of the managerial contract is reduced at the margin, which is captured by (47), the large shareholder will choose to hold a lower equity stake, which suggests negative relationships between the large shareholder's ownership and stock return variables such as stock return volatility and expected excess dollar return from holding a share of the stock. In contrast, the owner-manager, who does not face an agency problem, chooses a higher equity stake as the wedge between the large shareholder's and small investors marginal valuations of a share increases with the underlying volatility. The owner-manager case shows the opposite relationships, that is, as the underlying risk of the firm changes, its large shareholder ownership and expected stock return and stock return volatility move in the same direction. Based on the discussion above, when the firm's underlying risk changes, the expected stock return and stock return volatility move in the opposite direction with the manager's pay-performance sensitivity, as the second set of graphs shows in Figure 1.

Figure 2 shows the variations of the equilibrium variables with the productivity of effort. Recall that it is the large shareholder who exerts effort in the owner-manager case that affects the firm's output process, whereas it is the manager in the contracting case who receives incentives to provide the recommended effort levels. As the productivity of effort increases, it is optimal to effectively increase the level of effort up to the point that equates marginal benefit to marginal cost. In the owner-manager case, however, the the cost of effort is solely borne by the large shareholder. Since the marginal cost of effort increases with her shareholdings, the large shareholder reduces her holdings of the stock, which in turn increases the shares available to small investors and thus lowers the stock price. This explains the negative correlation between the large shareholder's ownership and the expected stock return in the owner-manager case. In the presence of optimal contracting, however, the large shareholder can raise incentives to induce the effort level that is optimal in response to an increase in the productivity. Note that the cost of inducing greater effort is shared with small investors, Further, as the increased incentives lowers the stock return volatility, the large shareholder can choose a higher equity stake in the firm and thus imposes a negative correlation between large shareholder ownership and expected stock returns and stock return volatility. The equilibrium correlations of the managerial incentives and stock return variables or the large shareholder ownership are similar to those shown in Figure 1.

We also report the quantitative effects of either of the risk aversion parameters in Figures 3-5. As noted earlier from (47), the large shareholder's optimal ownership stake is mostly affected by her risk sharing incentives both with small shareholders and the manager. The large shareholder's optimal ownership stake thus declines with γ_L, but increases with the risk aversions of other players (γ_S and γ_M). Also, each coefficient of risk aversion affects the level of managerial incentives: a higher level of risk aversion of each group of investors (γ_L or γ_S) induces higher incentives to the manager, whereas that of the manager induces lower incentives as the costs of risk sharing with the manager becomes higher. These observations imply the corresponding correlations between the large shareholder ownership and managerial incentives. As higher incentives received by the manager can lower the volatility of the dividend process and therefore the stock return volatility, we can obtain clear implications for the stock return volatility. The predictions on the expected stock return, however, are relatively subtle in the sense that the expected stock return is determined by both the large shareholder's ownership choice and the stock return volatility.

In addition, it is clear that both the expected stock return and stock return volatility are higher in the owner-manager case than in the presence of optimal contracting. As emphasized above, the optimal incentive provision to the manager plays a significant role of risk-sharing, which effectively lowers the firm's stock return volatility. The observation that the owner-manager case involves a more volatile stock return and a higher risk premium partially explains the higher equity risk premium in emerging markets, in which owner-manager firms, that is, family-controlled public firms are more prevalent, than in developed markets. As noted above, the expected stock return increases as either group of investors becomes more risk averse, but its volatility decreases. It is also interesting to see that the manager's risk aversion can increase the expected stock return and stock return volatility. By considering the manager's trading for managerial incentives, Gorton et al. (2013) show that managerial trading results in more volatile stock prices and higher risk premiums only when the manager is more risk-averse than investors. However, in their model, this finding only holds with dispersed small investors, not with large shareholders.

Further, it is worth emphasizing that the key endogenous variables in equilibrium — large shareholder ownership, managerial incentives, expected stock return and stock return volatility — can be positively or negatively associated with each other, depending on which parameter value changes. First, the equilibrium correlation between the large shareholder ownership (or outside block ownership) and the pay-performance sensitivity of CEO compensation can be positive when the underlying firm risk, the productivity of effort or the risk aversion of small shareholders varies. But their correlation can be negative when the risk aversion of the large shareholder or that of the manager changes. The empirical evidence is mixed regarding their relationships. Mehran (1995) and Bertrand and Mullainathan (2001), on the one hand, argues by showing a negative association between incentive-based compensation and the size of external block ownership that monitoring by large shareholders appears to be a substitute for incentive compensation for aligning incentives. More recent studies such as Almazan, Hartzell and Starks (2005) and Kim (2010), on the other hand, show that the sensitivity of CEO compensation to firm performance is positively associated with the level of outside block ownership. Whether these two major governance mechanisms are complements or substitutes should be addressed in a framework of general equilibrium interactions as in our study.

Lastly, we find that the large shareholder ownership is negatively related to the expected stock return and stock return volatility when the firm's underlying risk or the productivity of effort changes, whereas their correlation can be positive when their is an exogenous shock that affects the risk aversions of the players. Whether the pay-performance sensitivity of the managerial contract is positively or negatively related to the firm's expected stock return also depends on the underlying parameter value changes. Given that we take into account the endogenous determination of corporate governance mechanisms rather than taking corporate governance quality as exogenous, we can address the the mixed empirical evidence in the literature initiated by Gompers, Ishii and Metrick (2003) that has studied the impact of corporate governance on stock returns. Some follow-up studies, including Johnson, Moorman and Sorescu (2009) and Bebchuk, Cohen and Wang (2013), show that the positive association between governance and abnormal returns identified by Gompers, Ishii and Metrick (2003) is not robust enough depending on industry clustering or time period. Our theoretical predictions at least suggest that some careful consideration is required when one empirically examines the relationships between endogenous equilibrium variables.

5.3 Large Shareholder's Optimal Ownership Dynamics

We now consider a trading environment in which the large shareholder can dynamically trade at discrete times. Figures 6-9 illustrate the large shareholder's ownership dynamics on a set of discrete dates in which the large shareholder is assumed to initially hold 80% of the firm's shares outstanding. Each figure shows the firm's ownership dynamics in response to variations in one of the model parameters both for the owner-manager case and for the contracting case.

As DeMarzo and Urošević (2006) emphasize, in the owner-manager case, the model parameters, in particular, the firm's underlying risk and the productivity of effort, affect the speed of adjustment at which the large shareholder trades the shares of the firm towards the competitive level over time that is determined at the perfect risk sharing between the two groups of investors. The more volatile the firm's cash flows or the less productive the firm is, the greater the owner-manager's desire to reduce her stake, thereby increasing the convergence rate. What matters more in the contracting case, however, is the competitive level itself rather than the rate of convergence. Interestingly, the large shareholder tends to hold a greater equity stake in the long-run when the firm's underlying risk is higher or when the firm is less productive. In contrast to the owner-manager case, the large shareholder can optimally choose the manager's contract which has an impact on the firm's stock return volatility and thus the large shareholder's risk premium. As the firm's cash flows become more volatile, the costs of risk-sharing between L and M increase so that the incentive intensities of M's contracts decrease. As a result, L holds a higher equity stake in the firm. The reverse is true when the cash flow volatility declines. In fact, our results show that, in some scenario, L may find it optimal to sell her stake entirely when volatility falls sufficiently far. This is a manifestation of the optimality of high-powered incentives for the agent when output risk is low. If the firm is less productive, on the other hand, the large shareholder needs to provide greater incentives to lead to the optimal effort level, which in turn lowers the stock return volatility so that she can hold more shares of the firm.

In Figure 7, one can see that the manger's reservation value is critical for the large shareholder's ownership dynamics. If the large shareholder needs to promise a greater continuation value to the manager (a higher $\underline{W}_M$), she tends to trade towards zero holdings, and vice versa. Although we have simplified the recontracting environment in our numerical exercise by assuming a constant continuation value over time, one can extend the analysis by incorporating the time varying contracting environment. As DeMarzo and Urošević (2006) point out, if the competitive equilibrium level of the large shareholder's ownership is time-varying, then the large shareholder's optimal ownership policy is not necessarily monotonic in time. The contracting environment, due either to the labor market dynamics or to the bargaining game between the large shareholder and the manger, can be a source of non-monotonic trades or trades towards zero holdings by the large shareholder. In relation to recontracting with the manager, we show in Figure 8 that the frequency of the large shareholder's trading and therefore recontracting also affects her long-term holdings of the stock. At the baseline value of the manager's continuation value ($\underline{W}_M = -0.90$), the manager's reservation value is set to this level on every recontracting date. Since this level renders a negative value to the large shareholder as shown in the first-order condition (43) for her optimal trading policy, a higher frequency of recontracting (that is, a smaller time interval between her successive trading dates), she chooses to hold small shareholdings or even immediately divest all of her shares of the company.

6 Discussions

There are several directions in which the relatively simple structure in this paper can be extended. First, we have considered a representative firm in the current paper so that there are only two assets available to investors — the risk-free bond and the firm's stock. We intend to extend this simple model to a general framework of multiple firms in a subsequent paper that would allow us to present a CAPM with optimal managerial contracting and and cross-sectional implications for large shareholder ownership, managerial incentives, and stock return characteristics (expected returns, volatilities, and stock betas). More importantly, these theoretical predictions should be supported by empirical asset pricing tests.

Second, we have simplified the implementation of optimal contracting by making the assumption of recontracting on the large shareholder's trading dates. It would be certainly interesting to see how the equilibrium outcome would change by assuming that the large shareholder commits to an optimal long-term contract, while at the same time trading significant blocks of shares in the firm.

Third, we adopt a simple CARA-normal setup in this paper. This enables us to obtain a closed-form solution for the optimal contract, equilibrium asset price and ownership structure. There are two theoretical limitations to the generalization of the framework. As Sannikov (2008) presents numerical solutions to the contracting problems with different non-CARA preferences, it would be difficult to find a closed-form solution even to the optimal contracting problem. In addition, under different preferences and cumulative payoff structures, it would not be plausible to guess and verify the specific form of the equilibrium price. More work would be necessary to obtain the general form of the price process in such a generalization.

Finally, we do not consider an endogenous distinction between the large shareholder and small shareholders as in Admati, et. al (1994) and DeMarzo and Urošević (2006). As our simple calibration exercise suggests risk preference differences between these two types of investors in order to match the empirically observed moments, it would be interesting to further explore an ex-ante investor's choice to be a large shareholder in a particular firm in equilibrium, possibly by taking into account a fee structure that the large shareholder charges small shareholders for the extra risk she bears.

7 Conclusion

We present a simple tractable dynamic framework that embeds a standard managerial moral hazard problem into an asset pricing model with a large shareholder. Using a simple CARA-normal continuous-time framework, we fully characterize the endogenous determination of a representative firm's ownership structure, managerial compensation and stock return characteristics and examine their equilibrium interactions. We compare the equilibrium solutions in the owner-manager and agency contracting cases and then discuss the impact of optimal contracting on the large shareholder's ownership dynamics and the firm's equity prices.

We show that, in contrast to the benchmark owner-manager case, incorporating the optimal contracting problem results in a stochastically evolving stock price and nontrivial dynamics for the large shareholder's ownership. Also, the additional sensitivity of the manager's contract that is required for the incentive provision implies a lower stock return volatility and a lower expected stock return than in the owner-manager case. In addition, it is plausible that both large shareholder trading policy and stock price can be affected by the contracting environment that varies due either to the labor market dynamics or to the bargaining power between the large shareholder and the manager. Finally, we predict the correlations among the equilibrium variables in response to variations of the model parameters, which can help to explain the mixed evidence in the empirical literature.

References

[1] Admati, A. R., P. Pfleiderer, and J. Zechner. 1994. Large Shareholder Activism, Risk Sharing, and Financial Market Equilibrium. *Journal of Political Economy* 102: 1097-1130.

[2] Almazan, A., J. C. Hartzell, and L. T. Starks. 2005. Active Institutional Shareholders and Costs of Monitoring: Evidence from Executive Compensation. *Financial Management* 34: 5-34.

[3] Arrow, K. J. 1971. Insurance, Risk and Resource Allocation. *Essays in the Theory of Risk Bearing.* Chicago: Markham.

[4] Bertrand, M. and S. Mullainathan. 2001. Are Executives Paid for Luck? The Ones without Principals Are. *The Quarterly Journal of Economics* 116: 901.932.

[5] Burkart, M., D. Gromb, and F. Panunzi. 1997. Large Shareholders, Monitoring, and the Value of the Firm. *The Quarterly Journal of Economics* 112: 693-728.

[6] Cremers, K. J. M. and V. B. Nair. 2005. Governance Mechanisms and Equity Prices. *Journal of Finance* 60: 2859-2894.

[7] Cuoco, D., and J. Cvitanic, 1998, Optimal Consumption Choices for a 'Large' Investor, *Journal of Economic Dynamics and Control*, 22: 401-436.

[8] Cvitanic, J., 1997, Nonlinear Financial Markets: Hedging and Portfolio Optimization. In *Mathematics of Derivative Securities*, M. A. H. Dempster and S. Pliska (editor). Proceedings of the Mathematical Finance Programme, Isaac Newton Inst. Math. Scie., Jan-Jun 1995. New York: Cambridge Univ. Press.

[9] Cvitanic, J., and J. Zhang. 2013. *Contract Theory in Continuous-Time Models.* Springer-Verlag, Berlin.

[10] DeMarzo, P., and B. Uroŝević. 2006. Optimal Trading and Asset Pricing with a Large Shareholder. *Journal of Political Economy* 114: 774-815.

[11] Edmans, A., X. Gabaix, and A. Landier. 2009. A Multiplicative Model of Optimal CEO Incentives in Market Equilibrium. *Rreview of Financial Studies* 22: 4881-4917.

[12] El Karoui, N., S. Peng, and M. C. Quenez, 1997, Backward Stochastic Differential Equations in Finance. *Mathematical Finance*, 7: 1-71.

[13] Gorton, G. B., P. He, and L. Huang. 2013. Agency-based Asset Pricing. *Journal of Economic Theory*, forthcoming.

[14] Grossman, S. J., and O. D. Hart. 1983. An Analysis of the Principal-Agent Problem. *Econometrica* 51: 7-45.

[15] Guvenen, F. 2009. A Parsimonious Macroeconomic Model for Asset Pricing. *Econometrica* 77: 1711-1750.

[16] Haddad, V. 2013. Concentrated Ownership and Equilibrium Asset Prices. *Working Paper*, Princeton University.

[17] He, Z.. 2009. Optimal Executive Compensation when firm size follows geometric Brownian Motion. *Review of Financial Studies* 22: 859-892.

[18] Holderness, C. G. 2009. The Myth of Diffuse Ownership in the United States. *Review of Financial Studies* 22: 1377-1408.

[19] Holmstrom, B., and P. Milgrom. 1987. Aggregation and Linearity in the Providsion of Intertemporal Incentives. *Econometrica* 55: 303-328.

[20] Ju, N. and X. Wan. 2012. Optimal Compensation and Pay-Performance Sensitivity in a Continuous-Time Principal-Agent Model. *Management Science* 58: 641-657.

[21] Kim, K. 2010. Blockholder monitoring and the efficiency of pay-performance benchmarking. *Journal of Corporate Finance* 16 : 748-766.

[22] Laffont, J. J., and D. Martimort. 2002. *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, New Jersey.

[23] Ou-Yang, H. 2005. An Equilibrium Model of Asset Pricing and Moral Hazard. *Review of Financial Studies* 18: 1253-1303.

[24] Parigi, B. M., L. Pelizzon and E. von Thadden. 2013. Stock Market Returns, Corporate Governance and Capital Market Equilibrium, *CESifo Working Paper*.

[25] Parlour, C. A., and J. Walden. 2011. General Equilibrium Returns to Human and Investment Capital under Moral Hazard. *Review of Economic Studies* 78: 394-428.

[26] Prescott, E. C., and R. M. Townsend. 1984. Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard. *Econometrica* 52: 21-46.

[27] Sannikov., Y. 2008. A Continuous-Time Version of the Principal-Agent Problem. *Review of Economic Studies* 74: 957-984.

[28] Shleifer, A., and R. Vishny. 1997. A survey of corporate governance. *Journal of Finance* 52: 737-783.

[29] Tirole, J. 2006. *The Theory of Corporate Finance*. Princeton University Press.

[30] Williams, N. 2009. On Dynamic Principal-Agent Problems in Continuous Time. *Working paper*, University of Wisconsin - Madison.

[31] Zame, W. R. 2007. Incentives, Contracts, and Markets: A General Equilibrium Theory of Firms. *Econometrica* 75: 1453-1500.

Appendix: Proofs

While small shareholders can trade shares of the firm continuously, the large shareholder trades its shares Θ_{t_i} on a finite set of dates with $i = 0, 1, \ldots, N$ (note that $t_N = T$). Consider the interval $t \in [t_i, t_{i+1})$ on which the large shareholder's equity stake is held constant: $\Theta_t = \Theta_{t_i}$. In the following we first prove the result for owner-manager case. It is almost identical to DeMarzo-Urosevic (2006), but serves as an illustration of proofs for our main model.

Owner-Manager Case

We first look at the owner-manager case with no contracting. The large shareholder's value function, that is, the expected utility at time t from her consumption and effort choices continuously adjusted over the period is

$$W_{L,t} = \max_{c_L, a} E_t \left[\int_t^{t_{i+1}} e^{-r(\tau - t)} u(c_L) d\tau \right]. \tag{48}$$

which we conjecture is of the form

$$F(B_{L,t}, t) = -\frac{1}{\gamma_L r} e^{-\gamma_L \left[r\left[B_{L,t} + G(t)\right] + \frac{\delta_L - r}{\gamma_L r} \right]}. \tag{49}$$

By solving the HJB equation, we have

$$G'(t) = rG(t) - k_L(\Theta_t, t) \tag{50}$$

where $k_L(\Theta_t, t)$ is the net benefit flow to the large shareholder from holding Θ_t at time t,

$$k_L(\Theta_t, t) = \max_{a_t} \ \Theta_t \mu(a_t, t) - f(a_t, \Theta_t, t) - \frac{1}{2}\gamma_L r \Theta_t^2 \sigma^2(a_t, t), \tag{51}$$

as in the D-U paper. For simplicity, we assume that the mean and volatility terms in the firm's cash flows as well as the effort cost are stationary, that is,

$$\mu(a_t, t) = \mu_0 + \mu_1 a_t; \ f(a_t, \Theta_t, t) = \frac{1}{2}\varphi a_t^2; \ \sigma(a_t, t) = \sigma \tag{52}$$

By integrating (50), we obtain

$$G(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1} - t)}\right) k_L(\Theta_{t_i}, t_i) + e^{-r(t_{i+1} - t)} G(t_{i+1}) \tag{53}$$

Now consider date $t = t_i$ when the large shareholder makes a trading decision to hold Θ_{t_i} over the interval $[t_i, t_{i+1})$ and thus realizes capital gains or losses. We can write the large shareholder's objective function for her optimal trading policy on date t_i as follows:

$$\begin{aligned} G(t_i) &= -(\Theta_{t_i} - \Theta_{t_{i-1}}) P_{t_i} + G(t_i^+) \\ &= -(\Theta_{t_i} - \Theta_{t_{i-1}}) P_{t_i} + \frac{1}{r}\left(1 - e^{-r(t_{i+1} - t_i)}\right) k_L(\Theta_{t_i}, t_i) + e^{-r(t_{i+1} - t_i)} G(t_{i+1}) \end{aligned} \tag{54}$$

where $G(t_i^+)$ represents the large shareholder's continuous payoff (53) on date t_i right after her new equity stake Θ_{t_i} is determined.

Now we look at how the equilibrium stock price is determined. Given an anticipated trading policy and effort choice of the large shareholder $\hat{\Theta}_t$ and $\hat{a}_t$, we can show that the equilibrium stock price P_t, which is determined by the market clearing condition for the firm's shares, is derived as a time-deterministic function

$$P_t = e^{rt}\omega(\hat{\Theta}_t, t) \tag{55}$$

where

$$\begin{aligned} d\omega &= -e^{-rt}k_S(\hat{\Theta}_t, t)dt \\ \text{and} \quad k_S(\hat{\Theta}_t, t) &= \mu(\hat{a}_t) - (1-\hat{\Theta}_t)\gamma_S r\sigma^2. \end{aligned} \tag{56}$$

The above can be rewritten as

$$\begin{aligned} P_t &= \int_t^\infty e^{-r(\tau-t)}\ k_S(\hat{\Theta}_\tau, \tau)d\tau \\ &= \int_t^\infty e^{-r(\tau-t)}\ (\mu(\hat{a}_\tau) - (1-\hat{\Theta}_\tau)\gamma_s r\sigma^2)d\tau. \end{aligned} \tag{57}$$

When the large shareholder trades on a finite set of dates, the equilibrium stock price jumps due to the change in her ownership stake which is rationally anticipated by investors. Thus, we have the following recursive form of the stock price

$$P_{t_i} = P(\hat{\Theta}_{t_i}, t_i) = \phi_i k_S(\hat{\Theta}_{t_i}, t_i) + (1 - r\phi_i)P(\hat{\Theta}_{t_{i+1}}, t_{i+1}). \tag{58}$$

where $\phi_i = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t_i)}\right)$.

We now go back to the large shareholder's decision problem for her optimal trading policy on date t_i that maximizes her continuation payoff (54) given her prior holdings $\Theta_{t_{i-1}}$ and the stock price process P_{t_i}. By taking into account the impact of her own trading policy on the stock price, we can solve the large shareholder's ownership dynamics as

$$G^*(\Theta_{t_{i-1}}, t_i) = \max_{\Theta}\ \phi_i k_L(\Theta, t_i) - (\Theta - \Theta_{t_{i-1}})P(\Theta, t_i) + (1 - r\phi_i)G^*(\Theta, t_{i+1}), \tag{59}$$

whose first order condition for the large shareholder's optimal holdings $\Theta^*_{t_i}(\Theta_{t_{i-1}})$ is derived as

$$k_{L1}(\Theta^*_{t_i}, t_i) - k_S(\Theta^*_{t_i}, t_i) - (\Theta^*_{t_i} - \Theta_{t_{i-1}})k_{S1}(\Theta^*_{t_i}) = 0. \tag{60}$$

In the simple stationary example with (52), the above condition directly derives the large shareholder's optimal ownership dynamics as

$$\Theta^*_{t_i}(\Theta_{t_{i-1}}) = \frac{\frac{\mu_1^2}{\varphi}\Theta_{t_{i-1}} + \gamma_S r\sigma^2(1 + \Theta_{t_{i-1}})}{\frac{\mu_1^2}{\varphi} + (2\gamma_S + \gamma_L)r\sigma^2}. \tag{61}$$

The D-U paper provides the recursive solution for a time-varying linear case.

Contracting case: Proposition 1-3

We now look at the agency contracting case. For simplicity, we assume that the manager's contract gets renewed in each of these dates which requires the feasibility of the contract that the manager's expected utility at date t_i is at least as great as his constant reservation utility $\underline{W}^M$. In a more general case, his reservation utility can be time varying or stochastic depending on the contractual environment (Sannikov, 2008). The derivation is similar to the owner-manager case, except now there are two state variables, the savings account balance $B_{L,t}$ and the manager's value function $W_{M,t}$. They follow the following two processes:

$$dB_{L,t} = (rB_{L,t} - c_{L,t} - \Theta c_{M,t} + \Theta a_t)\,dt + \Theta\sigma dZ_t.$$
$$dW_{M,t} = \Big(\delta_M W_{M,t} - e^{\delta_M t} u_M(t, c_{M,t}, a_t)\Big)\,dt - e^{\delta_M t}\partial_a u_M(t, c_{M,t}, a_t)\sigma dZ_t$$
$$= \left(\delta_M W_{M,t} + \frac{1}{\gamma_M} e^{-\gamma_M (c_{M,t} - \frac{1}{2}\varphi a_t^2)}\right) dt + \varphi a_t e^{-\gamma_M (c_{M,t} - \frac{1}{2}\varphi a_t^2)}\sigma dZ_t,$$

We follow similar steps through Proposition 1-3 as in the owner-manager case. Given the large shareholder's equity stake Θ_{t_i} on date $t \in [t_i, t_{i+1})$, we guess that the large shareholder's value function is of the form

$$F(B_{L,t}, W_{M,t}, t) = -\frac{1}{\gamma_L r} e^{-\gamma_L \left[r\left[B_{L,t} + G(t)\right] + \frac{\Theta_t}{\gamma_M}\ln(-W_{M,t}) + \frac{\delta_L - r}{\gamma_L r}\right]}. \quad (62)$$

By solving the HJB equation,

$$G'(t) = rG(t) - k_L(\Theta_t, t) \quad (63)$$

where $k_L(\Theta_t, t)$ is given by

$$k_L(\Theta_t, t) = \max_{a_t}\ \Theta_t\left(\mu(a_t) + \frac{1}{\gamma_M}\ln\beta - f(a_t)\right) + \frac{\Theta_t}{\gamma_M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) - \frac{1}{2}\gamma_L r \Theta_t^2\left(\sigma - \frac{\sigma_W}{\gamma_M r}\right)^2 \quad (64)$$

where $C_M = -\frac{1}{\gamma_M}\ln\beta - \frac{1}{\gamma_M}\ln(-W_{M,t}) + f(a_t)$, $\mu_W = \delta_M - \frac{\beta}{\gamma_M}$, and $\sigma_W = \frac{f'(a_t)}{\mu'(a_t)}\beta\sigma$. This proves Proposition 1.

We verify that the equilibrium stock price in the agency contracting case has the following form of

$$P_t = e^{rt}\omega(\hat{\Theta}_t, t) + \frac{1}{\gamma_M r}\ln(-W_{M,t}) \quad (65)$$

where

$$d\omega = -e^{-rt}k_S(\hat{\Theta}_t, t)dt$$
$$\text{and}\quad k_S(\hat{\Theta}_t, t) = \mu(\hat{a}_t) + \frac{1}{\gamma_M}\ln\beta - f(\hat{a}_t) + \frac{1}{\gamma_M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) - (1 - \hat{\Theta}_t)\gamma_S r\left(\sigma - \frac{\sigma_W}{\gamma_M r}\right)^2. \quad (66)$$

This proves Proposition 2.

The large shareholder needs to choose the optimal trading policy on date t_i that solves

$$G^*(\Theta_{t_{i-1}}, t_i) = \max_{\Theta}\ \phi_i k_L(\Theta, t_i) - (\Theta - \Theta_{t_{i-1}})P(\Theta, t_i) + (1 - r\phi_i)G^*(\Theta, t_{i+1}) + \frac{\Theta}{\gamma_M r}\ln(-W_{M,t}). \quad (67)$$

By assuming a recontracting case with the constant reservation payoff $\underline{W_M}$, we derive the first order condition for the above problem by

$$k_{L1}(\Theta_{t_i}^*, t_i) - k_S(\Theta_{t_i}^*, t_i) - (\Theta_{t_i}^* - \Theta_{t_{i-1}})k_{S1}(\Theta_{t_i}^*) + \frac{(1 - r\phi_i)}{\phi_i}\frac{1}{\gamma_M r}\ln(-\underline{W}_M) = 0. \quad (68)$$

Given that k_L and k_S are non-linear in the the large shareholder's ownership stake Θ, it is not easy to find an analytical solution even in the simple stationary case. So, we obtain the large shareholder's ownership dynamics by numerically solving the first order condition (68). This proves Proposition 3.

Table I: Baseline Parameters

Model Parameter	Baseline Value
Risk-free rate (r)	0.02
Firm output (cash flow) volatility (σ)	0.25
Subjective discount rate ($\delta_L = \delta_M = \delta_S$)	0.0513
Time interval between trading dates (Δ_t)	1
Unit cost of effort (ψ)	1
Constant term (μ_0) in the linear mean cash flow function $\mu(a_t)$	1
Productivity of effort (μ_1) in the linear mean cash flow function $\mu(a_t)$	0.48
Large shareholder's absolute risk aversion (γ_L)	133.35
Manager's absolute risk aversion (γ_M)	207.77
Small shareholders' absolute risk aversion (γ_S)	133.35
Manager's reservation value ($\underline{W}_M$)	-0.90

Table II: Observed and Model-Predicted (Steady-state) Moments

	Large shareholder's ownership stake ($\overline{\Theta}^*$)	Sensitivity of CEO pay to stock return ($\overline{PPS}_2^*$)	Sharpe ratio $\overline{\mu}_R^*/\overline{\sigma}_R^*$	Market-to-book ratio ($\overline{MB}^*$)
Observed	0.37	0.003	0.32	1.183
Predicted	0.37	0.0063	0.3196	1.1818

Table III: Baseline (Steady-state) Equilibrium in the Actual and Benchmark Scenarios

	$\overline{\Theta}^*$	$\overline{a}^*$	μ_W^*	σ_W^*	$\overline{PPS}_1^*$	$\overline{PPS}_2^*$	$\overline{\mu}_R^*$	$\overline{\sigma}_R^*$	$\overline{\mu}_R^*/\overline{\sigma}_R^*$	$\overline{MB}^*$
Owner-Manager Case	0.5	0.24					0.0833	0.25	0.3334	1.1257
Agency Contracting Case	0.37	0.4723	0.0464	0.2485	0.0048	0.0063	0.0608	0.1902	0.3196	1.1818

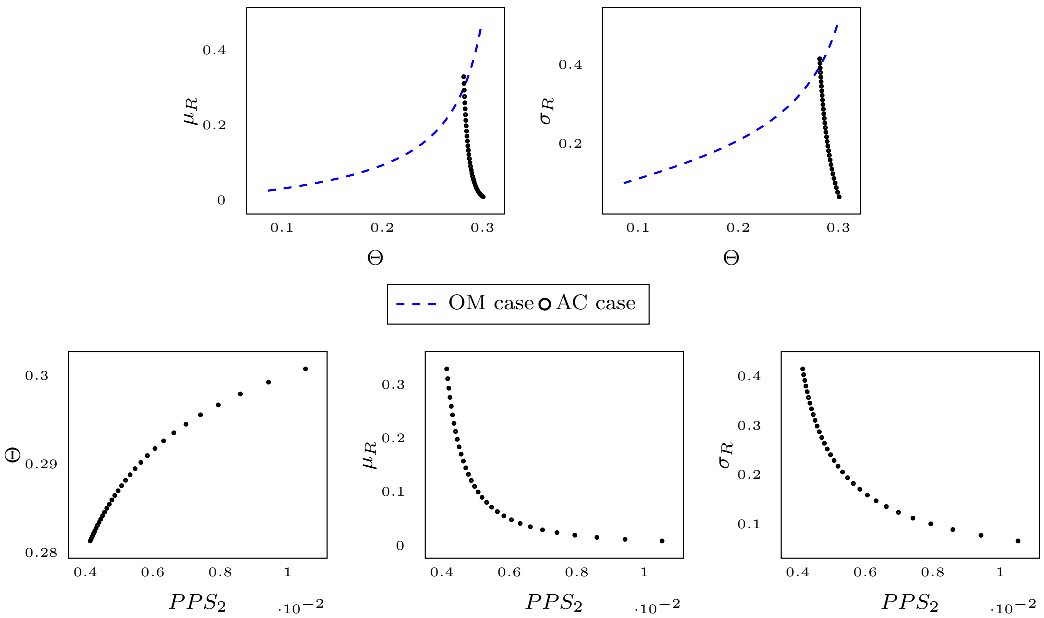

Figure 1: Effects of Firm Cash Flow Volatility σ

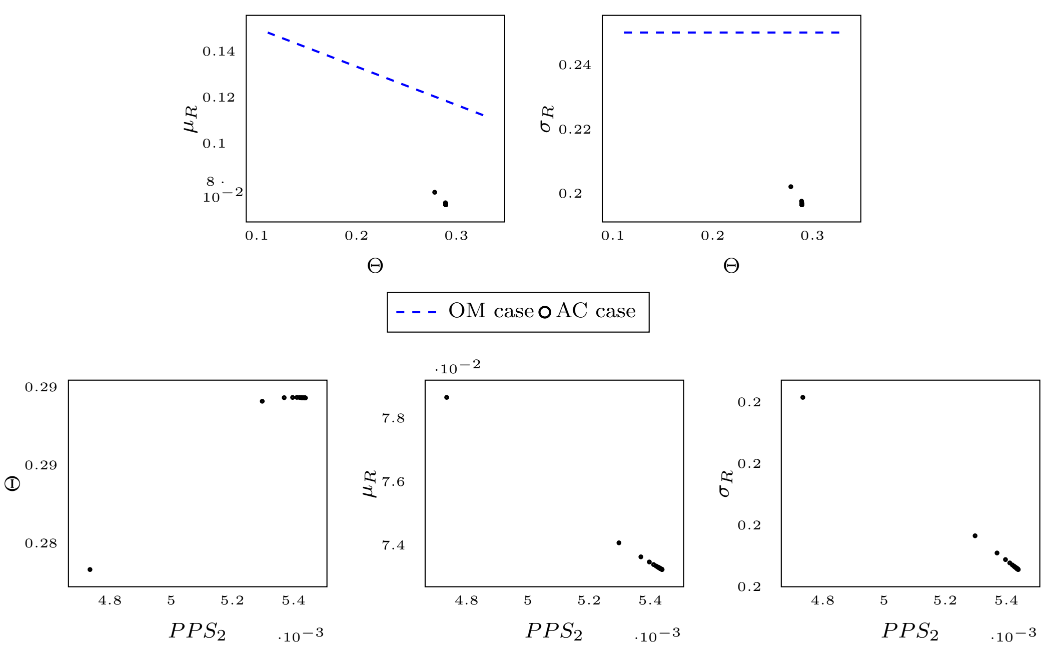

Figure 2: Effects of Productivity of Effort μ_1

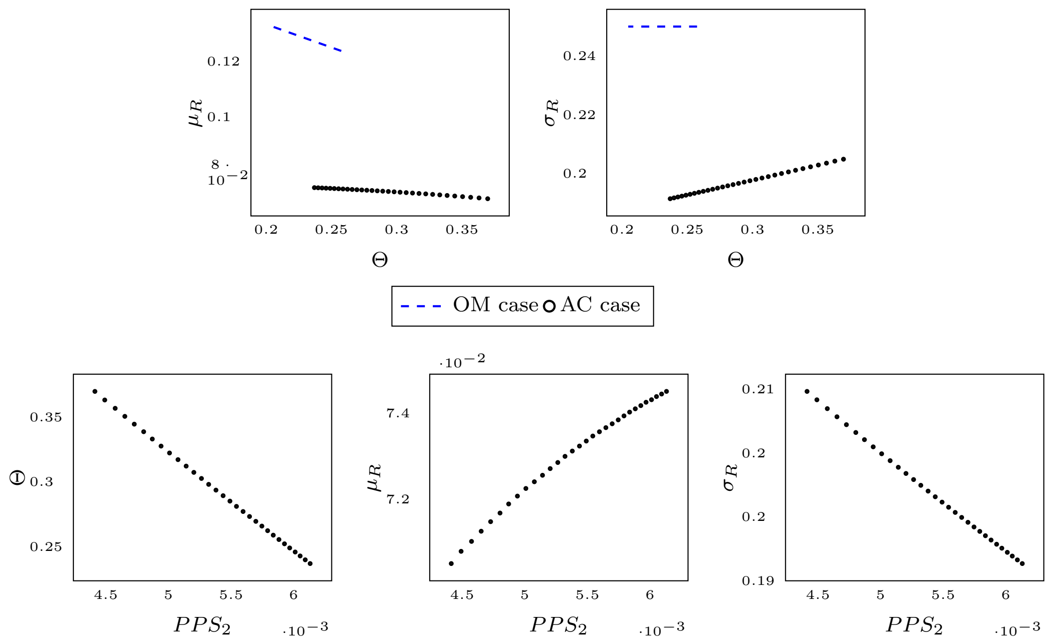

Figure 3: Effects of Large Shareholder's Risk Aversion γ_L

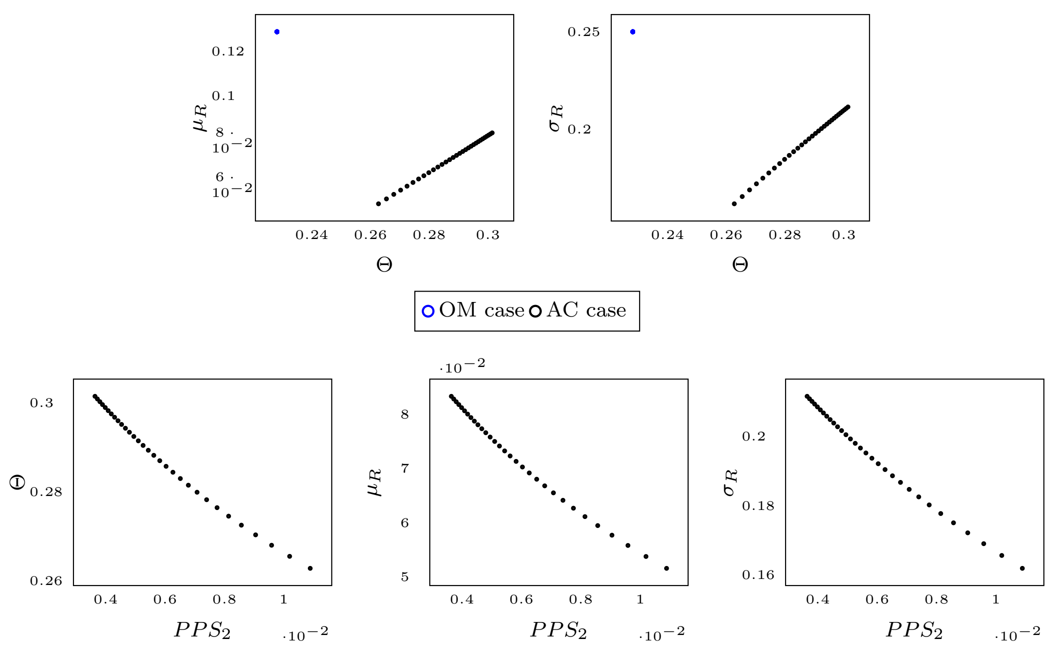

Figure 4: Effects of Manager's Risk Aversion γ_M

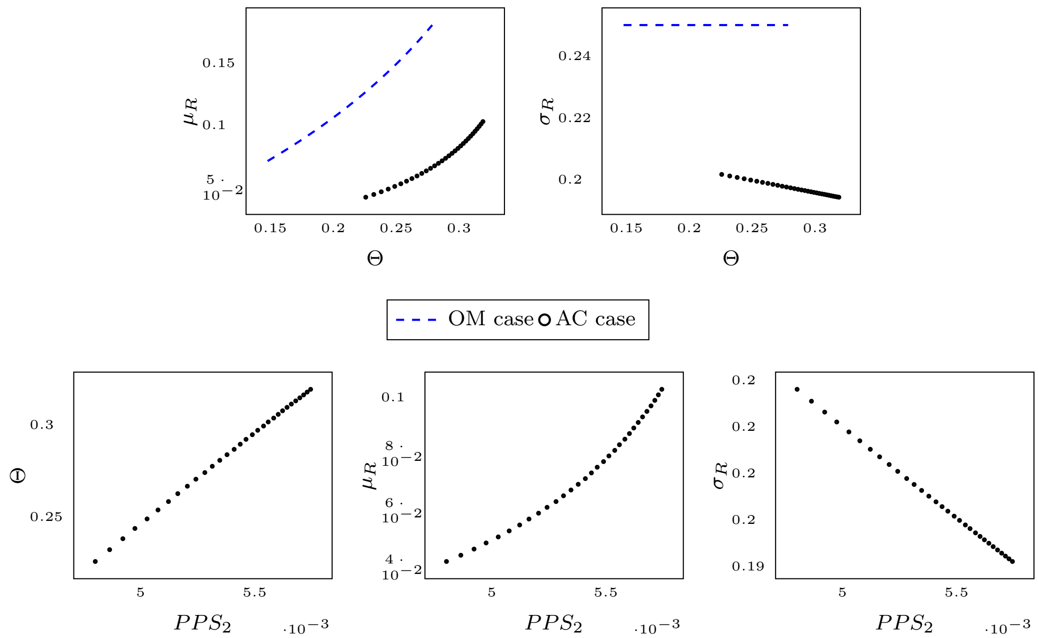

Figure 5: Effects of Small Shareholders' Risk Aversion γ_S

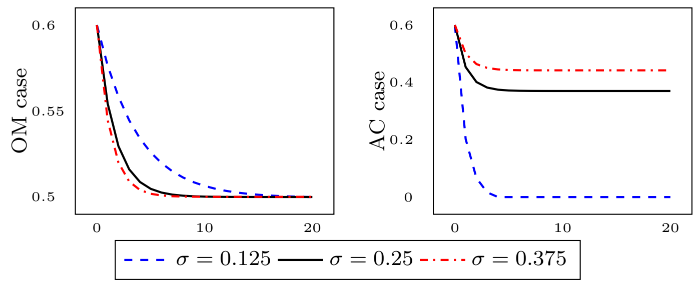

Figure 6: Large Shareholder's Ownership Dynamics over Time (in response to σ)

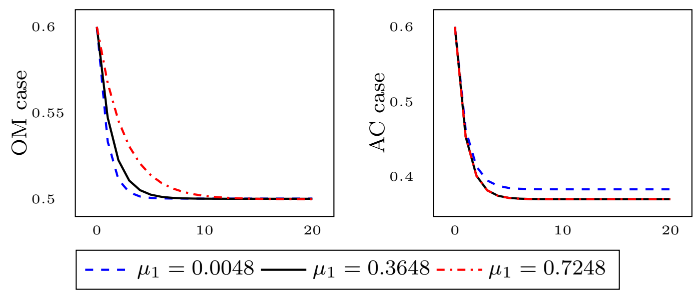

Figure 7: Large Shareholder's Ownership Dynamics over Time (in response to μ_1)

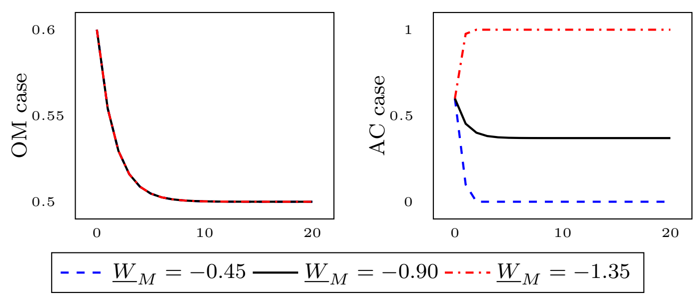

Figure 8: Large Shareholder's Ownership Dynamics over Time (in response to $\underline{W}_M$)

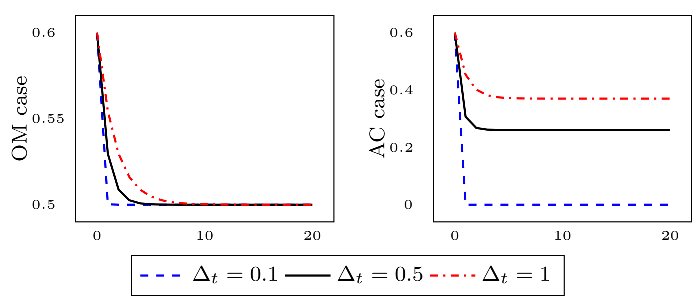

Figure 9: Large Shareholder's Ownership Dynamics over Time (in response to Δ_t)