Partial Moment Momentum

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This version: 09 August, 2017

Abstract

Despite the fact that momentum strategies have been shown to generate significant positive returns by many previous studies, momentum can experience extreme weakness during periods of economic upheaval, especially during financial crises. Consistent with the literature, we find that momentum profits benefit from persistent trends of the market which can be predicted by market volatility. We propose partial moments-based (PM-based) momentum trading strategies and find that our partial moments-based (PM-based) momentum trading strategies outperform plain momentum and volatility-scaled momentum strategies. Our best performing PM-based strategy shows an annualized Sharpe ratio of 1.32 during the financial crisis period of 2008-2012, compared to an annualized Sharpe ratio of -0.54 from a benchmark 6×6 plain momentum strategy. An explanation of this strong profitability is that investors can distinguish between good and bad risk. Our results are robust across different momentum strategies and multiple time periods.

JEL classification: G11, G12, G17

Keywords: Downside risk; Momentum; Partial moments; Portfolio performance

Acknowledgement: We thank seminar participants at OMGI and J.P. Morgan London for helpful comments and suggestions.

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1. Introduction

Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) present evidence that scaling the weights of momentum portfolios increases the Sharpe ratio of the plain momentum strategy. Rather than scaling plain momentum portfolios, we construct two partial moments-based (PM-based) strategies, the partial moment momentum (PMM) strategy and the partial moment-decomposed (PM-decomposed) strategy, which enable momentum traders to better capture market trends and avoid huge losses during market turbulence using market decomposed variances. We find that both PM-based strategies significantly outperform the plain momentum strategy. We further introduce an adapted-Sortino ratio to better capture the performances of strategies, particularly, during market downturns.

Momentum strategies are employed by buying previous winners and selling previous losers¹ (henceforth plain momentum strategies). The literature shows that cross-sectional momentum strategies are profitable in different markets and asset classes outside the US across different sample periods². However, evidence on time-series momentum profits is inconclusive. Chordia and Shivakumar (2002) demonstrate that economic expansionary periods may be important in explaining the profits in the US equities market, but the literature is inconclusive whether momentum profits are positive or negative during contractionary periods. Ali and Trombley (2006) find that the level of momentum returns of US stocks for the period from 1984 to 2001 is positively related to short sales constraint, and that loser portfolios rather than winner portfolios drive this result. In contrast, Gao and Leung (2017) show that momentum returns of Australian stocks are negatively correlated to short sale restrictions and they are less profitable during the global financial crisis (GFC) compared to the pre-GFC period. The explanation is that the imposition of short selling restrictions by the Australian regulatory authority during the GFC may have moderated the ability for momentum traders to profit from the short sale of loser portfolios. This is supported by Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), who show that momentum strategies experience extreme losses during periods of economic upheaval following market crashes and high market volatility.

¹ Jegadeesh and Titman (1993) find that momentum strategies are profitable in the US equities markets over the short to medium horizons (3 to 12 months) from 1965 to 1989. Jegadeesh and Titman (2001) continue to show similar results for the period 1990 to 1998. Israel and Moskowitz (2013) extend momentum evidence to two periods from 1927 to 1965 and from 1990 to 2012.

² See Richards (1997) for evidence of momentum in stock market indices and Asness, Liew, and Stevens (1997) in country indices; Rouwenhorst (1998, 1999) in emerging stock markets; Chan, Hameed and Tong (2000) and Hameed and Yuanto (2002) for momentum in international equity markets. See Okunev and White (2003) for momentum in exchange rates; Erb and Harvey (2006) in commodities and Moskowitz, Ooi and Pedersen (2012) in futures contracts. Consistent with Asness, Moskowitz and Pedersen (2013), Daniel and Moskowitz (2016) find momentum returns in markets across regions (European Union, Japan, United Kingdom and United States) and asset classes (fixed income, commodities, foreign exchange and equity) from 1972 through 2013.

Recent literature has introduced a number of volatility-scaled momentum strategies, in both a crosssectional momentum setting (Barroso and Santa-Clara (2015)) and in a time-series momentum setting (Daniel and Moskowitz (2016)). In both cases, there is the notion of the use of a target volatility to scale the risk exposure of plain momentum returns to produce risk-managed momentum returns. For instance, Barroso and Santa-Clara (2015) reveal that gains in momentum returns can be wiped out by momentum crashes over short periods. For example, US stocks delivered -91.59% momentum returns in 1932 over just two months, and -73.42% over three months in 2009. What is more surprising is the high level of predictability of the risk of momentum returns. They show that the autoregression of monthly realized variance of daily returns of US stocks from 1927 through 2012 produces an out-ofsample R-square of 57.82%. They proceed to scale the long-short winners-minus-losers (WML) momentum portfolio by its prior six months' realized volatility to implement a constant volatility strategy which avoids forward-looking bias. This risk managed momentum strategy results in negligible negative returns during the crashes, a doubling of the Sharpe ratio and a reduction in both excess kurtosis and left skewness. Daniel and Moskowitz (2016) implements a dynamic momentum strategy based on conditional moments (mean and variance) and as a result achieve twice the alpha and Sharpe ratio compared to the traditional static WML strategy over multiple time periods and different equity markets³. This evidence suggests that momentum strategies which dynamically account for past volatility act as a hedging mechanism to the extreme momentum losses subsequent to sudden market downturns. However, the unconstrained leverage implicit in such a strategy makes these strong results questionable as a practical investment strategy.

Barndorff-Nielsen, Kinnebrock, and Shephard (2010) show that future volatility is related more to past negative returns than past positive returns. In doing so, they develop a volatility measure called realized semi-variance which decomposes the direction of the quadratic variation in asset prices, termed realized variance⁴. As such, the negative and positive returns of asset prices are used to compute a downside (RS-) and an upside (RS+) realized semi-variance, respectively. A review of the semi-variance literature is presented in Sortino and Satchell (2001). Further development of the economic theory underpinning this risk measure is set out in Pedersen and Satchell (2002). These directional volatility measures are found to capture the asymmetrical properties of volatility experienced by asset prices. Hedge funds may employ downside realized semi-variance in the context of risk management. These investors may have short positions in the market and therefore a drop in

³ Both Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) construct momentum portfolio with a 11-month formation period and a 1-month holding period. Particularly, there is a one-month gap between the formation and holding period and this strategy uses value-weighted holding period returns without rebalancing. We refer this strategy in particular to (static) *WML* strategy.

⁴ Andersen, Bollerslev, Diebold and Ebens (2001) and Barndorff-Nielsen and Shephard (2002) define the realized variance as the sum of the squared returns to estimate the quadratic variation in high frequency asset prices.

price realizes a positive return with the corresponding measure of risk being RS-. Baruník, Kočenda and Vácha (2016) extend this idea to construct asymmetric volatility spillover indices and reveal high levels of asymmetrical spillover amongst the most liquid U.S. stocks in seven sectors. As we shall argue later, it is more appropriate that this semi-variance be called partial moments of order 2. Upper and lower partial moments are the upside and downside realised semi-variance, respectively.

The literature has further shown that momentum returns are related to partial moments, especially lower partial moments⁵. We employ the idea of downside realized partial moment (RPM-) and upside realized partial moment (RPM+) in the context of momentum trading strategies. We hypothesize that if volatility based momentum strategies such as those in Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) have been shown to counter the extreme momentum losses in times of market crashes, then further momentum profits and better risk management can be achieved by utilizing the connection between RS- and future volatility due to Barndorff-Nielsen, Kinnebrock, and Shephard (2010).

We introduce two partial moment-based (PM-based) momentum strategies. We call our first PMbased momentum strategy the PMM strategy. It involves switching positions of the winner and loser portfolios during the holding periods depending upon current estimates of partial moments. Our second PM-based momentum strategy, an extended PM-decomposed momentum strategy, can be viewed as an extension of the dynamic volatility-based momentum strategy in Barroso and Santa-Clara (2015). The key idea is that they do not differentiate between upside or downside risk. We extend this class of strategies by tilting our strategy long or short towards favourable/unfavourable volatility signals and holding an off-setting position in cash.

We show strong evidence that our two PM-based strategies outperform benchmark plain momentum strategies⁶, in particular, during financial turbulences. Our results reveal an annualized adapted-Sortino ratio (defined on page 8) of 1.35 from the Global Financial Crisis (GFC) period, 2008 to 2012, of the PMM Strategy 5 (PMM_S5) compared to an annualized adapted-Sortino ratio of -0.23 of the benchmark 6×6 plain momentum strategy. Our robustness test results show that an annualized Sharpe ratio of 1.48 from 2000 to 2016 of a PM-decomposed WML strategy compared to an annualized Sharpe ratio of 0.22 in Daniel and Moskowitz (2016) for dynamic WML strategies from

⁵ See Menkhoff and Schmeling (2006), Daniel, Jagannathan and Kim (2012) and Baltas and Kosowski (2012).

⁶ We use a 6×6 plain momentum strategy, which represents a 6-month formation period and a 6-month holding period plain momentum strategy with one month gap between formation and holding periods, as benchmark. We use it as the benchmark strategy since this plain strategy yields the highest returns over the whole sample period among all four plain momentum strategies. Previous findings also reveal this phenomenon. See, for example, Jegadeesh and Titman (1993, 2001).

2000 to 2013, confirming the persistence of momentum returns beyond 2013⁷. Further out of sample analyses show consistent results.

The remainder of the paper is organised as follows. Section 2 describes the data and plain momentum construction and shows the performances of plain momentum strategies in US equity markets. Section 3 provides a theoretical development of the PMM strategies and examines their performance. Section 4 examines the performances of extended PM-decomposed momentum strategies. Section 5 further tests the out of sample performances of PM-based momentum strategies and studies the persistence of these strategies over financial turbulences. Section 6 assesses the robustness of our findings. Section 7 concludes.

2. Momentum in the US equity market

2.1 Data

The data used in this paper are sourced from Centre for Research in Security Prices (CRSP) via Wharton Research Data Services (WRDS), Datastream and Kenneth French Data Library (French Library)⁸. The monthly and daily US equity data are sourced from CRSP over the period January 1964 to December 2016. Our sample includes common stocks (CRSP share code 10 or 11) of all firms listed on NYSE, Amex and Nasdaq (CRSP exchange code 1, 2 or 3). We use S&P 500 Index and one-month Treasury bill rate as the proxy for the market portfolio and the risk-free rate, respectively. We obtain daily and monthly S&P 500 price index from Datastream and one-month Treasury bill rate from French Library over the same period as that of US equity data.

2.2 Momentum portfolio construction

We follow a similar methodology to the Jegadeesh and Titman (1993) $J \times K$ Trading Model to construct our momentum portfolio with zero-net position. Firstly, in month *t*, all sample stocks are ranked and sorted into deciles based on their past *J-month* formation period returns from month *t-J- I* to *t-1*. Subsequently, we buy the best performed portfolio (winners) and short the worst performed portfolio (losers) for the *K-month* holding period from month *t* to *t+K*. The equally-weighted holding period returns are calculated for computing our momentum profits. In the meantime, this strategy rebalances every month. Thus, in any month t, one certain strategy holds not only the winner and loser portfolios constructed in month t, but also those portfolios in the previous *K-2* months. We also use the one month lag between the formation period and the holding period to allow us to be consistent

⁷ We use a 11×1 plain momentum WML strategy, consistent with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), as our benchmark for performance comparison.

⁸ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016)⁹, whilst noting that they differ from us in other respects.

2.3 Performances of plain momentum strategies in the US equity markets

[Insert Table 1 about here]

From Table 1, we can compare the performances of four plain momentum strategies: 3×3 , 6×6 , 9×9 and 12×12 . During our whole sample period P1 and Jegadeesh and Titman 1993 period P2, the 6×6 strategy, which represents a momentum strategy has 6-month formation (J) and holding (K) periods with a one-month gap between J and K, can earn 6.29% and 12.01% annualised return, respectively. It outperforms all other 3 strategies during these two periods by having higher positive Sharpe ratios. 3×3 and 9×9 strategies also report positive returns during P1 and P2. However, none of these four plain strategies can generate profits during periods of P4 financial turbulence.

3. Partial moment momentum

Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) present evidence that, by scaling the weights of momentum portfolio, this new scaling strategy increases the Sharpe ratio of the plain momentum strategy.

Rather than scaling plain momentum portfolios, we firstly construct a PMM strategy by switching positions of the winner and loser portfolios during the holding periods. Our PMM strategy enables momentum traders to better capture market trends and hopefully avoid huge losses during market rebounds using market decomposed variances.

3.1 Partial moments and realised semi-variance

We can construct the sample realised volatility using equi-spaced data over [t-1, t] which we define as RV. This is defined over n+1 prices or their logarithms to define n returns, r_i , i=1,...,n. We define RV as

$$RV_t = \sum_{i=1}^{n} r_{i,t}^2$$
(1)

We call this the realised variance over [t-1, t] and is known to be a consistent estimator of the quadratic variation if we assume the prices are generated by a particular class of processes, see Baranik et al (2016) for more details. Properties of *RV* and related measures can be found in Andersen

⁹ By so doing, it can avoid short-term reversals. See Jegadeesh (1990) and Lehmann (1990).

et al (2001) and Barndorff-Nielson (2002). Following the literature, we define two statistics RS^- and RS^+

where

$$RS^{-} = \sum_{i=1}^{n} r_{i}^{2} I(r_{i} < 0)$$
⁽²⁾

and

$$RS^{+} = \sum_{i=1}^{n} r_{i}^{2} I(r_{i} \ge 0)$$
(3)

where I() is the indicator function, these being sample lower and upper partial moments of order 2 with truncation at zero in both cases. There is an identity,

$$RV = RS^- + RS^+ \tag{4}$$

If the population has a mean of zero, then we may hope to treat these quantities as estimators of the unconditional population variance and the unconditional lower and upper semi-variances of the process over the interval [t-1, t], which we could denote by σ^2 , σ^{2-} , σ^{2+} respectively. Hopefully, $\sigma^2 = \sigma^{2-} + \sigma^{2+}$; However, the difficulty here is that $E(RS^+)$, for example, is not equal to σ^{2+} but rather $\sigma^{2+} + E^2(\sum_{i=1}^n r_i l(r_i \ge 0))$. This means that descriptions in terms of partial moments seems more appropriate. Thus, we refer these two statistics as upper and lower partial moments (RPM^+ and RPM^-).

3.2 Some theoretical considerations

It is argued by Barroso and Santa-Clara (2015) that momentum volatility is strongly forecastable relative to other styles, see in particular Table 2, page 114. This suggests that momentum volatility, or momentum partial moments may be useful in forecasting momentum returns. We note a caution in that we believe that part of the forecastability is artificial. To see this, we will oversimplify and assume that a momentum strategy can be described as a position in an asset where the "weight" of the asset is equal to last period's return. Readers will identify this as version of single variable time-series momentum. Thus, R_t , the return to the strategy is equal to $r_t r_{t-1}$.

We first show that momentum is forecastable even when returns are iid. Assume that r_t is iid (μ , σ^2). It follows by elementary calculation that

$$Var(R_t) = E(r_t^2 r_{t-1}^2) - E^2(r_t r_{t-1})$$
(5)

and

$$Cov(R_t, R_{t-1}) = E(r_t r_{t-1}^2 r_{t-2}) - E(r_t r_{t-1}) E(r_{t-1} r_{t-2})$$
(6)

This leads to an autocorrelation coefficient $\rho = \frac{\mu^2 \sigma^2}{\sigma^4 + 2\mu^2 \sigma^2}$. If we interpret the signal to noise ratio of the strategy $SN = \frac{\mu}{\sigma}$; then $\rho = \frac{SN^2}{1+2SN^2}$. Thus, if the strategy has a signal to noise ratio of 0.5, then returns will appear to have an autocorrelation coefficient of 0.17 although the underlying data are pure white noise.

Furthermore, as SN becomes large we reach an upper bound for ρ of $\frac{1}{2}$.

We now assume that $E(r_t) = 0$. If we now turn to

$$Var(R_t^2) = E(r_t^4 r_{t-1}^4) - E^2(r_t^2 r_{t-1}^2)$$
(7)

and

$$Cov(R_t^2, R_{t-1}^2) = E(r_t^2 r_{t-1}^4 r_{t-2}^2) - E\left((r_t^2 r_{t-1}^2) E\left((r_{t-2}^2 r_{t-1}^2)\right)\right)$$
(8)

Let
$$\mu_j = E(r_t^j)$$
. Then $\rho = \frac{1}{\frac{\mu_4}{\mu_2^2} + 1}$.

We see that, when underlying returns are white noise, the forecastibility of momentum volatility is connected to underlying kurtosis, the higher the kurtosis the lower the forecastibility. If we take this to be the kurtosis of six-monthly index returns we might expect a number near 5 and so the (spurious) autocorrelation might be of a similar magnitude to before. Thus we do not use momentum volatility in guiding our strategies but use instead a market index. It follows if we define $E(RV) = E(\sum_{i=1}^{n} r_i^2)$ and $E(RPM^-) = E(\sum_{i=1}^{n} r_i^2 I(r_i < 0))$ and $E(RPM^+) = E(\sum_{i=1}^{n} r_i^2 I(r_i \ge 0))$.

3.3 Partial moment momentum strategies

Whilst we continue to report Sharpe ratios, we also provide a performance measure called the Adapted-Sortino ratio. We define the Adapted-Sortino ratio as

$$Adapted - Sortino\ ratio = \frac{Excess\ Return}{2 * Downside\ SemiDeviation}$$
(9)

where

$$Excess Return_t = R_t - Desired Target Return_t$$
(10)

Sortino and Price (1994), for example, defines sample Downside Semi-Deviation as¹⁰

¹⁰ Third paragraph, Page 61, Sortino and Price (1994). Sortino and Price (1994) refers to this "downside risk" measure a Downside Deviation, which we thought the name Downside Semi-Deviation seems more appropriate.

Downside SemiDeviation

$$= \sqrt{\frac{\sum_{i=1}^{N} (Excess \ Return_i - \overline{Excess \ Return})^2}{N}} I(Excess \ Return_i < 0)$$
(11)

where *Excess Return* is the portfolio's excess return on *Desired Target Return* which we use a one-month Treasury bill. *I*() is the indicator function, this being sample target excess return lower than zero. Our adaptation differs from the standard Sortino ratio with target return equal to the riskless rate, as, in the event that downside and upside standard deviations are equal, we recover the Sharpe ratio. Thus, we have put our version of the Sortino ratio into a broadly similar scale as the Sharpe ratio.

3.3.1 Partial moments and boundaries

[Insert Figure 1a and 1b about here]

Figures 1a and 1b show the histograms of upper and lower partial moments $(RPM_t^+ \text{ and } RPM_t^-)$ with kernel density curves. Values of 10th percentile of RPM_t^+ , 75th percentile of RPM_t^- and maximum observations of both RPM_t^+ and RPM_t^- are reported. For the upper partial moment (RPM_t^+) , the normal kernel estimate for c=0.7852 has a bandwidth of 0.0002 and an approximate mean integrated square error (AMISE) of 3.3237; For the lower partial moment (RPM_t^-) , the normal kernel estimate for c=0.7852 has a bandwidth of 0.0002 and an AMISE of 3.1071.

[Insert Table 2 about here]

Table 2 reports the distributions of monthly realised variance (RV_t) , upper and lower market partial moments $(RPM_t^+ \text{ and } RPM_t^-)$ throughout the whole sample period from January 1964 to December 2016. In this paper, we use 10th percentile of RPM_t^+ and 75th percentile of RPM_t^- as the critical values for upper and lower market partial moments. ¹¹Although our results are not sensitive to our choice of boundaries¹², we thought that the point where we would have a behavioural shift should be lower for positive risk so 90 per cent of the time we carry out the same gain strategy, differing only when positive risk is low. For negative risk, we carry out the same strategy 75 per cent of the time differing only when negative risk is high. Given these are the cases when momentum performs very badly, it seems appropriate to have an area of larger probability.

¹¹ All analyses are repeated using 2 other pairs of boundaries, medians of RPM_t^+ and RPM_t^- and 10^{th} percentile of RPM_t^+ and 75^{th} percentile of RPM_t^- in addition to the original critical values 10^{th} percentile of RPM_t^+ and 75^{th} percentile of RPM_t^- . Results are found to be consistent to those based on the original boundaries.

¹² It is not sensitive to variation away from the medians but when we run all tests using median boundaries, our results are insignificant.

The previous approach used past values evaluated in different ways as our forecast of partial moments. An alternative to this approach would be to assume that the partial moments satisfy a statistical model such as a vector-autoregressive process of order p (VAR(p)). We shall assume, for simplicity, that monthly partial moments satisfy a VAR (1) given by Regressions (12) and (13)

$$RPM_t^+ = \alpha_1 + \beta_{11}RPM_{t-1}^+ + \beta_{12}RPM_{t-1}^- + \varepsilon_{1t}$$
(12)

$$RPM_{t}^{-} = \alpha_{2} + \beta_{21}RPM_{t-1}^{+} + \beta_{22}RPM_{t-1}^{-} + \varepsilon_{2t}$$
(13)

Suppose that the conditional properties of μ_{2t}^+ and μ_{2t}^- , these being the upper and lower partial moments of degree 2, can be described by the following equations

$$\mu_{2t}^{+} = \alpha_1 + \beta_{11} RPM_{t-1}^{+} + \beta_{12} RPM_{t-1}^{-}$$
(14)

$$\mu_{2t}^{-} = \alpha_2 + \beta_{21} RPM_{t-1}^{+} + \beta_{22} RPM_{t-1}^{-}$$
(15)

Equations (14) and (15) can be interpreted as a matrix analogue of an ARCH (1) model except that it is a model for conditional partial moments. Replacing population moments by sample counterparts together with the errors involved gives us (12) and (13).

Such a model has certain features we can exploit for analytic purposes. We can compute the conditional and unconditional means of the partial moments and use potentially better forecasts.

Writing (12) and (13) in terms of vectors and matrices,

$$RPM_t = \alpha + \beta RPM_{t-1} + \varepsilon_t \tag{16}$$

Then, $E(RPM_t) = (I - \beta)^{-1} \alpha$ is the unconditional mean and the one period-ahead forecast for time t+1 at time t is given by $\alpha + \beta RPM_t$. The stationarity condition is that all the roots of β are less than one in absolute value.¹³ This is satisfied in all cases in this paper, based on estimated β .

[Insert Table 3 about here]

We present the estimated model in Table 3. We see that the past lower partial moment does not forecast the current partial lower moment otherwise all coefficients are significant and positive. We note also that the stationarity conditions are satisfied as well as the unconditional means being positive.

3.3.2 Rules and performances of PMM strategies

[Insert Table 4 about here]

¹³ We run standard unit root test on our VAR process given by regression (9) and (10). We find that the VAR process is stationary.

The idea behind our switching strategies is to change momentum strategies depending upon our current (RPM_t^+, RPM_t^-) estimates of partial moments. We could describe this as a regional PMM strategy. We refer to if simply as a PMM strategy.

Our switching methodologies are shown in Table 4 and Figure 2. Panel A of Table 4 shows four conditions of upper and lower partial moments based on their boundaries. The corresponding switching methodology (-ies) during holding periods under each of these four conditions are presented. Panel B illustrates different holding periods actions and returns to each of the four conditions presented on Panel A of our six partial moment momentum strategies, which we called PMM Strategy 1 to 6, represented by PMM_S1 to PMM_S6.

[Insert Figure 2a and 2b about here]

Figure 2a illustrates the four PMM strategies conditions in the coordinate plane. The origin point represents both the critical values for RPM_t^+ and RPM_t^- ($CV(RPM_t^+)$) and $CV(RPM_t^-)$) in month *t*. Each quadrant represents a PMM condition based on upper and lower partial moments and their critical values. For example, if in month *t*, both upper and lower partial moments are higher than their critical values, the point of PMM condition in month t is in the first quadrant in this coordinate plane. Then condition 1 applies in month *t* and the corresponding trading methods to condition 1 are employed as shown on Panel B, Table 4.

Figure 2b shows the estimation of four PMM strategies conditions when the 10th percentile of RPM_t^+ and the 75th percentile of RPM_t^- are used as critical values for RPM_t^+ and RPM_t^- , respectively. Apparently, the possibility of condition 4 occurs, when the upper partial moment is higher than the 10th percentile of RPM_t^+ while the lower partial moment being lower than the 75th percentile of RPM_t^- , is the most frequent of these four conditions.

In this paper, we use a 6×6 plain momentum strategy with a one month gap between formation and holding periods as the benchmark strategy to demonstrate the effectiveness of our PM-based strategies since this strategy yields the highest returns over the whole sample period among all four plain momentum strategies.¹⁴ Thus, we set our benchmark as high as possible.

[Insert Figure 3 about here]

Figure 3 demonstrates the timeline of a 6×6 PMM strategy. In any month *t*, all sample stocks are ranked and sorted into deciles based on their past *6-month* formation period returns from month *t*-7 to *t*-1 (due to a one month gap, month *t*-1 to *t*, between formation and holding periods for returns).

¹⁴ See Table 1.

Then we classify holding strategies into four conditions based on partial moments determined in the period month *t*-1 to *t* and their boundaries analysed in section 4.3.1. Then during the *6-month* holding period from month *t* to *t*+6, we continue to compare upper and lower partial moments with their boundaries in each month and switch positions of winners, losers and the cash asset to keep zero-net position based on Panel A, Table 4. That is to say, for any PMM strategies, we keep rebalancing and switching positions of all assets held. For instance, in month *t*+2, to a 6×6 PMM strategy which was constructed at time t, PMM conditions in month *t*+2 are determined based on partial moments during the period [*t*+1, *t*+2].

In particular, all six PMM strategies switch to the same positions if conditions 2 or 3 occur, but act differently if conditions 1 or 4 occur. Besides, we hold cash long or short cash to keep our PMM strategies net zero positions. $r_{w,t+1}$, $r_{t,t+1}$, $r_{f,t+1}$ represent return of winners, losers and risk-free asset in month t+1, respectively. For example, for PMM Strategy 4 (PMM_S4), if condition 1 applies during the period [*t*-*1*, *t*] where upper and lower partial moments are all higher than their boundaries, then we close out our positions in both winners and losers. The PMM return for month *t*+*1* is 0; If condition 2 applies during the period [*t*-*1*, *t*] where the upper partial moment is lower than its boundary and the lower partial moment is higher than its boundary, then we short losers only, liquidating our longs and holding cash long. The PMM return for month *t*+*1* is $r_{f,t+1} - r_{l,t+1}$; If condition 3 applies during the period [*t*-*1*, *t*] where upper and lower partial moments are all lower than their boundaries, then we carry on momentum strategy by buying winners and short selling losers. The PMM return for month *t*+*1* is r_{w,t+1} - $r_{l,t+1}$; If condition 4 applies during the period [*t*-*1*, *t*] where the upper partial moment is boundary, then we buy winners only and short cash. The PMM return for month *t*+*1* is $r_{w,t+1} - r_{l,t+1}$.

The key characteristic of our PMM strategies is that we change positions of winners, losers and a cash asset based on market partial moments during holding periods. For instance, if we construct an a4 PMM strategy on a J × K basis with zero-net position in month t, we rank all sample stocks and sort into deciles based on their past *J*-month formation period returns from month *t*-*J*-1 to *t*-1. Then during the *K*-month holding period from month t to t+K, we compare upper and lower partial moments with their boundaries in each month and switch positions of winners, losers and the cash asset to keep a zero-net position based on Panel A, Table 4. However, if the current partial moment condition persists over consecutive months, then we hold the same positions based on this condition's methodology and keep rebalancing until other conditions apply. Assume in month t, this $J \times K$ based PMM Strategy 4 (PMM_S4) meets Condition 4, then we buy winners only and short cash; If in month t+1 Condition 4 still holds, then we hold current positions and rebalance winners' portfolio based on past *J*-month performances from month t-1 to t, which is the same rebalancing method to our plain

momentum strategy in Section 3.2; If in month t+2 Condition 1 holds, then we close out long positions by selling winners and close out short positions in cash for the month. And so repeatedly.

[Insert Table 5 about here]

Table 5 shows a comparison of the performances of six PMM strategies on a 6×6 basis and the benchmark strategy over three sample periods: P1, whole sample period (1964-01 to 2016-12); P2, Jegadeesh and Titman (1993) period (1965-01 to 1989-12); P3, the Global Financial Crisis (GFC) period (2008-01 to 2012-12) and P4, the Era of Turbulence (2000-01 to 2016-12).

From Table 5, almost all six PMM strategies beat the benchmark strategy. In particular, PMM Strategies 4 and 5 (PMM_S4 and PMM_S5) persistently outperform the benchmark over the whole sample period and two sub-periods by generating higher Sharpe ratios and yielding higher-thanbenchmark significant returns. Even during the GFC, four PMM strategies earn positive returns and high Sharpe ratios and the adapted-Sortino ratios. Specifically, PMM Strategy 5 (PMM_S5) generates the highest 28.52% annualised return which is also the most significant among all 7 strategies including the benchmark.

4. An extended partial moment-decomposed momentum strategy

Recent papers have enhanced momentum strategies by weighting by volatility in various ways.¹⁵ These do not differentiate between upside or downside risk and, typically are, by construction net zero funds. We extend this class of strategies by tilting our strategy long or short towards favourable/unfavourable volatility signals and holding an off-setting position in cash.

We define a ($\varphi_1(RPM_t^+, RPM_t^-), \varphi_2(RPM_t^+, RPM_t^-)$) strategy as a net zero portfolio long $\varphi_1(RPM_t^+, RPM_t^-)$ in winners, short $\varphi_2(RPM_t^+, RPM_t^-)$ in losers with an offsetting position of $\varphi_1(RPM_t^+, RPM_t^-) - \varphi_2(RPM_t^+, RPM_t^-)$ in cash; we call this portfolio *p*. Denote its return at time t+1 as

$$r_{p,t+1} = \varphi_1(RPM_t^+, RPM_t^-)r_{w,t+1} - \varphi_2(RPM_t^+, RPM_t^-)r_{l,t+1} + (\varphi_2(RPM_t^+, RPM_t^-)) - \varphi_1(RPM_t^+, RPM_t^-)) r_{f,t+1}$$
(17)

Where $r_{w,t+1}$, $r_{l,t+1}$, and $r_{f,t+1}$ are the returns at time t+1 to the winners, losers and "cash" portfolios. There is no obvious guidance as to the functional form of $\varphi_1(RPM_t^+, RPM_t^-)$ and $\varphi_2(RPM_t^+, RPM_t^-)$ but we might expect $\varphi_1(RPM_t^+, RPM_t^-)$ to be increasing in its first argument and decreasing in its

¹⁵ See from Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016).

second argument and $\varphi_2(RPM_t^+, RPM_t^-)$ to have the opposite properties. As we discuss later, we might wish to have different assumptions on $\varphi_2(RPM_t^+, RPM_t^-)$

Furthermore, we might want to normalise them, as in Barroso and Santa-Clara (2015) in terms of some target volatility σ_{tar} . We pick the same target annualised volatility of 12% to keep consistency. With these considerations in mind we might also want the constraint

$$\varphi_1(RPM_t^+, RPM_t^-) + \varphi_2(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}}$$
(18)

which is broadly in accord with Barroso and Santa-Clara (2015). We choose $\varphi_1(RPM_t^+, RPM_t^-)$ and $\varphi_2(RPM_t^+, RPM_t^-)$ as follows:

$$\varphi_1(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}} \left(\frac{RPM_t^+}{RPM_t^+ + RPM_t^-} \right)$$
(19)

$$\varphi_2(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}} \left(\frac{RPM_t^-}{RPM_t^+ + RPM_t^-} \right)$$
(20)

If $RPM_t^+ = RPM_t^-$, then we have a conventional long-short portfolio with scaling $\frac{\sigma_{tar}}{\sqrt{RV_t}}$ as in Barroso and Santa-Clara (2015), formula (5), page 115.

There are many alternative ways we could specify our strategy and (18) to (20) could be criticised that we might want shrink our exposure on the downside when RPM_t^- is large. Such an approach might be more consistent with the empirical result that momentum profits mainly result from the long-side of the portfolio.¹⁶ Also it might be argued that choosing functions that are homogeneous of degree zero loses some of the partial moment information. The argument in favour of increasing one's exposure on the downside is that, in prospect theory, agents are usually deemed to be risk-loving on the downside.¹⁷

In practise leverage is an issue in long-short portfolios. Many institutional hedge funds have strict restrictions on leverage, where 200% leverage is a typical upper bound. The previous popularity of

¹⁶ Jegadeesh and Titman (1993) state that "the abnormal performance of the zero-cost (momentum) portfolio is due to the buy side of the transaction rather than the sell side." Moskowitz and Grinblatt (1999) argue that "industry momentum strategies appear to profit mostly on the buy side". See, for instance, Chan, Jegadeesh and Lakonishok (1996), Jegadeesh and Titman (2001) and Israela and Moskowitz (2013) for further evidence.

¹⁷ Ang, Chen and Xing (2006) conclude that "...the behavioural framework of Kahneman and Tversky's (1979) loss aversion preferences and the axiomatic approach taken by Gul's (1991) disappointment aversion preferences allow agents to place greater weights on losses relative to gains in their utility functions." They also argue that "agents who place greater weight on downside risk demand additional compensation for holding stocks with high sensitivities to downside market movements."

130-30 funds is evidence that leverage is not unconstrained in practise.¹⁸ Leverage is defined as the sum of absolute value of long and short weights (ignoring cash positions) so that our strategies above have a leverage as in (18).

To rescale our weights to obey a 200% leverage condition, we need to change our scaling to

$$\varphi_1(RPM_t^+, RPM_t^-) = 2\left(\frac{RPM_t^+}{RPM_t^+ + RPM_t^-}\right)$$
(21)

$$\varphi_2(RPM_t^+, RPM_t^-) = 2\left(\frac{RPM_t^-}{RPM_t^+ + RPM_t^-}\right)$$
(22)

with corresponding positions in cash.

[Insert Table 6 about here]

Table 6 shows the performance of PM-decomposed momentum strategy over four sample periods. Our PM-decomposed momentum strategy (M66*) dramatically outperform the plain momentum strategy over all four sample periods by generating higher and more significant positive returns, and having larger Sharpe ratios and Adapted-Sortino ratios. Even during P3, the GFC period and P4, the Era of Turbulence when holding a plain 6×6 momentum strategy earns negative returns, our PM-decomposed momentum strategy persistently yields an annualised return of about 40%. In particular, during the GFC, this strategy has an annualised Sharpe ratio of 1.32 and an annualised adapted-Sortino ratio of 1.02. Besides, our leveraged PM-decomposed momentum strategy (M66#) reveals similar good performances even though it slightly underperforms the unleveraged strategy.

5. Out of sample analysis

Our PMM performances in the previous sections are conditional on the realised semi-variance boundaries over our whole sample period over January 1964 to December 2016. Concerns may be raised that this involves using out-of-sample information and lacks realism.

To strengthen the effectiveness of our PMM model and to provide practical investment insights for momentum investors, we further construct PMM strategies using parameters and boundaries computed within an in-sample period from January 1964 to December 1999. We call the out-of-sample period, which is from January 2000 to December 2016, the Era of Turbulence since during which, there was the IT bubble (early 2000s), the Hedge Fund Crisis (2006), the GFC (Since 2008) and the European Debt Crisis (Since late 2009). Therefore, it may be more convincing if our selected

¹⁸ "130-30" represents a leveraged long-short mutual fund strategy that allows fund managers to hold short positions up to 30% of the initial investment and use the corresponding funds to take long positions. Other popular strategies of this type include "150-50", "120-20" and so on.

PMM and PM-decomposed momentum strategies outperform plain momentum during these financial turbulences.

[Insert Table 7 about here]

Table 7 shows the use of critical values of 10^{th} percentile for RPM_t^+ and 75^{th} percentile for RPM_t^- during in-sample periods for the VAR of upper and lower partial moments, respectively, in order to maintain consistency with previous sections.

[Insert Table 8 about here]

Table 8 presents the frequencies of the four PMM switching conditions based on in-sample partial moments over the whole out-of-sample periods, the Era of Turbulence, from January 2000 to December 2016. It shows that in any given month *t* over the out-of-sample period, there is a high probability that condition 1 or 4 occurs. Compared to the estimated frequencies shown in Table 8 Panel B and Figure 2b, the major variation is that the frequency of condition 1 more than doubled during the Era of Turbulence (actual frequency of 50.49% compared to an expected frequency of 22.5%). One possible explanation is that the Era of Turbulence contains a lower proportion of market upturns compared to periods of market downturns, in contrast to our in-sample period from 1964 to 1999. Thus, condition 1, where upper and lower partial moments are greater than the in-sample critical values, is fulfilled more frequently during this period.

5.1 Performances of out of sample PMM strategies

[Insert Table 9 about here]

Table 9 presents a comparison of the performances of six PMM strategies and the benchmark strategy over the whole out-of-sample period and the GFC period. Similar to the results presented in Table 5, five out of six PMM strategies persistently outperform the benchmark strategy over these two periods. Particularly, strategies a4 and a5 both earn significant positive returns during the GFC while the benchmark strategy makes plain momentum investors wounded by losing approximately 1.2% a month.

5.2 A dynamic PM-decomposed strategy

By using the estimated coefficients over in-sample periods from Table 7, we generate a dynamic outof-sample PM-decomposed momentum strategy. During the out-of-sample periods, we employ estimated partial moments using formulae (21) and (22) into the models developed in Section 5. The estimated model is:

$$RPM_t^+ = 0.00056 + 0.34015 * RPM_{t-1}^+ - 0.01652 * RPM_{t-1}^-$$
(21)

$$RPM_t^- = 0.00056 + 0.37669 * RPM_{t-1}^+ + 0.00449 * RPM_{t-1}^-$$
(22)

[Insert Table 10 about here]

Whilst the results, presented in Table 10, show that partial moments are forecastable, we find the performance of the PM strategies based on using these forecasts to scale positions does not improve performance. This is possibly due to estimation errors swamping any forecasting benefits.

6. Robustness Check: a WML strategy

Consistent with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), we construct a WML portfolio with a 11-month formation period and a 1-month holding period over January 1964 to December 2016. Particularly, there is a one-month gap between the formation and holding period and this strategy uses value-weighted holding period returns without rebalancing.

6.1 PM-based momentum strategies on a WML basis

We follow the switching rules presented in Table 3 and construct six PMM strategies on a WML basis over the whole sample period (1964 to 2016) which are presented in Table 11.

[Insert Table 11 about here]

From Table 11, apart from PMM Strategy 6 (PMM_S6), all other five PMM strategies outperform the benchmark strategy (a WML strategy) over both periods: P3, the GFC period (2008 to 2012) and P4, the Era of Turbulence (2000-01 to 2016-12). Particularly, all six PMM strategies based on WML earn positive returns during the GFC.

[Insert Table 12 about here]

We further test WML-based PMM on out-of-sample analyses same as Section 5.1 as shown in Table 12. These results show that all six WML-based PMM strategies surpass the performance of a plain WML strategy during Pa, the Era of Turbulence (2000 to 2016) and Pb, the GFC period (2008 to 2012).

[Insert Table 13 about here]

We then conduct two PM-decomposed momentum strategies, with and without a 200% leverage condition, based on WML as shown in Table 13. Same to the results in Table 6, PM-decomposed momentum strategies on a WML basis consistently outperform the plain WML strategy over all sample periods.

[Insert Table 14 about here]

Moreover, we test the dynamic PM-decomposed momentum strategies on a WML basis over the outof-sample period as in Section 5.2 as shown in Table 14. These results provide further evidence that PM-decomposed momentum strategies outperform plain momentum during financial turbulences. The robustness checks reveal that our PM-based momentum strategies are robust across different momentum strategies and multiple time periods. Whilst we do not claim that investors could earn these returns in practise, we suggest that PM-based strategies seem a good way of managing momentum risk.

6.2 Comparison between PM-decomposed momentum strategies and Barroso and Santa-Clara (2015) volatility-scaled momentum strategy

[Insert Table 15 about here]

Table 15 presents a comparison of the performances of our PM-decomposed momentum strategies and the volatility-scaled momentum strategy constructed by Barroso and Santa-Clara (2015). We construct the volatility-scaled momentum portfolio following the same methodology as Barroso and Santa-Clara (2015) use for their "risk-managed momentum". ¹⁹ Results reveal that our PM-decomposed momentum strategies outperform the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy for all four sample periods, including those periods of financial booming (P2) and financial turbulences (P3 and P4). In particular, our PM-decomposed momentum strategy earns an annualised return of 12.93%, which is significant at 1% level, with an annualised Sharpe ratio of 1.58 during the GFC, compared to an insignificant annualised return of 4.40% with an annualised Sharpe ratio of 0.36 for the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy during the same period.

We might find that, by tilting our strategy long or short towards favourable/unfavourable volatility signals, our extended partial moment-decomposed momentum strategy is more efficient and profitable, during both good and bad times.

7. Conclusion

We have demonstrated good performance for cross-sectional momentum-strategies, using information in past partial moments. We investigate two types; Firstly, either a portfolio choice based on the region that upper and lower partial moments lie in; Secondly, based on partial moment scaling whereby we load up on longs if upper partial moments are forecast to be relatively large or load up on shorts when lower partial moments are forecast to be relatively large.

¹⁹ They referred this volatility-scaled momentum strategy as the "risk-managed momentum". See from section 4, page 115, Barroso and Santa-Clara (2015).

Our approach, relative to what a fund manager might do, is conservative in the sense of using minimal calculation. Whilst we use CRSP data, which involves many stocks too small or illiquid to be considered by a fund of reasonable size, we do not update our models, estimating them once based on in sample data and using them, with fixed parameters, when we carry out our out-of-sample back testing.

The success of this strategy occurs because of the forecastability of partial moments, see Table 3. This immediately raises the question. Is this finding evidence of market inefficiency? We can best answer this by quoting Timmermann and Granger (2004).

"There is now substantial evidence that volatility of asset returns varies over time in a way that can be partially predicted. For this reason there has been considerable interest in improved volatility forecasting models in the context of option pricing, see e.g. Engle, Hong, Kane, and Noh (1993). Does this violate market efficiency? Clearly the answer is no unless a trading strategy could be designed that would use this information in the options markets to identify under- and over-valued options. "

Whilst we note possible subtleties in that options price directly off volatility; we also note that partial moments are barely traded at all. Thus, it seems, to us at least, that the presentation of a trading strategy that generates significant returns by forecasting partial moments is an argument against market efficiency.

The source of this inefficiency, whether it be behavioural or predictable risk premia or investors unwilling to embrace a more challenging measure of risk such as partial moments, awaits further research.

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Figure 1a Histograms of upper partial moments with kernel density curves

Figure 1a shows histograms of upper partial moment (RS_t^+) with kernel density curves. Values of 10th percentile and maximum observations of RS_t^+ are reported. The normal kernel estimate for c=0.7852 has a bandwidth of 0.0002 and an approximate mean integrated square error (AMISE) of 3.3237. Since the distribution of upper partial moment has a fat tail, we combine all upper partial moments with values between 99th percentile (0.0079066) and the maximum observation (0.027443).



Figure 1b Histograms of lower partial moments with kernel density curves

Figure 1b shows histograms of lower partial moment (RS_t^-) with kernel density curves. Values of 75th percentile and maximum observations of RS_t^- are reported. The normal kernel estimate for c=0.7852 has a bandwidth of 0.0002 and an approximate mean integrated square error (AMISE) of 3.1071. Since the distribution of lower partial moment has a fat tail, we combine all lower partial moments with values between 99th percentile (0.0087403) and the maximum observation (0.066847).



Figure 2a PMM strategies conditions in the coordinate plane

Figure 2a illustrates the four PMM strategies conditions in the coordinate plane. The origin point represents both the critical values for RPM_t^+ and RPM_t^- ($CV(RPM_t^+)$) and $CV(RPM_t^-)$) in month *t*. Each quadrant represents a PMM condition based on upper and lower partial moments and their critical values. For example, if in month *t*, both upper and lower partial moments are higher than their critical values, the point of PMM condition in month t is in the first quadrant in this coordinate plane. Then condition 1 applies in month *t* and the corresponding trading methods to condition 1 are employed as shown on Panel B, Table 4.



Figure 2b PMM strategies conditions estimation

Figure 2b shows the estimation of four PMM strategies conditions when 10^{th} percentile of RPM_t^+ and 75^{th} percentile of RPM_t^- are used as critical values for RPM_t^+ and RPM_t^- , respectively. Estimated percentage reports the theoretical possibilities of each of these four PMM strategies conditions occur.

Condition	Estimated Percentage
1	22.50%
2	2.50%
3	7.50%
4	67.50%
Total	100.00%

Figure 3 Timeline of a 6 × 6 PMM strategy

Figure 3 demonstrates the timeline of a 6×6 PMM strategy. In any month *t*, all sample stocks are ranked and sorted into deciles based on their past 6-month formation period returns from month *t*-7 to *t*-1 (due to a one month gap, month *t*-1 to *t*, between formation and holding periods for returns). Then we classify holding strategies into four conditions based on partial moments determined in the period month *t*-1 to *t* and their boundaries analysed in section 4.3.1. Then during the 6-month holding period from month *t* to *t*+6, we continue to compare upper and lower partial moments with their boundaries in each month and switch positions of winners, losers and the cash asset to keep zero-net position based on Panel A, Table 4. That is to say, for any PMM strategies, we keep rebalancing and switching positions of all assets held. For instance, in month *t*+2, to a 6×6 PMM strategy which was constructed at time t, PMM conditions in month *t*+2 are determined based on partial moments during the period [*t*+1, *t*+2].



Table 1 Performances of plain momentum strategies in the US equity markets

Table 1 shows the performances of 4 plain momentum strategies over four sample periods: P1 whole sample period (1964 to 2016), P2 Jegadeesh and Titman 1993 period (1965 to 1989), P3 the Global Financial Crisis (GFC) period (2008 to 2012) and P4 the Era of Turbulence (2000 to 2016). Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. M33, M66, M99 and M1212 represent the 3×3 , 6×6 , 9×9 and 12×12 plain momentum strategies with one month gap between formation and holding periods, respectively. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio
Panel A: P1, Whole	e sample period: 196	4-01 to 2016-12		
M33	4.18	1.48	-0.03	-0.01
M66	6.23	1.97(**)	0.06	0.03
M99	2.08	0.71	-0.13	-0.06
M1212	-3.49	-1.24	-0.40	-0.20
Panel B: P2, Jegaa	leesh and Titman (19	93) period: 1965-01 to 1	1989-12	
M33	8.96	2.95(***)	0.12	0.06
M66	12.03	3.59(***)	0.29	0.15
M99	9.10	2.68(***)	0.12	0.06
M1212	3.11	0.90	-0.22	-0.12
Panel C: P3, The C	Global Financial Cris	tis: 2008-01 to 2012-12		
M33	-10.59	-1.09	-0.50	-0.23
M66	-13.45	-1.17	-0.54	-0.23
M99	-14.59	-1.28	-0.59	-0.27
M1212	-15.02	-1.37	-0.63	-0.30
Panel D: P4, The E	Era of Turbulence: 20	000-01 to 2016-12		
M33	-0.88	-0.13	-0.09	-0.04
M66	-0.92	-0.12	-0.08	-0.04
M99	-6.17	-0.98	-0.30	-0.14
M1212	-9.51	-1.68	-0.47	-0.24

Table 2 Distributions of partial moments of the US equity market

Table 2 reports the distributions of monthly realised variance (RV_t) , upper and lower market partial moments $(RPM_t^+ \text{ and } RPM_t^-)$ throughout the whole sample period from January 1964 to December 2016. 10th Pctl, 25th Pctl, 75th Pctl, 90th Pctl and Std Dev represent 10th, 25th, 75th, 90th percentile and standard deviation of each variable. Median, mean, skewness and kurtosis are also reported. *SDEQV_t* stands for annual standard deviation equivalent (in percentage) of monthly realised variance. For example, *RV_t* mean average of 22.03 corresponds to an annual standard deviation of 16.26%. Values of all percentiles, median, mean and standard deviation are in 0.0001.

Variable	10th Pctl	25th Pctl	Median	75th Pctl	90th Pctl	Mean	Std Dev	Skewness	Kurtosis
RV_t	4.61	7.21	12.04	22.27	42.06	22.03	47.37	11.01	156.45
RPM_t^+	1.94	3.57	6.48	11.29	20.04	10.64	17.76	8.01	93.17
RPM_t^-	1.11	2.52	6.44	10.78	21.58	11.39	32.86	14.41	263.72
SDEQV _t	7.44	9.30	12.02	16.35	22.47	16.26			

Table 3 VAR results of partial moments

Table 3 reports the VAR (1) process results of partial moments throughout the whole sample period from January 1964 to December 2016. For Equation (12), the dependent variable is RPM_t^+ which represents the upper partial moment at month *t*; For Equation (13), the dependent variable is RPM_t^- which represents the lower partial moment at month *t*. For each regression, α represents the coefficient of the intercept; RPM_{t-1}^+ and RPM_{t-1}^- represent the value of the upper partial moment and the *lower* partial moment at month *t*-1, respectively. R_{adj}^2 reports the adjusted R squared value. The Newey-West (1987) t-test statistics are marked as *, **, ***. These indicate significance at the 10%, 5% and 1% level, respectively.

Variable	Estimated coeffic	ients (t-statistics)
	(12)	(13)
	RPM_t^+	RPM_t^-
1	0.00051	
	(7.37) ***	
RPM_{t-1}^+	0.44978	
· -	(9.24) ***	
RPM_{t-1}^{-}	0.02631	
	(2.42) **	
1		0.00046
		(3.22) ***
RPM_{t-1}^+		0.55273
		(6.53) ***
RPM_{t-1}^{-}		0.07961
		(1.47)
	0.2912	0.1274
	Variable1 RPM_{t-1}^+ RPM_{t-1}^- 1 RPM_{t-1}^+ RPM_{t-1}^-	Variable Estimated coeffic (12) RPM_t^+ 1 0.00051 (7.37) *** $(9.24) ***$ RPM_{t-1}^- 0.02631 (2.42) ** $(2.42) **$ 1 RPM_{t-1}^+ 0.2912 (0.2912)

Table 4 PMM strategies construction

Table 4 presents how we construct partial moment momentum (PMM) strategies. Panel A shows four conditions of upper and lower partial moments based on their boundaries: 10th percentile of RPM_t^- . In any given month *t*, we classify holding strategies into four conditions based on partial moments in the period [*t*-1, *t*] (there is a one month gap between the formation and holding periods for returns) and their boundaries analysed in section 3.3.1. Correspondingly, switching methodologies during holding periods under each of these four conditions are presented. Panel B illustrates different holding periods actions and returns to each of the four conditions presented on Panel A of our six partial moment momentum strategies, which we called PMM Strategy 1 to 6, represented by PMM_S1 to PMM_S6. In particular, all six PMM strategies switch to the same positions if condition 2 or 3 occur, but act differently if conditions 1 or 4 occur. Besides, we hold cash long or short cash to keep our PMM strategies net zero positions. $r_{w,t+1}$, $r_{t,t+1}$, $r_{f,t+1}$ represent return of winners, losers and risk-free asset in month *t*+1, respectively. For example, for PMM strategy 4 (PMM_S4), if condition 1 applies during the period [*t*-1, *t*] where upper partial moment is lower than its boundary, then we short losers only, liquidating our longs and holding cash long. The PMM return for month *t*+1 is $r_{t,t+1} - r_{t,t+1}$; If condition 3 applies during the period [*t*-1, *t*] where upper and lower partial moment is lower than its boundary, then we short losers only, liquidating our longs and holding cash long. The PMM return for month *t*+1 is $r_{w,t+1} - r_{t,t+1}$; If condition 3 applies during the period [*t*-1, *t*] where upper and lower partial moment is lower than its boundary, then we short losers only, liquidating our longs and holding cash long. The PMM return for month *t*+1 is lower than its boundary and short cash. The PMM return for month *t*+1 is $r_{w,t+1} - r_{t,t+1}$.

Panel A: Four conditions of partial moments based on their boundaries

Condition 1	if	RPM_t^+	>	$CV(RPM_t^+)$	and if	RPM_t^-	>	$CV(RPM_t^-)$ then	Method 1.1: Close out
									Method 1.2: Go Contrarian
									Method 1.3: Short losers only
Condition 2		RPM_t^+	<	$CV(RPM_t^+)$		RPM_t^-	>	$CV(RPM_t^-)$	Method 2: Short losers only
Condition 3		RPM_t^+	<	$CV(RPM_t^+)$		RPM_t^-	<	$CV(RPM_t^-)$	Method 3: Go momentum
Condition 4		RPM_t^+	>	$CV(RPM_t^+)$		RPM_t^-	<	$CV(RPM_t^-)$	Method 4.1: Go momentum
									Method 4.2: Buy winners only

Panel B: Methodologies and returns of six partial moment momentum strategies (In any given month t, conditions are classified based on partial moments in the previous month t-1 due to one-month gap between the formation and holding periods; $\mathbf{r}_{w,t+1}$, $\mathbf{r}_{f,t+1}$, represent returns of winners, losers and the risk-free asset in month t+1, respectively. Correspondingly, returns of momentum and contrarian strategies are $\mathbf{r}_{w,t+1}$ - $\mathbf{r}_{t,t+1}$ - $\mathbf{r}_{w,t+1}$, respectively.)

DMM Stratagios	Condi	tion 1	Condition 2		Condi	tion 3	Condition 4	
Piviivi Strategies —	Method	Return	Method Return		Method	Return	Method	Return
PMM_S1	1.1	0	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.1	$r_{w,t+1} - r_{l,t+1}$
PMM_S2	1.2	$r_{l,t+1} - r_{w,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.1	$r_{w,t+1} - r_{l,t+1}$
PMM_S3	1.3	$r_{f,t+1} - r_{l,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.1	$r_{w,t+1} - r_{l,t+1}$
PMM_S4	1.1	0	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.2	$r_{w,t+1} - r_{f,t+1}$
PMM_S5	1.2	$r_{l,t+1} - r_{w,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.2	$r_{w,t+1} - r_{f,t+1}$
PMM_S6	1.3	$r_{f,t+1} - r_{l,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.2	$r_{w,t+1} - r_{f,t+1}$

Table 5 Performances of PMM strategies on a 6 × 6 basis

Table 5 shows performances of partial moment momentum (PMM) strategies on a 6×6 basis over four sample periods: P1, whole sample period (1964-01 to 2016-12), P2, Jegadeesh and Titman (1993) period (1965-01 to 1989-12), P3, the Global Financial Crisis (GFC) period (2008-01 to 2012-12) and P4, the Era of Turbulence (2000-01 to 2016-12). Partial moments boundaries are computed over whole sample period P1. M66 (strategy) represents a 6×6 plain momentum strategy with one month gap between formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies which are on a 6×6 basis and constructed following the switching rules presented in Table 4. Return reports the annualised return of each strategy in percentage. Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio
Panel A: P1, Whole s	ample period: 1964-	01 to 2016-12		
M66	6.23	1.97(*)	0.06	0.03
PMM_S1	6.08	3.19(***)	0.09	0.05
PMM_S2	5.90	1.86(*)	0.05	0.03
PMM_S3	4.23	1.07	-0.02	-0.01
PMM_S4	13.67	5.29(***)	0.46	0.31
PMM_S5	13.47	3.66(***)	0.31	0.26
PMM_S6	11.70	2.65(***)	0.21	0.11
Panel B: P2, Jegadee	sh and Titman (199.	3) period: 1965-01 to 1	989-12	
M66	12.03	3.59(***)	0.29	0.15
PMM_S1	9.30	3.46(***)	0.16	0.10
PMM_S2	6.64	1.99(**)	-0.02	-0.02
PMM_S3	11.32	2.69(***)	0.20	0.10
PMM_S4	14.70	3.43(***)	0.34	0.23
PMM_S5	11.91	2.53(**)	0.20	0.15
PMM_S6	16.81	3.10(***)	0.35	0.21
Panel C: P3, The Glo	bal Financial Crisis	: 2008-01 to 2012-12		
M66	-13.45	-1.17	-0.54	-0.23
PMM_S1	3.57	0.94	0.38	0.24
PMM_S2	23.62	1.77(*)	0.78	1.59
PMM_S3	-8.57	-0.51	-0.24	-0.13
PMM_S4	7.74	1.32	0.56	0.35
PMM_S5	28.52	2.00(**)	0.88	1.35
PMM_S6	-4.85	-0.27	-0.13	-0.07
Panel D: P4, The Era	of Turbulence: 200	0-01 to 2016-12		
M66	-0.92	-0.12	-0.08	-0.04
PMM_S1	4.77	1.64	0.26	0.15
PMM_S2	10.85	1.38	0.28	0.24
PMM_S3	-1.47	-0.16	-0.08	-0.04
PMM_S4	10.95	2.95(***)	0.60	0.42
PMM_S5	17.36	2.07 (**)	0.45	0.41
PMM_S6	4.38	0.44	0.07	0.03

Table 6 Performances of PM-decomposed momentum strategy on a 6 × 6 basis

Table 6 shows performances of two partial moments-decomposed momentum strategies over four sample periods: P1, whole sample period (1964-01 to 2016-12), P2, Jegadeesh and Titman (1993) period (1965-01 to 1989-12), P3, the Global Financial Crisis (GFC) period (2008-01 to 2012-12) and P4, the Era of Turbulence (2000-01 to 2016-12). M66 (strategy) represents a 6×6 plain momentum strategy with one month gap between formation and holding periods. M66* and M66# represent a PM-decomposed momentum strategy and a leveraged PM-decomposed momentum strategy, respectively. Both strategies are on a 6×6 basis and constructed following the methodologies described in Section 4. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	t-value Sharpe ratio	
Panel A: P1, Whole				
M66	6.23	1.97(**)	0.06	0.03
M66*	18.61	14.65(***)	1.46	1.31
M66#	17.70	13.32(***)	1.31	0.89
Panel B: P2, Jegad	eesh and Titman (19	993) period: 1965-01 to 1	989-12	
M66	12.03	3.59(***)	0.29	0.15
M66*	23.47	11.49(***)	1.54	2.17
M66#	21.44	12.14(***)	1.58	1.59
Panel C: P3, The G	lobal Financial Cri	sis: 2008-01 to 2012-12		
M66	-13.45	-1.17	-0.54	-0.23
M66*	9.42	3.07(***)	1.32	1.02
M66#	11.34	2.44(**)	1.05	0.61
Panel D: P4, The E	ra of Turbulence: 2	000-01 to 2016-12		
M66	-0.92	-0.12	-0.08	-0.04
M66*	13.44	6.56 (***)	1.39	1.05
M66#	14.43	5.21(***)	1.11	0.65

Table 7 VAR results of partial moments over January 1964 to December 1999

Table 7 reports the VAR (1) process results of partial moments throughout the in-sample period from January 1964 to December 1999. For Equation (12), dependent variable is RPM_t^+ which represents upper partial moment at month *t*; For Equation (13), dependent variable is RPM_t^- which represents lower partial moment at month *t*. For each regression, α represents the coefficient of the intercept; RPM_{t-1}^+ and RPM_{t-1}^- represent the value of upper partial moment and lower partial moment at month *t*-1, respectively. R_{adj}^2 reports adjusted R squared value. The Newey-West (1987) t-test statistics are marked as *, **, ***. These indicate significance at the 10%, 5% and 1% level, respectively.

Coefficient	Variable	Estimated coefficients (t-statistics)					
		(12)	(13)				
		RPM_t^+	RPM_t^-				
α_1	1	0. 00056					
		(8.67) ***					
β_{11}	RPM_{t-1}^+	0.34015					
		(6.28) ***					
β_{12}	RPM_{t-1}^{-}	-0.01652					
		(-0.79)					
α2	1		0. 00056				
			(2.68) ***				
β_{21}	RPM_{t-1}^+		0.37669				
	v 1		(1.81) *				
β_{22}	RPM_{t-1}^{-}		0.00449				
			(0.07)				
R_{adj}^2		0. 0898	0. 0109				

Table 8 Conditions switching over the out-of-sample periods

Table 8 reports the frequencies of the four PMM switching conditions based on in-sample partial moments over the whole out-of-sample periods, the Era of Turbulence, from January 2000 to December 2016. Panel A shows the actual out-of-sample frequencies of PMM conditions based on in-sample estimates. The variable "Sab" represents a situation when condition in the current month *t* is a and condition in the next month t+1 is b. N represents the number of each situation. Pct represents percentage of each situation per each of the four conditions. Cond X represents the total numbers of months when condition X applies over the whole out-of-sample periods. For example, Cond 1 equals 103 and it means that there are 103 months over out-of-sample periods when condition 1 applies. N of S14 is 31 means that there are 31 months over out-of-sample periods when condition 1 applies in the current month, there is a 30.1% chance that condition 4 will apply in the next month. Panel B compares the estimations of four PMM strategies to the actual observations. Estimated percentage reports the theoretical possibilities of each of these four PMM strategies conditions occur which is consistent with Figure 2b.

Panel A:	Panel A: Frequencies of the four PMM switching conditions										
Variable	Ν	Pct	Variable	Ν	Pct	Variable	Ν	Pct	Variable	Ν	Pct
S11	70	67.96%	S21	0	0.00%	S31	2	66.67%	S41	30	30.93%
S12	0	0.00%	S22	0	0.00%	S32	0	0.00%	S42	0	0.00%
S13	2	1.94%	S23	0	0.00%	S33	0	0.00%	S43	1	1.03%
S14	31	30.10%	S24	1	100.00%	S34	1	33.33%	S44	66	68.04%
Cond 1	103	100.00%	Cond 2	1	100.00%	Cond 3	3	100.00%	Cond 4	97	100.00%
Panel B:	Estim	nated and a	ictual freq	uenc	cies compa	rison					
	Cond	lition		Es	timated Pe	rcentage		Ac	tual Percent	age	
	1	[22.50	%		50.49%			
2 2.50%						0.49%					
	3			7.50%			1.47%				
	4			67.50%				47.55%			
	То	tal			100.00	%			100.00%		

Table 9 Out-of-sample performances of PMM strategies on a 6 × 6 basis

Table 9 shows out-of-sample performances of partial moment momentum (PMM) strategies on a 6×6 basis over two sample periods: Pa, whole out-of-sample period (Era of Turbulence, 2000 to 2016) and Pb, the Global Financial Crisis (GFC) period (2008 to 2012). Partial moments boundaries are computed over in-sample sample period from January 1964 to December 1999. M66 (strategy) represents a 6×6 plain momentum strategy with one month gap between formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies which are on a 6×6 basis and constructed following the switching rules presented in Table 4. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio
Panel A: Pa, The Era	of Turbulence: 2000	-01 to 2016-12		
M66	-0.92	-0.12	-0.08	-0.04
PMM_S1	2.31	0.96	0.07	0.04
PMM_S2	5.11	0.66	0.11	0.10
PMM_S3	-0.48	-0.05	-0.05	-0.03
PMM_S4	10.22	3.28(***)	0.66	0.50
PMM_S5	13.22	1.61	0.34	0.31
PMM_S6	7.23	0.71	0.13	0.07
Panel B: Pb, The Glob	al Financial Crisis:	2008-01 to 2012-12		
M66	-13.45	-1.17	-0.54	-0.23
PMM_S1	1.68	0.56	0.19	0.11
PMM_S2	19.20	1.45	0.64	1.12
PMM_S3	-5.92	-0.34	-0.16	-0.09
PMM_S4	10.82	2.31(**)	1.00	0.86
PMM_S5	29.77	2.12(**)	0.93	1.58
PMM_S6	2.60	0.14	0.05	0.03

Table 10 Performances of the dynamic PM-decomposed momentum strategies on a 6 × 6 basis

Table 10 shows performances of two out-of-sample partial moments-decomposed momentum strategies over two sample periods: Pa, whole out-of-sample period (Era of Turbulence, 2000 to 2016) and Pb, the Global Financial Crisis (GFC) period (2008 to 2012). M66 (strategy) represents a 6×6 plain momentum strategy with one month gap between formation and holding periods. M66* and M66# represent a PM-decomposed momentum strategy and a leveraged PM-decomposed momentum strategy, respectively. Both strategies are on a 6×6 basis and constructed following the methodologies described in Section 4 and Section 5.2. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value Sharpe ratio		Adapted-Sortino ratio
Panel A: Pa, The Er	a of Turbulence: 2000)-01 to 2016-12		
M66	-0.92	-0.12	-0.08	-0.04
M66*	0.88	0.50	-0.11	-0.05
M66#	-0.51	-0.19	-0.19	-0.09
Panel B: Pb, The Gl	obal Financial Crisis:	· 2008-01 to 2012-12		
M66	-13.45	-1.17	-0.54	-0.23
M66*	-2.03	-0.81	-0.43	-0.21
M66#	-4.65	-1.07	-0.52	-0.22

Table 11 Performances of PMM strategies on a WML basis

Table 11 shows performances of partial moment momentum (PMM) strategies on a WML basis (11×1) over four sample periods: P1, whole sample period (1964-01 to 2016-12), P2, Jegadeesh and Titman (1993) period (1965-01 to 1989-12), P3, the Global Financial Crisis (GFC) period (2008-01 to 2012-12) and P4, the Era of Turbulence (2000-01 to 2016-12). Partial moments boundaries are computed over whole sample period P1. WML (strategy) represents an 11×1 plain momentum strategy with one month gap between formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies which are on a WML basis and constructed following the switching rules presented in Table 4. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio	
Panel A: P1, Whole sample period: 1964-01 to 2016-12					
WML	16.51	4.60(***)	0.44	0.24	
PMM_S1	14.79	6.36(***)	0.58	0.39	
PMM_S2	13.11	3.68(***)	0.31	0.24	
PMM_S3	14.08	3.63(***)	0.32	0.18	
PMM_S4	12.61	5.35(***)	0.44	0.31	
PMM_S5	10.96	3.07(***)	0.23	0.19	
PMM_S6	11.92	3.07(***)	0.25	0.15	
Panel B: P2, Jegadee	sh and Titman (199.	3) period: 1965-01 to 1	989-12		
WML	21.61	5.13(***)	0.67	0.40	
PMM_S1	18.01	5.07(***)	0.59	0.40	
PMM_S2	14.52	3.46(***)	0.34	0.26	
PMM_S3	20.29	4.49(***)	0.57	0.34	
PMM_S4	15.67	3.98(***)	0.42	0.29	
PMM_S5	12.23	2.72(***)	0.22	0.17	
PMM_S6	17.90	3.71(***)	0.43	0.28	
Panel C: P3, The Glo	bal Financial Crisis	: 2008-01 to 2012-12			
WML	-1.79	-0.10	-0.05	-0.03	
PMM_S1	13.56	1.95(*)	0.85	0.73	
PMM_S2	31.08	1.56	0.69	0.81	
PMM_S3	3.46	0.17	0.07	0.04	
PMM_S4	7.28	1.51	0.64	0.39	
PMM_S5	23.92	1.27	0.56	0.65	
PMM_S6	-2.31	-0.12	-0.06	-0.03	
Panel D: P4, The Era of Turbulence: 2000-01 to 2016-12					
WML	4.89	0.61	0.10	0.05	
PMM_S1	7.49	1.96(*)	0.37	0.24	
PMM_S2	10.24	1.25	0.25	0.21	
PMM_S3	5.20	0.59	0.10	0.06	
PMM_S4	6.65	2.20(**)	0.40	0.27	
PMM_S5	9.38	1.20	0.24	0.20	
PMM_S6	4.37	0.52	0.08	0.04	

Table 12 Out-of-sample performances of PMM strategies on a WML basis

Table 12 shows out-of-sample performances of partial moment momentum (PMM) strategies on a WML basis (11×1) over two sample periods: Pa, whole out-of-sample period (Era of Turbulence, 2000 to 2016) and Pb, the Global Financial Crisis (GFC) period (2008 to 2012). Partial moments boundaries are computed over insample sample period from January 1964 to December 1999. WML (strategy) represents an 11×1 plain momentum strategy with one month gap between formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies which are on a WML basis and constructed following the switching rules presented in Table 4. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio	
Panel A: Pa, The Era o	of Turbulence: 2000	-01 to 2016-12			
WML	4.89	0.61	0.10	0.05	
PMM_S1	4.25	1.28	0.19	0.12	
PMM_S2	3.69	0.46	0.06	0.05	
PMM_S3	5.23	0.58	0.10	0.06	
PMM_S4	6.69	2.55(**)	0.46	0.32	
PMM_S5	6.12	0.78	0.14	0.11	
PMM_S6	7.70	0.87	0.17	0.10	
Panel B: Pb, The Global Financial Crisis: 2008-01 to 2012-12					
WML	-1.79	-1.10	-0.05	-0.03	
PMM_S1	7.90	1.60	0.68	0.47	
PMM_S2	18.47	0.96	0.42	0.44	
PMM_S3	4.89	0.23	0.09	0.05	
PMM_S4	8.76	2.09(**)	0.90	0.66	
PMM_S5	19.41	1.02	0.45	0.47	
PMM_S6	5.72	0.27	0.11	0.06	

Table 13 Performances of PM-decomposed momentum strategy on a WML basis

Table 13 shows performances of two partial moments-decomposed momentum strategies on a WML basis (11 \times 1) over four sample periods: P1, whole sample period (1964-01 to 2016-12), P2, Jegadeesh and Titman (1993) period (1965-01 to 1989-12), P3, the Global Financial Crisis (GFC) period (2008-01 to 2012-12) and P4, the Era of Turbulence (2000-01 to 2016-12). WML (strategy) represents an 11 \times 1 plain momentum strategy with one month gap between formation and holding periods. WML* and WML# represent a PM-decomposed momentum strategy and a leveraged PM-decomposed momentum strategy, respectively. Both strategies are on a WML basis and constructed following the methodologies described in Section 4. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio		
Panel A: P1, Whole sample period: 1964-01 to 2016-12						
WML	16.51	4.60(***)	0.44	0.24		
WML*	22.30	17.00(***)	1.79	1.62		
WML#	21.43	15.38(***)	1.61	1.12		
Panel B: P2, Jegad	Panel B: P2, Jegadeesh and Titman (1993) period: 1965-01 to 1989-12					
WML	21.61	5.13(***)	0.67	0.40		
WML*	31.71	13.88(***)	1.97	2.22		
WML#	28.74	13.99(***)	1.93	1.70		
Panel C: P3, The Global Financial Crisis: 2008-01 to 2012-12						
WML	-1.79	-0.10	-0.05	-0.03		
WML*	12.93	3.64(***)	1.58	1.00		
WML#	18.05	2.77(***)	1.21	0.63		
Panel D: P4, The Era of Turbulence: 2000-01 to 2016-12						
WML	4.89	0.61	0.10	0.05		
WML*	13.54	6.36(***)	1.34	0.92		
WML#	16.01	5.47(***)	1.18	0.73		

Table 14 Performances of the dynamic PM-decomposed momentum strategies on a WML basis

Table 14 shows performances of two out-of-sample partial moments-decomposed momentum strategies on a WML basis (11×1) over two sample periods: Pa, whole out-of-sample period (Era of Turbulence, 2000 to 2016) and Pb, the Global Financial Crisis (GFC) period (2008 to 2012). WML (strategy) represents an 11×1 plain momentum strategy with one month gap between formation and holding periods. WML* and WML# represent a PM-decomposed momentum strategy and a leveraged PM-decomposed momentum strategy, respectively. Both strategies are on a WML basis and constructed following the methodologies described in Section 4 and Section 5.2. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio	
Panel A: Pa, The Era of Turbulence: 2000-01 to 2016-12					
WML	4.89	0.61	0.10	0.05	
WML*	2.00	1.07	0.05	0.03	
WML#	1.57	0.57	-0.01	0.00	
Panel B: Pb, The Global Financial Crisis: 2008-01 to 2012-12					
WML	-1.79	-0.10	-0.05	-0.03	
WML*	0.89	0.23	0.06	0.03	
WML#	-0.46	-0.07	-0.06	-0.03	

Table 15 Performances comparison between PM-decomposed momentum strategies and the

Barroso and Santa-Clara (2015) volatility-scaled momentum strategy

Table 15 compares the performances between our PM-based momentum strategies and the volatility-scaled momentum strategies introduced by Barroso and Santa-Clara (2015) over four in-sample periods: P1, whole sample period (1964-01 to 2016-12), P2, Jegadeesh and Titman (1993) period (1965-01 to 1989-12), P3, the Global Financial Crisis (GFC) period (2008-01 to 2012-12) and P4, the Era of Turbulence (2000-01 to 2016-12). To keep consistency with Barroso and Santa-Clara (2015), all momentum portfolios are constructed based on a WML basis. BSC represents the volatility-scaled momentum strategy constructed by Barroso and Santa-Clara (2015). WML (strategy) represents an 11×1 plain momentum strategy with one month gap between formation and holding periods. PMD and PMD L represent a PM-decomposed momentum strategy and a leveraged PM-decomposed momentum strategy, respectively. Both strategies are constructed following the methodologies described in Section 4 and Section 5.2. PMM S1 to PMM S6 represent six PMM strategies which are on a WML basis and constructed following the switching rules presented in Table 4. Return reports the annualised return of each strategy in percentage. Sharpe ratio reports the annualised Sharpe ratio of each strategy. It is calculated following the methodology in Sharpe (1994), as the excess return divided by its standard deviation. Adapted-Sortino ratio reports the annualised adapted-Sortino ratio of each strategy. It is calculated following the methodology in formula (9) to (11), as the excess return divided by twice its downside semi-deviation. The Newey-West (1987) t-test statistics are provided. *, **, *** indicate significances at the 10%, 5% and 1% level, respectively.

Strategy	Return	t-value	Sharpe ratio	Adapted-Sortino ratio	
P1: Whole sample period: 1964-01 to 2016-12					
WML	16.51	4.60(***)	0.44	0.24	
BSC	18.33	7.47(***)	0.89	1.21	
PMD	24.30	17.00(***)	1.79	1.62	
PMD_L	22.43	15.38(***)	1.61	1.12	
P2: Jegadeesh and	Titman (1993): 196	5-01 to 1989-12			
WML	21.61	5.13(***)	0.67	0.40	
BSC	28.20	6.32(***)	1.14	1.56	
PMD	33.71	13.88(***)	1.97	2.22	
PMD_L	30.84	13.99(***)	1.93	1.70	
P3: The Global Fin	ancial Crisis: 2008-				
WML	-1.79	-0.10	-0.05	-0.03	
BSC	4.40	0.82	0.36	0.45	
PMD	12.93	3.64(***)	1.58	1.00	
PMD_L	18.05	2.77(***)	1.21	0.63	
P4: The Era of Turbulence: 2000-01 to 2016-12					
WML	4.89	0.61	0.10	0.05	
BSC	4.81	1.64	0.36	0.50	
PMD	13.54	6.36(***)	1.34	0.92	
PMD_L	16.01	5.47(***)	1.18	0.73	