# A Demand-Based Equilibrium Model of Volatility Trading*

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# A Demand-Based Equilibrium Model of Volatility Trading


**Abstract**

This paper is the first to provide a demand-based equilibrium model of volatility trading with three kinds of traders – dealers, asset managers and leveraged funds – which complements Eraker and Wu's (2017) consumption-based equilibrium model. Our theoretical results are consistent with existing empirical observations, and two endogenous cases reach the same conclusion. Our novel model links together risk aversion, market price of the volatility risk, variance risk premium, VIX futures price and return and futures trading activities. This allows us to test empirically the impact of the three traders' net positions on the VRP and the VIX futures return.



**Keywords:** Volatility trading; Demand-based equilibrium; VIX futures; Heston model.
**JEL Classifications:** D53; G11; G12; G13.

# 1. Introduction

This paper is the first to provide a demand-based equilibrium model of volatility trading with three kinds of traders (i.e., dealers, asset managers and leveraged funds), which complements Eraker and Wu's (2017) consumption-based equilibrium model. According to Mixon and Onur (2015), volatility derivatives are mainly traded by three kinds of traders: dealers, asset managers and leveraged funds. Dealers as market makers balance the orders of volatility derivatives. Asset managers as hedgers prefer to take long positions, while leveraged funds as speculators prefer to take short positions. Mixon and Onur (2015) collect the daily volatility derivatives transaction data from the Swap Data Repositories (SDR) reported by the Commodity Futures Trading Commission (CFTC) and find that the gross vega notional outstanding for variance swaps, in 2014, is over USD 2 billion, with USD 1.5 billion in S&P 500 variance swaps. From Bollen, O'Neill, and Whaley (2016), the dollar value of open interest of the CBOE Market Volatility Index (VIX) futures in 2013 is around USD 7 billion, and the dollar market value of VIX Exchange Traded Products (ETPs) linked to the short-term S&P 500 VIX futures index is around USD 2 billion. The market for volatility trading has become an important new avenue of financial markets in addition to equity and fixed income securities over last decade.

However, since 2009, there has been a huge loss of investing in positive multiplier VIX Exchange Traded Notes (ETNs) (e.g., iPath S&P 500 VIX Short-Term Futures ETN (VXX)). This is because of the negative return of VIX futures (Eraker and Wu (2017)). In this paper, we mainly investigate the question: How do volatility trading activities affect the variance risk premium (VRP) and the VIX futures' price and return? By using an equilibrium model of volatility trading, we provide an economic theory to explain that the lower positions (more net short) of dealers, the lower short positions of leverage funds and the higher long positions of asset managers lead to a higher VIX futures price and a more negative VIX futures return, so that the positive multiplier VIX ETNs produce a huge loss. This is different than the explanation in

Eraker and Wu (2017), which states that the negative futures return is only determined by the investors risk aversion. Besides considering the risk aversion, adding the trading behaviour of the three main traders complement their consumption-based equilibrium model. The novel model proposed in this paper is our main contribution. Furthermore, empirically we use the weekly Traders in Financial Futures (TFF) reports data to test the impact of trading on the VIX futures return. These results are new.

In terms of demand-based equilibrium models, Garleanu, Pedersen, and Poteshman (2009) propose a demand-based equilibrium model for option pricing, with two kinds of agents: dealers and end users. Dong (2016) extends their model to explain how the demand for ETPs demand affects the VIX futures price. In contrast to the previous literature, our model considers three kinds of traders (i.e., dealers, asset managers and leveraged funds) and analyses how traders demand for VIX-type futures demand of traders influences the volatility market. Based on the daily trading data in Mixon and Onur (2015) and the weekly data used in this paper strongly support that we should use a three-trader model instead of the two-trader model.

There are a huge number of papers studying the VRP and its predictive power. For example, Carr and Wu (2009) find that there exists a large and negative mean of the VRP on five stock indexes and 35 individual stocks. Recently, González-Urteaga and Rubio (2016) have discussed and tested the volatility risk premium at the individual and portfolio level. Barras and Malkhozov (2016) formally compare the market VRP inferred from equity and option markets.[1] However, there is a paucity of research on predicting the VRP empirically. Konstantinidi and Skiadopoulos (2016) compare four predicting models and find that the trading activity model is the best performing. Fan, Imerman, and Dai (2016) claim that the magnitude of VRP is significantly affected

[1]Furthermore, Todorov (2010); Bollerslev and Todorov (2011) use the rare events to account for the large average VRP. Aït-Sahalia, Karaman, and Mancini (2015); Li and Zinna (2016) examine the term structures of the VRP. Choi, Mueller, and Vedolin (2017) study variance risk premiums in the bond market. Bollerslev, Todorov, and Xu (2015); Jin (2015) and others study the predictive power of the VRP for the stock return, while Londono and Zhou (2017) recently provide evidence of the predictive power of the VRP for the currency return.

by investors' demand for hedging tail risk. Two papers do not use the explicit trading positions of dealers, asset managers and leveraged funds as predictive variables. For example, Konstantinidi and Skiadopoulos (2016) use the trading volume of all S&P 500 futures contracts and the TED spread to explain the high negative VRP. In this paper, we test the impacts of the trading positions of the three main traders and find that the high negative VRP is driven by the higher short positions of dealers, the lower short positions of leverage funds and the higher long positions of asset managers in variance swaps. This is new empirical evidence. In addition, our theoretical model also gives a very neat economic theory to explain the new empirical evidence.[2]

This paper makes at least two contirubutions. First, it is the first paper to provide a demand-based equilibrium model of volatility trading with three kinds of traders (i.e., dealers, asset managers and leveraged funds) that fully supports the existing empirical results. Second, due to our novel model, this paper is the first to test the impact of the three main traders net positions on the VRP and the VIX futures return.

The remainder of this article is organized as follows. Section 2 presents the model and results, and Section 3 provides two endogenous cases. Section 4 gives the empirical analysis. Section 5 concludes. Appendix A collects all proofs, and Appendix B gives solutions for the endogenous cases.

# 2. Models and results

## 2.1 Heston model

We set up our demand-based equilibrium model starting from the Heston (1993) model, which is the most popular stochastic volatility model in the literature. We adopt it to

[2]Bollerslev, Tauchen, and Zhou (2009); Drechsler and Yaron (2011); Bollerslev, Sizova, and Tauchen (2012); Drechsler (2013); Jin (2015) adopt the long-run risks model (i.e., long-run risks and investor preferences) to explain the negative VRP and Buraschi, Trojani, and Vedolin (2014) use a two-tree Lucas (1978) economy with two heterogeneous investors (i.e., disagreement). In contrast to them, we use the trading positions of dealers, asset managers and leveraged funds to explain the high VRP.

describe the dynamicS of the stock price (i.e., S&P 500 Index (SPX)) in the physical measure $\mathbb{P}$ as follows:

$$\begin{cases} \frac{dS_t}{S_t} = \mu dt + \sqrt{v_t} dB_{S,t}, \\ dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dB_{v,t}, \end{cases} \tag{1}$$

where $v_t$ is the instantaneous variance; $B_{S,t}$ and $B_{v,t}$ are a pair of correlated Brownian motions with correlation coefficient $\rho$. Empirical evidence documents that $\rho$ is negative for SPX, so that here we assume $-1 < \rho < 0$.

Developing Heston (1993), we assume the stock price in the risk-neutral measure $\mathbb{Q}$ as follows:[3]

$$\begin{cases} \frac{dS_t}{S_t} = r dt + \sqrt{v_t} dB^{\mathbb{Q}}_{S,t}, \\ dv_t = \left[\kappa(\theta - v_t) - \lambda(S, v, \Theta, t)\right] dt + \sigma_v \sqrt{v_t} dB^{\mathbb{Q}}_{v,t}, \end{cases} \tag{2}$$

where $\lambda(S, v, \Theta, t)$ represents the market price of the volatility risk and $\Theta$ captures the volatility trading activities,[4] and

$$dB^{\mathbb{Q}}_{S,t} = dB_{S,t} + \frac{\mu - r}{\sqrt{v_t}} dt; \quad dB^{\mathbb{Q}}_{v,t} = dB_{v,t} + \frac{\lambda(S, v, \Theta, t)}{\sigma_v \sqrt{v_t}} dt. \tag{3}$$

The above transformation between the physical measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$ indicates that the pricing kernel $\pi_t$ must satisfy,

$$\frac{d\pi_t}{\pi_t} = -r dt - \frac{\mu - r}{\sqrt{v_t}} dB_{S,t} - \frac{\lambda(S, v, \Theta, t)}{\sigma_v \sqrt{v_t}} dB_{v,t}. \tag{4}$$

In Heston (1993), he assumes $\lambda(S, v, \Theta, t)$ as $\lambda(S, v, t)$.[5] It means that the market

[3]To be clear, all notions (e.g., Brownian motion, conditional expectation, conditional variance and conditional covariance) with superscript $\cdot^{\mathbb{Q}}$ throughout the paper are in the risk-neutral measure $\mathbb{Q}$, while all notions without superscript $\cdot^{\mathbb{Q}}$ are in the physical measure $\mathbb{P}$.

[4]Strictly speaking, the market price of the volatility risk is $\frac{\lambda(S,v,\Theta,t)}{\sigma_v \sqrt{v_t}}$. As $\sigma_v \sqrt{v_t}$ is fixed, then $\lambda(S, v, \Theta, t)$ is able to measure the magnitude of the market price of the volatility risk.

[5]Furthermore, Heston (1993) just assumes $\lambda(S, v, t) = \lambda v_t$, where $\lambda$ is a constant.

price of the volatility risk is determined only by the stock price and its volatility. In our setting, besides the stock price and its volatility, the volatility trading activities contributes to the market price of the volatility risk as well.

We simplify the Heston (1993) model based on the following assumptions.

**Assumption 1.** *To simplify the model, we set*

$$r = 0, \quad \kappa = 0. \tag{5}$$

*In addition,*

$$-1 < \rho < 0, \quad 0 < \sigma_v < 1, \tag{6}$$

*which leads to* $Var_t(R_T) > |Cov_t(R_T, v_T)|$ *where* $R_T = \log \frac{S_T}{S_t}$ *and the conditional variance and covariance are given in Appendix A.*

We note that Assumption 1 is made for notational simplicity only, and is unimportant for the conclusions we get below. The results can be extended if we relax the above assumption.

Under Assumption 1, we rewrite the dynamics of the stock price in the physical measure $\mathbb{P}$ as

$$\begin{cases} \frac{dS_t}{S_t} = \mu dt + \sqrt{v_t} dB_{S,t}, \\ dv_t = \sigma_v \sqrt{v_t} dB_{v,t}, \end{cases} \tag{7}$$

and in the risk-neutral measure $\mathbb{Q}$ as

$$\begin{cases} \frac{dS_t}{S_t} = \sqrt{v_t} dB^{\mathbb{Q}}_{S,t} \\ dv_t = -\lambda(S, v, \Theta, t)dt + \sigma_v \sqrt{v_t} dB^{\mathbb{Q}}_{v,t}. \end{cases} \tag{8}$$

## 2.2 Variance risk premium and the CBOE VIX Index

Given SPX in (7), we are able to obtain the *annualized realized variance* (RV) at time $t$ during period $[t, t+\tau]$ as

$$RV_t = E_t\left[\frac{1}{\tau}\int_t^{t+\tau} (\log S_u)^2\, du\right] = E_t\left[\frac{1}{\tau}\int_t^{t+\tau} v_u du\right] = v_t. \tag{9}$$

Similarly, the *annualized implied variance* (IV) at time $t$ during period $[t, t+\tau]$ can be written as

$$IV_t = E_t^{\mathbb{Q}}\left[\frac{1}{\tau}\int_t^{t+\tau} (\log S_u)^2\, du\right] = E_t^{\mathbb{Q}}\left[\frac{1}{\tau}\int_t^{t+\tau} v_u du\right] = v_t - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{Q}}[\lambda(S, v, \Theta, u)]du. \tag{10}$$

According to Carr and Wu's (2009) definition for the *variance risk premium* (VRP), $VRP_t = RV_t - IV_t$, we have the following theorem.

**Theorem 1 (Variance risk premium).** *The VRP at time $t$ during period $[t, t+\tau]$ can be defined as*

$$VRP_t \equiv RV_t - IV_t = \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{Q}}[\lambda(S, v, \Theta, u)]du. \tag{11}$$

Based on the above theorem, the sign of $VRP_t$ depends on the sign of $\lambda$ the more negative the $\lambda$, the more negative the $VRP_t$. The VRP on average is empirically negative, so that $\lambda$ should have a negative mean (e.g., Carr and Wu (2009)).

Following the definition of the CBOE VIX Index,[6] the VIX Index at time $t$ can be defined as the product of the square root of the implied variance during period $[t, t + 21/252]$ and the notional amount, 100, i.e.,

$$VIX_t = \sqrt{IV_t} \times 100 = 100 \times \sqrt{v_t - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{Q}}[\lambda(S, v, \Theta, u)]du}, \tag{12}$$

[6] The CBOE VIX white paper can be found at https://www.cboe.com/micro/vix/vixwhite.pdf.

where $\tau = 21/252$.

**Lemma 1.** *Based on simplified Heston (1993), we have the following:*

*(i) The implied volatility and the CBOE VIX increase with the more negative $\lambda$.*

*(ii) The negative VRP is caused by the negative $\lambda$.*

*(iii) In addition, more negative $\lambda$ leads to more negative VRP.*

In Heston's (1993) setup, the VRP is determined by the average market price of the volatility risk $\lambda$ during a future period $(t, t, +\tau)$. In addition, the implied volatility and CBOE VIX are contributed by $\lambda$. This is the key to link the VRP and the existing risks in the financial market. In this paper, the risks are from the traders' holdings in volatility products, i.e., $\Theta$. However, for different purposes, we could set is so that $\lambda$ is related to other market risks.

## 2.3 Volatility market

As Heston (1993) mentioned, any derivatives with the particular payoff function $U_T$ must satisfy the following partial differential equation (PDE):

$$\frac{1}{2}vS^2\frac{\partial^2 U}{\partial S^2} + \rho\sigma_v vS\frac{\partial^2 U}{\partial S\partial v} + \frac{1}{2}\sigma_v^2 v\frac{\partial^2 U}{\partial v^2} - \lambda\frac{\partial U}{\partial v} + \frac{\partial U}{\partial t} = 0. \tag{13}$$

The value of the derivative is determined by the market price of the volatility risk $\lambda$. Here we assume that there is a volatility derivative, i.e., $VIX^2$ futures, written on the square of the CBOE VIX Index with the maturity date $T$.[7] Then, its fair price at time

[7] $VIX^2$ futures can be regarded as a proxy of VIX futures because they have similar properties. The reason we use $VIX^2$ futures instead of VIX futures is that the pricing formula of $VIX^2$ futures is more tractable. The tractable formula produces a lot of intuitions in Section 2.5. It can be extended into VIX futures without affecting the main results in the paper.

$t$ is given by

$$
\begin{aligned}
F_{t,T}^{VIX^2} &= E_t^{\mathbb{Q}}[VIX_T^2] = E_t^{\mathbb{Q}}\left[v_T - \frac{1}{\tau}\int_T^{T+\tau} E_T^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du\right] \times 100^2 \\
&= \left(v_t - \int_t^T E_t^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du - \frac{1}{\tau}\int_T^{T+\tau} E_t^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du\right) \times 100^2. \quad (14)
\end{aligned}
$$

Applying Itô's Lemma to $F_{t,T}^{VIX^2}$, we get

$$
d\frac{F_{t,T}^{VIX^2}}{100^2} = dv_t + \lambda(S,v,\Theta,t)dt = \sigma_v\sqrt{v_t}dB_{v,t}^{\mathbb{Q}}, \quad (15)
$$

which is a martingale in the risk-neutral measure $\mathbb{Q}$. It can be also rewritten as

$$
\frac{d\frac{F_{t,T}^{VIX^2}}{100^2}}{\frac{F_{t,T}^{VIX^2}}{100^2}} = \frac{1}{\frac{F_{t,T}^{VIX^2}}{100^2}}\sigma_v\sqrt{v_t}dB_{v,t}^{\mathbb{Q}} = \frac{1}{\frac{F_{t,T}^{VIX^2}}{100^2}}\left[\lambda(S,v,\Theta,t)dt + \sigma_v\sqrt{v_t}dB_{v,t}\right]. \quad (16)
$$

From (16), we can get the expected return of $VIX^2$ futures as follows.

**Theorem 2 ($\boldsymbol{VIX^2}$ futures return).** *The expected return of $VIX^2$ futures is*

$$
R_{t,T}^{VIX^2} \equiv \frac{1}{dt}E_t\left[\frac{d\frac{F_{t,T}^{VIX^2}}{100^2}}{\frac{F_{t,T}^{VIX^2}}{100^2}}\right] = \frac{\lambda(S,v,\Theta,t)}{v_t - \int_t^T E_t^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du - \frac{1}{\tau}\int_T^{T+\tau} E_t^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du}. \quad (17)
$$

This indicates that the return of $VIX^2$ futures depends on the average $\lambda$ during future periods $(t, t+\tau)$ and $(T, T+\tau)$; the negative $\lambda$ leads to a negative return of $VIX^2$ futures. In addition, the value of $VIX^2$ futures basis at time $t$ can be solved in the following.

**Theorem 3 ($\boldsymbol{VIX^2}$ futures basis).** *The value of $VIX^2$ futures basis at time $t$ is*

$$Basis_{t,T}^{VIX^2} \equiv F_{t,T}^{VIX^2} - VIX_t^2 = \Bigg[ -\int_t^T E_t^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du + \frac{1}{\tau}\left( \int_t^{t+\tau} E_t^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du - \int_T^{T+\tau} E_t^{\mathbb{Q}}[\lambda(S,v,\Theta,u)]du \right) \Bigg] \times 100^2. \tag{18}$$

**Lemma 2.** *Based on simplified Heston (1993), we have the following:*

*(i) The price of the $VIX^2$ futures increases with more negative $\lambda$.*

*(ii) The negative return of the $VIX^2$ futures is caused by the negative $\lambda$.*

*(iv) More negative $\lambda$ leads to more negative return of $VIX^2$ futures.*

*(v) In addition, more negative $\lambda$ leads to more positive $VIX^2$ futures basis.*

In Heston's (1993) framework, the variables related to the volatility of the underlying are affected by the market price of the volatility risk $\lambda$. In other words, once we determine the value of $\lambda$, we are able to price any volatility derivatives in Heston's (1993) model.

## 2.4 The market price of the volatility risk and traders

In the economy, there are three kinds of traders: dealers (market makers), asset managers (hedgers) and leveraged funds (speculators). The trading data of the three main traders are reported by the Commitments of Traders (COT) reports and the TFF reports published by CFTC. We consider only a single-period model. There are 2 dates, $t$ and $T$, where $0 \leq t < T + \tau$. All traders make their decisions at time $t$ and hold it until time $T$. In detail, the *optimal futures positions* in volatility markets (i.e., $VIX^2$ futures market) of dealers, asset managers and leveraged funds are $x_t, y_t$ and $z_t$.

**Assumption 2.** *We assume the market price of the volatility risk is only related to the volatility trading activities,*

$$\lambda(S, v, \Theta, t) = \lambda(\Theta, t) := \lambda_t. \tag{19}$$

*In addition, the market price of the volatility risk is related to the trading strategies of dealers, $x_t$, i.e.,*[8]

$$\Theta = \{x\}. \tag{20}$$

*Based on the single-period model, the trading strategies $x_t$ will never change until date $T$, so that we assume $\lambda_t$ is time-homogeneous.*[9]

Under Assumption 2, for any time $u$ where $t \leq u \leq T$, the implied variance becomes $IV_u = v_u - \lambda_t$, and then the VRP is

$$VRP_u = \lambda_t; \tag{21}$$

the price of the $VIX^2$ futures becomes

$$\frac{F_{u,T}^{VIX^2}}{100^2} = v_u - \lambda_t \left(T - u + 1\right); \tag{22}$$

the expected return of $VIX^2$ futures,

$$R_{t,T}^{VIX^2} = \lambda_t \frac{100^2}{F_{u,T}^{VIX^2}}; \tag{23}$$

[8] In reality, the net position of dealers is the sum of net positions of the asset managers and leveraged funds. This is why we set $\Theta = \{x\}$ rather than $\Theta = \{x, y, z\}$.

[9] In other words, $x_t$ is time-homogeneous. That is, $x_u = x_t$ and $\lambda_u = \lambda_t$ for any $u$, where $t \leq u \leq T$, so that $E_t^{\mathbb{Q}}[\lambda_u] = E_t^{\mathbb{Q}}[\lambda_t] = \lambda_t$ where $t \leq u \leq T$. As the formulas of futures price and return are involved in the term $\int_T^{T+\tau} E_t^{\mathbb{Q}}[\lambda_u] du$, we further assume $\lambda_u = \lambda_t$ for any $u$ where $T < u \leq T + \tau$.

and the value of $VIX^2$ futures basis,

$$Basis_{u,T}^{VIX^2} = -\lambda_t(T-u) \times 100^2. \tag{24}$$

We consider the following special case in which the market price of volatility $\lambda_t$ is in proportion to dealers' positions.

**Assumption 3.** *In particular, we assume*

$$\lambda_t = ax_t + b, \tag{25}$$

*where $a$ and $b$ will be solved in equilibrium.*[10]

**Assumption 4.** *The trading strategies of dealers $x_t$ are exogenous.*[11]

*Leveraged funds (speculators).* To take the advantage of the negative return of $VIX^2$ futures, leveraged funds (speculators) prefer to short $VIX^2$ futures.[12] Thus, we assume their demand of $VIX^2$ futures is $y_t < 0$ at time $t$.

*Asset managers (hedgers).* In order to hedge their long positions in the stock market, asset managers (hedgers) prefer to long $VIX^2$ futures. Thus, we assume their demand of $VIX^2$ futures is $z_t > 0$ at time $t$.

*Dealers (market makers).* Dealers are risk-averse. They choose the *optimal order $\phi_t$ in stocks* and $x_t$ in $VIX^2$ futures to maximize the mean-variance preferences with the risk aversion coefficient $\gamma_D$, i.e.,

$$\max_{\phi,x} E_t[W_{D,T}] - \frac{\gamma_D}{2} Var_t(W_{D,T}), \tag{26}$$

[10]For the multiple-period model, $a$ can be recursively solved; see Garleanu et al. (2009).

[11]Here we assume the optimal trading orders of dealers $x_t$ are exogenous (actually, $x_t, y_t$ and $z_t$ are all exogenous in this case). As similar idea can be found in, for example, Bansal and Yaron (2004). The optimal consumption in Bansal and Yaron (2004) is solved from an equilibrium, but it is exogenously given.

[12]As Mixon and Onur (2015) mentions, leveraged funds always short negative-return VIX futures. Our empirical results also support this assumption.

with terminal wealth process $W_{D,T}$ given by[13]

$$W_{D,T} = W_{D,t} + \phi_t S_t R_T + x_t \left( F_{T,T}^{VIX^2} - F_{t,T}^{VIX^2} \right), \tag{27}$$

where $W_{D,t}$ is their initial wealth and the stock return $R_T = \log \frac{S_T}{S_t}$.

Using the $VIX^2$ futures price formula (22), the terminal wealth processes of dealers can be rewritten as

$$W_{D,T} = W_{D,t} + \phi_t S_t R_T + x_t \left[ v_T - v_t + (a x_t + b)(T - t) \right] \times 100^2. \tag{28}$$

Thus, their optimization problem becomes

$$\begin{aligned} \max_{\phi,x} \quad & W_{D,t} + \phi_t S_t E_t[R_T] + x_t \left[ (a x_t + b)(T - t) \right] \times 100^2 \\ & - \frac{\gamma_D}{2} \left[ \phi_t^2 S_t^2 Var_t[R_T] + x_t^2 Var_t[v_T] \times 100^4 + 2\phi_t x_t S_t Cov_t[R_T, v_T] \times 100^2 \right]. \end{aligned}$$

The first-order conditions (FOCs) lead to

$$x_t = \frac{b(T - t) - \gamma \phi_t S_t Cov_t[R_T, v_T]}{\gamma Var_t[v_T] \times 100^2 - 2a(T - t)}, \tag{29}$$

and

$$\phi_t = \frac{E_t[R_T] - x_t \gamma Cov_t[R_T, v_T] \times 100^2}{\gamma S_t Var_t[R_T]}, \tag{30}$$

where $Cov_t[R_T, v_T]$, $Var_t[v_T]$, $E_t[R_T]$ and $Var_t[R_T]$ are shown in Appendix A.

We rearrange Equation (29) and get

$$a x_t = \frac{\gamma Var_t[v_T] \times 100^2}{2(T - t)} x_t - \frac{b}{2(T - t)} + \frac{\gamma \phi_t S_t Cov_t[R_T, v_T]}{2(T - t)}. \tag{31}$$

[13] In reality, $W_{D,T} = W_{D,t} + \phi_t (S_T - S_t) + x_t \left( F_{T,T}^{VIX^2} - F_{t,T}^{VIX^2} \right)$. Here we use the continuously compounded return $\log \frac{S_T}{S_t}$ to approximate the simple return $\frac{S_T - S_t}{S_t}$, then we get the wealth process (27). This approximation is due to the simplicity of calculating $E_t[R_T], Var_t[R_T]$ and $Cov_t[R_T, v_T]$. However, whether we use this approximation does not change the main results in the paper.

Plugging Equation (30) into (31),

$$0 = \left( a - \frac{\gamma \left( Var_t^2[R_T] - Cov_t^2[R_T, v_T] \right) \times 100^2}{2(T-t)Var_t[R_T]} \right) x_t + \frac{E_t[R_T]Cov_t[R_T, v_T] - bVar_t[R_T]}{2(T-t)Var_t[R_T]}. \tag{32}$$

Under Assumption 4, $x_t$ is an exogenously given random variable. Thus, the coefficient of $x_t$ should be zero, and then we get

$$a = \frac{\gamma \left( Var_t^2[R_T] - Cov_t^2[R_T, v_T] \right) \times 100^2}{2(T-t)Var_t[R_T]} > 0, \tag{33}$$

and

$$b = \frac{E_t[R_T]Cov_t[R_T, v_T]}{Var_t[R_T]} < 0. \tag{34}$$

## 2.5 Equilibrium

**Definition 1 (Volatility market equilibrium I).** *Equilibrium in our economy is defined in a standard way: The equilibrium $VIX^2$ futures order $x_t$ of dealers is such that dealers maximize their mean-variance preferences, and the $VIX^2$ futures market is clear, i.e., $x_t + y_t + z_t = 0$.*

In equilibrium, we summarize all trading activities in futures market as the following theorem.

**Theorem 4 (Benchmark).** *Under Assumption 1-4, the equilibrium solutions are solved as*

$$\begin{cases} \lambda_t = a x_t + b, \\ a = \frac{\gamma \left( Var_t^2[R_T] - Cov_t^2[R_T, v_T] \right) \times 100^2}{2(T-t)Var_t[R_T]} > 0, \\ b = \frac{E_t[R_T]Cov_t[R_T, v_T]}{Var_t[R_T]} < 0, \\ x_t = -y_t - z_t, \\ y_t < 0, z_t > 0, \end{cases} \tag{35}$$

*where $x_t, y_t$ and $z_t$ are all exogenous.*

By using Theorem 4, we get the following lemma.

**Lemma 3 (Benchmark).** *In equilibrium, under Assumption 1-4,*

*(i) The larger short position of dealers leads to more negative $\lambda_t$.*

*(ii) The larger short position of leverage funds, $y_t$, leads to lower negative $\lambda_t$.*

*(iii) The larger long position of asset managers, $z_t$, leads to more negative $\lambda_t$.*

*(iv) Higher risk aversion, $\gamma$, more negative $\lambda_t$, if $x_t < 0$, and vice versa.*

Lemma 3 shows how the volatility trading activities influence the market price of the volatility risk $\lambda_t$. We plug $\lambda_t$ into Equation (21)-(24) and summarize as

$$\begin{cases} VRP_t = ax_t + b = -ay_t - az_t + b, \\ \frac{F_{t,T}^{VIX^2}}{100^2} = -\left(ax_t + b\right)\left(T - t + 1\right) + v_t = \left(ay_t + az_t - b\right)\left(T - t + 1\right) + v_t, \\ \frac{Basis_{t,T}^{VIX^2}}{100^2} = -\left(ax_t + b\right)\left(T - t\right) = \left(ay_t + az_t - b\right)\left(T - t\right), \\ \frac{R_{t,T}^{VIX^2}}{100^2} = \frac{ax_t + b}{F_{t,T}^{VIX^2}} = \frac{-ay_t - az_t + b}{F_{t,T}^{VIX^2}}. \end{cases} \tag{36}$$

By using Equation (36) and Lemma 1-3, we get the following proposition.

**Proposition 1 (Benchmark).** *In equilibrium, under Assumption 1-4*

*(i) The larger short positions of dealers lead to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price (equivalently, higher $VIX^2$ futures basis).*

*(ii) The larger short positions of leverage funds lead to less negative VRP and $VIX^2$ futures return, and a lower $VIX^2$ futures price.*

*(iii) The larger long positions of asset managers leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price.*

*(iv) The higher risk aversion of dealers leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price if dealers are in a short position.*

Mixon and Onur (2015) empirically test part of the results (i)-(iii) in Proposition 1; i.e., the futures price is negatively (positively) related to the level of positioning by dealers (asset managers and leveraged funds). Furthermore, dealers as market makers balance the futures positions. If buyers (i.e., asset managers) need more hedging demand than sellers (i.e., leveraged fundss) supply, dealers will issue new futures for asset managers. In this case, the futures price will be higher and the return will be negatively lower. The higher risk aversion of dealers leads to more negative futures return and higher futures price only if dealers are in a short position.

The economic mechanism is very clear and intuitive. Asset managers as hedgers have a high demand for volatility derivatives, which raises the volatility derivatives prices. At the same time, due to the their high demand, asset managers are willing to pay the high risk premium to hold volatility derivatives. On the sellers side, if leveraged funds short more volatility derivatives (i.e., higher supply), it leads to lower prices of volatility products. Facing the higher supply and lower prices, buyers, of course, are willing to pay a lower risk premium to hold these risky products. As the market makers, dealers positions indicate the balance of the supply and demand of volatility derivatives. If the demand is higher than the supply, dealers will issue new volatility contracts for buyers. The higher net short positions of dealers imply the higher demand for volatility derivatives, so that their prices will be higher. In the same scenario, dealers need to short more volatility contracts in order to cater to the needs of buyers. If dealers are more risk-averse, however, they will prefer to short less. Then the supply will be lower and consequently the price will be higher. Finally, higher prices always lead to a lower return.

If we treat the $VIX^2$ futures as variance swaps, from (i)-(iii) in Proposition 1, the model well explains the empirical results in Konstantinidi and Skiadopoulos (2016); i.e., volatility trading activities strongly predict the market VRP. In contrast to Konstantinidi and Skiadopoulos (2016), who claim that this is because for dealers holding a short position in index options based on Garleanu et al. (2009), the model suggests that the high negative VRP is driven by the larger short positions of dealers, the lower short positions of leverage funds and the larger long positions of asset managers in variance swaps.

To summarize, the volatility market is affected contemporaneously by three types of traders: asset managers, leveraged funds and dealers.

# 3. Endogenous trading strategies

In this section, we endogenize the trading strategies and find that the main conclusions are not changed.

## 3.1 Case I: One-market equilibrium with endogenous trading strategies

In contrast to Assumption 4, we give a new assumption as follows.

**Assumption 5.** *The trading strategies of dealers $x_t$ are endogenous.*[14]

As $x_t$ is endogenous and there is only one equilibrium in the volatility market, we can not determine the parameters $a$ and $b$ in Assumption 3. Thus, there are two solutions to overcome this issue: (i) reducing parameters (e.g., $b$) and (ii) defining additional equilibrium in the stock market. We analyse the former in this subsection and discuss the latter in the next subsection. Then, in order to decrease the number of unknowns, we change Assumption 3 as follows.

**Assumption 6.** *We assume*

$$\lambda_t = ax_t, \tag{37}$$

[14] Actually, $x_t, y_t$ and $z_t$ are endogenous, while $\psi_t$ is exogenous in this case.

*where a will be solved in equilibrium.*

In addition, we relax the assumption that the trading strategies of leveraged funds and asset managers are endogenous as well. We show the details of their trading behaviours as follows.

*Leveraged funds* (*speculators*). By taking advantage of the negative return of $VIX^2$ futures, leveraged funds (speculators) prefer to short $VIX^2$ futures and only speculate on the $VIX^2$ futures market.[15] They choose the optimal order $y_t$ in $VIX^2$ futures to maximize the mean-variance preferences with the risk-aversion coefficient $\gamma_L$, i.e.,

$$\max_y E_t[W_{L,T}] - \frac{\gamma_L}{2} Var_t(W_{L,T}), \tag{38}$$

with terminal wealth process $W_{L,T}$ given by

$$W_{L,T} = W_{L,t} + y_t \left( F_{T,T}^{VIX^2} - F_{t,T}^{VIX^2} \right), \tag{39}$$

where $W_{L,t}$ is their initial wealth. The terminal wealth processes of dealers can be rewritten as

$$W_{L,T} = W_{L,t} + y_t \left[ v_T - v_t + \lambda_t (T-t) \right] \times 100^2. \tag{40}$$

Thus their optimization problem becomes

$$\max_y W_{L,t} + y_t \left[ \lambda_t (T-t) \right] \times 100^2 - \frac{\gamma_L}{2} y_t^2 Var_t[v_t] \times 100^4.$$

The FOC leads to

$$y_t = \frac{\lambda_t (T-t)}{\gamma_L Var_t[v_T] \times 100^2} < 0 \quad \text{if} \quad \lambda_t = a x_t < 0. \tag{41}$$

The position of leveraged funds in the futures market depends on the sign of $\lambda_t$, which

[15] As Mixon and Onur (2015) mentions, leveraged funds always short negative-return VIX futures. Our empirical results also support this assumption.

indicates the sign of the $VIX^2$ futures returns. Thus, the short positions in $VIX^2$ futures of leveraged funds are due to the negative return of futures (i.e., $\lambda_t < 0$).

*Asset managers (hedgers).* In order to hedge their *long positions* $\psi_t > 0$ *in the stock market*, asset managers (hedgers) prefer to long $VIX^2$ futures, due to the negative correlation between stock return and volatility derivatives. Thus, given $\psi_t > 0$ in stocks, they choose optimal $z_t$ in $VIX^2$ futures in order to maximize the mean-variance preferences with the risk aversion coefficient $\gamma_A$, i.e.,

$$\max_z E_t[W_{A,T}] - \frac{\gamma_A}{2} Var_t(W_{A,T}), \tag{42}$$

with terminal wealth process $W_{A,T}$ given by

$$W_{A,T} = W_{A,t} + \psi_t S_t R_T + z_t \left( F_{T,T}^{VIX^2} - F_{t,T}^{VIX^2} \right), \tag{43}$$

where $W_{D,t}$ is their initial wealth. Similarly, the terminal wealth processes of dealers can be rewritten as

$$W_{A,T} = W_{A,t} + \psi_t S_t R_T + z_t \left[ v_T - v_t + \lambda_t (T-t) \right] \times 100^2. \tag{44}$$

Thus their optimization problem becomes

$$\begin{aligned} \max_z \quad & W_{A,t} + \psi_t S_t E_t[R_T] + z_t \left[ \lambda_t (T-t) \right] \times 100^2 \\ & - \frac{\gamma_A}{2} \left[ \psi_t^2 S_t^2 Var_t[R_T] + z_t^2 Var_t[v_t] \times 100^4 + 2\psi_t z_t S_t Cov_t[R_T, v_T] \times 100^2 \right]. \end{aligned}$$

The FOC leads to

$$z_t = \frac{\lambda_t (T-t) - \gamma_A \psi_t S_t Cov_t[R_T, v_T]}{\gamma_A Var_t[v_T] \times 100^2} = \underbrace{\frac{\lambda_t (T-t)}{\gamma_A Var_t[v_T] \times 100^2}}_{\text{negative return of futures}} + \underbrace{\frac{-\psi_t S_t Cov_t[R_T, v_T]}{Var_t[v_T] \times 100^2}}_{\text{negative correlation}}. \tag{45}$$

The positions of asset managers in $VIX^2$ futures are contributed from two components.

Due to the negative return of $VIX^2$ futures, they try to take short positions. However, as they hold a bunch of stocks, they have to long $VIX^2$ futures to hedge their long positions in stocks by using the negative correlation between the stock return and the futures price (i.e., $Cov_t[R_T, v_T] < 0$). Finally, their positions are determined by the size of the initial wealth in stocks. If $\psi_t S_t$ is large, then $z_t > 0$, and vice versa. This explains why sometimes asset managers have net short positions in the volatility market. It is because they may reduce their position in the stock market and the return of volatility derivatives is deeply negative.

*Dealers (market makers)*. Dealers (market makers) are the same here as in Section 2.4. Thus, their optional endogenous trading strategies are

$$x_t = \frac{-\gamma_D \phi_t S_t Cov_t[R_T, v_T]}{\gamma_D Var_t[v_T] \times 100^2 - 2a(T-t)}, \tag{46}$$

and

$$\phi_t = \frac{E_t[R_T] - x_t \gamma_D Cov_t[R_T, v_T] \times 100^2}{\gamma_D S_t Var_t[R_T]}. \tag{47}$$

**Definition 2 (Volatility market equilibrium II).** *Equilibrium in our economy is defined in a standard way: The equilibrium $VIX^2$ futures orders of dealers, $x_t$, and the stock orders of dealers, $\phi_t$, are such that they maximize their mean-variance preferences; the equilibrium $VIX^2$ futures orders of leveraged funds, $y_t$ , and the equilibrium $VIX^2$ futures orders of asset managers, $z_t$, are such that they maximize their mean-variance preferences, and the $VIX^2$ futures market is clear, i.e., $x_t + y_t + z_t = 0$.*

In this case, the competition among the three traders is close to in a von Stackelberg game with the dealers as the leader. Dealers propose an "$a$" in the market and then asset managers and leveraged funds submit their trading positions of the $VIX^2$ futures. Finally, dealers as market makers maximize their expected utilities with the market clearing constriction, $x_t = -y_t + z_t$, so that an equilibrium "$a$" is determined.

Based on Definition 2, we summarize the equilibrium as the following theorem.[16]

[16]Even though, in the equilibrium, there are three traders. Asset managers and leveraged funds

**Theorem 5 (Case I).** *Under Assumption 1-2 and 5-6, the equilibrium solutions can be solved from the following system,*

$$\begin{cases} \lambda_t = a x_t, \\ x_t = \frac{-\gamma_D \phi_t S_t Cov_t[R_T, v_T]}{\gamma_D Var_t[v_T] \times 100^2 - 2a(T-t)}, \\ \phi_t S_t = \frac{E_t[R_T] - x_t \gamma_D Cov_t[R_T, v_T] \times 100^2}{\gamma_D Var_t[R_T]}, \\ y_t = \frac{\lambda_t (T-t)}{\gamma_L Var_t[v_T] \times 100^2}, \\ z_t = \frac{\lambda_t (T-t) - \gamma_A \psi_t S_t Cov_t[R_T, v_T]}{\gamma_A Var_t[v_T] \times 100^2}, \\ x_t + y_t + z_t = 0. \end{cases} \tag{48}$$

*The analytical solutions are provided in Appendix B.*

The equilibrium system (48) has six equations with six unknowns, $a, \lambda_t, x_t, y_t, z_t, \phi_t S_t$. Given parameters $v_t, T-t, \mu, \rho, \sigma_v, \psi_t S_t, \gamma_D, \gamma_L, \gamma_A$, we can easily solve them by most solvers. All solutions are provided in Appendix B. For example, the key parameter $a$ is solved as

$$a = \frac{\gamma_A \gamma_L \left(\gamma_D \psi_t S_t Cov_t^2[R_T, v_T] - \gamma_D \psi_t S_t Var_t[R_T] Var_t[v_t] - E_t[R_T] Var_t[v_t]\right) \times 100^2}{(T-t)\left((\gamma_L + \gamma_A) E_t[R_T] - 2\gamma_A \gamma_L \psi_t S_t Var_t[R_T]\right)}. \tag{49}$$

We assume their risk aversion as $\gamma_L < \gamma_D < \gamma_A$. This is because asset managers are the most risk-averse and want to hedge their risks as much as they can, while leveraged funds, as speculators, are risk takers, who prefer to sell $VIX$ futures to gamble on more profits. The risk aversion of dealers lies somewhere between them. To simplify, we fix $\gamma_D = \gamma$, and then we set $\gamma_L = \gamma/\delta, \gamma_A = \gamma\delta$, where $\delta > 1$ can be a measure of risk-averse heterogeneity. Following Aït-Sahalia and Kimmel (2007), we set $\rho = -0.75$, $\sigma_v = 0.5$. In addition, we set $\mu = 0.06, v_t = 0.2^2, T-t = 1, \gamma = 2, \delta = 2$ and $\psi_t S_t = 1$. Then we have $a = 199.85 > 0, \lambda_t = -0.0086 < 0, x_t = -0.43 \times 10^{-4} < 0, y_t = -0.86 \times 10^{-4} <$

submit their orders of volatility products and deals as market makers clear the volatility market. Our model is essential different to Kyle (1985)-type model which derives equilibrium security prices when traders have asymmetric information. All traders in our model have symmetric information, while they have different hedging needs of volatility products.

$0, z_t = 1.29 \times 10^{-4} > 0, \phi_t S_t = 0.29$.

We analyse the sensitivity of $a$ to other parameters, e.g., the risk-averse heterogeneity $\delta$, investment horizon $T-t$ and the initial wealth of asset managers in stocks $\psi_t S_t$ (equivalently, the demands of asset managers in $VIX^2$ futures). In Figure 1, we find the value of $a$ is always positive. Now we can conclude that, under reasonable parameter settings, the value of $a$ can be always positive.

[Insert Figure 1]

In this case, the sign of $\lambda$ is purely determined by the net positions of dealers. By using the positive $a$ and the market clearing condition $x_t = -y_t - z_t$, we can get the following lemma, which is similar to Lemma 3.

**Lemma 4 (Case I).** *In equilibrium, under Assumption 1-2 and 5-6,*

*(i) The larger short position of dealers leads to more negative $\lambda_t$.*

*(ii) The larger short position of leverage funds, $y_t$, leads to lower negative $\lambda_t$.*

*(iii) The larger long position of asset managers, $z_t$, leads to more negative $\lambda_t$.*

Lemma 4 shows that under Assumption 1-2 and 5-6, the conclusions in Lemma 4 are not changed, compared with the results (i)-(iii) in Lemma 3. Similarly, we plug $\lambda_t$ into Equation (21)-(24) and summarize as

$$\begin{cases} VRP_t = ax_t = -ay_t - az_t, \\ \frac{F_{t,T}^{VIX^2}}{100^2} = -ax_t\,(T-t+1) + v_t = (ay_t + az_t)\,(T-t+1) + v_t, \\ \frac{Basis_{t,T}^{VIX^2}}{100^2} = -ax_t(T-t) = (ay_t + az_t)\,(T-t), \\ \frac{R_{t,T}^{VIX^2}}{100^2} = \frac{ax_t}{F_{t,T}^{VIX^2}} = \frac{-ay_t - az_t}{F_{t,T}^{VIX^2}}. \end{cases} \tag{50}$$

By using Equation (50) and Lemma 1-2 and 4 , we get the following proposition.

**Proposition 2 (Case I).** *In equilibrium, under Assumption 1-2 and 5-6,* *(i) The larger short positions of dealers lead to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price (equivalently, higher $VIX^2$ futures basis).*

*(ii) The larger short positions of leveraged funds lead to less negative VRP and $VIX^2$ futures return, and a lower $VIX^2$ futures price.*

*(iii) The larger long positions of asset managers leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price.*

The results in Proposition 2 are same as the results (i)-(iii) in Proposition 1. Thus, the economic mechanism is the same as in Section 2.5. Here, we are interested in how risk aversion, risk-averse heterogeneity, investment horizon and the hedging demand of assert managers affect the VRP, the $VIX^2$ futures return and the level of the $VIX^2$ futures price.

[Insert Figure 2]

Panel A in Figure 2 shows higher risk-averse heterogeneity leads to less negative $\lambda$, which means the less negative VRP and $VIX^2$ futures return, and lower $VIX^2$ futures price, while Panel B-D in Figure 2 shows that the higher risk aversion of dealers (or total social risk aversion), shorter horizon and larger hedging demand of assert managers generate more negative $\lambda$. We summarize these results in the following proportion.

**Proposition 3 (Case I).** *In equilibrium, under Assumption 1-2 and 5-6,*

*(i) Higher risk-averse heterogeneity leads to a less negative VRP and $VIX^2$ futures return, and a lower $VIX^2$ futures price.*

*(ii) The larger risk aversion of dealers leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price (equivalently, higher $VIX^2$ futures basis).*

*(iii) The shorter investment horizon leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price.*

*(iv) The larger hedging demand of assert managers leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price.*

## 3.2 Case II: Two-market equilibrium with endogenous trading strategies

In this case, we take Assumption 1-3 and 5 and then we have one more unknown, i.e., $b$, so that the equilibrium in the stock market has to be defined, in order to determine the additional unknown $b$.[17] The behaviours of traders are same as in Case I, except for asset managers' trading activities in the stock market. Thus, the optimal positions in $VIX^2$ futures of leveraged funds is same as (41), and the optimal portfolios of dealers in $VIX^2$ futures and stocks are same as (29) and (30). Asset managers choose both the optimal order $\psi_t$ in stocks and $z_t$ in $VIX^2$ futures to maximize the mean-variance preferences, i.e.,

$$\max_{\psi,z} E_t[W_{A,T}] - \frac{\gamma_A}{2} Var_t(W_{A,T}). \tag{51}$$

Here, in the volatility market, asset managers are hedgers and dealers are the market makers, while, in the stock market, asset managers are the market makers (clearing the stock market) and dealers are hedgers (hedging their long or short position in $VIX^2$ futures). Leveraged funds only speculate in the volatility market.

Similarly, we can solve the optimal order $\psi_t$ in stocks and $z_t$ in $VIX^2$ futures as

$$z_t = \frac{\lambda_t(T-t) - \gamma_A \psi_t S_t Cov_t[R_T, v_T]}{\gamma_A Var_t[v_T] \times 100^2}, \tag{52}$$

and

$$\psi_t = \frac{E_t[R_T] - z_t \gamma_A Cov_t[R_T, v_T] \times 100^2}{\gamma_A S_t Var_t[R_T]}. \tag{53}$$

[17] Actually, $x_t, y_t, z_t$ and $\psi_t$ are all endogenous in this case.

The definition of the equilibrium in volatility and stock markets is given as follows.

**Definition 3 (Two-market equilibrium).** *Equilibrium in our economy is defined in a standard way: Equilibrium $VIX^2$ futures orders $x_t, y_t, z_t$ and stock orders $\phi_t$ and $\psi_t$ maximize all traders' mean-variance preferences, and $VIX^2$ futures and stock markets clear, i.e., $x_t + y_t + z_t = 0$ and $\phi_t + \psi_t = Z$, where $Z$ is the total amount of stocks.*[18]

Based on Definition 3, we summarize the equilibrium as the following system.

**Theorem 6 (Case II).** *Under Assumption 1-3 and 5, the equilibrium solutions can be solved from the following system,*

$$
\begin{cases}
\lambda_t = a x_t + b, \\
x_t = \frac{b(T-t) - \gamma_D \phi_t S_t Cov_t[R_T, v_T]}{\gamma_D Var_t[v_T] \times 100^2 - 2a(T-t)}, \\
\phi_t S_t = \frac{E_t[R_T] - x_t \gamma_D Cov_t[R_T, v_T] \times 100^2}{\gamma_D Var_t[R_T]}, \\
y_t = \frac{\lambda_t (T-t)}{\gamma_L Var_t[v_T] \times 100^2}, \\
z_t = \frac{\lambda_t (T-t) - \gamma_A \psi_t S_t Cov_t[R_T, v_T]}{\gamma_A Var_t[v_T] \times 100^2}, \\
\psi_t S_t = \frac{E_t[R_T] - z_t \gamma_A Cov_t[R_T, v_T] \times 100^2}{\gamma_A Var_t[R_T]}, \\
x_t + y_t + z_t = 0, \\
\phi_t S_t + \psi_t S_t = Z S_t.
\end{cases}
\tag{54}
$$

*The analytical solutions are provided in Appendix B.*

The equilibrium system (54) has eight equations with six unknowns $a, b, \lambda_t, x_t, y_t, z_t, \phi_t S_t$ and $\psi_t S_t$. Given parameters $v_t, T-t, \mu, \rho, \sigma_v, ZS_t, \gamma_D, \gamma_L$ and $\gamma_A$, we can easily solve them (see Appendix B). Similarly, we set $\rho = -0.75$, $\sigma_v = 0.5, \mu = 0.06, v_t = 0.2^2, T-t = 1, \gamma = 2, \delta = 2$ and $ZS_t = 1$. Then we have $a = 97.03 > 0, b = -0.023 < 0, \lambda_t = -0.012 < 0, x_t = 1.12 \times 10^{-4} > 0, y_t = -1.17 \times 10^{-4} < 0, z_t = 0.044 \times 10^{-4} > 0, \phi_t S_t = 0.78$ and $\psi_t S_t = 0.22$. We find that, in this model, the positive $x_t = 1.12 \times 10^{-4} > 0$ can produce the negative $\lambda_t = -0.012 < 0$. This is consistent with the position data in our paper (i.e., Table 1) and Mixon and Onur (2015).

[18] In a more general case, $Z$ can be regarded as the remainder of the total supply traded by purely-stock-trading traders.

We analyse the sensitivities of $a$ and $b$ against other parameters in Figure 3, which shows that the value of $a$ is always positive and the value of $b$ is always negative. So, under reasonable parameter settings, the value of $a$ can be always positive, while $b$ can be always negative. The signs of them are same as the signs in the exogenous case (i.e., Equation (33) and (34)).

[Insert Figure 3]

In this model, the sign of $\lambda$ is determined by not only the positions of dealers in $VIX^2$ futures, but also the value of $b$, which measures the impacts of the position in stocks. This explains why the positive $x_t$ can produce the negative $\lambda_t$.

[Insert Figure 4]

The effects in Figure 4 are similar to Figure 3, except the investment horizon. In Panel C, Figure 4, a longer horizon leads to more negative $\lambda_t$. We get the following proposition, which is the same as Proposition 3.

**Proposition 4 (Case II).** *In equilibrium, under Assumption 1-3 and 5,*

*(i) The higher risk-averse heterogeneity most likely leads to less negative VRP and $VIX^2$ futures return, and a lower $VIX^2$ futures price.*

*(ii) The larger the risk aversion of dealers leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price (equivalently, higher $VIX^2$ futures basis).*

*(iii) The longer investment horizon leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price.*

*(iv) The larger hedging demand of asset managers leads to more negative VRP and $VIX^2$ futures return, and a higher $VIX^2$ futures price.*

# 4. Empirical analysis

In this section, first, Konstantinidi and Skiadopoulos (2016) test the trading activity model by using the trading volume of all S&P 500 futures contracts and the TED spread (which measures traders funding liquidity), while we extend the model with the net positions of dealers, asset managers and leveraged funds, which reveals the relation between the VRP and the net positions of three main traders. Second, inspired by Eraker and Wu (2017), we newly investigate the impact of the net positions on the VIX futures return. Finally, we empirically study the results in Mixon and Onur (2015) by using obtainable data.[19]

Based on Proposition 1, we propose the following six hypothesises:

(i-a) VRP is positively related to the level of positioning by dealers.

(i-b) VRP is negatively related to the level of positioning by asset managers and leveraged funds.

(ii-a) VIX futures return is positively related to the level of positioning by dealers.

(ii-b) VIX futures return is negatively related to the level of positioning by asset managers and leveraged funds.

(iii-a) VIX futures basis is negatively related to the level of positioning by dealers.

(iii-b) VIX futures basis is positively related to the level of positioning by asset managers and leveraged funds.

Hypothesis (i-a) and (i-b) are designed to explicitly explain the observation in Konstantinidi and Skiadopoulos (2016); hypothesis (ii-a) and (ii-b) are extended from Eraker and Wu (2017); and hypothesis (i-a) and (i-b) correspond to the empirical findings in Mixon and Onur (2015).

[19] The daily trading data used in Mixon and Onur (2015) is not available to the public. Thus, we have to use weekly data to test our model. Our empirical results are consistent with Mixon and Onur (2015).

## 4.1 Data

CFTC began to publish weekly TFF reports on 4 September 2009 to add further transparency to the financial futures markets, together with the disaggregated data in the CFTC's weekly COT reports. Supporting the legacy COT reports, the TFF reports provide a breakdown of each Tuesday's open interest for markets in which 20 or more traders hold positions equal to or above the reporting levels established by the CFTC and separates large traders in the financial markets into the following four categories: dealers, asset managers, leveraged funds and other reportables.[20] We download TFF Futures Only Reports weekly data from the CFTC website.[21] The available time period is from 13 June 2006 to 25 Oct 2016.

The TFF reports disclose the long and short open interest for four categories. We are interested in their net positions in VIX futures. So, we convert the long and short open interest variables into net positions, which are defined as the long open interest minus the short open interest. Here the net positions are the total aggregated open interest in VIX futures across different maturities for each type of trader. The statistics are given in Table 1.

[Insert Table 1]

Table 1 shows that the sum of the average net positions of asset manager and leveraged funds is 24199.63, which is close to the average net position of dealers, 26031.73. The average net position of other reportables is very small, around 3% of leveraged funds. Thus, contributions from other reportables can be omitted. In addition, Table 1 documents that we have to consider at least three main traders in the equilibrium instead of using the two-trader equilibrium model in Garleanu et al. (2009); Dong (2016). Our equilibrium model is more realistic than others.

[20]TFF Explanatory Notes can be found at http://www.cftc.gov/idc/groups/public/ @commitmentsoftraders/documents/file/tfmexplanatorynotes.pdf.

[21]See http://www.cftc.gov/marketreports/commitmentsoftraders/historicalcompressed/index.htm. Variable names for TFF reports are introduced in http://www.cftc.gov/MarketReports/ CommitmentsofTraders/HistoricalViewable/cotvariablestfm.

[Insert Figure 5]

Furthermore, from Figure 5, the trading in VIX futures is not very active before 2012. In order to make an easy comparison with the empirical results in Mixon and Onur (2015), we consider the last four years trading data, i.e., from 23 October 2012 to 25 October 2016.

We download VIX, S&P 500 index and VIX futures daily data from Bloomberg. As the TFF reports provide only the total open interest of VIX futures markets held by the three main traders across different maturity contracts, we examine only the futures basis for the first and second nearest futures contracts, which are the most active. Table 2 also supports our treatment. The average open interest of the first and second nearest VIX futures is two times more than the rest of contracts.

[Insert Table 2]

Following Mixon and Onur (2015), we calculate the daily futures basis as

$$Basis_t^i = Price\_future_t^i - VIX_t \tag{55}$$

where $Price\_future_t^i$ is VIX futures last price at date $t$ for the first nearest ($i = 1$) and the second nearest ($i = 2$).[22] In addition, according to Bollerslev et al. (2009) and González-Urteaga and Rubio (2016), the daily VRP is defined as

$$VRP_t^{t\to t+21} = RV_t^{t\to t+21} - VIX_t^2, \tag{56}$$

where $RV_t^{t\to t+21}$ is calculated as the variance of the daily percentage returns of the S&P 500 over 21-day windows at day $t$; $VIX_t^2$ is the daily squared VIX index divided by 12 as one-month horizon at time $t$.

Now we merge the daily $Basis_t^i$ and $VRP_t$ with weekly net position data and then

[22] We only consider the last price because the settlement price is the same as the last price in the VIX futures data downloaded from Bloomberg.

we calculate the weekly returns of VIX futures $i$,

$$Ret\_future_t^i = \log\left(Price\_future_t^i\right) - \log\left(Price\_future_{t-1}^i\right). \tag{57}$$

Finally, we provide summary statistics for all of the variables in Table 3.

[Insert Table 3]

In Table 3, the average future basis is positive and the means of the VRP and the futures return are negative, which implies that the market price of the volatility risk $\lambda$ in our model should be negative in the real world. The mean of leveraged funds' net positions is negative and that of asset managers' net positions is positive. This is consistent with our model assumptions in Section 2.4. Using the weekly data, we are able to analyse the impact of trading on futures basis, the VRP and the VIX futures return.

## 4.2 Empirical results: the impact of trading on VRP

Konstantinidi and Skiadopoulos (2016) compare the predictive ability of four models and conclude that the trading activity model is the best to predict the VRP. They claim that the greater VRP is due to dealers' greater short positions in index options. However, their trading activity variables are the trading volume of all S&P 500 futures contracts and the TED spread. They do not explicitly test the impacts of the net positions of dealers, asset managers and leveraged funds on the VRP. To fill this gap, we further develop their trading activity model into a VIX futures trading model by using the net positions of the three main traders. Then we are able to investigate the impacts of volatility trading activities on the VRP. Corresponding to Konstantinidi and Skiadopoulos (2016), we run the following regressions:[23]

$$VRP_t^{t\to t+21} = \alpha + \beta_1 NP\_d_t + \varepsilon_t, \qquad \text{Hypothesis (i-a)} \tag{58}$$

and

$$VRP_t^{t\to t+21} = \alpha + \beta_1 NP\_am + \beta_2 NP\_lf_t + \varepsilon_t, \qquad \text{Hypothesis (i-b).} \tag{59}$$

In contrast to the impact of trading on futures basis, based on (36), the sign of the coefficient $\beta$ signifies whether the VRP is positively (negatively) related to the level of positioning by dealers (asset managers and leveraged funds). The results are given in Table 4.

[Insert Table 4]

Panel A of Table 4 shows that the coefficient is significantly positive. In other words, the VRP is positively related to the level of positioning by dealers. The larger the short position of dealers, the more negative the VRP. Panel B shows that both the coefficients are negative, where the coefficient of leveraged funds' net positions is significantly negative. Similarly, we can empirically conclude that the VRP is negatively related to the level of positioning by asset managers and leveraged funds. The larger long positions of asset managers in VIX futures lead to more negative VRP, while the larger short positions of leveraged funds lead to the more negative VRP. All results correspond to Proposition 1 in this paper.

Our demand-based equilibrium model provides a channel to explain the high negative VRP. Based on our empirical and theoretical results, the high negative VRP is driven by the large hedging demand of asset managers in VIX futures to hedge the underlyings they hold. It is very intuitive that the buyers of volatility derivatives are willing to pay some risk premium (i.e., $-VRP$) to protect their long positions in underlyings. On the other hand, if leveraged funds increase (decrease) their short position,

[23]The better way to examine the impact of volatility trading on VRP is to use S&P 500 variance swaps trading data for the three major traders and to calculate the VRP based on variance swaps rates. However, the data are unobtainable.

the supply of volatility derivatives will increase (decrease), so that volatility derivatives buyers are willing to pay less (more) to buy these derivatives. This is why the VRP decreases with a short position of leveraged funds.

The large negative mean of the VRP is mainly captured by the constant term $\alpha$ in Regression (58) and (59), which essentially is the solution $b$ in our benchmark model. Table 4 shows that the constants are significantly negative with a large magnitude. Based on Equation (34), we know that the negative sign of $b$ comes from the negative correlation between the stock return and its volatility. Therefore, we observe that the large negative VRP is caused by the volatility trading activities and the negative correlation between the stock return and its volatility. Actually, due to the negative correlation, volatility derivatives provide a channel for investors to hedge the position in the stock market or speculate the position in the volatility market.

### 4.3 Empirical results: the impact of trading on VIX futures return

In order to investigate the impact of trading on the VIX futures return, we run the following regressions,

$$Ret\_future_t^i = \alpha^i + \beta_1^i NP\_d_t + \varepsilon_t, \qquad \text{Hypothesis (ii-a)} \tag{60}$$

and

$$Ret\_future_t^i = \alpha^i + \beta_1^i NP\_am + \beta_2^i NP\_lf_t + \varepsilon_t, \qquad \text{Hypothesis (ii-b).} \tag{61}$$

Consistent with Eraker and Wu (2017), the negative VIX futures return is contemporaneously related to the negative VRP. The results in Table 5 are similar to the results in Table 4. In other words, the VIX futures return and the VRP are positively (negatively) related to the level of positioning by dealers (asset managers and leveraged funds); see Proposition 1-2. The larger the short position of dealers, the more negative the return of VIX futures (see Panel A of Table 5). In addition, the larger long positions of asset managers in VIX futures lead to more negative return, while the large short positions of leveraged funds lead to the more negative return (see Panel B of Table 5).

[Insert Table 5]

Our demand-based equilibrium model very intuitively explains the mechanism. As hedgers, the high demand of asset managers in volatility derivatives acts to increase the prices of volatility derivatives and make the returns more negative. On the other hand, the leveraged funds, as speculators, selling (shorting) more volatility derivatives brings a high supply, so that the prices of volatility derivatives decrease and their returns become less negative. The balance of the supply and demand determines the prices of volatility derivatives and their returns.

## 4.4 Empirical results: the impact of trading on futures basis

In order to test whether the VIX futures (i.e., VIX futures basis) varies according to the level of different types of traders' net positions in VIX futures, following Mixon and Onur (2015), we run the following regressions:

$$Basis^i = \alpha^i + \eta^i VIX + \beta_1^i NP\_d + \varepsilon^i, \qquad \text{Hypothesis (iii-a)} \tag{62}$$

and

$$Basis^i = \alpha^i + \eta^i VIX + \beta_1^i NP\_am + \beta_2^i NP\_lf + \varepsilon^i, \qquad \text{Hypothesis (iii-b).} \tag{63}$$

The signs of the coefficient $\beta^i$ signifies whether the futures price (equivalently, the futures price basis) is negatively (positively) related to the level of positioning by dealers (asset managers and leveraged funds). The results are given in Table 6.

[Insert Table 6]

The results in both Panel A and Panel B of Table 6 are consistent with Mixon and Onur (2015), even though we use weekly data. Panel A shows that the coefficient is significantly negative for the first nearest futures contracts. Even though the coefficient of the second nearest contracts is not significant, the sign is negative. Thus, we can see the futures basis is negatively related to the level of positioning by dealers. In Panel B, the coefficients of asset managers and leveraged funds net positions for the first and second nearest contracts are all significantly positive. This means that the futures basis is positively related to the level of positioning by asset managers and leveraged funds. As on average, the asset managers net positions are positive and leveraged funds net positions are negative, the larger long positions of asset managers act to bring about higher VIX futures prices, while the larger short positions of leveraged funds act to lower VIX futures prices. Our empirical results are consistent with Proposition 1-2 and Mixon and Onur (2015).

# 5. Conclusions

We provide a very neat demand-based equilibrium model of volatility trading, which reveals an intuitive economic mechanism of how asset managers, leveraged funds and dealers' volatility trading activities affect the volatility market. Our model complements Eraker and Wu's (2017) consumption-based equilibrium model. After solving the equilibrium, we get several theoretical results which are consistent with the empirical tests in Mixon and Onur (2015) and the observation in Konstantinidi and Skiadopoulos (2016). Our empirical tests significantly support the theoretical results implied by our equilibrium model. As this is the first paper to model the volatility trading flows, our model can be easily extended into more complicated settings.

# Appendix A

Denoting $R_T = \log \frac{S_T}{S_t} = \int_t^T \left(\mu - \frac{1}{2}v_u\right) du + \int_t^T \sqrt{v_u} dB_{S,u}$, we have

$$E_t[R_T] = \mu(T-t) - \frac{1}{2}v_t(T-t). \tag{64}$$

By using $v_T = v_t + \sigma_v \int_t^T \sqrt{v_u} dB_{V,u}$, we have

$$\begin{aligned}
Var_t[R_T] =& E_t\left[R_T - E_t[R_T]\right]^2 \\
=& E_t\left[\int_t^T \sqrt{v_u} dB_{S,u} - \frac{1}{2}\left(\int_t^T v_u du - E_t \int_t^T v_u du\right)\right]^2 \\
=& E_t \int_t^T v_u du - E_t\left[\int_t^T \sqrt{v_u} dB_{S,u}\left(\int_t^T v_u du - E_t \int_t^T v_u du\right)\right] \\
&+ \frac{1}{4}E_t\left[\left(\int_t^T v_u du - E_t \int_t^T v_u du\right)\right]^2 \\
=& v_t(T-t) - \rho\sigma_v v_t \int_t^T (s-u) du + \frac{1}{4}\sigma_v^2 v_t^2 \int_t^T (s-u)^2 du \\
=& v_t(T-t) - \rho\sigma_v v_t \frac{1}{2}(T-t)^2 + \frac{1}{12}\sigma_v^2 v_t^2 (T-t)^3.
\end{aligned}$$

In addition,

$$E_t[v_T] = v_t, \tag{65}$$

and

$$Var_t[v_T] = E_t\left[\sigma_v^2 \int_t^T v_u du\right] = \sigma_v^2 v_t (T-t). \tag{66}$$

Furthermore, we calculate the conditional covariance between $R_T$ and $v_T$ as

$$\begin{aligned} Cov_t[R_T, v_T] &= E_t\left[(R_T - E_t[R_T])\,(v_T - E_t[v_T])\right] \\ &= \sigma_v E_t\left[\int_t^T \sqrt{v_u}dB_{S,u} \int_t^T \sqrt{v_u}dB_{V,u}\right] \\ &= \rho\sigma_v E_t\left[\int_t^T v_u du\right] \\ &= \rho\sigma_v v_t(T-t). \end{aligned}$$

By using the conditions, $-1 < \rho < 0$ and $0 < \sigma_v < 1$, we have

$$|Cov_t[R_T, v_T]| < v_t(T-t) < Var_t[R_T]. \tag{67}$$

# Appendix B

Given the equilibrium system (48), we solve the equilibrium as

$$a = \frac{\gamma_A\gamma_L \left(Cov_t^2[R_T, v_T]\psi_t S_t\gamma_D - Var_t[R_T]Var_t[v_T]\psi_t S_t\gamma_D - E_t[R_T]Var_t[v_T]\right) \times 100^2}{(T-t)\left(-2Var_t[R_T]\psi_t S_t\gamma_A\gamma_L + E_t[R_T]\left(\gamma_A + \gamma_L\right)\right)}, \tag{68}$$

$$\lambda_t = \frac{\gamma_A\gamma_L \left(Cov_t^2[R_T, v_T]\psi_t S_t\gamma_D - Var_t[R_T]Var_t[v_T]\psi_t S_t\gamma_D - E_t[R_T]Var_t[v_T]\right) Cov_t[R_T, v_T]}{(T-t)\left(Cov_t^2[R_T, v_T]\gamma_A\gamma_D + Cov_t^2[R_T, v_T]\gamma_D\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_A\gamma_D - 2Var_t[R_T]Var_t[v_T]\gamma_A\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_D\gamma_L\right)}, \tag{69}$$

$$\phi_t S_t = \frac{2Cov_t^2[R_T, v_T]\psi_t S_t\gamma_A\gamma_D\gamma_L - E_t[R_T]Var_t[v_T]\gamma_A\gamma_D - 2E_t[R_T]Var_t[v_T]\gamma_A\gamma_L - E_t[R_T]Var_t[v_T]\gamma_D\gamma_L}{\left(Cov_t^2[R_T, v_T]\gamma_A\gamma_D + Cov_t^2[R_T, v_T]\gamma_D\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_A\gamma_D - 2Var_t[R_T]Var_t[v_T]\gamma_A\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_D\gamma_L\right)\gamma_D}, \tag{70}$$

$$x_t = \frac{1}{100^2}\frac{Cov_t[R_T, v_T]\left(-2Var_t[R_T]\psi_t S_t\gamma_A\gamma_L + E_t[R_T]\gamma_A + E_t[R_T]\gamma_L\right)}{Cov_t^2[R_T, v_T]\gamma_A\gamma_D + Cov_t^2[R_T, v_T]\gamma_D\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_A\gamma_D - 2Var_t[R_T]Var_t[v_T]\gamma_A\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_D\gamma_L}, \tag{71}$$

$$y_t = \frac{Cov_t[R_T, v_T]\,\gamma_A\left(Cov_t^2[R_T, v_T]\psi_t S_t\gamma_D - Var_t[R_T]Var_t[v_T]\psi_t S_t\gamma_D - E_t[R_T]Var_t[v_T]\right)}{100^2 Var_t[v_T]\left(Cov_t^2[R_T, v_T]\gamma_A\gamma_D + Cov_t^2[R_T, v_T]\gamma_D\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_A\gamma_D - 2Var_t[R_T]Var_t[v_T]\gamma_A\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_D\gamma_L\right)}, \tag{72}$$

$$z_t = -\frac{Cov_t[R_T, v_T]\left(Cov_t^2[R_T, v_T]\psi_t S_t\gamma_A\gamma_D - Var_t[R_T]Var_t[v_T]\psi_t S_t\gamma_A\gamma_D - 2Var_t[R_T]Var_t[v_T]\psi_t S_t\gamma_A\gamma_L + E_t[R_T]Var_t[v_T]\gamma_L\right)}{100^2 Var_t[v_T]\left(Cov_t^2[R_T, v_T]\gamma_A\gamma_D + Cov_t^2[R_T, v_T]\gamma_D\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_A\gamma_D - 2Var_t[R_T]Var_t[v_T]\gamma_A\gamma_L - Var_t[R_T]Var_t[v_T]\gamma_D\gamma_L\right)}. \tag{73}$$

Similarly, given the equilibrium system (54), we solve the equilibrium as

$$a = \frac{\left(Cov_t^2[R_T, v_T] - Var_t[R_T]Var_t[v_T]\right) \times 100^2}{T-t} \frac{\begin{array}{r} Cov_t^2[R_T, v_T]ZS_t\gamma_A^2\gamma_D^2 - Var_t[R_T]Var_t[v_T]ZS_t\gamma_A^2\gamma_D^2 - Var_t[R_T]Var_t[v_T]ZS_t\gamma_A^2\gamma_D\gamma_L \\ -Var_t[R_T]Var_t[v_T]ZS_t\gamma_A\gamma_D^2\gamma_L + E_t[R_T]Var_t[v_T]\gamma_A^2\gamma_D + E_t[R_T]Var_t[v_T]\gamma_A^2\gamma_L \\ + E_t[R_T]Var_t[v_T]\gamma_A\gamma_D^2 + 2E_t[R_T]Var_t[v_T]\gamma_A\gamma_D\gamma_L + E_t[R_T]Var_t[v_T]\gamma_D^2\gamma_L \end{array}}{\begin{array}{l} -Cov_t^2[R_T, v_T]Var_t[R_T]ZS_t\gamma_A^2\gamma_D + Var_t^2[R_T]Var_t[v_T]ZS_t\gamma_A^2\gamma_D + Var_t^2[R_T]Var_t[v_T]ZS_t\gamma_A\gamma_D\gamma_L \\ +Cov_t^2[R_T, v_T]E_t[R_T]\gamma_A^2 - E_t[R_T]Var_t[R_T]Var_t[v_T]\gamma_A^2 \\ - E_t[R_T]Var_t[R_T]Var_t[v_T]\gamma_A\gamma_D - E_t[R_T]Var_t[v_T]\gamma_A\gamma_L Var_t[R_T] - E_t[R_T]Var_t[v_T]\gamma_D\gamma_L Var_t[R_T] \end{array}}, \tag{74}$$

$$b = -\frac{\begin{array}{r} Cov_t^2[R_T, v_T]ZS_t\gamma_A^2\gamma_D^2 - Var_t[R_T]Var_t[v_T]ZS_t\gamma_A^2\gamma_D^2 - 2Var_t[R_T]Var_t[v_T]ZS_t\gamma_A^2\gamma_D\gamma_L \\ -Var_t[R_T]Var_t[v_T]ZS_t\gamma_A\gamma_D^2\gamma_L + E_t[R_T]Var_t[v_T]\gamma_A^2\gamma_D + 2E_t[R_T]Var_t[v_T]\gamma_A^2\gamma_L \\ + E_t[R_T]Var_t[v_T]\gamma_A\gamma_D^2 + 3\,E_t[R_T]Var_t[v_T]\gamma_A\gamma_D\gamma_L + E_t[R_T]Var_t[v_T]\gamma_D^2\gamma_L \end{array}}{(T-t)\,Cov_t[R_T, v_T]\,\gamma_A^2\gamma_D}, \tag{75}$$

$$\lambda_t = -\frac{Var_t[v_T]\gamma_L\left(-Var_t[R_T]ZS_t\gamma_A\gamma_D + E_t[R_T]\gamma_A + E_t[R_T]\gamma_D\right)}{\gamma_A\,(T-t)\,Cov_t[R_T, v_T]\,\gamma_D}, \tag{76}$$

$$\phi_t S_t = \frac{\begin{array}{r} Cov_t^2[R_T, v_T]ZS_t\gamma_A^2\gamma_D - Var_t[R_T]Var_t[v_T]ZS_t\gamma_A^2\gamma_D - Var_t[R_T]Var_t[v_T]ZS_t\gamma_A\gamma_D\gamma_L + E_t[R_T]Var_t[v_T]\gamma_A\gamma_D \\ + E_t[R_T]Var_t[v_T]\gamma_A\gamma_L + E_t[R_T]Var_t[v_T]\gamma_D\gamma_L \end{array}}{\left(Cov_t^2[R_T, v_T] - Var_t[R_T]Var_t[v_T]\right)\gamma_D\gamma_A^2}, \tag{77}$$

$$\psi_t S_t = -\frac{Var_t[v_T]\left(-Var_t[R_T]ZS_t\gamma_A\gamma_D\gamma_L + E_t[R_T]\gamma_A\gamma_D + E_t[R_T]\gamma_A\gamma_L + E_t[R_T]\gamma_D\gamma_L\right)}{\left(Cov_t^2[R_T, v_T] - Var_t[R_T]Var_t[v_T]\right)\gamma_D\gamma_A^2}, \tag{78}$$

$$x_t = \frac{\begin{array}{r} -Cov_t^2[R_T, v_T]Var_t[R_T]ZS_t\gamma_A^2\gamma_D + Var_t^2[R_T]Var_t[v_T]ZS_t\gamma_A^2\gamma_D + Var_t^2[R_T]Var_t[v_T]ZS_t\gamma_A\gamma_D\gamma_L \\ +Cov_t^2[R_T, v_T]E_t[R_T]\gamma_A^2 - E_t[R_T]Var_t[R_T]Var_t[v_T]\gamma_A^2 - E_t[R_T]Var_t[R_T]Var_t[v_T]\gamma_A\gamma_D \\ - E_t[R_T]Var_t[v_T]\gamma_A\gamma_L Var_t[R_T] - E_t[R_T]Var_t[v_T]\gamma_D\gamma_L Var_t[R_T] \end{array}}{100^2 Cov_t[R_T, v_T]\,\gamma_A^2\gamma_D\left(Cov_t^2[R_T, v_T] - Var_t[R_T]Var_t[v_T]\right)}, \tag{79}$$

$$y_t = -\frac{-Var_t[R_T]ZS_t\gamma_A\gamma_D + E_t[R_T]\gamma_A + E_t[R_T]\gamma_D}{100^2\gamma_A Cov_t[R_T, v_T]\,\gamma_D}, \tag{80}$$

$$z_t = \frac{-Var_t^2[R_T]Var_t[v_T]ZS_t\gamma_A\gamma_D\gamma_L + Cov_t^2[R_T, v_T]E_t[R_T]\gamma_A\gamma_D + E_t[R_T]Var_t[v_T]\gamma_A\gamma_L Var_t[R_T] + E_t[R_T]Var_t[v_T]\gamma_D\gamma_L Var_t[R_T]}{100^2 Cov_t[R_T, v_T]\,\gamma_A^2\gamma_D\left(Cov_t^2[R_T, v_T] - Var_t[R_T]Var_t[v_T]\right)}. \tag{81}$$

**Table 1: Summary statistics on net positions.** We report the summary statistics of net positions of the different types of traders. $NP\_i$ where $i = d, am, lf, or$ represents the net positions of dealers, asset managers, leveraged funds and other reportables. The time period is from 13 June 2006 to 25 October 2016.

| Variable | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|
| $NP\_d$ | 26,031.73 | 46,864.47 | -58,735 | 144,190 |
| $NP\_am$ | 16,056.04 | 19,204.95 | -13,396 | 90,706 |
| $NP\_lf$ | -40,255.67 | 755,660.03 | -195,486 | 63,753 |
| $NP\_or$ | -1,235.41 | 7,186.36 | -38,950 | 33,621 |

**Table 2: Open interest of VIX futures across different maturities.** We report the summary statistics of open interest of VIX futures across different maturities (first six maturities). The time period is from 23 October 2012 to 25 October 2016.

| Contract Expiry | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|
| 1 | 142,654.0 | 46,600.07 | 44,360 | 295,871 |
| 2 | 111,317.4 | 45,966.74 | 32,357 | 295,930 |
| 3 | 43,304.3 | 12,214.04 | 16,890 | 95,476 |
| 4 | 31,663.4 | 7,533.20 | 14,389 | 56,336 |
| 5 | 25,618.2 | 7,035.65 | 9,940 | 47,991 |
| 6 | 18,398.0 | 5,298.13 | 7,609 | 35,450 |

**Table 3: Summary statistics on all variables.** We report the summary statistics of variables in weekly frequency. The time period is from 23 October 2012 to 25 October 2016.

| Variable | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|
| $Basis^1$ | 0.58 | 1.13 | -10.70 | 2.66 |
| $Basis^2$ | 1.46 | 1.83 | -13.47 | 4.50 |
| $VIX$ | 15.43 | 3.54 | 10.99 | 36.02 |
| $VRP_t^{t\rightarrow t+21}$ | -6.64 | 13.33 | -56.54 | 64.91 |
| $NP\_lf$ | -74,160.72 | 61,307.91 | -195,486 | 63,753 |
| $NP\_am$ | 29,727.25 | 20,625.31 | -13,396 | 90,706 |
| $NP\_d$ | 48,697.36 | 52,010.26 | -58,735 | 144,190 |
| $Ret\_future^1$ | -0.0011152 | 0.11429 | -0.37194 | 0.57328 |
| $Ret\_future^2$ | -0.0008592 | 0.07553 | -0.18382 | 0.39609 |

**Table 4: Results from regressing VRP on dealer and non-dealer VIX futures positions.** The table displays estimation results for the regressions in Equation (58) shown in Panel A and in Equation (59) shown in Panel B. T-statistics are based on Newey-West (1987) standard errors with 3 Newey-West lags. The regressions are estimated on weekly data spanning the period 23 October 2012 to 25 October 2016. $^{***}$, $^{**}$ and $^{*}$ represent statistical significance at the 1, 5 and 10% level.

| Panel A | | | |
|---|---|---|---|
| $NP\_d$ ($\times 10^{-5}$) | | $Cons.$ | Adj. $R^2$ (%) |
| 4.92$^{**}$ | | -9.00$^{***}$ | 3.24 |
| (2.04) | | (-4.83) | |

| Panel B | | | |
|---|---|---|---|
| $NP\_am$ ($\times 10^{-5}$) | $NP\_lf$ ($\times 10^{-5}$) | $Cons.$ | Adj. $R^2$ (%) |
| -8.86 | -5.19$^{*}$ | -7.84$^{***}$ | |
| (-1.07) | (-1.83) | (-2.89) | 2.18 |

**Table 5: Results from regressing VIX futures return on VIX and dealer and non-dealer VIX futures positions.** The table displays estimation results for the regressions in Equation (60) shown in Panel A and in Equation (61) shown in Panel B. Regressions are estimated separately for each contract ( $i = 1$ and 2). T-statistics are based on Newey-West (1987) standard errors with 3 Newey-West lags. The regressions are estimated on weekly data spanning the period 23 October 2012 to 25 October 2016. $^{***}$, $^{**}$ and $^{*}$ represent statistical significance at the 1, 5 and 10% level.

Panel A

| Contract Expiry | $NP\_d$ ($\times 10^{-7}$) | | $Cons.$ ($\times 10^{-2}$) | Adj. $R^2$ (%) |
|---|---|---|---|---|
| 1 | 2.50$^{**}$ | | 1.32 | 0.81 |
| | (2.32) | | (-1.43) | |
| 2 | 1.06 | | 0.60 | 0.05 |
| | (1.34) | | (-0.91) | |

Panel B

| Contract Expiry | $NP\_am$ ($\times 10^{-7}$) | $NP\_lf$ ($\times 10^{-7}$) | $Cons.$ ($\times 10^{-3}$) | Adj. $R^2$ (%) |
|---|---|---|---|---|
| 1 | -7.57$^{*}$ | -3.54$^{***}$ | 4.75 | 0.94 |
| | (-1.88) | (-2.80) | (-0.39) | |
| 2 | -3.51 | -1.55$^{*}$ | 1.87 | 0.13 |
| | (-1.36) | (-1.75) | (-0.22) | |

**Table 6: Results from regressing VIX futures basis on VIX and dealer and non-dealer VIX futures positions.** The table displays estimation results for the regressions in Equation (62) shown in Panel A and in Equation (63) shown in Panel B. Regressions are estimated separately for each contract ($i = 1$ and 2). T-statistics are based on Newey-West (1987) standard errors with 3 Newey-West lags. The regressions are estimated on weekly data spanning the period 23 October 2012 to 25 October 2016. $^{***}$, $^{**}$ and $^{*}$ represent statistical significance at the 1, 5 and 10% level.

Panel A

| Contract Expiry | $VIX$ | $NP\_d$ ($\times 10^{-6}$) | | $Cons.$ | Adj. $R^2$ (%) |
|---|---|---|---|---|---|
| 1 | -0.24$^{***}$ | -5.50$^{***}$ | | 4.61$^{***}$ | 47.52 |
| | (-4.30) | (-2.82) | | (5.04) | |
| 2 | -0.44$^{***}$ | -1.66 | | 8.34$^{***}$ | 69.28 |
| | (-7.36) | (-0.61) | | (8.53) | |

Panel B

| Contract Expiry | $VIX$ | $NP\_am$ ($\times 10^{-5}$) | $NP\_lf$ ($\times 10^{-6}$) | $Cons.$ | Adj. $R^2$ (%) |
|---|---|---|---|---|---|
| 1 | -0.23$^{***}$ | 1.91$^{***}$ | 7.56$^{***}$ | 4.15$^{***}$ | 50.97 |
| | (-4.09 ) | (4.28) | (3.89) | (4.43) | |
| 2 | -0.42$^{***}$ | 3.49$^{***}$ | 6.54$^{***}$ | 7.35$^{***}$ | 77.45 |
| | (-7.20) | (6.39) | (2.73) | (7.55) | |

**Figure 1: The value of parameter $a$ in Case I**. This figure shows the value of $a$. The benchmark model parameters are as follows: $\rho = -0.75$, $\sigma_v = 0.5, \mu = 0.06, v_t = 0.2^2, T - t = 1, \gamma = 2, \delta = 2$ and $\psi_t S_t = 1$.

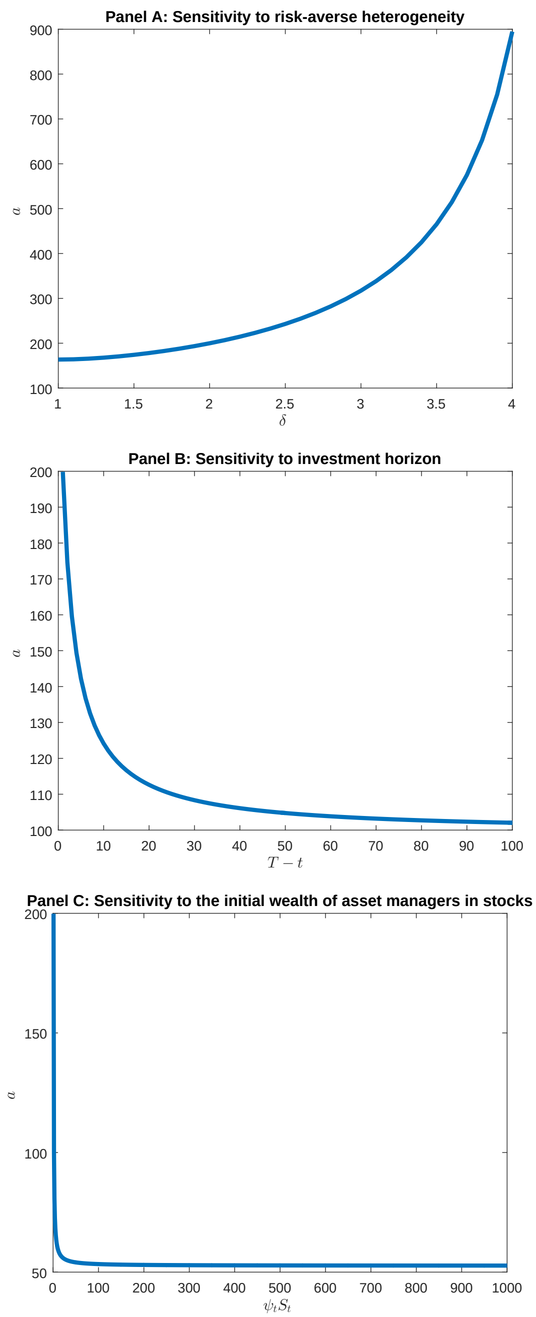


**Figure 2: The value of $\lambda$ in Case I**. This figure shows the value of $\lambda$. The benchmark model parameters are as follows: $\rho = -0.75$, $\sigma_v = 0.5, \mu = 0.06, v_t = 0.2^2, T - t = 1, \gamma = 2, \delta = 2$ and $\psi_t S_t = 1$.

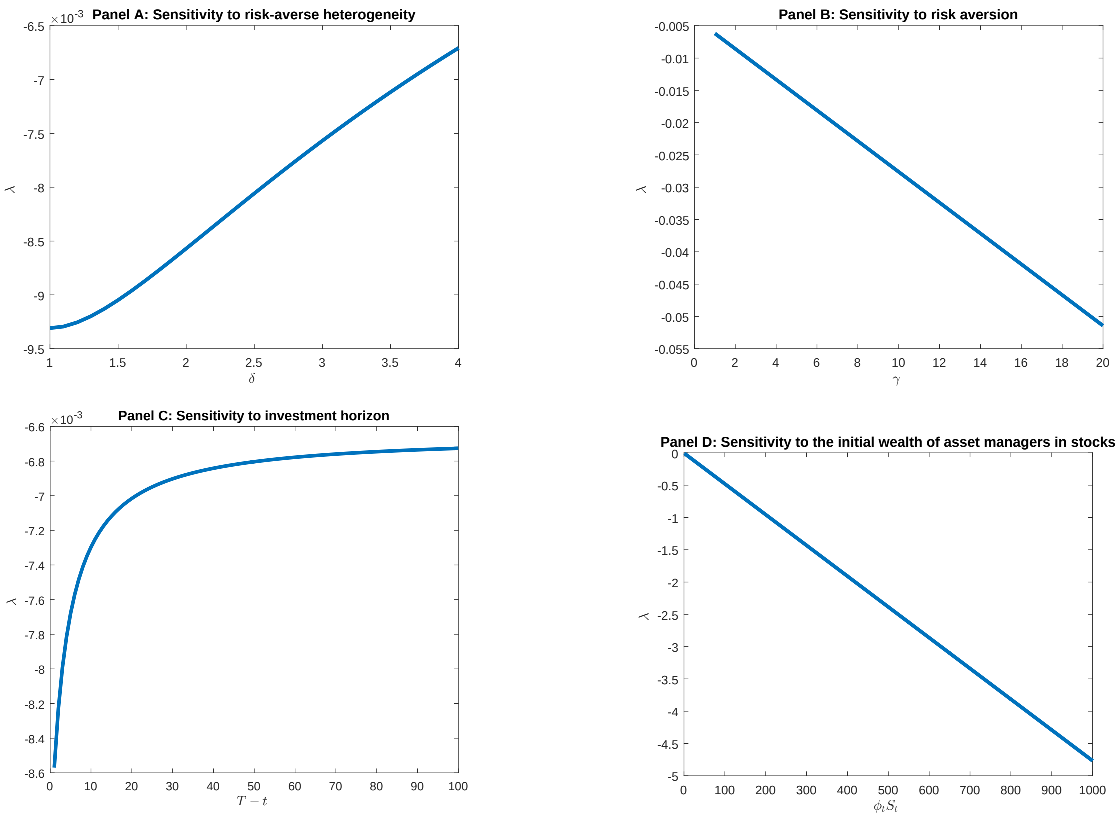


**Figure 3: The value of parameter $a$ in Case II**. This figure shows the value of $a$. The benchmark model parameters are as follows: $\rho = -0.75$, $\sigma_v = 0.5, \mu = 0.06, v_t = 0.2^2, T - t = 1, \gamma = 2, \delta = 2$ and $ZS_t = 1$.

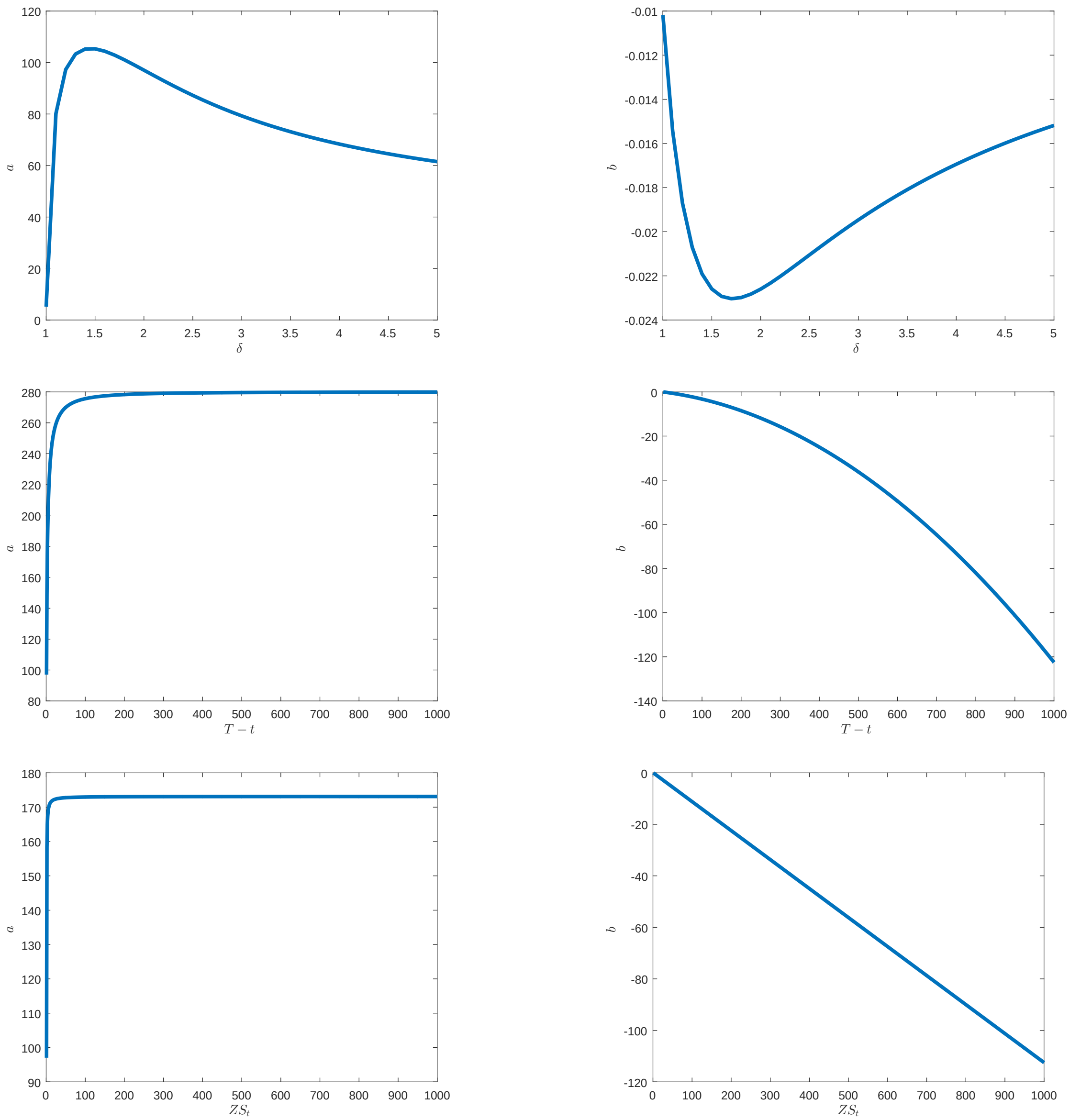


**Figure 4: The value of $\lambda$ in Case II**. This figure shows the value of $\lambda$. The benchmark model parameters are as follows: $\rho = -0.75$, $\sigma_v = 0.5, \mu = 0.06, v_t = 0.2^2, T - t = 1, \gamma = 2, \delta = 2$ and $ZS_t = 1$.

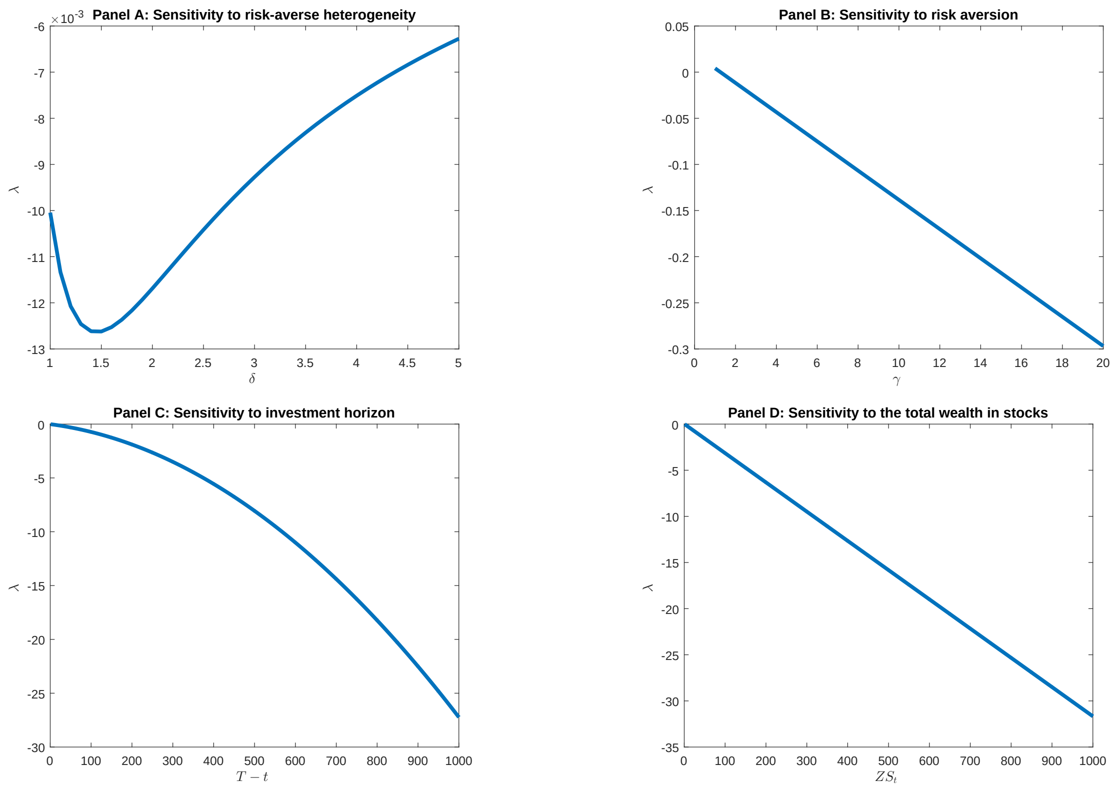
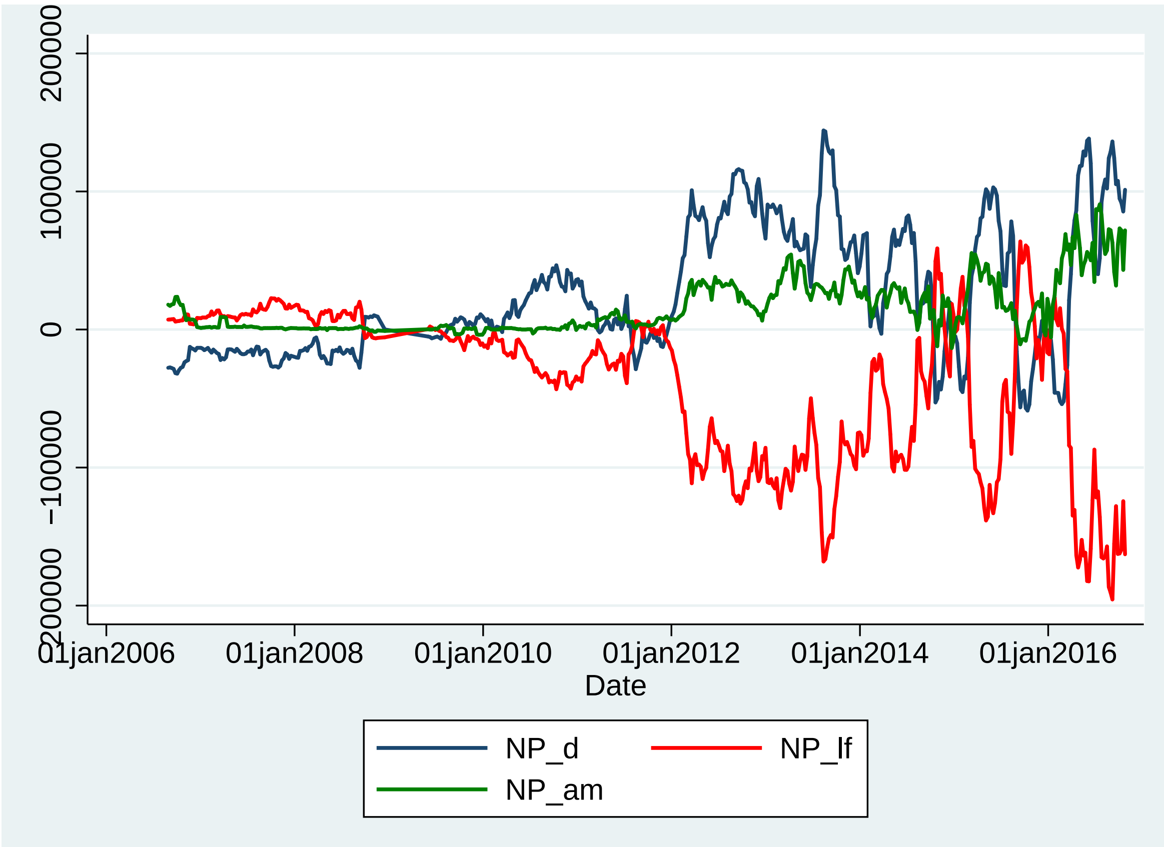


**Figure 5: Net positioning in VIX futures by asset managers, leveraged funds, and dealers.** The figure displays the net positions, aggregated within each of the dealers, asset managers and leveraged funds. $NP^i$ where $i = d, am, lf, or$ represents the net positions of dealers, asset managers, leveraged funds and other reportables. The time period is from 13 June 2006 to 25 October 2016.



# References

Aït-Sahalia, Yacine, Mustafa Karaman, and Loriano Mancini, 2015, The term structure of variance swaps and risk premia. Available at SSRN 2136820.

Aït-Sahalia, Yacine, and Robert Kimmel, 2007, Maximum likelihood estimation of stochastic volatility models, *Journal of Financial Economics* 83, 413–452.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Barras, Laurent, and Aytek Malkhozov, 2016, Does variance risk have two prices? Evidence from the equity and option markets, *Journal of Financial Economics* 121, 79–92.

Bollen, Nicolas PB, Michael J O'Neill, and Robert E Whaley, 2016, Tail wags dog: Intraday price discovery in VIX markets, *Journal of Futures Markets,* forthcoming.

Bollerslev, Tim, Natalia Sizova, and George Tauchen, 2012, Volatility in equilibrium: Asymmetries and dynamic dependencies, *Review of Finance* 16, 31–80.

Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected stock returns and variance risk premia, *Review of Financial Studies* 22, 4463–4492.

Bollerslev, Tim, and Viktor Todorov, 2011, Tails, fears, and risk premia, *Journal of Finance* 66, 2165–2211.

Bollerslev, Tim, Viktor Todorov, and Lai Xu, 2015, Tail risk premia and return predictability, *Journal of Financial Economics* 118, 113–134.

Buraschi, Andrea, Fabio Trojani, and Andrea Vedolin, 2014, When uncertainty blows in the orchard: Comovement and equilibrium volatility risk premia, *Journal of Finance* 69, 101–137.

Carr, Peter, and Liuren Wu, 2009, Variance risk premiums, *Review of Financial Studies* 22, 1311–1341.

Choi, Hoyong, Philippe Mueller, and Andrea Vedolin, 2017, Bond variance risk premiums, *Review of Finance,* forthcoming.

Dong, Xiaoyang Sean, 2016, Price impact of ETP demand on underliers. Available at SSRN 2788084.

Drechsler, Itamar, 2013, Uncertainty, time-varying fear, and asset prices, *Journal of Finance* 68, 1843–1889.

Drechsler, Itamar, and Amir Yaron, 2011, What's vol got to do with it, *Review of Financial Studies* 24, 1–45.

Eraker, Bjørn, and Yue Wu, 2017, Explaining the negative returns to volatility claims: An equilibrium approach, *Journal of Financial Economics* 125, 72–98.

Fan, Jianqing, Michael B Imerman, and Wei Dai, 2016, What does the volatility risk premium say about liquidity provision and demand for hedging tail risk?, *Journal of Business & Economic Statistics* 34, 519–535.

Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M Poteshman, 2009, Demand-based option pricing, *Review of Financial Studies* 22, 4259–4299.

González-Urteaga, Ana, and Gonzalo Rubio, 2016, The cross-sectional variation of volatility risk premia, *Journal of Financial Economics* 119, 353–370.

Heston, Steven L, 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327–343.

Jin, Jianjian, 2015, Jump-diffusion long-run risks models, variance risk premium, and volatility dynamics, *Review of Finance* 19, 1223–1279.

Konstantinidi, Eirini, and George Skiadopoulos, 2016, How does the market variance risk premium vary over time? evidence from S&P 500 variance swap investment returns, *Journal of Banking and Finance* 62, 62–75.

Kyle, Albert S, 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1336.

Li, Junye, and Gabriele Zinna, 2016, The variance risk premium: Components, term structures and stock return predictability, *Journal of Business & Economic Statistics* 1–44.

Londono, Juan M, and Hao Zhou, 2017, Variance risk premiums and the forward premium puzzle, *Journal of Financial Economics,* forthcoming.

Lucas, Robert E, 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

Mixon, Scott, and Esen Onur, 2015, Volatility derivatives in practice: activity and impact. Available at SSRN 2554745.

Todorov, Viktor, 2010, Variance risk-premium dynamics: The role of jumps, *Review of Financial Studies* 23, 345–383.