Forecasting Liquidity-Adjusted VaR: A conditional EVT-copula approach

Abstract

This study models the joint distribution of individual stock returns and bid-ask spreads using combined EGARCH-EVT and combined GP-INGARCH-EVT processes for the marginals, and bivariate copulas for the dependence structure. We use the proposed approach to first simulate returns and spreads of individual stocks from different countries and then forecast the Liquidity-adjusted Value-at-Risk (L-VaR) measure according to three types of L-VaR models. The backtesting results suggest that the proposed simulation-based L-VaR models perform better in forecasting L-VaR than the same three L-VaR models which use original returns and spreads and the traditional VaR model which uses original returns.

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1. Introduction

Liquidity risk is an important issue for investors, portfolio managers and policymakers. At times of market debacle, the liquidity in the market suddenly dries up, forcing investors to square off their positions at a higher cost. In fact, on looking back two or three decades, we find that liquidity risk has been the cause of several major market crises. The Russian financial crisis and the collapse of the Long Term Capital Management (LTCM) firm in 1998, and the unprecedented subprime mortgage crisis in the US in 2007 are the consequences of ignoring the impacts of illiquidity. All market crashes can be attributed to the absence of liquidity. Evidence shows that liquidity is a critical factor in financial markets, especially when trading takes place.

The anecdote cited above, no doubt, supports the fact that liquidity needs to be considered when measuring market risk, yet it is not addressed in the standard Value-at-Risk (VaR) concept. Hence, to plug the gap, researchers have introduced several liquidity-adjusted VaR (L-VaR) models in the literature. In a seminal paper, Bangia et al. (1998) develop an L-VaR model which serves as a basic structure to integrate liquidity risk into the traditional VaR. By focusing on the exogenous illiquidity, they build an L-VaR measure using both mid-price and bid-ask spread. Following Bangia et al. (1998), several other researchers also incorporated liquidity risk in the VaR framework (Berkowitz, 2000; Heude and Wynendaele, 2001; Angelidis and Benos, 2006; Cosandey, 2012 etc.). While Bangia et al. (1998) and Angelidis and Benos (2006) simply add liquidity cost to price risk, assuming that liquidity and price are perfectly correlated, others, such as Heude and Wynendaele (2001) adjust the liquidity risk without making such an assumption.

Although researchers have focused on how to integrate liquidity risk to total risk, they have given less attention to model the marginal distributions of returns and bid-ask spreads, which need an appropriate parametric approach. Bangia et al. (1998) use a standard parametric VaR to estimate the price risk assuming a normal distribution of return. While estimating the liquidity risk, they empirically model the distribution of bid-ask spread. However, empirical studies show that both return and spread distributions exhibit non-zero skewness, excess kurtosis, and volatility clustering and, thus, are far from normal. Ernst et al. (2012) propose a parametric approach following the Cornish-Fisher technique to deal with non-normality in price and liquidity risk. To adequately characterize the fat tail of the return and spread distributions, Muela et al. (2017) use the GARCH type model with extreme value theory (EVT), popularly known as conditional EVT, to measure the L-VaR and then compare their results with that of the Ernst et al. (2012) approach. The findings suggest that the conditional EVT outperforms the Ernst et al. (2012) approach in estimating the L-VaR. While estimating the marginal distribution based on the conditional EVT, they first use the EGARCH and GARCH models to capture the volatility clustering of the return and the relative spread series, respectively, and then apply the EVT on the standardized residuals of both the series. Stock return is a continuous series and so its empirical properties such as volatility clustering can be parameterized using the EGARCH type model. However, the bid-ask spread belongs to the class of discrete count data and, hence, it's not appropriate to model the conditional volatility of relative spreads based on a simple GARCH model. Further, research on modeling the L-VaR can be observed, among others, by Weiß and Supper (2013) and Gong et al. (2018). While Gong et al. (2018) model the bid-ask spreads by the autoregressive conditional duration (ACD) method proposed by Engle and Russell (1998) assuming the series to be continuous rather than discrete, Weiß and Supper (2013) use the more appropriate Autoregressive Conditional Double Poisson (ACDP) approach to model the discrete time series of spreads. Of course, since addressing the dependence between returns and spreads is important for liquidity risk management, Weiß and Supper (2013) and Gong et al. (2018) appropriately use different copulas to model the joint distribution of returns and spreads of individual stocks. However, while estimating the margins of both return and spread series, they have not used the EVT and simply ignored the fat tails of the distributions.

The above studies have no doubt contributed to the L-VaR literature by incorporating liquidity cost in the standard VaR model. However, most studies have ignored some empirical properties of the return and spread series. Not considering any one of the properties could lead to a misspecification of the L-VaR models, and consequently, an inaccurate estimate of the L-VaR. Thus, a holistic approach is needed to appropriately capture all important characteristics of the returns and spread series to more accurately measure the L-VaR. This is missing in the above studies. This study aims to fill this gap by tackling all these properties with appropriate tools while forecasting the L-VaR.

In this article, we propose an econometric model to estimate liquidity-adjusted risk measures and use the proposed model on daily individual stock returns and spreads taken from different countries across the globe. We start our analysis by investigating the stylized empirical properties, if any, present in the return and spread series for the selected stocks. We also perform different diagnostic tests to gain a preliminary idea of the dependence pattern between returns and spreads for the individual stocks. These preliminary analyses suggest evidences of many of the above-mentioned empirical stylized facts including a strong linear correlation as well as tail dependence between stock returns and spreads. Thereafter, we model the margin of both the stock returns and spreads. From the descriptive statistics it appears that excess kurtosis is present in both the return and spread series. This suggests that both the series have “fat tails” making their distributions far from normal. Hence, to adequately characterize the fat tail of the distribution, we use a method based on the conditional EVT. Following the two-stage approach of McNeil and Frey (2000), we apply the EGARCH model on return series and generalized Poisson Integer valued GARCH (GP-INGARCH) model on spread series, which is a time series of count data, in stage one, in order to filter the return and spread series to get their (nearly) identical and independently distributed (iid) innovations. In stage two, we safely apply the EVT framework to their iid standardized innovations. As mentioned earlier, the combination of the EVT with appropriate GARCH specification such as EGARCH for return series and GP-INGARCH for spread series, is known as the conditional EVT. The advantage of this conditional EVT is that it can capture the volatility clustering in the data with the GARCH framework, and simultaneously also model the fat tail behavior with the EVT method. Finally, since return-liquidity dependence is an important consideration for L-VaR estimation, we use different bivariate copulas to estimate their dependence structure. Thus, our proposed model is the EGARCH/GP-INGARCH-EVT-copula combined approach.

Using our proposed EGARCH/GP-INGARCH-EVT-copula combined approach, we first simulate the returns and spreads of individual stocks from different countries and then estimate the L-VaR measures according to three different L-VaR models (Bangia et al., 1998; Heude and Wynendaele, 2001; and Weiß and Supper, 2013) using the simulated returns and spreads. We also estimate the L-VaR measures according to the same three L-VaR models using the original returns and spreads directly. Further, we estimate the VaR measure according to a traditional VaR method as a benchmark model using the original returns. Then, we evaluate the relative performance of all the models in forecasting L-VaR/VaR by performing backtesting analysis. We investigate whether the L-VaR models based on simulated series are superior to the L-VaR models based on the original series. It is expected that the simulation-based L-VaR models, which capture the dependence between return and liquidity, can perform better than the L-VaR models, which do not capture the dependence. We also examine whether the L-VaR models are superior to the benchmark VaR model which neglects the liquidity risk.

This study makes two specific contributions to the L-VaR literature. We compute daily L-VaR / VaR for twelve countries across Asia, Australia, Europe, and North America. Using individual stock data from twelve different markets, we avoid the dependency of our results on a specific stock market. Thus, the present study is different from earlier ones which measure L-VaR using data mostly from a single country. This is the first contribution of the present study. The second contribution is that it proposes a new conditional EVT-copula combined approach first to simulate the return and spread series, then measure the L-VaR of three different models using the simulated data, and finally compare the accuracy of the proposed L-VaR models with the other competing models. So far as we know, this is the first empirical study conducted in the L-VaR literature to apply a combined approach where the EGARCH model filters the return series, the GP-INGARCH model filters the discrete spread series, EVT captures fat tails, and the copula function measures the nonlinear as well as asymmetric dependency between return and liquidity. Notably, while two previous studies (Weiß and Supper, 2013; Gong et al., 2018) have used the ACDP and ACD approaches, respectively, to model the discrete spread series, this study is the first to use the more appropriate GP-INGARCH model, which is more flexible and allows for predicting conditional mean and variance of the spread. The conditional mean and variance are then used to estimate the iid standardized residuals on which the EVT can safely be applied to capture the fat tails. Since the proposed combined approach takes into account all the statistical features of the data appropriately, it is expected to accurately estimate the L-VaR.

The remainder of the article is organized as follows. Section 2 presents the Econometric Methodology while Section 3 focuses on the data. Section 4 discusses the empirical findings and finally, Section 5 concludes the study by summarizing the findings.

2. Econometric Methodology

In this section, we first introduce the econometric models for daily returns and spreads along with their dependence structure. Then, we briefly describe the steps for simulating returns and spreads. Finally, we present three different L-VaR models.

As mentioned in the introduction, we use the conditional EVT to estimate the margins of both return and spreads. More clearly, we use the EGARCH-EVT combined approach to model the margin of daily returns and the GP-INGARCH-EVT combined approach to model the margin of daily spreads.

2.1. Marginal distribution of daily returns with the EGARCH-EVT combined approach

To capture the volatility dynamics of return series, we apply the standard EGARCH (1, 1) model where the mean equation is given by

$$r_t = c + a\ r_{t-1} + b\ \varepsilon_{t-1} + \varepsilon_t = \mu_t + \varepsilon_t = \mu_t + Z_t \sqrt{h_t} \tag{1}$$

and the variance equation is given by

$$\log h_t = \omega + \alpha\ \frac{|\varepsilon_{t-1}| + \gamma\ \ \varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta\ \log h_{t-1} \tag{2}$$

where μ_t is the expected return and Z_t is the standardized innovation that should follow iid.

Using the EGARCH (1, 1) process described above, we get the standardized residuals (Z_t) and then model them by the generalized Pareto distribution (GPD) of EVT. The tail estimator may be represented as follows:

$$F(z) = 1 - \frac{k}{n}\left[1 + \xi \frac{(z-u)}{\psi}\right]^{-\frac{1}{\xi}}, \quad \text{for } Z > u \tag{3}$$

where k is the number of observations exceeding the threshold u, n specifies the total number of observations, ξ indicates the shape parameter, and ψ depicts the scale parameter.

By inverting the tail estimation formula in Eq. (3), we can obtain the tail quantile for a given probability q>F(u) (see Embrechts et al., 1997):

$$z_q = u + \frac{\psi}{\xi}\left[\left(\frac{1-q}{k/n}\right)^{-\xi} - 1\right] \tag{4}$$

2.2. Marginal distribution of daily spreads with the GP-INGARCH-EVT combined approach

As bid-ask spreads represent the sum of the number of ticks between bid and ask prices, they can be converted to a time series of discrete count data. The Poisson distribution is a popular framework for modeling the count data, however, the pre-requisite condition for the equality of mean and variance is too restrictive in reality. Quite often data are overdispersed or underdispersed, with the mean less or greater than the variance. Extending the Poisson distribution, Consul and Jain (1973) proposed the more flexible generalized Poisson (GP) distribution, which allows for overdispersion or underdispersion.

Ferland et al. (2006) propose an integer-valued GARCH (INGRACH) model with Poisson deviates. As the INGARCH process can capture the serial dependence of the Poisson variates, we can estimate the conditional mean which happens to be the conditional variance based on the past values of the series and on its own past values. Although the INGARCH model allows for overdispersion in count data, it can't deal with underdispersion. Extending the Ferland et al. (2006) model, Zhu, F. (2012) introduces an INGARCH model based on the GP distribution that can tackle both overdispersion and underdispersion. The model is known as generalized Poisson INGARCH (GP-INGARCH) model, which is briefly introduced below.

Assume that { X_t } is a count data series[1] and the random variables $X_1,...,X_n$ are independent. We also assume that the conditional distribution of X_t is specified by a GP distribution, i.e.,

$$X_t \mid F_{t-1} : gp(\lambda_t^*, \kappa), \qquad \frac{\lambda_t^*}{1-\kappa} = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i} + \sum_{j=1}^{q} \beta_j \lambda_{t-j} \tag{5}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $i = 1,..., p$, $j = 1,..., q$, $p \geq 1, q \geq 0$, $\max(-1, -\lambda_t^* / 4) < \kappa < 1$, F_{t-1} is the σ -field generated by { $X_{t-1}, X_{t-2},...$}.

The Eq. (5) represents the GP-INGARCH (p, q) model. If $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$, then $\{X_t\}_{t \in Z}$ in Eq.(5) represents a stationary time series.

When p = q = 1, GP-INGARCH (1, 1) model becomes

$$X_t \mid F_{t-1} : gp(\lambda_t^*, \kappa), \qquad \frac{\lambda_t^*}{1-\kappa} = \alpha_0 + \alpha_1 X_{t-1} + \beta_1 \lambda_{t-1}. \tag{6}$$

The conditional mean and variance of X_t are defined by

$$E(X_t \mid F_{t-1}) = \frac{\lambda_t^*}{1-\kappa} = \lambda_t, \quad Var(X_t \mid F_{t-1}) = \frac{\lambda_t^*}{(1-\kappa)^3} = \phi^2 \lambda_t, \tag{7}$$

where $\phi = 1/(1-\kappa)$. Then the unconditional mean and variance are defined by

$\mu = E(X_t) = \dfrac{\alpha_0}{1-\alpha_1-\beta_1}$ and

$$\begin{aligned} Var(X_t) &= E\{(Var(X_t \mid F_{t-1})\} + Var\{E(X_t \mid F_{t-1})\} = \phi^2 \mu + Var(\lambda_t) \\ &= \frac{\phi^2 \mu [1-(\alpha_1+\beta_1)^2 + \alpha_1^2]}{1-(\alpha_1+\beta_1)^2} \end{aligned} \tag{8}$$

[1] Since spread is nothing but the multiples of ticks, it is expressed as the time series of count data.

The unconditional distribution exhibits its overdispersion (i) if $\phi > 1$, and (ii) if $\phi < 1$, we have unconditional underdispersion.

The GP-INGARCH (1, 1) model can be estimated by maximising the following log-likelihood function:

$$l(\theta) = \sum_{t=2}^{n} l_t(\theta) = \sum_{t=2}^{n} \left\{ In\lambda_t + (X_t - 1) In[\lambda_t + (\phi - 1)X_t] - X_t \, In\phi - \frac{\lambda_t + (\phi - 1)X_t}{\phi} - In(X_t!) \right\}. \quad (9)$$

The variables in Eq. (9) are explained in Zhu, F. (2012). The standardized residuals Z_t is defined as $\frac{X_t - \lambda}{\phi\sqrt{\lambda}}$ which should follow iid.

Using the GP-INGARCH (1, 1) model described above, we get the standardized residuals (Z_t) of the count data which are then modelled by the GPD explained earlier.

2.3. Copula function and copula models

The Sklar (1973) theory connects the marginal distributions and copulas to the joint distribution. When F_X and F_Y represent the two marginal distributions, there exists a bivariate copula cumulative distribution function C on [0, 1], such that for all (x, y) $\in \mathbb{R}^2$.

$$F_{XY}(x, y) = C\,[F_X\,(x), F_Y\,(y)] \quad (10)$$

For continuous F_X and F_Y, C is uniquely determined by:

$$C(u, v) = F_{XY}[F^{-1}X(u), F^{-1}Y(v)] \quad (11)$$

where $u = F_X(x)$, and $v = F_Y(y)$ belong to a class of uniform distribution [0,1], and $F_X^{-1}(u)$ and $F_Y^{-1}(y)$ are the generalized inverse distribution functions of the marginal F_X and F_Y, respectively.

Using different bivariate copulas such as Gaussian, student-*t*, Clayton, Gumbel, Frank, BB1, and BB7, we model the dependence structure between individual stock return and spreads. Different copulas characterize different dependence structures. Gaussian and Frank copulas have no tail dependence, while Student *t*-copula has symmetric lower and upper tail dependence. Clayton copula has left tail dependence and no right tail dependence. On the contrary, Gumbel copula has right tail dependence and no left tail dependence. BB1 and BB7 copulas capture both upper and lower tail dependence. Reader may refer to Joe (1997) and Nelsen (1999) for further details of the above copulas.

2.4. Simulation steps

Based on the conditional EVT-copula combined approach explained above, we simulate returns and spreads of each individual stock, using the following steps:

1. Fit the EGARCH (1, 1) model to return series, assuming skewed Student-*t* innovations, and the GP-INGARCH (1, 1) model to the spread series. Obtain the standardized innovations of EGARCH fitted to return series and GP-INGARCH fitted to spread series.
2. Apply the GPD on each of the standardized series after selecting the suitable threshold value for both tails of the distribution. Convert each innovation series into a uniform variate between 0 and 1, using the probability integral transformation.
3. For each pair of uniform variates, fit a suitable copula and obtain its parameters by using the two step-estimation procedure called the Inference Function for Margins (IFM).

4. Simulate *N* times from the estimated copula to form *N* standardized residuals of return and spread series, and then calculate the simulated return and spread series[2] using the respective mean and variance forecast of EGARCH (1, 1) and GP-INGARCH (1, 1) models applied in Step 1.

2.5. Liquidity-adjusted daily VaR

After modelling the marginal distributions of the returns and spreads, and their bivariate dependence structure to obtain simulated returns and spreads, we present below three different L-VaR models. The models are developed by Bangia et al. (1998), Heude and Wynendaele (2001), and Weiß and Supper (2013).

2.5.1. The L-VaR model of Bangia et al. (1998)

Focussing on the exogenous liquidity, Bangia et al. (1998), hereafter BDSS, develop a framework that incorporates cost of liquidity estimated using spread and its volatility to standard VaR. Thus, they provide the L-VaR model as given below:

$$L-VaR_t = P_t\left[1-\exp\left(\mu_t - z_{1-\alpha}\sqrt{h_t}\right)\right] + \frac{1}{2}P_t\left({}_r\bar{S} + s_\alpha\sigma_s\right) \tag{12}$$

where P_t denotes the mid-price on day *t*, μ is the expected mean return, h_t is the return volatility, $z_{1-\alpha}$ represents the 1-α quantile of the standard normal distribution, ${}_r\bar{S}$ is the mean relative spread, s_α indicates the α quantile of the relative spread, and σ_s is the standard deviation of the relative spread which is defined as ${}_rS = \frac{S}{P_t}$.

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[2] Since spread series are converted to time series of count data which is obtained dividing the spread by the tick size and then the GP-INGARCH (1, 1) model is fitted to the count series, the simulated count data are then converted to simulated spreads by multiplying it by the tick size.

Hence, the first addend in (12) is given by

$$VaR_{price} = P_t \left[1 - \exp\left(\mu_t - z_{1-\alpha} \sqrt{h_t} \right)\right] \tag{13}$$

and it yields the standard VaR of the mid-prices.

The second addend is given by

$$COL = \frac{1}{2}.P_t({}_r\bar{S} + s_\alpha \sigma_s) \tag{14}$$

and it explains the exogenous cost of liquidity in terms of liquidity risk which is estimated using relative spread and its volatility.

Thus, we can break up the total risk into price and liquidity risk and rewrite Eq. (12) as

$$L - VaR = VaR_{price} + COL \tag{15}$$

This model has some limitations: first, it's difficult to estimate the scaling factor when spread distribution is not specified, as the distribution is far from normal; second, it assumes the perfect correlation between extreme returns and spreads, which may not be true in reality. Next, we present a model developed by Heude and Wynendaele (2001), which aims to overcome the drawbacks of BDSS.

2.5.2. The L-VaR model of Heude and Wynendaele (2001)

Heude and Wynendaele (2001), hereafter HW, developed the L-VaR model, as given below:

$$L - VaR_t = P_t \left[\left\{ 1 - \left(1 - \frac{{}_r\bar{S}}{2} \right).\exp\left(\mu_t - z_{1-\alpha} \sqrt{h_t} \right) \right\} + \frac{1}{2}\left({}_rS_t - {}_r\bar{S} \right) \right] \tag{16}$$

The first addend of (16) which is given by

$$P_t\left\{1-\left(1-\frac{{}_r\bar{S}}{2}\right).\exp\left(\mu_t - z_{1-\alpha}\sqrt{h_t}\right)\right\} \tag{17}$$

and it explains the basic VaR adjusted to liquidity, in which the price risk and (exogenous) liquidity risk are jointly captured in a single expression.

The second addend is given by

$$\frac{1}{2}P_t\left({}_rS_t - {}_r\bar{S}\right) \tag{18}$$

which increases (decreases) the VaR number, if the immediate relative spread (${}_rS_t$) is greater (less) than the average relative spread.

The next sub-section presents the third model proposed by Weiß and Supper (2013) that is based on liquidity-adjusted returns.

2.5.3. The L-VaR model of Weiß and Supper (2013)

As mentioned earlier, BDSS assume that extreme returns and spreads are perfectly correlated. However, in reality this assumption may not be true. Weiß and Supper (2013), hereafter WS, circumvent the problem of liquidity-return correlation and develop the L-VaR model on the basis of liquidity-adjusted returns. They first incorporate the (exogenous) liquidity cost into returns and then use the standard VaR to the liquidity-adjusted returns. As liquidity cost reduces realize returns and due to the capacity of relative spreads as normalizing devices, the returns are adjusted as ${}_{adj}r_t = r_t - \frac{1}{2}\frac{S_t}{P_t}$. Using the VaR concept to liquidity adjusted returns (${}_{adj}r_t$), they define

$$L-VaR_t = VaR_t({}_{adj}r_t) = P_t\left[1-\exp\left({}_{adj}\mu_t - {}_{adj}z_{1-\alpha}\sqrt{{}_{adj}h_t}\right)\right] \tag{19}$$

where ${}_{adj}\mu_t$ and ${}_{adj}h_t$ are the expected mean and volatility of the liquidity adjusted return, and ${}_{adj}z_{1-\alpha}$ represents the 1-α quantile of the ${}_{adj}z$, which is defined as $\frac{{}_{adj}r_t - {}_{adj}\mu}{\sqrt{{}_{adj}h_t}}$.

3. Dataset and preliminary analysis

The data set used in the study is taken from twelve countries selected on the basis of highest trading volumes recently reported by the World Federation of Exchanges. From each country, we take five stocks. The daily mid-price and average bid-ask spread data of each individual stocks are downloaded from the Bloomberg terminal for a period spanning from 12 June 2012 to 30 October 2020[3]. The daily mid-price P_t is calculated as the average of closing ask and bid prices on day *t*. For each stock, daily returns are calculated as the logarithmic difference of mid-prices.

The descriptive statistics of the daily return and spread for all individual stocks selected from different countries are reported in Table 1. The mean daily returns are zero for 37 stocks and positive for the remaining 23 stocks, justifying an upward movement in their daily price. The average return calculated over five individual stocks from each country suggest that the average return is the highest for Australia (0.0008) and lowest for Hong Kong (0). The standard deviation which measures the volatility varies from country to country. Interestingly, Australia, which has the highest mean return, has the highest average standard deviation of 0.0258, while Hong Kong, which has the lowest mean return, has the lowest average standard deviation of 0.0126.

[3] The beginning date of the sample period is 12 June 2012, as the daily average bid-ask spread data are available from this date only.

[Insert Table 1 about here]

The daily average spread over five individual stocks varies considerably among different countries with South Korea having the highest average spread (228.165), and China having the lowest average spread (0.011). The average standard deviation also varies from country to country. South Korea has the highest average standard deviation (22.609), while China has the lowest average standard deviation (0.002).

The skewness of returns is negative for most of the stocks, while the skewness of spreads is positive for all the stocks except Sun Hung Kai Properties from Hong Kong, Shenzhen Airport from China, and Credit Suisse Group from Switzerland. This suggests that negative (positive) shocks are more frequent for returns (spread) series. The kurtosis values of returns are above 3 for all the stocks except Bharti Airtel from India and NongShim from South Korea. The maximum value of Kurtosis is 56.667 for Bank of Montreal from Canada and the minimum value is 2.703 for the NongShim. The kurtosis values of spreads are more than 3 for 41 stocks and less than 3 for the remaining 19 stocks. The highest kurtosis value is 72.545 for Takara Holdings from Japan and the lowest value is -1.234 for Credit Suisse Group from Switzerland. The non-zero skewness, and the high kurtosis values of return and spread series implies the non-normality of both the return and spread distributions, a fact which is corroborated by the high Jarque-Bera test statistics values of both the series. Furthermore, the Ljung–Box Q statistic rejects the conjecture that all correlation coefficients up to lag 16 are jointly zero for most of the returns and all the spread series. Thus, evidences suggest that some linear dependence is present in a majority of returns and all the spread series. Moreover, the Ljung–Box Q^2 statistic suggests the presence of non-linear dependencies in most of the return and all the spread series. The evidence shows that volatility clustering is present in both the return and the spread series for the individual stocks. The stylized properties observed in each of the return and spread series, thus, motivate the application of the EGARCH-EVT and the GP-INGARCH-EVT to estimate the margin of return and spread series, respectively.

We also perform different diagnostic tests to understand the dependence between returns and spreads for individual stocks. First, to get a basic idea of the systematic relationship between the return and spread, we do the Pearson's linear correlation tests, reported in the 2nd column of Table 2. The results reveal that the correlation coefficient between the return and spread of the majority of stocks is negative suggesting a decrease in returns followed by an increase in illiquidity and vice versa. These co-movements of return and liquidity are in line with the findings of Chordia et al. (2000), Amihud (2002) and Acharya & Pedersen (2005).

[Insert Table 2 about here]

The presence of linear correlation preliminarily supports the need for considering the dependence between returns and liquidity while estimating L-VaR. However, the non-linear and extremal dependence between returns and spreads may be present in addition to the observed linear dependence which may require the use of copula. Hence, to explore further, we estimate the lower and upper tail dependences between the returns and spreads using the nonparametric estimator proposed by Schmidt & Stadtmüller (2006). Columns 3 and 4 of Table 2 show the coefficients of the lower and upper tail dependences, respectively. The results indicate substantial proof of a nonlinear dependence between return and spread of individual stocks. Interestingly, the coefficient of upper tail dependence appears to be more prominent than the lower tail dependence, which apparently supports the fact that extreme rise in illiquidity is associated with an extreme increase in the risk premium in contemporaneous returns. On the contrary, it is observed that the lower tail dependence coefficients are constantly less than the upper tail dependence coefficients and zero for most of the stocks. This suggests that the extreme decreases in illiquidity do not concurrently happen with extreme falls in returns.

The analysis of dependence made above provides substantial evidence of non-linear and asymmetric dependence between returns and spreads, the neglecting of which may result in biased estimates of L-VaR. This motivates us to use the different bivariate copulas to measure the dependence pattern between the returns and bid-ask spreads of individual stocks.

Thus, while the volatility clustering and fat-tailed distributions observed in the data suggest us to use the EGARCH-EVT combined approach to model the margin of return, and the GP-INGARCH-EVT combined approach to model the margin of spreads, the evidence of non-linear and asymmetric dependence motivates us to apply bivariate copulas to model their joint distributions.

4. Empirical Findings

Now, we estimate L-VaR according to the three models presented in the sub-section 2.5, using the simulated spreads and returns series. We also estimate L-VaR according to these three models using original return and spread series, and measure VaR according to the benchmark VaR model using original return. Then, we evaluate the relative performance of all models in forecasting L-VaR/VaR by performing backtesting analysis. We investigate whether the simulation-based L-VaR models, which take the return-liquidity dependence into account, are superior to the L-VaR models, which are based on original series and do not take return and liquidity dependence into account. We also examine whether the L-VaR models perform better than the benchmark VaR model which neglects the liquidity risk.

The empirical analysis is divided into two parts: an in-sample analysis where models are estimated using the in-sample data, and an out-of-sample analysis in which we compare the accuracy of the competing models in forecasting L-VaR/VaR. In doing so, we split the full sample period into an in-sample period from 12 June 2012 to 11 April 2016, containing 1000 observations and an out-of-sample period from 12 April 2016 to 31 October 2020, containing the remaining 1189 observations.

4.1. In-sample analysis

First, we use the EGARCH (1, 1) and the GP-INGARCH (1, 1) models to estimate the conditional volatility of the in-sample return and spread series, respectively. The estimated parameters of the fitted models are reported in Table 3.

[Insert Table 3 about here]

Most of the coefficients in the mean equation of the return series are significant. Similarly, the constant (ω), the ARCH (α) and the GARCH (β) coefficients in the variance equation of the return series are mostly significant. The high values of GARCH coefficients imply that the return volatility is predictable and persistent over a long period. The values of γ which capture the leverage effects are also significant in majority of the cases, which indicates that the variance rises relatively more when the market declines in majority of the stocks in different countries.

The significant values of coefficients (α_1) and (β_1) of the GP-INGARCH (1, 1) model imply that the mean and variance of the bid-ask spreads are predictable. The $\alpha_1 + \beta_1$ value being less than 1 indicates stationary process $\{X_t\}_{t\in Z}$ that satisfies the GP-INGARCH (1, 1) model. Moreover, the significant values of parameter ϕ exhibit that both underdispersion as well as overdispersion are fitted with the GP-INGARCH (1, 1) model. The majority of the discrete count data are underdispersed and only in case of seven stocks (five from South Korea, one from Japan and one from the UK) the data are overdispersed.

Now, we extract the standardized residuals Z_t from the returns series using the EGARCH (1, 1) model, and from spreads the series, using the GP-INGARCH (1, 1) model, following step 1 in sub-section 2.4. We also perform the diagnostics tests to investigate whether there is still any autocorrelation present in the standardized residuals of the EGARCH (1, 1) and GP-INGARCH (1, 1) models. The test results are not shown for brevity but are available with the authors. The unreported results suggest the absence of autocorrelation for most of the EGARCH (1, 1) model fitted to return series. The corresponding results of the standardized residual of the GP-INGARCH (1, 1) model additionally confirm that it is well-fitted for the bid-ask spreads.

Then, following step 2, the standardized residuals Z_t are modelled by the GPD distribution. To fit the GPD to the standardized residuals, we need to select the lower and upper thresholds. Following Vaz de Melo Mendes (2005), we reserve 10% of the residuals for each of the lower and upper tails.

Having fitted the GPD to each standardized residual series, we transform each standardized residual series into a uniform (0,1) one, using the probability-integral transformation. Next, following step 3, we fit the best-fitted copula to each pair of the transformed data vectors and estimate parameters by applying the IFM method. We now follow step 4 and simulate *M*=10,000 times from the estimated copula to form 10,000 standardized residuals of returns and spreads series, and then calculate the M=10,000 simulated pairs of returns and spread (r_t^m, S_t^m) using the respective mean and variance forecasts of the marginal models, $m = 1, ..., M$. We also estimate simulated relative spread, which is computed as ${}_rS_t^m = \left\lfloor \frac{S_t^m}{P_t^m} \right\rfloor, P_t^m = P_{t-1} \exp(r_t^m), P_{t-1}$ is the actual mid-price at time t-1.

Using Eq. (12), we estimate L-VaR according to the BDSS model on simulated pairs of returns and spread (r_t^m, S_t^m). Using Filtered Historical Simulation, we estimate the value of

$(\mu_t - z_{1-\alpha}\sqrt{h_t})$ from the first addend of Eq. (12) on simulated return (r_t^m) and the value of $\left({}_r\bar{S} + s_\alpha \sigma_s\right)$ from the second addend on simulated relative spread $({}_rS_t^m)$.

Using Eq. (16), we estimate L-VaR according to the HW model on simulated returns (r_t^m) and relative spreads $({}_rS_t^m)$. Using Filtered Historical Simulation, the value of $(\mu_t - z_{1-\alpha}\sqrt{h_t})$ from the first addend of Eq. (16) is estimated on simulated return (r_t^m). The value of ${}_r\bar{S}$ used in both the addends is the average of simulated relative spreads, $({}_rS_t^m)$.

Using Eq. (19), we estimate L-VaR according to the WS model on simulated liquidity adjusted return ${}_{adj}r_t$. Using the simulated pairs of return and spread $(r_t^m,\ S_t^m)$ the liquidity adjusted return is computed as ${}_{adj}r_t^m = \left[r_t^m - \frac{1}{2}\ {}_rS_t^m\right]$. Using Filtered Historical Simulation, the value of $({}_{adj}\mu_t - {}_{adj}z_{1-\alpha}\sqrt{{}_{adj}h_t})$ from first addend of Eq. (19) is estimated on the adjusted return $({}_{adj}r_t^m)$.

4.2. Out-of-sample analysis

Now, we evaluate the relative performance of all the models in forecasting L-VaR/VaR by performing backtesting of each model on the out-of-sample return and spread series using the following procedure.

Initially, the parameters for each model are estimated using the most recent n = 1000 observations. The magnitude of n is set to be equal to the length of the in-sample period. Following the methods described in sub-section 4.1, we simulate returns and spreads from the proposed model, and estimate the daily L-VaR for day 1 of the out-of-sample period. Then, fixing the length of the window at n =1000, we roll forward the estimation procedure and estimate the next day L-VaR. More specifically, we fit a new EGARCH (1,1) model to return series and a new GP-INGARCH (1, 1) model to spread series on each interval t ∈ T. We then estimate the new GPD tail parameters for both the margins. Then, we use the appropriate copulas to model the joint distribution. We simulate 10,000 times from the best-fitted copula and form 10,000 simulated pairs of returns and spreads to estimate L-VaR. This process is repeated until the last day.

First, we estimate L-VaR according to the three models using simulated return and spread series, as explained above. Then, we estimate L-VaR according to these three models using original return and spread; and traditional VaR according to the benchmark VaR model using original return on the basis of the same rolling window explained above[4]. As mentioned earlier, we evaluate the relative performance of all models in forecasting L-VaR/VaR by performing backtesting analysis. In this procedure, we compare the L-VaR/VaR forecasts with actual realized liquidity-adjusted profits and losses (PL_t). Focussing on the downward risk, PL_t at time *t* is based on the mid-price P_{t-1} and bid price B_t.

$$PL_t = -[(P_{t-1} - P_t) + (P_t - B_t)] = B_t - P_{t-1} \tag{20}$$

The relative performances of competing models are assessed on the basis of violation ratio. A violation takes place if the actual loss exceeds the predicted L-VaR/VaR. We use two

[4] As mentioned in the text we have estimated L-VaR according to the three models using original returns and spread series. Using Eq. (12) we estimate L-VaR according to BDSS model. Using the EGARCH-EVT combined approach, the value of $(\mu_t - z_{1-\alpha}\sqrt{h_t})$ from the first addend of Eq. (12) is estimated on original return, where μ_t and h_t are the one step ahead forecast for the conditional mean and variance of Eqs. (1) and (2), respectively, and z_{1-q} is given by Eq.(4). Using the GP-INGARCH-EVT combined approach, the value of $\left({}_r\bar{S} + s_\alpha \sigma_s\right)$ from the second addend is estimated on relative spread $({}_rS)$, where ${}_r\bar{S}$ and σ_s are the one step ahead forecast for the conditional mean and conditional standard deviation of relative spread estimated from Eq. (7) and s_α is given by Eq.(4). Using Eq. (16) we estimate L-VaR according to the HW model. Using the EGARCH-EVT combined approach, the value of $(\mu_t - z_{1-\alpha}\sqrt{h_t})$ from the first addend of Eq. (16) is estimated on original return, where μ_t and h_t are the one step ahead forecast for the conditional mean and variance of Eqs. (1) and (2), respectively, and z_{1-q} is given by Eq.(4). The value of ${}_r\bar{S}$ used in both the addends of Eq. (16) is the one step ahead forecast for the conditional mean of relative spread estimated from Eq. (7). Using Eq. (19) we estimate L-VaR according to the WS model based on net adjusted return ${}_{adj}r_t$ defined in the text, where ${}_{adj}\mu_t$ and ${}_{adj}h_t$ are the one step ahead forecast for the conditional mean and variance of ${}_{adj}r_t$ estimated from Eqs. (1) and (2), respectively, and ${}_{adj}z_{1-\alpha}$ is the $z_{1-\alpha}$ quintile of ${}_{adj}r_t$, which is given by Eq. (4). The traditional VaR according to the benchmark VaR model is also estimated using EGARCH-EVT combined approach.

backtesting criteria to investigate the statistical validity of each model. First, we examine if the actual number of violations is as per the predicted confidence level L-VaR/VaR on the basis of unconditional coverage test of Kupiec (1995). Second, we use the conditional coverage test of Christoffersen (1998), which is a joint test of unconditional coverage and independence property. The technical details of these two tests are explained in Marimoutou et al. (2009).

Tables 4 and 5 report statistics of the above two tests, respectively, for each model at three different confidence levels of 95%, 99% and 99.5%. The 'asterisks' symbol in the tables signifies that the theoretical violation ratio is statistically different from the empirical violation ratio. Thus, the presence of 'asterisks' suggests that the model fails to predict L-VaR/VaR appropriately.

[Insert Table 4 about here]

[Insert Table 5 about here]

Table 6 reports the summarized results of the number of times each model fails based on the presence of 'asterisks' shown in Tables 4 and 5. Panel A of Table 6 which reports the unconditional coverage test statistics shows that out of the 180 cases (12 markets ×5 stocks × 3 quantiles) analysed, the benchmark VaR model fails 39 times, the BDSS model 31 times, the HW model 27 times, the WS model 28 times, the simulated BDSS models 24 times, the simulated HW model 26 times, and the simulated WS model 20 times. Thus, under this test, the three L-VaR models using simulated series perform better than the three L-VaR models using the original series in L-VaR forecasting. Within the proposed models, the WS model which applies the VaR concept on liquidity adjusted returns performs best followed by the BDSS and the HW models. The three L-VaR models that use the original return and spread series perform better than the benchmark VaR model which neglects the liquidity risk. The models that fail in the maximum number of cases are Taiwan followed by China.

[Insert Table 6 about here]

For the conditional coverage test reported in panel B of Table 6 out of 180 cases analysed, the benchmark VaR model fails 41 times, the BDSS model 29 times, the HW model 25 times, the WS model 28 times, the simulated BDSS model 24 times, the simulated HW model 25 times and the simulated WS model 25 times. Under this test too the two L-VaR models (WS and BDSS) that use simulated series again perform better than the corresponding L-VaR models that use the original series in L-VaR forecasting. On the other hand, the simulated HW and the HW based on original series perform equally. Within the proposed models, the BDSS model performs best while both the HW and the WS models perform equally. The traditional VaR model again becomes the least performing model. Out of the twelve countries under study, Germany is the only country where all the models successfully pass every time. On the other hand, here too, the models failing in maximum number of cases are Taiwan followed by China.

The combined results of unconditional and conditional coverage tests reveal that the simulated WS model performs best followed by the simulated BDSS and the simulated HW models. When we measure L-VaR using original return and spread series, the HW model performs best followed by the WS and the BDSS model, thus, supporting the fact that the HW model successfully overcomes the drawbacks of the BDSS model. The superior performance of all three simulation-based models suggests that if return-liquidity dependence is taken into account, the model forecasts the L-VaR more accurately. Finally, the least performance of the benchmark VaR model reveals that if the liquidity risk is neglected, the model cannot perform well.

5. Conclusion

This study proposes an econometric model to estimate and forecast the L-VaR/VaR measures. We use the EGARCH-EVT combined approach for modelling the margin of daily returns and the GP-INGARCH-EVT combined approach for modeling the margin of daily spreads. Finally, since return-liquidity dependence is an important consideration for L-VaR estimation, we use different bivariate copulas to measure the dependence between the returns and spreads of individual stocks. Thus, our proposed model is the EGARCH/GP-INGARCH-EVT-copula combined approach. Using this approach, we first simulate the returns and spreads of individual stocks from different countries and then estimate the L-VaR measures according to the three different L-VaR models using the simulated returns and spreads. We also estimate the L-VaR measures according to the same three L-VaR models using the original returns and bid-ask spreads. Further, we estimate the traditional VaR according to the benchmark model using the original returns.

We evaluate the relative performance of all models in forecasting L-VaR/VaR by performing backtesting analysis. We investigate whether the simulation-based L-VaR models which capture the return-liquidity dependence are superior to the L-VaR models which use original series and do not consider the return-liquidity dependence. The backtesting results reveal that the simulated based L-VaR models perform better than the competing L-VaR models, suggesting that if return-liquidity dependence is taken into account, the models can forecast the L-VaR more accurately. We also examine whether the L-VaR models perform better than the benchmark VaR model, which neglects the liquidity risk. The least performance of the benchmark VaR model reveals that if the liquidity risk is neglected, the model cannot perform well. The superior performance of the proposed models are possibly due to the ability of the combined approach to properly capture the important characteristics of data, including volatility clustering and fat-tailedness in the distribution of returns and spreads series as well as their nonlinear and asymmetric dependence relationship.

The findings have implications for the policymakers and investors who want to manage the market risk based on the L-VaR models. They can use our proposed models to accurately estimate the L-VaR. The results presented in this study are fairly robust, as we have used a large sample size, implying that they may be generalized for L-VaR forecasting.

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Table 1: Descriptive statistics for log-returns and bid-ask spreads.

Panel A: India		Mean	SD	Skewness	Kurtosis	JQ Bera	Q(16)	$Q^2(16)$
Bajaj Auto	returns	0.000	0.016	-0.258	8.564	6727.6***	8.585	334.93***
	spreads	1.866	0.483	0.787	3.473	1329.5***	16640***	12151***
Bharti Airtel	returns	0.000	0.020	0.363	2.891	812.96***	18.84	191.58***
	spreads	0.237	0.051	0.786	4.564	2130.5***	11743***	9936.4***
Hindustan Unilever	returns	0.001	0.014	0.969	9.164	8017.4***	13.965	58.148***
	spreads	0.667	0.323	0.798	0.466	252.35***	27726***	23332***
Infosys	returns	0.001	0.018	-1.442	29.438	79924***	11.519	21.996
	spreads	0.226	0.075	0.478	0.434	100.77***	22188***	18943***
Kotak Mahindra Bank	returns	0.001	0.017	0.005	6.422	3769.6***	44.985***	1034.3***
	spreads	0.593	0.189	0.679	1.237	308.87***	22058***	18651***
Panel B: Germany								
Allianz	returns	0.001	0.014	-0.724	14.122	18414***	43.776***	953.99***
	spreads	0.048	0.011	1.087	14.639	20013***	17370***	9450.9***
BASF	returns	0.000	0.016	-0.341	4.306	1738.5***	22.793	631.03***
	spreads	0.017	0.003	1.702	8.970	8410.5***	11999***	8345.6***
Bayerische Motoren Werke	returns	0.000	0.017	-0.248	6.401	3767.6***	29.035**	982.62***
	spreads	0.024	0.010	2.738	7.712	8172.9***	25927***	24695***
Deutsche Post	returns	0.001	0.015	-0.238	6.269	3613.4***	42.066***	1549.3***
	spreads	0.010	0.002	2.166	7.946	7483.2***	21211***	18728***
Fresenius Medical Care	returns	0.000	0.015	-1.078	13.502	17082***	14.714	78.332***
	spreads	0.030	0.008	1.669	9.576	9397.3***	15345***	8449.4***
Panel C: United States								
Caterpillar Inc.	returns	0.000	0.017	-0.576	6.594	4095.4***	37.402***	866.61***
	spreads	0.029	0.022	3.327	20.330	41801***	22275***	9939.3***
JPMorgan Chase & Co	returns	0.001	0.016	-0.345	16.685	25478***	210.84***	2728.7***
	spreads	0.012	0.005	5.882	49.800	239172***	18245***	10306***
3M Co.	returns	0.000	0.013	-0.998	15.047	21050***	70.234***	696.46***
	spreads	0.041	0.025	2.037	8.029	7407.4***	21745***	12358***
Walt Disney	returns	0.000	0.015	-0.208	15.499	21964***	90.111***	1756.6***
	spreads	0.017	0.008	2.747	12.172	16294***	20063***	13998***
Boeing Co	returns	0.000	0.023	-1.056	31.672	92038***	141.96***	3244.5***
	spreads	0.067	0.058	1.533	2.419	1393.7***	27900***	18156***
Panel D: Hong Kong								
Henderson Land Development	returns	0.000	0.014	0.095	3.366	1039.5***	13.344	219.67***
	spreads	0.045	0.010	0.233	-0.175	22.544***	18290***	16655***
	returns	0.000	0.013	-0.118	5.905	3192.8***	25.7*	65.601***

Sun Hung Kai Properties	spreads	0.102	0.022	-0.316	0.146	38.544***	17410***	15321***
Swire Pacific	returns	0.000	0.013	-0.582	7.232	4903.7***	46.583***	1031.2***
	spreads	0.098	0.028	1.214	1.892	866.63***	8131.9***	7003.5***
CK Infrastructure Holdings	returns	0.000	0.012	-0.834	12.604	14770***	22.806	616.19***
	spreads	0.064	0.015	2.193	12.552	16152***	3092***	1901.5***
Hang Seng Bank	returns	0.000	0.011	0.246	6.639	4051.1***	31.251**	361.03***
	spreads	0.118	0.032	1.925	4.673	3350***	17771***	17857***
Panel E: United Kingdom								
Ashtead Group	returns	0.001	0.022	-0.555	14.298	18791***	55.267***	952.37***
	spreads	1.201	0.470	2.233	11.507	13921***	24274***	17296***
AVEVA Group	returns	0.001	0.022	-1.589	51.828	246284***	35.888***	6.8047
	spreads	3.328	1.985	3.646	22.364	50548***	11506***	3763.6***
BHP Group	returns	0.000	0.021	-0.315	8.439	6544.3***	75.971***	967.67***
	spreads	0.483	0.117	0.667	3.698	1413.5***	19478***	14529***
HSBC Holdings	returns	0.000	0.014	-0.355	5.498	2809.6***	46.204***	624.42***
	spreads	0.131	0.025	1.888	12.007	14475***	12010***	6571***
British American Tobacco	returns	0.000	0.014	-0.352	5.628	2941.3***	34.904***	469.86***
	spreads	0.931	0.240	4.043	30.855	92936***	15249***	11575***
Panel F: Australia								
A2B Australia	returns	0.000	0.022	-0.357	11.972	13143***	31.847**	190.41***
	spreads	0.011	0.009	2.939	10.552	13329***	10756***	4707***
Adbri	returns	0.000	0.019	-2.903	38.326	137254***	18.932	40.414***
	spreads	0.009	0.001	0.300	-0.181	35.684***	28348***	27648***
Bell Financial Group	returns	0.001	0.025	0.207	12.805	14997***	96.227***	1226.1***
	spreads	0.009	0.005	3.268	21.342	45513***	2778.5***	2183.9***
EML Payments	returns	0.002	0.045	0.532	22.901	48014***	106.26***	1095.3***
	spreads	0.013	0.010	5.687	42.375	175837***	12054***	12703***
Fisher & Paykel Healthcare Corp	returns	0.001	0.018	0.430	7.524	5241.6***	54.862***	297.23***
	spreads	0.020	0.014	4.159	29.453	85563***	7030.7***	1632.5***
Panel G: Canada								
Agnico Eagle Mines	returns	0.000	0.026	-0.039	3.930	1413.2***	19.865	232.32***
	spreads	0.028	0.015	2.855	13.457	19523***	21375***	12582***
AltaGas	returns	0.000	0.019	-2.270	38.227	135367***	135.4***	2745.4***
	spreads	0.017	0.007	1.624	3.180	1888.5***	20381***	16677***
Algonquin Power & Utilities	returns	0.001	0.014	-1.067	20.628	39291***	119.53***	2755.9***
	spreads	0.009	0.001	4.120	34.008	111846***	19917***	15422***
Bank of Montreal	returns	0.000	0.012	-1.555	56.667	294198***	212.52***	2375.6***
	spreads	0.015	0.006	3.420	21.765	47546***	18643***	14617***

Cascades	returns	0.001	0.021	-0.667	9.143	7802.7***	12.066	74.147***
	spreads	0.019	0.006	1.269	2.638	1225.4***	12326***	10572***
Panel H: China								
Ping An Bank	returns	0.001	0.021	0.213	4.703	2038.7***	23.767*	277.5***
	spreads	0.009	0.002	-0.468	-0.710	125.8***	30817***	28613***
Shenzhen Airport	returns	0.000	0.021	-0.050	6.296	3624.5***	69.335***	2704.5***
	spreads	0.011	0.001	2.861	12.702	17730***	8059.4***	6666.7***
TCL Technology Group	returns	0.001	0.024	0.021	4.484	1838.9***	20.93	1243.3***
	spreads	0.009	0.001	0.598	-0.423	146.78***	32148***	31995***
Xuji Electric	returns	0.000	0.027	0.027	3.062	858.2***	37.35***	855.74***
	spreads	0.014	0.004	2.059	5.892	4721.4***	13520***	10438***
Shenzhen Agricultural Products Group	returns	0.000	0.026	-0.087	3.747	1287.2***	38.474***	1077.6***
	spreads	0.012	0.003	4.374	27.615	76649***	17081***	13410***
Panel I: Japan								
COMSYS	returns	0.001	0.017	-0.061	3.701	1253.9***	40.024***	227.62***
	spreads	2.135	1.381	1.889	2.039	1683.9***	21316***	20357***
Takara Holdings	returns	0.000	0.021	0.971	16.160	24203***	32.684***	197.63***
	spreads	1.415	0.391	7.277	72.545	500037***	11243***	8619.9***
Kikkoman	returns	0.001	0.018	-0.020	3.307	1000.4***	16.87	255.4***
	spreads	6.596	3.146	0.070	-1.093	110.4***	29840***	27952***
Ajinomoto	returns	0.000	0.016	0.093	5.771	3048.3***	27.566**	72.412***
	spreads	1.281	0.651	2.179	7.884	7416.1***	23456***	14366***
Kyowa Kirin	returns	0.001	0.018	0.182	3.725	1281.2***	17.349	277.43***
	spreads	1.658	0.526	2.645	9.334	10516***	18122***	14718***
Panel J: South Korea								
Samsung Fire & Marine Insurance	returns	0.000	0.016	0.038	5.900	3182.3***	24.395*	476.43***
	spreads	464.002	27.970	0.825	0.761	301.9***	18409***	18156***
Bukwang Pharmaceutical	returns	0.001	0.028	1.705	17.921	30402***	50.154***	461.92***
	spreads	33.489	11.470	0.494	-1.071	193.25***	32385***	31847***
NongShim	returns	0.000	0.018	0.347	2.703	712.51***	14.295	175.67***
	spreads	528.575	40.903	2.637	19.143	36019***	2352.5***	1899.4***
GS Retail	returns	0.000	0.022	-0.282	6.288	3643.1***	14.314	37.667***
	spreads	59.312	16.914	1.636	0.974	1064.7***	25083***	25573***
Lotte Chemical Corp	returns	0.000	0.023	-0.096	6.316	3650.2***	27.286**	629.35***
	spreads	55.448	15.788	2.705	6.019	5983.9***	26997***	27289***
Panel K: Switzerland								
Credit Suisse Group	returns	0.000	0.020	-0.778	9.206	7965.7***	32.907***	698.52***
	spreads	0.010	0.003	-0.252	-1.234	161.55***	31077***	29803***

Givaudan	returns	0.001	0.011	-0.682	8.554	6856.5***	27.014**	386.76***
	spreads	1.279	0.282	1.650	6.916	5367.3***	22969***	19491***
Partners Group	returns	0.001	0.014	-1.279	23.863	52619***	37.904***	267.13***
	spreads	0.385	0.143	0.326	0.694	83.216***	28111***	23808***
Swiss Life	returns	0.001	0.014	-0.571	18.550	31555***	42.628***	821.49***
	spreads	0.148	0.034	1.278	3.877	1971.3***	22117***	18156***
Swatch Group	returns	0.000	0.017	-0.836	9.410	8346.8***	41.815***	130.33***
	spreads	0.244	0.161	1.462	0.504	804.04***	31738***	31165***
Panel L: Taiwan								
Fwusow Industry	returns	0.000	0.010	0.950	26.036	62254***	46.536***	620.71***
	spreads	0.105	0.041	1.725	5.921	4292***	3386.8***	1873.7***
Sun Yad Construction	returns	0.000	0.020	0.398	7.138	4715.1***	100.79***	556.48***
	spreads	0.075	0.028	2.547	11.337	14114***	6883.8***	3306.5***
Universal Inc	returns	0.001	0.019	1.308	12.740	15455***	153.65***	4656.5***
	spreads	0.114	0.164	6.127	53.397	274150***	22365***	8101.7***
BioLASCO Taiwan	returns	0.000	0.019	0.232	7.725	5474.7***	52.917***	548.55***
	spreads	0.197	0.115	2.476	8.338	8592.9***	15055***	12399***
Lily Textile	returns	0.000	0.017	0.850	8.039	6170.9***	31.247**	284.19***
	spreads	0.145	0.093	1.670	6.011	4322.1***	6269.5***	1735.7***

Note: The asterisks *, **, and *** denote significance at 10%, 5%, and 1% levels, respectively.

Table 2: Linear correlation, lower tail and upper tail dependence between log-returns and bid-ask spreads.

Panel A: India	Correlation	Lower tail dependence	Upper tail dependence
Bajaj Auto	-0.0741	0.0435	0.0217
Bharti Airtel	-0.0451	0.0435	0.0870
Hindustan Unilever	-0.0084	0.0435	0.0000
Infosys	0.0000	0.0652	0.0652
Kotak Mahindra Bank	-0.0540	0.0652	0.1087
Panel B: Germany			
Allianz	-0.0470	0.0000	0.0652
BASF	-0.0567	0.0435	0.0435
Bayerische Motoren Werke	-0.0152	0.0000	0.0435
Deutsche Post	-0.0062	0.0000	0.1522
Fresenius Medical Care	-0.0572	0.0000	0.0870
Panel C: United States			
Caterpillar Inc.	-0.0268	0.0000	0.2174
JPMorgan Chase & Co.	-0.0480	0.0000	0.3043
3M Co.	-0.1049	0.0000	0.1522
Walt Disney Co.	-0.0403	0.0000	0.1739
Boeing	-0.0583	0.0000	0.1087
Panel D: Hong Kong			
Henderson Land Development Co.	-0.0624	0.0870	0.0000
Sun Hung Kai Properties	-0.0175	0.0217	0.0217
Swire Pacific	-0.0128	0.0217	0.0217
CK Infrastructure Holdings.	-0.0608	0.0000	0.0000
Hang Seng Bank	-0.0104	0.0000	0.0217
Panel E: United Kingdom			
Ashtead Group	0.0171	0.0000	0.1957
AVEVA Group	0.0115	0.0435	0.1522
BHP Group	0.0079	0.1087	0.0870
HSBC Holdings	-0.0031	0.0217	0.0870
British American Tobacco	-0.0314	0.0000	0.1739
Panel F: Australia			
A2B Australia Ltd	-0.0193	0.0000	0.0217
Adbri Ltd	-0.0479	0.0000	0.2826
Bell Financial Group	0.0220	0.0000	0.0870
EML Payments	0.0533	0.0000	0.1304
Fisher & Paykel Healthcare Corp	0.0235	0.0000	0.0435
Panel G: Canada			
Agnico Eagle Mines Ltd	0.0168	0.0217	0.0652
AltaGas Ltd	0.0069	0.0000	0.0652
Algonquin Power & Utilities	-0.0823	0.0000	0.1739
Bank of Montreal	-0.0551	0.0000	0.2391
Cascades	0.0128	0.0000	0.0652
Panel H: China			
Ping An Bank	-0.0069	0.0000	0.0435

Shenzhen Airport Co Ltd	-0.1199	0.0000	0.0435
TCL Technology Group Corp	-0.0004	0.0000	0.0652
Xuji Electric	-0.1089	0.0000	0.1087
Shenzhen Agricultural Products Group	-0.1198	0.0000	0.1087
Panel I: Japan			
COMSYS	-0.0003	0.0000	0.0000
Takara Holdings	-0.0149	0.0000	0.0435
Kikkoman	-0.0398	0.0000	0.0435
Ajinomoto	-0.0009	0.0000	0.0652
Kyowa Kirin	-0.0258	0.0000	0.0217
Panel J: South Korea			
Samsung Fire & Marine Insurance	-0.0108	0.0000	0.0652
Bukwang Pharmaceutical	-0.0263	0.0000	0.0435
NongShim	0.0068	0.0217	0.0435
GS Retail	0.0134	0.0870	0.0000
Lotte Chemical Corp	0.0093	0.0435	0.0652
Panel K: Switzerland			
Credit Suisse Group	-0.0077	0.0435	0.0217
Givaudan	-0.0052	0.0000	0.2174
Partners Group	-0.0166	0.0000	0.2174
Swiss Life	-0.0939	0.0217	0.0870
Swatch Group	0.0087	0.0870	0.0000
Panel L: Taiwan			
Fwusow Industry	0.0039	0.0000	0.0435
Sun Yad Construction	-0.1111	0.0435	0.0000
Universal Inc	0.0003	0.0000	0.3043
BioLASCO Taiwan	-0.0509	0.0217	0.0000
Lily Textile	0.0294	0.0000	0.0435

Table 3: Parameter estimates for the EGARCH and GP-INGARCH model (in-sample period).

Panel A: India		Marginal model	c	a	b	ω	ϕ	α_0	α / α_1	β / β_1	γ
Bajaj Auto	returns	ARMA(1,1)-EGARCH(1,1)	0.0005	0.0932***	-0.2695	-0.4179***			-0.0621**	0.9498***	0.2735
	spread	GP-INGARCH(1,1)					0.8006***	1.2757***	0.3886***	0.5707***	
Bharti Airtel	returns	ARMA(1,1)-EGARCH(1,1)	0.0001	0.0674***	0.8311***	-0.1514***			-0.0319*	0.9808***	-0.8589***
	spread	GP-INGARCH(1,1)					0.8***	0.8408***	0.3745***	0.586***	
Hindustan Unilever	returns	ARMA(1,1)-EGARCH(1,1)	0.0007**	0.5564***	0.1326***	-3.339**			0.0362	0.6038***	-0.1525***
	spread	GP-INGARCH(1,1)					0.8***	0.06***	0.3248***	0.6638***	
Infosys	returns	ARMA(1,1)-EGARCH(1,1)	0.0009*	0.2544***	0.2595***	-6.2097**			0.0342	0.244	-0.2091***
	spread	GP-INGARCH(1,1)					0.8***	0.265***	0.3914***	0.5934***	
Kotak Mahindra	returns	ARMA(1,1)-EGARCH(1,1)	0.0008***	0.0995***	-0.9573***	-0.2799***			-0.0401**	0.9656***	0.9279***
Bank	spread	GP-INGARCH(1,1)					0.8***	0.414***	0.4245***	0.5236***	
Panel B: Germany											
Allianz	returns	ARMA(1,1)-EGARCH(1,1)	0.0007	0.1436	0.3299***	-0.556			-0.1552**	0.9362***	-0.308***
	spread	GP-INGARCH(1,1)					0.8***	0.5052**	0.5554***	0.4329***	
BASF	returns	ARMA(1,1)-EGARCH(1,1)	0.0003	0.0936***	-0.4055***	-0.3255***			-0.1181***	0.9614***	0.3947***
	spread	GP-INGARCH(1,1)					0.8***	0.4406***	0.2917***	0.6831***	
Bayerische Motoren	returns	ARMA(1,1)-EGARCH(1,1)	-0.0001	0.0705***	-0.2364***	-0.1168***			-0.0962***	0.9856***	0.2767***
Werke	spread	GP-INGARCH(1,1)					0.8***	0.3167***	0.512***	0.4761***	
Deutsche Post	returns	ARMA(1,1)-EGARCH(1,1)	0.0006	0.045***	-0.4836***	-0.203***			-0.0801***	0.976***	0.4512***
	spread	GP-INGARCH(1,1)					0.8***	0.2716***	0.2481***	0.7161***	
Fresenius Medical Care	returns	ARMA(1,1)-EGARCH(1,1)	0.0001	0.3505***	-0.9039***	-1.2045*			-0.0983**	0.861***	0.8901***
	spread	GP-INGARCH(1,1)					0.879***	1.0315***	0.5354***	0.4294***	
Panel C: United States											
Caterpillar Inc.	returns	ARMA(1,1)-EGARCH(1,1)	-0.0001	0.0546***	-0.3387***	-0.0877***			-0.0701***	0.9899***	0.3585***
	spread	GP-INGARCH(1,1)					0.8***	0.1613***	0.2956***	0.6374***	
JPMorgan Chase & Co	returns	ARMA(1,1)-EGARCH(1,1)	0.0007***	0.1428***	-0.94***	-0.9423***			-0.1436***	0.8919***	0.9127***
	spread	GP-INGARCH(1,1)					0.8***	0.287***	0.3292***	0.6372***	
3M Co.	returns	ARMA(1,1)-EGARCH(1,1)	0.0008***	0.0771***	0.1905***	-0.4912***			-0.1512***	0.9476***	-0.25***
	spread	GP-INGARCH(1,1)					0.8***	0.0069	0.415***	0.5744***	
Walt Disney	returns	ARMA(1,1)-EGARCH(1,1)	0.0008***	0.082***	-0.8209***	-0.534***			-0.1093***	0.9402***	0.7928***
	spread	GP-INGARCH(1,1)					0.8***	0.4867***	0.4644***	0.4951***	
Boeing Co	returns	ARMA(1,1)-EGARCH(1,1)	0.0006	0.1976***	0.3652***	-1.1423**			-0.0872**	0.8687***	-0.3374***
	spread	GP-INGARCH(1,1)					0.8***	0.0205***	0.2834***	0.7072***	
Panel D: Hong Kong											
Henderson Land	returns	ARMA(1,1)-EGARCH(1,1)	0	0.1446**	0.6276***	-0.3215***			-0.0479*	0.9613***	-0.5927***
Development	spread	GP-INGARCH(1,1)					0.9946***	2.4581***	0.3495***	0.5848***	
Sun Hung Kai	returns	ARMA(1,1)-EGARCH(1,1)	0.0003	0.2379***	0.3174***	-0.7102***			-0.017	0.9183***	-0.2735***
Properties	spread	GP-INGARCH(1,1)					0.8***	0.4476***	0.3974***	0.5763***	
Swire Pacific	returns	ARMA(1,1)-EGARCH(1,1)	0.0001	0.1072***	0.5135***	-0.2665***			-0.0748***	0.971***	-0.4964***
	spread	GP-INGARCH(1,1)					0.9191***	0.6596***	0.2219***	0.7448***	
CK Infrastructure	returns	ARMA(1,1)-EGARCH(1,1)	0.0008***	0.2647***	-0.5287***	-1.4776**			0.0036	0.8368***	0.438***
Holdings	spread	GP-INGARCH(1,1)					0.8***	1.5645**	0.3465***	0.5249***	

Hang Seng Bank	returns	ARMA(1,1)-EGARCH(1,1)	0.0005*	0.2291***	0.2682	-0.368***			-0.1027***	0.9608***	-0.2672
	spread	GP-INGARCH(1,1)					0.8***	1.19***	0.2822***	0.6577***	
Panel E: United Kingdom											
Ashtead Group	returns	ARMA(1,1)-EGARCH(1,1)	0.0013***	0.1691**	-0.9026***	-1.1761*			-0.1452***	0.8491***	0.8713***
	spread	GP-INGARCH(1,1)					0.8***	0.9086***	0.5476***	0.4028***	
AVEVA Group	returns	ARMA(1,1)-EGARCH(1,1)	0.0002***	-0.0346***	0.1755***	-0.0192***			-0.0614***	0.9975***	-0.2009***
	spread	GP-INGARCH(1,1)					1.1168***	2.5979**	0.3959***	0.4535***	
BHP Group	returns	ARMA(1,1)-EGARCH(1,1)	-0.0004***	-0.0226***	-0.5658***	-0.0166***			-0.0898***	0.9981***	0.5652***
	spread	GP-INGARCH(1,1)					0.8***	1.678**	0.6129***	0.3534***	
HSBC Holdings	returns	ARMA(1,1)-EGARCH(1,1)	-0.0001	0.155***	-0.0493	-0.5126***			-0.1082***	0.942***	0.0006
	spread	GP-INGARCH(1,1)					0.8***	1.5002***	0.4341***	0.5113***	
British American Tobacco	returns	ARMA(1,1)-EGARCH(1,1)	0.0003	0.0892***	0.215***	-0.3123***			-0.0837***	0.9658***	-0.2403***
	spread	GP-INGARCH(1,1)					0.8***	1.6681**	0.4951***	0.4051***	
Panel F: Australia											
A2B Australia	returns	ARMA(1,1)-EGARCH(1,1)	0.0002	0.2087***	0.8314***	-0.356***			-0.0363	0.9533***	-0.8595***
	spread	GP-INGARCH(1,1)					0.8***	0.1011***	0.2082***	0.7726***	
Adbri	returns	ARMA(1,1)-EGARCH(1,1)	0.0009**	0.0982	-0.6726***	-2.4583			-0.0409	0.7126***	0.5895***
	spread	GP-INGARCH(1,1)					0.8***	0.6829***	0.2256***	0.7658***	
Bell Financial Group	returns	ARMA(1,1)-EGARCH(1,1)	-0.0015***	1.4764***	-0.4301***	-2.6553***			-0.3125**	0.543***	0.31***
	spread	GP-INGARCH(1,1)					1.7818***	5.8713***	0.5957***	0.1197***	
EML Payments	returns	ARMA(1,1)-EGARCH(1,1)	0.0006**	0.6177***	-0.4887***	-0.5341***			-0.0121	0.9021***	0.4444***
	spread	GP-INGARCH(1,1)					2.1843***	4.6233***	0.4743***	0.3813***	
Fisher & Paykel Healthcare Corp	returns	ARMA(1,1)-EGARCH(1,1)	0.0016***	0.3951***	-0.6892***	-10***			0.0556	-0.2469	0.6096***
	spread	GP-INGARCH(1,1)					2.2816***	2.954***	0.2955***	0.5893***	
Panel G: Canada											
Agnico Eagle Mines	returns	ARMA(1,1)-EGARCH(1,1)	0.0001	0.0357***	-0.3422***	-0.0245***			-0.0569***	0.9965***	0.2494***
	spread	GP-INGARCH(1,1)					0.9827***	0.3745***	0.2435***	0.7395***	
AltaGas	returns	ARMA(1,1)-EGARCH(1,1)	0.0006**	0.1247***	0.1154***	-0.1426***			-0.0796***	0.9837***	-0.0237
	spread	GP-INGARCH(1,1)					0.9629***	0.6334***	0.3251***	0.6463***	
Algonquin Power & Utilities	returns	ARMA(1,1)-EGARCH(1,1)	0.0007***	0.378***	-0.9042***	-1.1114*			-0.0069	0.8712***	0.8522***
	spread	GP-INGARCH(1,1)					0.8***	0.1364***	0.1345***	0.856***	
Bank of Montreal	returns	ARMA(1,1)-EGARCH(1,1)	0.0007**	0.1584*	-0.2045***	-0.372***			-0.0988***	0.9626***	0.265***
	spread	GP-INGARCH(1,1)					0.8994***	0.4332***	0.2865***	0.6961***	
Cascades	returns	ARMA(1,1)-EGARCH(1,1)	0.0013***	0.3283***	-0.0358	-2.2784**			-0.0469	0.7164***	-0.0059
	spread	GP-INGARCH(1,1)					1.2246***	2.6197**	0.3126***	0.6064***	
Panel H: China											
Ping An Bank	returns	ARMA(1,1)-EGARCH(1,1)	0.0002	0.3331***	0.9699***	-0.3226***			-0.0278	0.9549***	-0.9793***
	spread	GP-INGARCH(1,1)					0.8***	0.5849***	0.6391***	0.3517***	
Shenzhen Airport	returns	ARMA(1,1)-EGARCH(1,1)	0.0002	0.2034***	-0.8761***	-0.0414***			0.0273	0.9938***	0.8564***
	spread	GP-INGARCH(1,1)					0.8***	0.7324***	0.4481***	0.5145***	
TCL Technology Group	returns	ARMA(1,1)-EGARCH(1,1)	0.0006***	0.2718***	-0.74***	-0.0514***			-0.0191	0.9922***	0.6322***
	spread	GP-INGARCH(1,1)					0.8***	0.7727	0.3688***	0.6217***	
Xuji Electric	returns	ARMA(1,1)-EGARCH(1,1)	0***	1.3657***	-0.1786***	-0.0215***			-1.3649***	1***	0.3056***

	spread	GP-INGARCH(1,1)					0.9994***	0.8378***	0.345***	0.6275***	
Shenzhen Agricultural Products Group	returns	ARMA(1,1)-EGARCH(1,1)	0.0004	0.1948***	0.9839***	-0.143***			0.0096	0.9787***	-0.9656***
	spread	GP-INGARCH(1,1)					0.8***	0.2843	0.4161***	0.5721***	
Panel I: Japan											
COMSYS	returns	ARMA(1,1)-EGARCH(1,1)	0.0009*	0.1004***	-0.0585	-0.5352***			-0.1022***	0.9338***	-0.0223
	spread	GP-INGARCH(1,1)					0.8081***	2.7612*	0.5519***	0.356***	
Takara Holdings	returns	ARMA(1,1)-EGARCH(1,1)	0.001*	0.2367***	0.3265***	-0.4333***			-0.1137***	0.9447***	-0.3745***
	spread	GP-INGARCH(1,1)					0.8776***	1.8572*	0.7431***	0.1966***	
Kikkoman	returns	ARMA(1,1)-EGARCH(1,1)	0.0015***	0.2026***	-0.4218***	-0.4772***			-0.0898***	0.9406***	0.3729***
	spread	GP-INGARCH(1,1)					0.9276***	0.3716***	0.5703***	0.4228***	
Ajinomoto	returns	ARMA(1,1)-EGARCH(1,1)	0.0008**	0.1749***	-0.1925	-0.3876***			-0.0781***	0.9529***	0.1088
	spread	GP-INGARCH(1,1)					1.0268***	0.9434***	0.4574***	0.5159***	
Kyowa Kirin	returns	ARMA(1,1)-EGARCH(1,1)	0.0011***	0.2377***	-0.4227***	-0.4946***			-0.1263***	0.9379***	0.3926***
	spread	GP-INGARCH(1,1)					0.9085***	1.5807***	0.6519***	0.3049***	
Panel J: South Korea											
Samsung Fire & Marine Insurance	returns	ARMA(1,1)-EGARCH(1,1)	0.0006	0.0889***	-0.4803***	-0.2359***			0.0252	0.9713***	0.3449***
	spread	GP-INGARCH(1,1)					0.8***	11.3763	0.206***	0.6661***	
Bukwang Pharmaceutical	returns	ARMA(1,1)-EGARCH(1,1)	0.0017***	0.1446***	-0.7755***	-0.0992***			0.078***	0.9864***	0.684***
	spread	GP-INGARCH(1,1)					0.8***	0.244*	0.284***	0.7104***	
NongShim	returns	ARMA(1,1)-EGARCH(1,1)	0.0008*	0.1116***	-0.4519***	-0.1493***			0.0489***	0.981***	0.3811***
	spread	GP-INGARCH(1,1)					0.8***	21.5603	0.4205***	0.2907***	
GS Retail	returns	ARMA(1,1)-EGARCH(1,1)	0.0006	0.0317***	-0.8716***	-0.0461***			0.0281***	0.9938***	0.826***
	spread	GP-INGARCH(1,1)					0.8***	1.1691***	0.4942***	0.4858***	
Lotte Chemical Corp	returns	ARMA(1,1)-EGARCH(1,1)	0	0.0506***	-0.1973	-0.0475***			-0.0182	0.9935***	0.2493
	spread	GP-INGARCH(1,1)					0.8***	4.456	0.1944	0.7657**	
Panel K: Switzerland											
Credit Suisse Group	returns	ARMA(1,1)-EGARCH(1,1)	-0.0002	0.1092***	0.8966***	-0.2324***			-0.0641***	0.9712***	-0.9***
	spread	GP-INGARCH(1,1)					0.8***	1.2191***	0.384***	0.5672***	
Givaudan	returns	ARMA(1,1)-EGARCH(1,1)	0.0007**	0.1712***	0.2054***	-0.7078***			-0.1105***	0.9212***	-0.1759***
	spread	GP-INGARCH(1,1)					0.8***	0.2354***	0.3845***	0.6047***	
Partners Group	returns	ARMA(1,1)-EGARCH(1,1)	0.0012***	0.1256	0.8897***	-1.0205			-0.1971***	0.8807***	-0.8965***
	spread	GP-INGARCH(1,1)					0.8006***	1.2757***	0.3886***	0.5707***	
Swiss Life	returns	ARMA(1,1)-EGARCH(1,1)	0.0009**	0.3582***	-0.384***	-1.4996***			-0.1762***	0.8244***	0.4479***
	spread	GP-INGARCH(1,1)					0.8006***	1.2757***	0.3886***	0.5707***	
Swatch Group	returns	ARMA(1,1)-EGARCH(1,1)	-0.0003	0.047***	0.3456***	0.0001			-0.0277***	1***	-0.221***
	spread	GP-INGARCH(1,1)					0.9927***	0.4891***	0.5111***	0.4818***	
Panel L: Taiwan											
Fwusow Industry	returns	ARMA(1,1)-EGARCH(1,1)	0.0002	1.6822***	-0.3013***	-0.5621**			0.4736***	0.9091***	0.1654**
	spread	GP-INGARCH(1,1)					1.0958***	1.4215***	0.2437***	0.6752***	
Sun Yad Construction	returns	ARMA(1,1)-EGARCH(1,1)	-0.0004	0.9387**	0.4009***	-0.5719***			-0.0817	0.9166***	-0.4266***
	spread	GP-INGARCH(1,1)					1.1456***	2.2067***	0.3435***	0.5289***	
Universal Inc	returns	ARMA(1,1)-EGARCH(1,1)	0.0005***	2.9099***	-0.2462***	-0.5384***			-0.4518**	0.9151***	0.1527***
	spread	GP-INGARCH(1,1)					0.8***	1.9423***	0.3425***	0.3726***	

BioLASCO Taiwan	returns	ARMA(1,1)-EGARCH(1,1)	0.001***	0.8681**	-0.6629***	-0.4655**			0.0034	0.9309***	0.5892***
	spread	GP-INGARCH(1,1)					0.8006***	1.2757***	0.3886***	0.5707***	
Lily Textile	returns	ARMA(1,1)-EGARCH(1,1)	0	0.4757***	0.864***	-0.8183*			0.0443	0.8921***	-0.8366***
	spread	GP-INGARCH(1,1)					0.8006***	1.2757***	0.3886***	0.5707***	

Note: The asterisks *, **, and *** denote significance at 10%, 5%, and 1% levels, respectively.

Table 4: Statistical tests of unconditional coverage.

	α	L-VaR-WS (GARCH-EVT-Copula)	L-VaR-BDSS (GARCH-EVT-Copula)	L-VaR-HW (GARCH-EVT-Copula)	L-VaR-WS (GARCH-EVT)	L-VaR-BDSS (GARCH-EVT)	L-VaR-HW (GARCH-EVT)	VaR (GARCH-EVT)
Panel A: India								
Bajaj Auto	5%	0.2087	0.2087	0.2087	0.003	0.103	0.035	0.118
	1%	0.7641	0.7641	0.7641	0.764	0.764	0.764	1.302
	0.5%	3.4578*	3.4578*	3.4578*	2.311	1.367	2.311	2.311
Bharti Airtel	5%	2.6427	2.6427	2.6427	3.996**	3.516*	3.516*	4.504**
	1%	1.9665	1.9665	1.9665	2.75*	1.966	1.966	1.966
	0.5%	2.3115	2.3115	2.3115	1.367	1.367	1.367	1.367
Hindustan Unilever	5%	0.3514	0.3514	0.2087	0.103	0.532	0.351	0.045
	1%	2.375	2.375	2.375	3.592*	3.592*	3.592*	1.446
	0.5%	1.7887	3.5389*	1.7887	0.72	3.539*	0.72	0.72
Infosys	5%	0.2254	0.2254	0.2254	0.045	0.225	0.225	0.366
	1%	3.6473*	3.6473*	3.6473*	2.75*	3.647*	3.647*	3.647*
	0.5%	2.3115	2.3115	2.3115	2.311	2.311	2.311	2.311
Kotak Mahindra Bank	5%	0.5399	0.3661	0.3661	0.366	0.118	0.225	0.985
	1%	0.7641	0.7641	0.7641	2.75*	1.302	1.302	1.302
	0.5%	2.3115	2.3115	2.3115	1.367	2.311	2.311	2.311
Panel B: Germany								
Allianz	5%	0.209	0.351	0.351	0.209	0.103	0.103	0.103
	1%	0.764	0.764	0.764	0.103	0.103	0.103	0.103
	0.5%	2.311	2.311	2.311	1.367	1.367	1.367	1.367
BASF	5%	0.985	0.746	0.985	0.985	0.985	0.985	0.985
	1%	0.103	0.103	0.103	0.361	0.361	0.361	0.361
	0.5%	0.647	0.647	0.647	0.647	0.647	0.647	0.647
Bayerische Motoren Werke	5%	0.225	0.118	0.118	0.225	0.225	0.225	0.225
	1%	0.361	0.361	0.361	0.361	0.361	0.361	0.361
	0.5%	2.311	2.311	2.311	2.311	2.311	2.311	3.458*
Deutsche Post	5%	0.366	0.366	0.366	0.54	0.746	0.746	0.746
	1%	0.103	0.103	0.103	0.764	0.764	1.302	1.966
	0.5%	0.18	0.18	0.18	0.647	0.647	0.647	0.647
Fresenius Medical Care	5%	2.25	1.888	2.25	1.888	1.888	1.888	2.25
	1%	1.302	1.302	1.302	1.302	1.302	1.302	1.302
	0.5%	1.367	1.367	1.367	3.458*	3.458*	3.458*	3.458*
Panel C: United States								
Caterpillar Inc.	5%	3.996**	3.996**	3.996**	0.985	0.985	0.985	0.985
	1%	2.75*	2.75*	2.75*	3.647*	3.647*	2.75*	3.647*
	0.5%	1.367	1.367	1.367	2.311	2.311	2.311	2.311
JPMorgan Chase & Co	5%	0.54	0.54	0.54	5.041**	4.504**	5.041**	5.041**
	1%	0.103	0.103	0.103	1.966	1.966	1.966	1.966
	0.5%	0.647	0.647	0.647	0.18	0.18	0.18	0.18
3M Co.	5%	0.54	0.366	0.746	0.985	0.746	0.985	1.556
	1%	0.001	0.001	0.001	1.302	1.302	1.302	1.302
	0.5%	0.647	0.647	0.647	1.367	1.367	1.367	1.367
Walt Disney	5%	1.255	1.255	1.255	0.225	0.118	0.118	0.225
	1%	1.966	1.966	1.966	0.103	0.103	0.103	0.103
	0.5%	2.311	2.311	2.311	0.647	0.647	0.647	0.647
Boeing Co	5%	0.746	0.746	0.746	0.54	0.54	0.54	1.255
	1%	2.75*	2.75*	2.75*	0.103	0.103	0.103	0.103
	0.5%	2.311	2.311	2.311	0.647	0.647	0.647	0.647

Panel D: Hong Kong								
Henderson Land Development	5%	0.532	0.532	0.351	1.255	0.746	0.985	3.996**
	1%	0.361	0.361	0.361	3.647*	3.647*	3.647*	6.96***
	0.5%	0.647	0.647	0.647	3.458*	3.458*	3.458*	3.458*
Sun Hung Kai Properties	5%	0.006	0.006	0.006	0.209	0.209	0.209	0.225
	1%	0.77	0.77	0.77	0.361	0.361	0.361	0.361
	0.5%	0.647	0.647	0.647	0.18	0.647	0.18	1.367
Swire Pacific	5%	3.516*	2.25	3.516*	0.006	0.006	0.006	0.225
	1%	2.75*	1.966	2.75*	0.77	0.77	0.77	0.318
	0.5%	1.367	1.367	1.367	0.18	0.18	0.18	0.647
CK Infrastructure Holdings	5%	0.118	0.118	0.118	3.516*	1.556	3.065*	6.197**
	1%	0.103	0.068	0.103	1.302	0.764	1.302	2.75*
	0.5%	0.18	0.18	0.18	1.367	0.18	1.367	2.311
Hang Seng Bank	5%	0.985	0.54	0.985	0.118	0.006	0.045	1.556
	1%	4.651**	3.647*	4.651**	0.001	0.068	0.068	0.764
	0.5%	2.311	1.367	2.311	0.18	0.18	0.18	0.18
Panel E: United Kingdom								
Ashtead Group	5%	0.351	0.351	0.351	0.209	0.351	0.351	0.351
	1%	0.764	0.764	0.764	0.361	0.361	0.361	1.302
	0.5%	0.647	0.647	0.647	0.647	0.647	0.647	0.647
AVEVA Group	5%	2.25	1.556	1.888	1.888	1.556	1.888	2.25
	1%	0.068	0.318	0.318	0.068	0.77	0.068	0.103
	0.5%	0.72	0.72	0.72	0.72	0.72	0.72	0.72
BHP Group	5%	0.045	0.045	0.045	4.504**	4.504**	4.504**	5.605**
	1%	0.103	0.103	0.103	5.757**	5.757**	5.757**	5.757**
	0.5%	0.103	0.103	0.103	0.647	0.647	0.647	0.647
HSBC Holdings	5%	0.225	0.225	0.225	0.045	0.045	0.045	0.045
	1%	0.647	0.647	0.647	0.103	0.103	0.103	0.103
	0.5%	0.647	0.647	0.647	1.367	1.367	1.367	1.367
British American Tobacco	5%	0.118	0.118	0.366	0.366	0.366	0.366	0.366
	1%	0.068	0.068	0.068	1.966	1.966	1.966	2.75*
	0.5%	0.18	0.18	0.18	0.001	0.001	0.001	0.18
Panel F: Australia								
A2B Australia	5%	0.5323	2.4578	0.1034	0.003	2.033	0.103	5.605**
	1%	0.0675	1.4461	0.0012	0.001	1.446	0.103	1.302
	0.5%	0.6473	0.6473	0.6473	2.311	0.18	0.647	1.367
Adbri	5%	1.556	1.556	1.888	1.888	2.643	2.643	7.461***
	1%	2.7505*	2.7505*	3.6473*	2.75*	1.966	1.966	2.75*
	0.5%	2.3115	2.3115	3.4578*	2.311	2.311	2.311	2.311
Bell Financial Group	5%	5.9504**	15.8722***	4.6**	13.428***	18.573***	3.996**	2.643
	1%	1.4461	3.5922*	1.4461	2.375	2.375	1.446	0.103
	0.5%	0.7199	1.7887	0.7199	0.158	1.789	0.72	0.001
EML Payments	5%	0.0028	0.752	0.045	2.033	1.011	0.118	3.065*
	1%	0.1035	0.0675	0.1035	11.108***	3.647*	0.103	39.109***
	0.5%	0.1798	0.0006	0.1798	0.001	0.72	0.18	0.647
Fisher & Paykel Healthcare Corp	5%	2.6427	2.6427	3.0647*	2.25	3.516*	3.996**	5.041**
	1%	1.9665	0.7641	2.7505*	1.302	0.764	1.966	2.75*

	0.5%	1.3672	1.3672	1.3672	0.647	0.647	0.647	0.647
Panel G: Canada								
Agnico Eagle Mines	5%	0.003	0.003	0.003	0.045	0.045	0.045	0.118
	1%	1.302	1.302	1.302	1.966	1.966	1.966	1.966
	0.5%	0.001	0.001	0.001	0.001	0.001	0.18	0.18
AltaGas	5%	0.035	0.035	0.045	0.103	0.103	0.003	0.006
	1%	0.103	0.103	1.302	0.001	0.001	0.001	0.103
	0.5%	0.647	0.647	3.458*	0.18	0.18	0.18	0.18
Algonquin Power & Utilities	5%	0.003	0.035	0.035	0.103	0.103	0.035	0.003
	1%	0.318	0.068	0.068	0.068	0.318	0.068	0.068
	0.5%	0.001	0.18	0.001	0.001	0.158	0.001	0.001
Bank of Montreal	5%	0.225	0.225	0.225	0.366	0.366	0.54	0.746
	1%	0.001	0.001	0.001	0.001	0.001	0.103	0.103
	0.5%	0.647	0.647	0.647	1.367	0.647	0.647	1.367
Cascades	5%	1.255	1.255	1.255	1.556	0.985	1.556	2.25
	1%	0.764	0.764	0.764	1.302	1.302	1.302	1.302
	0.5%	2.311	2.311	2.311	3.458*	3.458*	3.458*	3.458*
Panel H: China								
Ping An Bank	5%	0.532	0.532	0.532	0.752	0.752	2.643	0.351
	1%	0.318	0.318	0.318	0.001	0.001	2.75*	0.001
	0.5%	0.001	0.001	0.001	0.001	0.158	3.458*	0.001
Shenzhen Airport	5%	0.225	0.225	0.225	0.006	0.118	0.366	1.888
	1%	0.77	0.77	0.77	0.77	0.77	0.77	0.77
	0.5%	0.158	0.158	0.158	0.158	0.158	0.158	0.158
TCL Technology Group								
	5%	0.351	0.209	60.311***	0.532	0.752	0.752	142.931***
	1%	1.302	6.96***	201.107***	0.001	4.651**	0.361	329.929***
	0.5%	3.458*	7.941***	283.384***	0.647	7.941***	4.788**	443.452***
Xuji Electric	5%	1.311	2.458	1.311	0.752	1.651	0.752	0.209
	1%	0.361	0.103	0.361	0.103	0.103	0.103	0.103
	0.5%	2.311	2.311	2.311	2.311	2.311	2.311	2.311
Shenzhen Agricultural Products Group	5%	0.045	0.225	30.298***	24.454***	0.045	22.267***	24.454***
	1%	1.302	2.75*	96.79***	68.726***	2.75*	74.689***	87.092***
	0.5%	6.286**	9.74***	153.319***	109.571***	6.286**	122.259***	135.33***
Panel I: Japan								
COMSYS	5%	0.752	0.752	0.752	0.351	0.351	0.209	0.006
	1%	0.103	0.001	0.103	0.361	0.361	0.361	0.361
	0.5%	0.18	0.18	0.18	0.001	0.001	0.001	0.647
Takara Holdings	5%	0.54	0.54	0.54	0.54	0.746	0.985	1.556
	1%	1.966	1.966	1.966	1.966	1.966	1.966	1.966
	0.5%	3.458*	3.458*	3.458*	4.788**	4.788**	6.286**	6.286**
Kikkoman	5%	0.035	0.103	0.035	0.035	0.035	0.035	0.118
	1%	0.764	0.103	0.764	0.001	0.001	0.001	0.764
	0.5%	0.001	0.001	0.001	0.001	0.001	0.001	0.18
Ajinomoto	5%	1.311	1.311	1.311	2.458	2.033	1.651	0.532
	1%	0.361	0.103	0.361	0.103	0.103	0.103	0.103
	0.5%	1.367	1.367	1.367	1.367	1.367	1.367	1.367
Kyowa Kirin	5%	1.651	2.033	1.651	2.033	1.651	1.311	1.011
	1%	0.77	1.446	0.318	1.446	1.446	1.446	0.77
	0.5%	0.001	0.001	0.001	0.158	0.158	0.158	0.158

Panel J: South Korea								
Samsung Fire & Marine Insurance	5%	2.25	2.25	2.25	3.065*	3.065*	3.516*	6.197**
	1%	0.068	0.068	0.068	0.068	0.068	0.068	0.361
	0.5%	0.001	0.001	0.001	0.001	0.001	0.001	0.18
Bukwang Pharmaceutical	5%	1.556	1.556	1.888	0.985	1.556	1.556	2.643
	1%	2.75*	2.75*	2.75*	1.966	2.75*	2.75*	3.647*
	0.5%	1.367	1.367	2.311	1.367	2.311	2.311	2.311
NongShim	5%	1.011	1.011	1.011	0.752	1.011	1.011	0.003
	1%	0.001	0.001	0.103	0.361	0.001	0.103	0.361
	0.5%	0.18	0.18	0.18	0.18	0.001	0.18	0.18
GS Retail	5%	0.985	0.985	0.985	0.746	1.255	1.255	3.065*
	1%	0.77	0.77	0.77	0.77	0.77	0.77	0.77
	0.5%	0.158	0.158	0.158	0.158	0.158	0.158	0.158
Lotte Chemical Corp	5%	0.985	0.985	0.985	1.888	1.255	1.255	2.643
	1%	0.361	0.361	0.361	0.361	0.361	0.361	0.361
	0.5%	0.18	0.18	0.18	0.18	0.18	0.18	0.647
Panel K: Switzerland								
Credit Suisse Group	5%	0.118	0.118	0.366	0.225	0.225	0.366	0.366
	1%	0.068	0.068	0.068	0.001	0.001	0.001	0.001
	0.5%	0.18	0.18	0.18	0.001	0.001	0.001	0.001
Givaudan	5%	0.006	0.006	0.006	0.045	0.045	0.045	0.118
	1%	0.001	0.001	0.001	0.103	0.001	0.001	0.103
	0.5%	0.18	0.18	0.18	0.18	0.18	0.18	0.18
Partners Group	5%	0.351	0.351	0.351	0.532	0.532	0.351	0.003
	1%	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	0.5%	1.367	1.367	1.367	0.647	0.647	0.647	0.647
Swiss Life	5%	0.985	0.746	0.985	0.746	0.54	0.54	0.54
	1%	0.361	0.361	0.361	0.103	0.103	0.103	0.103
	0.5%	0.158	0.158	0.158	0.001	0.001	0.001	0.001
Swatch Group	5%	3.516*	3.516*	3.516*	3.516*	3.516*	3.516*	3.516*
	1%	0.068	0.068	0.001	0.318	0.77	0.77	0.318
	0.5%	0.158	0.158	0.001	0.158	0.158	0.158	0.158
Panel L: Taiwan								
Fwusow Industry	5%	4.6**	18.573***	2.926*	6.699***	15.872***	2.926*	40.726***
	1%	0.77	2.375	0.318	0.77	2.375	0.318	1.966
	0.5%	0.18	3.539*	0.18	1.789	3.539*	0.18	1.367
Sun Yad Construction	5%	23.349***	13.556***	25.579***	14.43***	14.43***	23.349***	68.658***
	1%	1.302	0.103	1.966	0.103	0.103	1.966	9.639***
	0.5%	2.311	2.311	2.311	2.311	2.311	2.311	2.311
Universal Inc	5%	5.95**	12.298***	1.011	2.458	12.298***	1.651	46.415***
	1%	0.103	0.318	0.361	0.103	0.77	0.103	5.757**
	0.5%	0.158	0.72	0.001	0.72	0.72	0.158	1.367
BioLASCO Taiwan	5%	1.311	3.996**	1.011	2.458	3.996**	1.311	0.746
	1%	0.068	1.446	0.068	1.446	1.446	0.318	0.068
	0.5%	0.18	0.001	0.18	0.158	0.001	0.001	0.001
Lily Textile	5%	38.915***	61.746***	20.026***	46.473***	61.746***	20.026***	102.166***
	1%	5.146**	7.104***	2.375	7.104***	7.104***	2.375	1.966
	0.5%	0.158	1.789	0.158	3.539*	3.539*	0.158	2.311

Note: The table presents statistical tests of unconditional coverage (uc). The test is asymptotically distributed as χ^2 with d.f. one. The asterisks *, **, and *** denote significance at 10%, 5% and 1% levels, respectively.

Table 5: Statistical tests of conditional coverage.

	α	L-VaR-WS (GARCH-EVT-Copula)	L-VaR-BDSS (GARCH-EVT-Copula)	L-VaR-HW (GARCH-EVT-Copula)	L-VaR-WS (GARCH-EVT)	L-VaR-BDSS (GARCH-EVT)	L-VaR-HW (GARCH-EVT)	VaR (GARCH-EVT)
Panel A: India								
Bajaj Auto	5%	2.0885	2.0885	2.0885	1.362	1.799	1.557	1.048
	1%	1.1481	1.1481	1.1481	1.148	1.148	1.148	1.739
	0.5%	3.6636	3.6636	3.6636	2.481	1.505	2.481	2.481
Bharti Airtel	5%	2.7636	2.7636	2.7636	4.021	3.565	3.565	4.513
	1%	2.4605	2.4605	2.4605	3.954	2.46	2.46	2.46
	0.5%	2.4814	2.4814	2.4814	1.505	1.505	1.505	1.505
Hindustan Unilever	5%	0.4907	0.4907	0.3949	0.13	0.631	0.491	0.291
	1%	2.458	2.458	2.458	3.653	3.653	3.653	1.555
	0.5%	1.8039	3.5456	1.8039	0.747	3.546	0.747	0.747
Infosys	5%	3.7266	3.7266	3.7266	4.104	3.727	3.727	5.442*
	1%	4.6891*	4.6891*	4.6891*	3.954	4.689*	4.689*	4.689*
	0.5%	5.6033*	5.6033*	5.6033*	5.603*	5.603*	5.603*	5.603*
Kotak Mahindra Bank	5%	3.5311	3.6065	3.6065	3.607	3.892	3.727	3.511
	1%	1.1481	1.1481	1.1481	7.613**	1.739	1.739	1.739
	0.5%	2.4814	2.4814	2.4814	1.505	2.481	2.481	2.481
Panel B: Germany								
Allianz	5%	0.882	1.138	1.138	0.882	0.673	0.673	0.673
	1%	1.148	1.148	1.148	0.391	0.391	0.391	0.391
	0.5%	2.481	2.481	2.481	1.505	1.505	1.505	1.505
BASF	5%	4.137	3.718	4.137	4.137	4.137	4.137	4.137
	1%	0.391	0.391	0.391	0.696	0.696	0.696	0.696
	0.5%	0.756	0.756	0.756	0.756	0.756	0.756	0.756
Bayerische Motoren Werke	5%	2.176	1.048	1.048	1.032	1.032	1.032	1.032
	1%	0.696	0.696	0.696	0.696	0.696	0.696	0.696
	0.5%	2.481	2.481	2.481	2.481	2.481	2.481	3.664
Deutsche Post	5%	1.058	1.058	1.058	1.127	1.238	1.238	1.238
	1%	0.391	0.391	0.391	1.148	1.148	1.739	2.46
	0.5%	0.263	0.263	0.263	0.756	0.756	0.756	0.756
Fresenius Medical Care	5%	2.394	2.084	2.394	2.084	2.084	2.084	2.394
	1%	1.739	1.739	1.739	1.739	1.739	1.739	1.739
	0.5%	1.505	1.505	1.505	3.664	3.664	3.664	3.664
Panel C: United States								
Caterpillar Inc.	5%	5.087*	5.087*	5.087*	5.129*	5.129*	5.129*	5.129*
	1%	3.954	3.954	3.954	4.689*	4.689*	3.954	4.689*
	0.5%	1.505	1.505	1.505	2.481	2.481	2.481	2.481
JPMorgan Chase & Co	5%	3.531	3.531	3.531	6.83**	5.461*	6.83**	6.83**
	1%	0.391	0.391	0.391	2.46	2.46	2.46	2.46
	0.5%	0.756	0.756	0.756	0.263	0.263	0.263	0.263
3M Co.	5%	5.293*	3.607	5.189*	7.067**	5.189*	7.067**	6.935**
	1%	0.246	0.246	0.246	1.739	1.739	1.739	1.739
	0.5%	0.756	0.756	0.756	1.505	1.505	1.505	1.505
Walt Disney	5%	5.113*	5.113*	5.113*	3.727	3.892	3.892	3.727
	1%	2.46	2.46	2.46	0.391	0.391	0.391	0.391
	0.5%	2.481	2.481	2.481	0.756	0.756	0.756	0.756
Boeing Co	5%	5.189*	5.189*	5.189*	2.127	2.127	2.127	9.138**
	1%	3.954	3.954	3.954	0.391	0.391	0.391	0.391
	0.5%	2.481	2.481	2.481	0.756	0.756	0.756	0.756

Panel D: Hong Kong								
Henderson Land								
Development	5%	2.813	2.813	2.426	1.581	0.779	0.999	4.352
	1%	2.397	2.397	2.397	4.266	4.266	4.266	7.79**
	0.5%	0.756	0.756	0.756	3.664	3.664	3.664	3.664
Sun Hung Kai								
Properties	5%	0.007	0.007	0.007	2.088	2.088	2.088	3.727
	1%	0.907	0.907	0.907	2.397	2.397	2.397	2.397
	0.5%	0.756	0.756	0.756	0.263	0.756	0.263	1.505
Swire Pacific	5%	9.102**	8.925**	9.102**	0.007	0.007	0.007	0.266
	1%	3.954	3.349	3.954	0.907	0.907	0.907	0.488
	0.5%	1.505	1.505	1.505	0.263	0.263	0.263	0.756
CK Infrastructure								
Holdings	5%	2.266	2.266	2.266	9.102**	6.935**	9.001**	11.903***
	1%	2.404	2.991	2.404	1.739	1.148	1.739	3.954
	0.5%	0.263	0.263	0.263	1.505	0.263	1.505	2.481
Hang Seng								
Bank	5%	0.999	0.599	0.999	2.266	2.582	2.401	5.14*
	1%	5.337*	4.266	5.337*	2.597	0.273	0.273	7.033**
	0.5%	2.481	1.505	2.481	0.263	0.263	0.263	0.263
Panel E: United Kingdom								
Ashtead Group	5%	1.138	1.138	1.138	0.26	0.435	0.435	0.435
	1%	1.148	1.148	1.148	0.696	0.696	0.696	1.739
	0.5%	0.756	0.756	0.756	0.756	0.756	0.756	0.756
AVEVA Group	5%	2.987	2.537	2.742	3.8	2.537	2.742	2.987
	1%	0.273	0.488	0.488	0.273	0.907	0.273	0.391
	0.5%	0.747	0.747	0.747	0.747	0.747	0.747	0.747
BHP Group	5%	1.108	1.108	1.108	4.788*	4.788*	4.788*	6.32**
	1%	0.391	0.391	0.391	6.513**	6.513**	6.513**	6.513**
	0.5%	3.664	3.664	3.664	0.756	0.756	0.756	0.756
HSBC								
Holdings	5%	0.361	0.361	0.361	1.108	1.108	1.108	1.108
	1%	3.954	3.954	3.954	0.391	0.391	0.391	0.391
	0.5%	0.756	0.756	0.756	1.505	1.505	1.505	1.505
British American								
Tobacco	5%	0.305	0.305	1.058	1.058	1.058	1.058	1.058
	1%	0.273	0.273	0.273	2.46	2.46	2.46	3.954
	0.5%	0.263	0.263	0.263	0.062	0.062	0.062	0.263
Panel F: Australia								
A2B Australia	5%	1.4412	3.0023	1.7991	1.362	3.706	1.799	5.768*
	1%	0.2733	1.5547	0.2463	2.597	1.555	2.404	2.881
	0.5%	0.7559	0.7559	0.7559	2.481	0.263	0.756	1.505
Adbri	5%	1.8132	1.8132	2.7423	2.084	2.743	2.743	10.934***
	1%	3.9542	3.9542	8.1093**	3.954	3.349	3.349	3.954
	0.5%	5.6033*	5.6033*	12.274***	5.603*	5.603*	5.603*	5.603*
Bell Financial								
Group	5%	6.1239**	17.046***	7.3139**	16.384***	20.07***	6.47**	5.472*
	1%	1.5547	3.6532	1.5547	2.458	2.458	1.555	0.391
	0.5%	0.7469	1.8039	0.7469	0.2	1.804	0.747	0.062
EML Payments	5%	1.4425	3.3642	2.5099	3.706	3.739	2.266	4.535
	1%	2.4045	2.9911	2.4045	13.81***	12.81***	2.404	46.27***
	0.5%	0.2628	0.0616	0.2628	0.062	0.747	0.263	0.756
Fisher & Paykel Healthcare								
Corp	5%	2.668	2.668	3.113	2.26	3.594	4.112	5.253*
	1%	2.4605	1.1481	3.3048	1.739	1.148	2.46	3.305

	0.5%	1.5047	1.5047	1.5047	0.756	0.756	0.756	0.756
Panel G: Canada								
Agnico Eagle Mines	5%	1.362	1.362	1.362	1.108	1.108	1.108	1.048
	1%	2.881	2.881	2.881	3.349	3.349	3.349	3.349
	0.5%	0.062	0.062	0.062	0.062	0.062	0.263	0.263
AltaGas	5%	9.971***	9.971***	8.562**	10.544***	10.54***	9.45***	8.98**
	1%	0.391	0.391	1.739	0.246	0.246	0.246	0.391
	0.5%	0.756	0.756	3.664	0.263	0.263	0.263	0.263
Algonquin Power & Utilities	5%	1.362	1.557	0.511	1.799	1.799	1.557	1.362
	1%	0.488	0.273	0.273	0.273	0.488	0.273	0.273
	0.5%	0.062	0.263	0.062	0.062	0.2	0.062	0.062
Bank of Montreal	5%	2.176	2.176	2.176	3.607	3.607	3.531	3.5
	1%	0.246	0.246	0.246	0.246	0.246	0.391	0.391
	0.5%	0.756	0.756	0.756	1.505	0.756	0.756	1.505
Cascades	5%	1.504	1.504	1.504	1.868	1.179	1.868	2.706
	1%	1.148	1.148	1.148	1.739	1.739	1.739	1.739
	0.5%	2.481	2.481	2.481	3.664	3.664	3.664	3.664
Panel H: China								
Ping An Bank	5%	2.813	2.813	2.813	3.251	3.251	5.472*	4.199
	1%	0.488	0.488	0.488	0.246	0.246	3.305	0.246
	0.5%	0.062	0.062	0.062	0.062	0.2	3.664	0.062
Shenzhen Airport	5%	0.872	0.872	0.872	0.445	0.677	1.107	3.319
	1%	0.907	0.907	0.907	0.907	0.907	0.907	0.907
	0.5%	0.2	0.2	0.2	0.2	0.2	0.2	0.2
TCL Technology Group	5%	0.454	0.938	0.225	1.673	0.952	0.952	451.28***
	1%	2.881	24.96***	326.96***	0.246	24.72***	2.397	830.38***
	0.5%	6.381**	14.76***	415.62***	0.756	14.76***	12.87***	968.63***
Xuji Electric	5%	2.646	4.315	2.646	1.793	3.15	1.793	0.882
	1%	0.696	0.391	0.696	0.391	0.391	0.391	0.391
	0.5%	2.481	2.481	2.481	2.481	2.481	2.481	2.481
Shenzhen Agricultural Products Group	5%	0.048	1.093	133.94***	103.08***	0.324	109.20***	117.61***
	1%	7.064**	12.571***	329.08***	262.57***	12.57***	279.61***	303.39***
	0.5%	13.709***	21.837***	402.01***	324.19***	13.70***	342.89***	361.78***
Panel I: Japan								
COMSYS	5%	0.817	0.817	0.817	0.491	0.491	0.395	0.445
	1%	0.391	0.246	0.391	0.696	0.696	0.696	0.696
	0.5%	0.263	0.263	0.263	0.062	0.062	0.062	0.756
Takara Holdings	5%	0.599	0.599	0.599	0.599	0.779	0.999	1.556
	1%	2.46	2.46	2.46	2.46	2.46	2.46	2.46
	0.5%	3.664	3.664	3.664	5.033*	5.033*	6.574**	6.574**
Kikkoman	5%	0.511	0.673	0.511	0.511	0.511	0.511	0.305
	1%	1.148	0.391	1.148	0.246	0.246	0.246	1.148
	0.5%	0.062	0.062	0.062	0.062	0.062	0.062	0.263
Ajinomoto	5%	1.329	1.329	1.329	2.46	2.033	1.657	1.441
	1%	2.397	2.404	2.397	2.404	2.404	2.404	2.404
	0.5%	5.075*	5.075*	5.075*	5.075*	5.075*	5.075*	5.075*
Kyowa Kirin	5%	1.657	2.033	1.657	2.033	1.657	1.329	1.05
	1%	0.907	1.555	0.488	1.555	1.555	1.555	0.907
	0.5%	0.062	0.062	0.062	0.2	0.2	0.2	0.2
Panel J: South Korea								

Samsung Fire & Marine Insurance	5%	9.119**	9.119**	9.119**	9.181**	9.181**	9.276***	10.359***
	1%	2.991	2.991	2.991	2.991	2.991	2.991	20.558***
	0.5%	0.062	0.062	0.062	0.062	0.062	0.062	0.263
Bukwang Pharmaceutical	5%	1.836	1.836	2.233	1.153	1.836	1.836	3.139
	1%	3.954	3.954	3.954	2.46	3.305	3.305	4.266
	0.5%	1.505	1.505	2.481	1.505	2.481	2.481	2.481
NongShim	5%	1.241	1.241	1.241	0.924	1.241	1.241	1.362
	1%	0.246	0.246	0.391	0.696	0.246	0.391	0.696
	0.5%	0.263	0.263	0.263	0.263	0.062	0.263	0.263
GS Retail	5%	14.561***	14.561***	14.561***	14.919***	14.25***	14.251***	13.404***
	1%	0.907	0.907	0.907	0.907	0.907	0.907	0.907
	0.5%	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Lotte Chemical Corp	5%	3.511	3.511	3.511	3.8	3.566	3.566	4.196
	1%	2.397	2.397	2.397	2.397	2.397	2.397	2.397
	0.5%	0.263	0.263	0.263	0.263	0.263	0.263	0.756
Panel K: Switzerland								
Credit Suisse Group	5%	0.305	0.305	1.058	0.361	0.361	1.058	1.058
	1%	0.273	0.273	0.273	0.246	0.246	0.246	0.246
	0.5%	0.263	0.263	0.263	0.062	0.062	0.062	0.062
Givaudan	5%	0.007	0.007	0.007	0.052	0.052	0.052	0.305
	1%	0.246	0.246	0.246	0.391	0.246	0.246	0.391
	0.5%	0.263	0.263	0.263	0.263	0.263	0.263	0.263
Partners Group	5%	6.391**	6.391**	6.391**	6.95**	6.95**	6.391**	4.669*
	1%	0.246	0.246	0.246	0.246	0.246	0.246	0.246
	0.5%	1.505	1.505	1.505	0.756	0.756	0.756	0.756
Swiss Life	5%	0.999	0.779	0.999	0.779	0.599	0.599	0.599
	1%	0.696	0.696	0.696	0.391	0.391	0.391	0.391
	0.5%	0.2	0.2	0.2	0.062	0.062	0.062	0.062
Swatch Group	5%	5.6*	5.6*	5.6*	5.6*	5.6*	5.6*	5.6*
	1%	0.273	0.273	0.246	0.488	0.907	0.907	0.488
	0.5%	0.2	0.2	0.062	0.2	0.2	0.2	0.2
Panel L: Taiwan								
Fwusow Industry	5%	5.61*	20.078***	3.572	6.93**	17.04***	3.572	43.594***
	1%	0.907	2.458	0.488	0.907	2.458	0.488	3.349
	0.5%	0.263	3.546	0.263	1.804	3.546	0.263	1.505
Sun Yad Construction	5%	32.235***	23.014***	33.61***	23.435***	23.43***	32.235***	72.046***
	1%	1.739	0.391	2.46	2.404	0.391	2.46	12.476***
	0.5%	2.481	2.481	2.481	2.481	2.481	2.481	2.481
Universal Inc	5%	9.168**	15***	2.195	6.226**	15***	3.15	47.031***
	1%	0.391	0.488	0.696	0.391	0.907	0.391	6.513**
	0.5%	0.2	0.747	0.062	0.747	0.747	0.2	1.505
BioLASCO Taiwan	5%	18.755***	22.001***	17.741***	18.281***	22.00***	18.755***	12.083***
	1%	0.273	1.555	0.273	1.555	1.555	0.488	0.273
	0.5%	0.263	0.062	0.263	0.2	0.062	0.062	0.062
Lily Textile	5%	39.533***	61.952***	20.138***	48.053***	61.95***	21.715***	103.82***
	1%	5.188*	7.131**	2.458	7.131**	7.131**	2.458	2.46
	0.5%	0.2	1.804	0.2	3.546	3.546	0.2	2.481

Note: The table presents statistical tests of conditional coverage (cc). The test is asymptotically distributed as χ^2 with d.f. two. The asterisks *, **, and *** denote significance at 10%, 5%, and 1% levels, respectively.

Table 6: Summarized results of the number of times each model fails.

	L-VaR-WS (GARCH-EVT-Copula)	L-VaR-BDSS (GARCH-EVT-Copula)	L-VaR-HW (GARCH-EVT-Copula)	L-VaR-WS (GARCH-EVT)	L-VaR-BDSS (GARCH-EVT)	L-VaR-HW (GARCH-EVT)	VaR (GARCH-EVT)
Panel A: Test of unconditional coverage							
India	2	3	2	5	4	3	2
Germany	0	0	0	1	1	1	2
United States	3	3	3	2	2	2	2
Hong Kong	3	1	3	3	2	3	5
United Kingdom	0	0	0	2	2	2	3
Australia	2	3	5	3	3	2	7
Canada	0	0	1	1	1	1	1
China	2	4	6	3	4	6	6
Japan	1	1	1	1	1	1	1
South Korea	1	1	1	1	2	2	3
Switzerland	1	1	1	1	1	1	1
Taiwan	5	7	3	5	8	3	6
Total number of fails	**20**	**24**	**26**	**28**	**31**	**27**	**39**
Panel B: Test of Conditional coverage							
India	2	2	2	2	2	2	3
Germany	0	0	0	0	0	0	0
United States	4	3	4	4	4	3	5
Hong Kong	2	1	2	1	1	1	4
United Kingdom	0	0	0	2	2	2	2
Australia	2	2	3	3	3	2	6
Canada	1	1	1	1	1	1	1
China	3	4	5	3	4	5	6
Japan	1	1	1	2	2	2	2
South Korea	2	2	2	2	2	2	3
Switzerland	2	2	2	2	2	2	2
Taiwan	6	6	3	6	6	3	7
Total number of fails	**25**	**24**	**25**	**28**	**29**	**25**	**41**
Grand Total	**45**	**48**	**51**	**56**	**60**	**52**	**80**