The Optimal Design of Green Securities *

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Abstract

We develop a model of green project financing which incorporates investors with green preferences into an otherwise standard framework of corporate financing with asymmetric information. Firms seek to finance green projects whose outcomes embed an uncertain, non-measurable component that is revealed only to the firm and which can be manipulated. Firms can raise funds using project-based non-contingent green debt contracts, such as green bonds, that restrict the set of projects to be financed using the proceeds, but make no commitment to green outcomes. Alternatively, they can use outcome-based contingent green debt contracts, such as sustainability-linked loans and bonds, that do not impose restrictions on the use of proceeds but embed contingencies which ensure commitment to outcomes. We demonstrate that the co-existence of the two green debt contracts is an equilibrium result when green outcomes are manipulable and firm types differ in their ability to manipulate. In the presence of asymmetric information about firms' type, non-contingent debt can be used as an expensive signaling device, and we find empirically that contingent green debt securities have lower credit rating, higher yields and are issued by more emissions intensive firms.

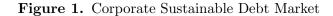
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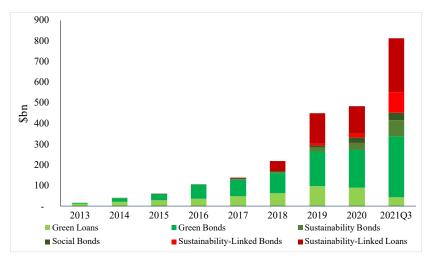
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1 Introduction

Financial markets are playing an increasingly important role in the fight against climate change and other sustainability issues by allowing sustainability-oriented investors to finance projects that have positive environmental and social benefits.¹ The corporate sustainable debt market opened slowly about a decade ago with the issuance of green bonds and has grown exponentially in recent years, reaching a cumulative volume of approximately 2.5\$tn as of the third quarter of 2021. While green bonds are still the most prevalent type of debt instrument in the market, notable is the spectacular rise in the issuance of sustainability-linked loans and bonds. These new types of debt instruments have only been introduced in 2017 but are now making up about 45% of the market for corporate sustainable debt (see Figure 1).



The figure shows cumulative issuance volume of corporate sustainable debt securities in \$ billions across years. Institutional details about the securities are reported in Section 3 and Appendix A.



Green bonds are fixed income instruments which earmark proceeds for specific green projects, that is, projects that have positive environmental and climate benefits - further referred to generically as green benefits or outcomes. They are differentiated from regular, plain vanilla bonds by a green label, which represents a commitment to exclusively use the funds raised to finance or re-finance green projects. The contract focuses solely on specifying ex-ante the activities that the borrower can allocate the proceeds to, but does not embed the mechanisms needed to ensure the delivery of costly green outcomes ex-post. In other words, a green bond is a debt contract that is defined by a

¹We use the term green to refer to environmentally related objectives, while the terms sustainability is wider and refers to environmental as well as social and potentially governance related issues. This terminology is in line with the ICMA standards governing the issuance of securities on the sustainable finance market.

green purpose, but which does not involve a commitment to deliver a green outcome. In contrast, the contract design underlying the newly emerging class of sustainable debt represented by the sustainability-linked loans (SLLs) and bonds (SLBs) does not impose ex-ante constraints on the projects that the proceeds can be allocated to, but instead embeds contingencies that ensure commitment to outcomes. In its most common design, these contingent contracts specify an outcome target with an associated base interest rate, and if the borrower fails to meet this target, it will be penalized by having to repay the debt at a higher interest rate, but if the borrower meets or exceeds the target, it will be financially rewarded by having to repay the debt at a lower interest rate.

The introduction of contingencies in securities' payoff addresses the limitations inherent in the design of green bonds by eliminating the need to restrict borrower's actions ex-ante and by making outcomes rather than intentions the focus of green projects financing. Importantly, this security design is in line with corporate finance theory which posits that optimal contracts should include all relevant contingencies (see, for example, Hart and Holmström [1987]). It is thus unclear why the market has started with, and is still dominated by, *project-based non-contingent* contracts such as green bonds. Our paper sets out not only to rationalize the design underlying the most prevalent type of green debt contract in the market, but also to explain why despite the successful implementation of alternatives that have long been deemed theoretically optimal, namely *outcome-based contingent contracts* such as SLLs and SLBs, we do not observe a complete switch to outcomebased contingent financing but instead, the observed market outcome points to the co-existence of project-based non-contingent contracts and outcome-based contingent contracts.

The model we propose in this paper rationalizes observed debt issuance patterns as equilibrium outcomes of a firm financing model which embeds verifiable moral hazard, manipulation and asymmetric information. The baseline model features two time periods, an investor, and a representative firm in the market. In the first time period, the firm has access to a business-as-usual investment which has a fixed cost and which will yield, in the second time period, a *certain* monetary return. In the first time period or at an interim date before the second time period, the firm can decide to upgrade to a green business model by investing in a green project. The green project yields the same monetary return as the business-as-usual project and, at some further cost, an *uncertain* green outcome, which can for instance be conceptualized as a reduction in carbon emissions. The firm seeks to maximize profits by choosing to finance its investment through the issuance of one

of the following three debt contract categories: a plain vanilla non-contingent contract, a projectbased non-contingent green contract (similar in spirit to GBs), or an outcome-based contingent green contract (similar in spirit to SLLS/SLBs).²

We take as given the existence of a market that deploys capital to fund green projects and focus solely on the firm's optimal debt financing choice (a similar assumption is also outlined in Pastor, Stambaugh, and Taylor [2020]). Specifically, we suppose that there is a risk-neutral investor with green preferences, that is an investor that equally values green and monetary outcomes, and we take as given that the investor absorbs the green security issuance, provided it generates at least zero return in expectation.³ We specify the green outcome delivered by the green project as the sum of a measurable and a non-measurable component. The measurable component represents the firm's costly action, which can be perfectly verified by the investor at a cost. The non-measurable component is uncertain and cannot be directly controlled by the firm, it can only be observed by the firm at an interim date, and its reported value can be manipulated at some distortion cost.⁴

The baseline single-firm model predicts that when the green outcome cannot be manipulated, the outcome-based contingent contract is always optimal and dominates the project-based noncontingent contract. This is because the project-based non-contingent contract requires the firm to make the investment choice at the first date before the uncertain state is realized, therefore creating an opportunity cost of ex-ante commitment compared to the more flexible, outcome-based contingent contract. Furthermore, contingencies eliminate the upfront verification costs associated with making credible the commitment to costly actions that deliver the green outcome.⁵ However, the optimality of outcome-based contingent contracts is jeopardised when contingencies depend on measurement systems which can be manipulated. This is because the investor anticipates that the reported green outcome on which the contingent payments are based contains a distortion, and as

²The plain vanilla non-contingent contract is the most basic form of corporate debt whereby the investor lends the money in the first time period and receives the principal plus a predefined interest rate in the second time period. Note that we focus solely on the firm debt financing problem and disregard capital structure considerations. In Appendix B we analyse the role of equity in a simple model extension which allows for uncertain monetary returns.

³In other words, we focus on how green projects should be optimally financed instead of studying whether investors should finance green projects in the first place. As far as the risk-neutrality assumption is concerned, we show in Appendix B that introducing risk-aversion does not alter the baseline predictions of the model.

⁴The measurable component can be conceptualized as the expected level of carbon emissions reduction which can be inferred from the scale of investment in the green technology. The non-measurable component can be interpreted, for example, as the uncertainty related to the true potential of the green technology to reduce carbon emissions, that becomes subsequently known to the firm.

⁵Actions are hidden unless a verification cost is borne, in the spirit of costly state verification by Townsend [1979].

a consequence demands a higher base interest rate to enter this contract. In other words, manipulable measurement systems give rise to a distortion discount that can dominate the costs associated with the project-based non-contingent contracts. When the measurability of the green outcome is very poor, which is likely to have been the case in the nascent phase of the sustainable market, the model predicts that project-based non-contingent debt contracts are preferred to outcome-based contingent contracts.

The trade-off between the opportunity cost of ex-ante commitment associated with project-based non-contingent contracts, and the distortion discount that comes with outcome-based contingent contracts, gives rise to an interesting non-monotonic relationship between the type of the green project and the firm's preference for issuing green debt to finance it. Specifically, in the model, the green project varies with its degree of materiality, defined as the relative magnitude of the uncertain, non-measurable component of the green outcome that cannot be controlled by the firm.⁶ Then everything else equal, the trade-off predicts that projects with very high/very low levels of materiality (i.e. projects whose cost of ex-ante commitment is very low/distortion discount is very high respectively), are those more likely to be financed using project-based non-contingent green debt. To the extent that a firm's green project varies fundamentally with its industry, this prediction is in line with evidence showing that the industries predominantly issuing non-contingent debt are those with green outcomes, as proxied by carbon emissions, that lie at the ends of the measurement/control spectrum, such as the financial and the utility sectors.

In addition to this measurement friction which sheds light on the broad evolution of the market, we extend the model along the firm type dimension to explain issuance patterns within industry. Firm types are differentiated with respect to the cost of action and the cost of distortion that they face. High type firms have a higher ability to invest in green projects and do not manipulate reported outcomes, while low type firms have a higher ability to manipulate outcomes and a lower ability to take costly action to deliver green outcomes.⁷ The model has markedly different predictions in terms of

⁶As detailed in the empirical section, we use the definition of materiality threshold as provided by the GHG Protocol standard, referring to the expected level of discrepancy between one firm's reported emissions and the verifier's belief about the firm's total emissions if all omitted sources where accounted for. Defined as such, materiality is increasing as the level of measurability/control over a firm's emissions increases, which is captured by the emission intensity scopes 1, 2 and 3 as defined by the GHG Protocol standard. Note that this is different, although related, to the concept of financial materiality in relation to carbon emissions used in Flammer [2021].

 $^{^{7}}$ We borrow this assumption from a work related to ours by Allen and Gale [1992], discussed in extent in the literature section, and test the validity of this assumption in the empirical section.

issuance choices across firm types depending on the degree of information available to the investor. When investors are perfectly informed about the firm type, the model predicts that, across possible choices of the model parameters, high firm types are more likely to issue outcome-based contingent green debt, intermediate types are more likely to issue project-based non-contingent green debt, whereas low firm types are more likely to issue vanilla non-contingent debt. This is because with perfect information, low firm types are correctly identified by investors and therefore penalized with a higher base interest rate on the contingent contract. Importantly, as a consequence of this prediction and the investor's zero returns requirement in equilibrium, outcome-based contingent contracts should vield lower monetary returns than project-based non-contingent contracts. On the other hand, when there is asymmetric information over firm types, the model predicts that high firm types are more likely to issue non-contingent green debt, intermediate types are more likely to issue outcome-based contingent green debt, whereas low firm types would continue to prefer noncontingent plain vanilla debt. This is because when there is asymmetric information, the investor learns something about the firm type from the financing contract proposed, and project-based non-contingent contracts act as a screening device allowing firms to credibly signal their ability to commit at issuance. Consequently, with asymmetric information project-based non-contingent green debt should yield lower monetary returns than outcome-based contingent green debt.

We therefore bring the model-implied testable predictions to the data to assess whether crosssectional patterns in the sustainable finance market are consistent with the presence of asymmetric information over firm types.⁸ Differentiating between firm types requires estimating the cost of action and the cost of manipulation that they face, which we do by merging security-level data from Bloomberg with firm-level data from S&P Trucost and Sustainalytics. We proxy the cost of action in delivering green outcomes as the physical cost of abating emissions,⁹ and measure it using historical abatement costs as reflected in the firm's historical emissions intensity, defined as total emissions scopes per unit of the firm's assets from S&P Trucost. On the other hand, we borrow from the greenwashing literature [Netto, Sobral, Ribeiro, and Soares, 2020, Yang, 2020] and proxy the cost of manipulation using the historical *discrepancy* between the firm's overall corporate sustainability image, as measured by the aggregate ESG score provided by Sustainalytics, and a

⁸We provide additional empirical evidence to validate our theoretical framework by testing an additional modelimplied prediction concerning issuance preferences across green project types, which vary at the industry level depending on the fundamental nature of the business as detailed in the empirical section.

⁹The focus on carbon emissions as a sensible proxy for green outcome is motivated by evidence that carbon emissions represent the most common metric underlying sustainability-linked debt targets (see appendix A).

credible signal of environmental commitment captured by the firm's actual adoption of an Environmental Management System (EMS).¹⁰ Regression results indicate that issuers of contingent green debt have higher cost of investment and lower cost of manipulation, a result that is statistically significant and robust to the introduction of industry and location fixed effects.

We complement this ex-ante evidence with evidence on the ex-post financial performance of green debt. Specifically, we follow the methodology in Zerbib [2017] and estimate green bond premia for contingent green bond (proxied by sustainability-linked bonds) and non-contingent green bond contracts (proxied by green, social and sustainable bonds), where green premium is defined as the negative yield differential between green bonds and virtually identical conventional bonds from the same issuer firm. We find a positive and statistical significant difference in green premium between non-contingent green bonds and contingent green bonds, in line with the model prediction that contingent debt issuers are not the best types. Additionally, we also document that contingent debt securities have lower credit rating at issuance than non-contingent green debt, consistently with the overall view that these securities are of worse quality relative to their non-contingent counterpart. Put together, this evidence supports the presence of an equilibrium market outcome affected by the interaction of measurement frictions and adverse selection, making us conclude that addressing information gaps in the sustainable finance market can significantly improve the allocation of funds towards the transition to a green economy.

2 Related Literature

Our paper is mainly related and contributes to the literature on contract design, and in particular the literature seeking to explain missing contingencies in optimal contracts. Contract theory suggests that optimal contracts should include many contingencies that take account of all relevant information [Hart and Holmström, 1987]. A number of papers study various frictions that explain empirically observed departures from this theoretical prediction. Holmstrom and Milgrom [1991] explain missing contingencies in employment contracts in a multitask principal-agent context in which the agent allocates limited effort among competing tasks and the principal monitors these tasks with different precisions. Nachman and Noe [1994] study a capital structure problem, and

¹⁰The EMS is a standardized framework that helps an organization achieve its environmental goals through consistent review, evaluation, and improvement of its environmental performance. A well functioning EMS both increases the firm's likelihood to achieve positive environmental outcomes and also makes it more difficult for the firm to manipulate the measurement system which monitors those outcomes (see also Lyon and Maxwell [2011]).

use asymmetric information and adverse selection to explain the optimality of issuing debt, i.e. non-contingent contract, as opposed to equity. The paper most related to ours is Allen and Gale [1992], which uses measurement distortions and adverse selection to explain missing contingencies in optimal contracts in the context of a generic transaction between a buyer and a seller. When the measurement systems on which contingencies are based can be manipulated and agents differ in their ability to manipulate, non-contingent contracts are chosen in equilibrium because they represent the contracts at which no type has an incentive to distort. Despite shared similarities, our model has two important distinctions with respect to the one proposed in Allen and Gale [1992]. First, our model embeds moral hazard in that costly actions to deliver green outcomes cannot be observed unless the firm pays a verification cost upfront. Second, in our model firms themselves are not perfectly informed in that they receive complete information about their green output only at an interim date after issuance of the security. Thus, it is flexibility, rather than private information, that plays a key role in driving the results. Whereas in our paper the key mechanism is that the non-contingent contract has the advantage of revealing information about firm's ability to credibly commit, in Allen and Gale [1992] non-contingent contracts are chosen in equilibrium because they do not reveal any information about the party proposing the contract.

This paper is also related to the literature on financial innovation, which deals with creating opportunities for risk sharing [Allen and Gale, 1994, Allen and Yago, 2010]. The literature has explored a large number of reasons behind agents' incentives to innovate such as completing markets, addressing information asymmetries, responding to regulatory and economic changes, or capitalizing on investment opportunities (see Tufano [2003] for a survey). In a similar spirit to the work of Allen and Gale [1988] and Allen and Gale [1991], in our model incentives to innovate come from changes in the value of pre-existing assets or firm value. The firm innovates to maximize its value by capitalizing on the fact that investors value the green benefit that the project under management has the potential to deliver and are willing to pay for it.¹¹ Monetizing investors' green preferences depends importantly on the possibility to measure green benefits, so it is the interaction between demand for green investing and advances in measurement systems that allow firms to innovate by incorporating contingencies in their green debt contracts. Although innovation is spurred by measurability, it is threatened by possibility to manipulate measurement systems. The novel co-existence result that we obtain is due to the fact that firms vary in their ability to manipulate, and there is asymmetric

¹¹This is in line with the evidence that the market for sustainable financed has had a bottom up development, being driven by investor demand.

information about firm types. A paper related to ours is Manso, Strulovici, and Tchistyi [2010] who study performance sensitive debt (PSD), an innovative debt instrument whereby the interest rate varies ex-post with some performance metric of the borrower. Despite sharing the same security payoff structure, theirs is a model model of risky debt valuation with endogenous costly bankruptcy which differs essentially from ours in that their performance metric is perfectly measurable by the investor and cannot be manipulated. Under perfect information their model predicts that PSD is sub-optimal, but when there asymmetric information between investors and the borrowing firm, PSD is optimally issued by best firm types because it can be used as a screening device.

Our paper is also related to the literature on sustainable investing, which mainly takes an investor focus and explores the conditions under and channels through which investing by sustainabilityoriented investors can have a real impact.¹² Notable papers in this literature stream include Heinkel. Kraus, and Zechner [2001] who study how exclusionary ethical investing impacts corporate behavior, Pastor et al. [2020] who study how shifts in customers' tastes for green products and investors' tastes for green holdings produce positive social impact, Oehmke and Opp [2020] who study the conditions for impact in a context in which investors can relax firms' financial constraints for responsible production, and Landier and Lovo [2020] who study how ESG funds should invest to maximize social welfare in a setup in which financing markets are subject to a search friction. Among the few works that take a firm perspective there is Ramadorai and Zeni [2019] who document and rationalize corporate commitment in reducing carbon emissions around a regulatory announcement with a strategic model of reputation, and Bolton and Kacperczyk [2021] who provide an empirical analysis voluntary disclosure initiatives driven by institutional investors, and show that while institutional pressure matters, firms that respond the most are the ones that are already less carbon intensive. The paper most related to ours in this literature strand is Chowdhry, Davies, and Waters [2019] who also make the case for introducing contingencies in financing contracts. In their model, firms that cannot commit to social goals are jointly financed by profit and socially-motivated investors, and thus face a trade-off regarding which output to emphasize. Financial contracting can be used to aligns incentives among these heterogenously motivated investor groups by making contracts contingent on realized social output. Like most existing papers in this literature stream, this paper has an investor focus and an important role is played by the existence and behavior of groups of investors with heterogeneous beliefs and tastes regarding non-pecuniary motives; the mass of

¹²There is no consensus on the terminology used to refer to investments that have non-pecuniary benefits. The terms impact, sustainable, responsible, or ESG investing tend to be used interchangeably.

sustainability-oriented investors plays an important role in achieving impact by creating incentives for firms to undertake sustainable projects. Our paper has a firm focus and takes as given the fact that investors with green preferences want to finance projects that have the capacity to yield green benefits. We do not explore the interaction between green and non-green investors but focus instead on the agency conflict between green investors and the firm that has incentives to maximize the profits from the security issuance at the expense of delivering green benefits.

3 The Sustainable Finance Market

This section provides some background on the evolution of the corporate sustainable debt market and the institutional details governing the issuance of the securities on the market, namely green bonds and loans, social bonds, sustainability bonds (categorized as project-based non-contingent securities), and sustainability-linked loans and bonds (categorized as outcome-based contingent securities). We also briefly discuss summary statistics on how these types of sustainable debt compare to traditional bonds and loans (further details are provided in Appendix A).

The market for sustainable finance started in 2007 with the issuance of the world's first green bond by the European Investment Bank, the so called Climate Awareness Bond.¹³ Green bonds (GB) are fixed income instruments which earmark proceeds for specific green projects, that is, projects that have positive environmental and climate benefits. A green bond is differentiated from a regular, plain vanilla bond by a green label, which signifies a commitment to exclusively use the funds raised to finance or re-finance green projects. The payoff to green bond investors is similar to that of plain vanilla bonds in the sense that the borrower commits to a fixed schedule of repayments of principal and interest. However, insofar as GB finance projects that are expected to yield green benefits, the capital raised depends on these expected green benefits, which are signalled ex-ante by the issuer and which effectively constitute a green promise that is monetised through the issuance of this security. Put differently, a firm issuing a green bond is basically receiving an upfront subsidy, which gives rise to an agency problem since the firm has no incentive to commit to delivering the promised green benefit once it has obtained the subsidy, given that it is costly to do so. An effective tool to mitigate this moral hazard problem is represented by the verification process associated with obtaining a green label, which is aimed at ensuring that ex-ante green promises are followed through.

¹³The first corporate green bond was issued in 2013 by Swedish housing company Vasakronan.

Whereas in the early days of the market it was common to issue a self-certified green bond, now issuers predominantly obtain a green label from a number of certification providers, most of which adhere to the Green Bond Principles (GBPs).¹⁴ The GBPs, which were introduced in January 2014 by the International Capital Market Association (ICMA), are voluntary process guidelines for issuing green bonds that were put together by a consortium of some of the largest investment banks worldwide. The ICMA GBPs provide issuers with high level guidance on the key components involved in launching a credible green bond, and place particular emphasis on ex-ante verification that all the necessary processes are in place to ensure that the proceeds will be used for the stated projects while making no reference to outcomes delivered by the projects.¹⁵ Alongside the development of GBs, the market has seen a proliferation of debt instruments that are similar in spirit but which serve to finance other purposes, such as Social Bonds and Sustainability Bonds. While Social Bonds raise funds for projects that address social issues and/or seek to achieve positive social outcomes, the proceeds obtained through the issuance of Sustainability Bonds are dedicated to financing a combination of both green and social projects. As for GBs, the ICMA has been developing principles to guide the issuance of Social and Sustainability Bonds, namely the Social Bond Principles (SBP) and the Sustainability Bond Guidelines (SBG), respectively. Consistently with earlier evidence documented in Baker, Bergstresser, Serafeim, and Wurgler [2018], these three types of bonds are on average larger than ordinary bonds in terms of amount issued, have a longer maturity, lower coupon rates, and higher credit ratings (Table A.4 in Appendix A). Green Loans are the private debt equivalent of Green Bonds and their issuance is also governed by a set of principles, the so-called Green Loan Principles put forward by the Loan Market Association (LMA). Relative to ordinary loans, green loans tranche sizes and coupon rates are smaller, their maturities are considerably larger, and their credit rating is marginally better although rating coverage is poorer than their public counterpart (Table A.4 in Appendix A).

Sustainability-linked Bonds (SLBs) and Loans (SSLs) represent new types of debt instruments which do not earmark proceeds for specific projects, but instead make the borrower's financing cost

¹⁴The role of the external certification providers is to confirm that the bond align with the principles, and their services or involvement range from second party opinion to rigorous verification against standardized scientific criteria and involving the appointment of approved 3rd party verifiers. The major certification providers include the Climate Bond Initiative (CBI) Climate Bonds Certification, MSCI Green Bond Indices, Moody's Green Bond Assessment and Standard & Poor's Green Evaluations.

¹⁵For example, Apple clearly states that there can be no assurance" that funded projects meet investor criteria or expectations regarding sustainability performance".

contingent on the borrower meeting specific targets, which reflect broad sustainability concerns, at predetermined dates throughout the life of the contract.¹⁶ A firm raising capital using these state-contingent debt contracts essentially commits to making a series of interest repayments that are linked to the deviation of its realized sustainability performance from the target. These debt contracts usually embed a two-way pricing structure whereby if the firm meets its target then the base interest rate decreases, but if the firm fails to meet its targets then the interest rate increases. The issuance of SLBs is governed by the ICMA Sustainability-Linked Bond Principles which are centred around specifying the performance targets and the ex-post reporting and verification of performance. The ex-post performance verification component is mandatory but is similar to an audit process so is less costly and less reliable compared to the ex-ante green label certification processes associated with green bonds.¹⁷ In the case of SLLs, which represent the private debt counterpart of SLBs and whose issuance is guided by the voluntary guidelines issued by the Loan Market Association (LMA), ex-post reporting and verification of performance is only *recommended*, and subject to negotiation between the borrower and lenders on a transaction-by-transaction basis.

Summary statistics reported in Appendix A show that the distribution of SLBs' holders is similar to that of Green, Social, and Sustainable bonds' holders, although SLBs are more likely to be held by investment advisors such as BlackRock and Allianz relative to Green, Social, and Sustainable bonds.¹⁸ SLBs are larger in terms of issuance volume and have slightly shorter maturity than Green, Social, and Sustainable bonds, which is not surprising given that SLBs are meant to finance a bundle of general purpose projects while committing to an aggregate sustainable outcome. Perhaps more interestingly, SLBs have distinctively *lower* credit ratings compared to Green, Social, and Sustainable bonds. Indeed, the appendix reports a clear difference in the skewness of the ratings distribution across the two green debt contract categories when compared to the universe of corporate bonds issued in the same period.¹⁹ As for the private debt counterpart of SLBs, we find that SLLs are bigger, have shorter maturity and a similar credit rating compared to green loans, although credit ratings are available only for few securities in the sample. While we cannot observe

¹⁶The first SLL was issued in April 2017 by the Dutch health technology company Koninklijke Philips.

¹⁷For a discussion about the difference between auditing reports and proper certifications of green securities see also the discussion in Baker et al. [2018].

¹⁸The bar plot A.9 shows that 85% of the SBLs issuance volume belong to investment advisors compared to 79% for Green, Social, and Sustainable bonds. In contrast, insurance companies and trust funds hold a higher share of Green, Social, and Sustainable bonds than SLBs (12% and 3% of Green, Social, and Sustainable bonds respectively against 9% and approximately 0% of SLBs).

¹⁹We find that a similar difference seems also to apply at the issuer level, in that a far larger share of SLBs issuers receive BBB or lower credit ratings compared to issuers of Green, Social, Sustainable bonds. We provide a detailed analysis on issuers later in the empirical section.

the specific pricing structure of SLBs and SLLs, we obtain from Bloomberg New Energy Finance (BNEF) information on the performance metrics on which these contracts are written. BNEF divides SLBs and SLLs' performance targets into four broad categories, namely aggregate ESG scores, environmental metrics, social metrics, and governance metrics respectively. The bar plot A.10 in Appendix A shows that performance metrics (as well as availability of information about these metrics) vary considerably across industries, consistently with the fact that these contracts are poorly standardized and mostly designed on a transaction-by-transaction basis.²⁰ However as Table A.5 reports, on average across all SLLs and SLBs contracts for which performance metrics are available, 75% of the targets are written on environmental metrics, and about half of these environmental metrics are GHG emissions. This figure matches the overall proportion of non-contingent green bonds and loans in the sustainable finance market (roughly 80% of non-contingent sustainable products are of environmental type), suggesting that SLLs and SLBs do not represent a complement but rather a substitute to the earlier forms of non-contingent sustainable debt in the market, and thus making the comparison between these two debt categories adequate.

4 Model

The baseline economy features two time periods, an investor, and a representative firm in the market. At time t = 0, the firm has access to a business-as-usual project which costs 1\$ and yields a certain monetary return of \$1+R at time t = 1. One can think of this project as the replacement of a firm's old technology with a new technology that delivers a predetermined higher level of output. At time t = 0 or at an interim date before t = 1, the firm can decide to upgrade to a green technology by investing in a green project. The green project delivers, at time t = 1, the same monetary return and an uncertain green outcome which can be conceptualized as a reduction in carbon emissions. The green outcome is the sum of two components

$$g(\tilde{z}, a) = a + \sigma \tilde{z} \tag{1}$$

the first component a denotes the firm's costly action choice, which can be thought of as the scale of investment in the green technology, whereas the second component $\tilde{z} \sim \mathcal{N}(0, 1)$ is an uncertain state

²⁰For example, we find that financial firms have the highest share of targets written on governance metrics as well as the highest share of targets for which metrics are unavailable, healthcare firms have the highest share of targets written on social metrics, whereas energy intensive manufacturing firms such as forest and paper products manufacturing, construction materials manufacturing, containers and packaging, and electrical equipment manufacturing issue contracts predominantly written on environmental metrics.

about the quality of the technology and its potential to deliver green benefits, that is revealed only to the firm at an interim date between t = 0 and t = 1. The action *a* encompasses the portion of the outcome that can be observed by the investor at some cost, is perfectly measurable and cannot be manipulated. The interpretation of this component is that based on public information about the technology, the investor can form an expectation about the average emission savings delivered by the project based on the scale of the technological investment i.e. the action can be backed out from the cost of action, which is expressed in monetary terms and thus measurable. The uncertain state \tilde{z} is the component of the outcome that can only be observed by the firm, is non-measurable and can be manipulated. The interpretation is that for a given scale of investment, there is residual uncertainty with respect to the emissions savings delivered by the project, which can for instance depend on hidden technology fundamentals that are privately revealed to the firm. The parameter σ controls the degree of *materiality* of the project, where materiality is defined as the expected level of discrepancy between the overall green outcome and its measurable component.²¹ The higher is σ the less material the project outcome, that is, the more uncertain and harder-to-assess the outcome.

The investor has a linear utility which equally values consumption (e.g. the monetary outcome) and the green outcome. Denoting $x = \{0, 1\}$ the firm's binary choice of whether to implement the green project, the investor's utility reads

$$\mathcal{U}^{I} = \mathcal{C}_{0}^{I} + \mathcal{C}_{1}^{I} + xg(\tilde{z}, a) \tag{2}$$

with endowments $n_0^I >> 1$ and $n_1^I = 0$ at time t = 0 and t = 1 respectively. The firm, on the other hand, has monetary preferences only and dislikes actions to deliver the green outcome

$$\mathcal{U}^f = \mathcal{C}^f_0 + \mathcal{C}^f_1 - xc(a) \tag{3}$$

with cost of action c(a) which takes the quadratic form $c(a) = \frac{1}{2}\theta a^2$, and endowments $n_0^f = n_1^f = 0$ at time t = 0 and t = 1 respectively. Since the firm has zero endowments while the investor has large endowments and a preference for both monetary and green outcomes at time t = 0, we focus the analysis on the firm and its project financing problem taking as given that the investor absorbs the security issuance provided it yields at least zero total returns in expectation. Before introducing

 $^{^{21}}$ When the green outcome is interpreted as a reduction in carbon emissions, this maps into the concept of materiality proposed by the GHG Protocol standard which is the maximum percentage difference between the company's *reported* emissions and the verifier's *belief* of what the company's emissions would be if all omitted sources were accounted for. More detail are provided in the empirical section.

the details of the financing problem though, it is useful to derive an efficient benchmark for project and investment choices in the case where the economy is centrally planned by a perfectly informed social planner.

4.1 Central Planner Problem

The first-best project and action choices, x and a respectively, are obtained by solving the problem of a perfectly informed social planner, indexed by s, that maximizes the aggregate utility

$$\max_{a,x} \mathcal{U}^I + \mathcal{U}^f = R + \max_{a,x} x(g(z,a) - c(a)).$$
(4)

Recalling the quadratic cost of action $c(a) = \frac{1}{2}\theta a^2$, the Euler conditions yield the following project and action choices

$$x^{s}(z) = 1\{\frac{1}{2}a^{s} + \sigma z > 0\}$$
 with $a^{s} = \frac{1}{\theta}$. (5)

Thus, the social planner finds it optimal to implement the green project provided that the realization of the uncertain state z is such that the green outcome delivered by the project is higher than the cost. The optimal action, interpreted as the level of investment, is conditional on the project implementation and can be thought of as the intensive margin of investment. Clearly, if the project is not implemented the optimal action is zero. Importantly, note that the social planner choices are state dependent.

4.2 Decentralized Problem

In the decentralized market, the firm seeks to maximize utility in (3) by proposing a debt contract y to the investor. The generic structure of the debt contract is as follows: at date t = 0, the investor lends an amount $b_0^y = \$1$ to the firm, so that the latter can afford the implementation of (at least) the business-as-usual project that has a positive certain monetary return.²² Depending on the design of the contract y and its associated characteristics, the firm will then decide the green project and action choices, x^y and a^y respectively, which depend on the realization z of the uncertain state variable \tilde{z} in ways that will be detailed below. At date t = 1, the firm will repay the investor an amount $b_1^y = \$1 + \rho^y$, with ρ^y denoting the interest rate associated with the debt contract.

²²The positive certain monetary return and the fact that the firm has zero endowments ensures that external financing is always profitable in equilibrium, e.g. there are no equilibrium outcomes where no contract y is chosen.

Conditional on issuance of a debt contract y, the firm problem can be specified as follows

$$\mathcal{U}_y^f = \max_{a,x} \mathcal{C}_{0,y}^f + \mathcal{C}_{1,y}^f - xc(a) \tag{6}$$

under the budget constraints

$$\mathcal{C}_{0,y}^{f} = b_{0}^{y} - 1
\mathcal{C}_{1,y}^{f} = 1 + R - b_{1}^{y}$$
(7)

and subject to the investor participation constraint, which features the contract specific optimal project and action choices, x^y and a^y , respectively²³

$$-b_0^y + \mathbb{E}[b_1^y + x^y g(\tilde{z}, a^y)] \ge 0$$
(8)

In what follows, we take a positive approach to studying green project financing in that we analyse the welfare implications of firm's issuance choices using a given set of debt contracts whose design is similar to that of securities currently observed in the market. Formally, we assume that the firm can choose one among a specified set of securities $y \in \{v, g, cg\}$ which vary with the interest rate specification, where v stands for a plain vanilla debt contract, g stands for a project-based non-contingent green debt contract, and cg stands for an outcome-based contingent green debt contract. The plain vanilla debt v is one which does not finance a green project, and which has a fixed interest rate that is independent of green outcomes. The project-based green debt g is a debt contract whose interest rate is fixed at issuance but which depends implicitly on the firm's commitment to implement a project that yields green outcomes.²⁴ The outcome-based contingent green debt cg is a debt contract whose interest rate varies ex-post with the deviation of the green outcome delivered from the contarctually agreed target.

The vanilla contract, indexed by v, is the most simple form of debt contract which repays the investor at date t = 1 a predefined amount

$$b_1^v = 1 + \rho^v \tag{9}$$

²³The participation constraint comes from the investor's zero return requirement $C_{0,y}^{I} + \mathbb{E}[C_{1,y}^{I} + xg(\tilde{z}, a)] \geq n_{0}^{I}$ where consumption is given by $C_{0,y}^{I} = -b_{0}^{y} + n_{0}^{I}$ and $C_{1}^{I} = b_{1}^{y}$, respectively.

²⁴This is in line with empirical evidence on the existence of a green premium, namely green bonds having lower yields than their plain vanilla counterparts, which increases with the credibility of the issuer [Kapraun and Scheins, 2019, Baker et al., 2018].

with ρ^v denoting the fixed interest rate which is determined at time t = 0 when the security is issued.

The project-based non-contingent green debt contract, indexed by g, involves ex-ante commitment to a project $x^g = 1$ and action a^g at the moment of issuing the security. Under this debt contract, a green outcome is delivered at date t = 1, along with the following monetary payoff

$$b_1^g = 1 + \rho^g \tag{10}$$

with ρ^g denoting the fixed interest rate which is determined at time t = 0 when the security is issued. At issuance, the firm also pays a verification cost α to certify its commitment to the project and action choices i.e. to let the investor observe the action choice a^g conditional on implementing the green project $x^g = 1$. The verification cost can be interpreted the green bond label that certifies the firm's commitment to dedicate the debt proceeds to green projects, e.g. the ex-ante certification of the firm's compliance with the GBPs.

The *outcome-based contingent* green debt contract, indexed by cg, does not involve ex-ante selection of projects nor commitment to actions, but incentivize commitment to outcomes through the introduction of a state-dependent interest rate ρ^{cg} which is contingent on the realization of the uncertain green outcome. Specifically, this contract repays the investor an amount

$$b_1^{cg} = 1 + \rho^{cg} \tag{11}$$

with ρ^{cg} state-contingent interest rate

$$\rho^{cg} = \bar{\rho}^{cg} - x^{cg}g(z_r^{cg}, a^{cg}) \tag{12}$$

where $\bar{\rho}^{cg}$ is a base interest rate set at date t = 0, x^{cg} and a^{cg} are the firm's optimal project and action choices decided at a later date after the security issuance, and z_r^{cg} is the *optimally reported* uncertain component of the green outcome. The specification (12) implies that the firm will pay the base interest rate $\bar{\rho}^{cg}$ if it reports no green outcome, and it will be rewarded (penalized) with a lower (higher) interest rate if it reports a positive (negative) green outcome. Note that by allowing the firm to report an uncertain state z_r^{cg} that is different from the true realized state z, this specification creates an incentive for ex-post manipulation of the non-measurable state, in that by reporting an outcome state $z_r^{cg} \geq z$, the firm can repay the debt at a lower interest rate than in the case of truthful reporting. The optimally reported state is function of an optimal level of distortion as

$$z_r^{cg} = z + d^{cg} \tag{13}$$

with d distortion choice variable that comes at a quadratic cost $\eta(d) = \frac{1}{2}\psi d^2$ (in the spirit of the literature on strategic communication with lying costs by Kartik [2009]).

5 Single Firm

This section considers a single firm model to highlight the key mechanisms that drive a firm's preferences for issuing a non-contingent or a contingent debt contract. The extended model with firm types, as well as its equilibrium predictions in presence of asymmetric information are considered in the next section.

5.1 No Manipulation

We begin with a setup in which measurement systems for contingencies are perfect, meaning that the reported green outcome cannot be manipulated, and then allow for the possibility that the reported green outcome can be manipulated by the firm at a cost.

5.1.1 Vanilla security

Substituting the budget constraint (7) into the firm problem (6) when b_1^y equates b_1^v in (9), the firm utility can be simplified to

$$\mathcal{U}_v^f = \max_{a,x} R - \rho^v - xc(a) \tag{14}$$

subject to the investor participation constraint

$$\rho^{v} \ge -\mathbb{E}[x^{v}g(\tilde{z}, a^{v})] \tag{15}$$

which states that the investor acquires this security if the interest rate associated with it is higher than the negative of the expected green outcome $\mathbb{E}[x^v g(\tilde{z}, a^v)]$, so that monetary and green returns add at least to zero in expectation. Given that project and action choices are not verified, any attempt to finance the green project with this security is affected by a standard moral hazard problem and the firm finds it optimal to chose the business-as-usual project. Specifically, the investor will anticipate that the firm has no incentive to implement the green project upon issuance of this contract and the firm's optimal project choice is

$$x^v = 0. (16)$$

The rational investor is not willing to pay a green premium by accepting a negative interest rate i.e. the participation constraint becomes $\rho^v = 0$, the green project is not implemented in equilibrium and the firm issues this plain vanilla security to finance the business-as-usual project. In such a case, firm's utility reads

$$\mathcal{U}_{v}^{f} = R. \tag{17}$$

5.1.2 Project-based, non-contingent security

Project-based non-contingent green debt contracts are those whereby project selection takes place ex-ante, at security issuance and thus prior to the realization of the uncertain state affecting the green outcome. Making ex-ante project selection a defining feature of this stylised security is in line with the green bond principles, which require ex-ante specification of the use of proceeds.

We capture this in the context of our model by making the firm choose the project x^g and commit to an action choice a^g at the moment of issuing the security and thus *prior* to the realization of the random state \tilde{z} . Importantly, the firm pays a verification cost α to make the commitment credible. This is interpreted as the cost that the firm incurs to set up the process by which the investor will be able to verify ex-post that the action it has committed to is effectively the same as the one actually implemented. This mechanism is again in line with the green bond principles which revolve around setting up the processes and mechanisms necessary to facilitate verification, such as placing the bond proceeds in a separate account that the investor can verify to make sure that they are used for projects aligned with the security purpose.

Substituting the budget constraint in (7) into the firm problem in (6) when b_1^y equates b_1^g in (10), the firm utility reads

$$\mathcal{U}_g^f = \max_{a,x} R - \rho^g - x(c(a) + \alpha) \tag{18}$$

subject to the participation constraint

$$\rho^g \ge -\mathbb{E}[x^g g(\tilde{z}, a^g)]. \tag{19}$$

Recalling that with this security there is credible commitment, meaning that the project and action choices revealed at the time of issuing the security are the same as those actually implemented by the firm, i.e. $x^g = x$ and $a^g = a$, and substituting the binding participation constraint (19) into the problem, the firm problem becomes

$$\mathcal{U}_g^f = R + \max_{a,x} x(\mathbb{E}[g(\tilde{z}, a)] - c(a) - \alpha)$$
(20)

from which we obtain optimal project and action choices

$$x^{g} = 1\{\frac{1}{2}a^{g} - \alpha > 0\}$$
 with $a^{g} = \frac{1}{\theta}$. (21)

Plugging back the optimal project and action choices into (19), it then follows that the interest rate on the project-based non-contingent contract is $\rho^g = -\frac{1}{\theta}$ if $x^g = 1$, and is $\rho^g = 0$ if $x^g = 0$. Thus, ex-ante commitment is important because insofar as it is credible, it influences the financing cost for the firm as reflected in a lower interest rate than the vanilla contract. Importantly though, since the project choice in (21) is determined at issuance and therefore independent of the realisation of the random state z, it is different from the first best outlined in (5). This is a first important implication of the model in that ex-ante credible commitment necessary to resolve the moral hazard problem intrinsic in non-contingent contracts implies sub-optimal investment choice.

Recalling (17) and plugging the optimal choices x^g and a^g and contract rate ρ^g back into (18), it is then simple to show that

$$\mathcal{U}_q^f > \mathcal{U}_v^f \quad \longleftrightarrow \quad x^g = 1 \tag{22}$$

meaning that the firm has a strict preference for contract g relative to contract v if and only if it commits to the implementation of a green project at issuance. On the other hand, by committing to a business-as-usual project, the firm's utility, as well as the interest rate on the contract, is the same as in the plain vanilla case.²⁵

²⁵This follows from the fact that the firm's utility if $x^g = 1$ is $\mathcal{U}_g^f = R + \frac{1}{2\theta} - \alpha$ and this is greater than \mathcal{U}_v^f if $2\alpha\theta < 1$, which is exactly the condition for $x^g = 1$. On the other hand, if $x^g = 1$ the firm utility is $\mathcal{U}_g^f = R = \mathcal{U}_v^f$.

5.1.3 Outcome-based, contingent security

Outcome-based contingent green debt contracts are those whereby the firm does not commit to projects ex-ante, but instead chooses them ex-post after the issuance of the security and thus after the observation of the random state \tilde{z} realisation. With this security, instead of ex-ante commitment we have ex-post reporting of realised green outcomes, which can be manipulated.

Substituting the budget constraint (7) into the firm problem in (6) when b_1^y equates b_1^{cg} in (12), the firm problem upon issuance of this contract can be simplified to

$$\mathcal{U}_{cg}^{f} = R - \bar{\rho}^{cg} + \max_{a,x,d} x(g(z_r, a) - c(a) - \eta(d))$$
(23)

where the base interest rate is now subject to the participation constraint

$$\bar{\rho}^{cg} \ge \mathbb{E}[x^{cg}g(\tilde{z}^{cg}, a^{cg}) - x^{cg}g(\tilde{z}, a^{cg})].$$

$$(24)$$

The participation constraint, which is obtained by substituting the rate ρ^{cg} in (12) into (8), effectively tells us that the base rate $\bar{\rho}^{cg}$ that the investor demands in order to enter the contract should be at least as high as the expected distortion imposed by the firm. Specifically, the minimum acceptable interest rate $\bar{\rho}^{cg}$ depends on the expected deviation of reported green outcome from the actual green outcome of the project, such that the investor effectively imposes a *distortion discount* in the pricing of this contract by raising the expected cost of financing for the firm. However, since is this case manipulation is not possible (e.g. the cost of distortion is prohibitively high $\eta(d) = +\infty$ such that $d^{cg} = 0$), the green outcome is truthfully reported $z_r^{cg} = z$ for each realization z of the uncertain state \tilde{z} . The minimum required interest rate $\bar{\rho}^{cg}$ is thus zero and the variable, state-contingent interest rate ρ^{cg} in (12) will depend on the reported green outcome; specifically, it will be set so as to perfectly offset the reported green performance across each state z. Making explicit the dependence on the realised state z, first-order conditions yield optimal choices

$$x^{cg}(z) = 1\{\frac{1}{2}a^{cg} + \sigma z > 0\}$$
 with $a^{cg} = \frac{1}{\theta}$ (25)

Note that when manipulation is not possible, the optimal state-dependent choices equate the first best in (5). The firm's utility in this case is

$$\mathcal{U}_{cg}^{f} = \mathcal{U}_{v}^{f} + \left(\frac{1}{2}\frac{1}{\theta} + \sigma\tilde{z}\right)^{+}$$
(26)

and its expected value is unambiguously higher than \mathcal{U}_v^f , as well as unambiguously higher than \mathcal{U}_g^f , as formalized in the proposition below. On the other hand, as shown in Appendix B, the green outcome obtained with the contingent green security can be higher or lower than that obtained with the non-contingent green security, i.e. $x^{cg}(z)g(z, a^{cg}) \leq x^g g(z, a^g)$. The intuition is that, when using the contingent security, the green project is both less likely to be implemented but also more likely to deliver a positive outcome conditional on implementation.

5.1.4 Optimal security choice

The firm problem is to maximize expected utility from issuing either the best among the noncontingent securities, namely the plain vanilla v or the non-contingent green security g, or the contingent green security cg. We define an indicator variable $\mathcal{I}(y) \in \{0, 1\}$ which takes the value 1 if the firm chooses to issue a contingent green security, y = cg, and 0 otherwise. Then the firm's contract choice problem reads

$$\mathcal{U}^{f} = \mathcal{U}_{nc}^{f} + \max_{y} \mathcal{I}(y) \left(\mathbb{E}[\mathcal{U}_{cg}^{f}] - \mathcal{U}_{nc}^{f} \right)$$
(27)

where $\mathcal{U}_{nc}^f = \max{\{\mathcal{U}_v^f, \mathcal{U}_g^f\}}$ is the firm's optimal choice between the two non-contingent contracts, namely than plain vanilla and the green contract.

Proposition 1. When the measurement system cannot be manipulated, for each pair (σ, θ) and verification cost α , the firm optimally chooses to issue the contingent green contract

$$y^* = cg \tag{28}$$

and the optimal green investment equates that one implemented in first-best

$$x^{y^*}(z) = x^s(z). (29)$$

The proof follows immediately from the Jensen's inequality and is reported in Appendix B. Proposition 1 implies that outcome-based contingent securities are always preferred to non-contingent securities. Furthermore, they allow for the decentralized economy to achieve a level of green investment that is the same as the first-best allocation outlined in (5). Therefore, non-contingent green securities such as green bonds are never optimal when reported green outcomes cannot be manipulated.

5.2 Manipulation

Let us consider now the situation in which contingencies depend on measurement systems which can be manipulated, or in other words, the reported green outcome on which the contingent payments are based on can be distorted.

Solving the firm problem (23) with respect to project choice, action and distortion yields

$$\hat{x}^{cg}(z) = 1\{\frac{1}{2}\hat{a}^{cg} + \frac{\sigma}{2}\hat{d}^{cg} + \sigma z > 0\} \text{ with } \hat{a}^{cg} = \frac{1}{\theta} \text{ and } \hat{d}^{cg} = \frac{\sigma}{\psi}$$
 (30)

where we use the hat superscript to refer to optimal choices when manipulation is possible. Result (30) states that when manipulation is possible, the firm's optimal distortion \hat{d}^{cg} increases with the non-materiality σ of the project green outcome, it decreases with the distortion cost ψ and given that it is conditional on project implementation it also varies with the realized state z.²⁶ Importantly, note that the firm may optimally spend more in distortion than in actual investment if the model parameters satisfy $\theta > \frac{\psi}{\sigma}$. Interpreted in a multiple project setting, this prediction implies that firms can achieve a higher reported level of green benefits by manipulating the reported green outcome of projects with a hard-to-assess impact instead of investing in costly projects with a measurable impact. This model feature speaks to the documented practice of greenwashing, discussed in more detail in the empirical section, which consists of engaging in selective disclosure and manipulative practices in order to inflate perceived sustainability performance.

Note that everything else equal, a project is more likely to be implemented when manipulation is possible than in the case of no manipulation i.e. comparing (30) with (25) we have $x^{cg}(z) \leq \hat{x}^{cg}(z) \forall z$, because of a further gain that comes from the possibility of distortion. On the other hand, as reported in Appendix B, the optimal expected green outcome under manipulation lies between the outcome obtained using the non-contingent green security g, and that obtained using the contingent green security cg with no manipulation.

²⁶This derives from the fact that $\eta(d)$ is independent of σ , hence distortion benefits increase with σ . A different specification where the distortion costs increase in σ does not alter the main predictions in the paper except for the comparative statics of the firm's preferences for contingent and non-contingent green debt contracts across varying σ , detailed below.

Plugging in optimal choices into the firm utility we have

$$\hat{\mathcal{U}}_{cg}^f = \mathcal{U}_v^f + \left(\frac{1}{2}\frac{1}{\theta} + \frac{1}{2}\frac{\sigma^2}{\psi} + \sigma\tilde{z}\right)^+ - \bar{\rho}^{cg}.$$
(31)

Note that if the minimum required rate $\bar{\rho}^{cg}$ was set to zero, then the firm would have a higher expected return relative to the case of no manipulation. However, the investor is aware that the reported green outcome is different from the actual green outcome, and so will require a higher base interest rate which, as outlined in the participation constraint under manipulation (24), will be adjusted so as to account for the expected distortion

$$\bar{\rho}^{cg} \ge \mathbb{E}\left[\frac{\sigma^2}{\psi} 1\left\{\frac{1}{2}\frac{1}{\theta} + \frac{1}{2}\frac{\sigma^2}{\psi} + \sigma\tilde{z} > 0\right\}\right]$$
(32)

which is given by plugging in the optimal distortion choice in (30) into (24). In other words, we assume that the investor is perfectly internalizing the distortion imposed by the firm by setting the base rate to satisfy the participation constraint outlined in (24).

Substituting the base rate into (31) and re-arranging, one gets that expected utility from issuing this contract reads

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^f] = \mathcal{U}_v^f + \mathbb{E}[(\frac{1}{2}\frac{1}{\theta} - \frac{1}{2}\frac{\sigma^2}{\psi} + \sigma\tilde{z})(\frac{1}{2}\frac{1}{\theta} + \frac{1}{2}\frac{\sigma^2}{\psi} + \sigma\tilde{z} > 0)].$$
(33)

As the proposition below formalizes, when the measurement system on which contingencies are based can be manipulated and the investor correctly internalizes this, the firm's utility obtained when financing is done using the contingent security is no longer unambiguously higher than that obtained when issuing non-contingent contracts.

5.2.1 Optimal security choice

Formally, the firm's contract choice can be written as

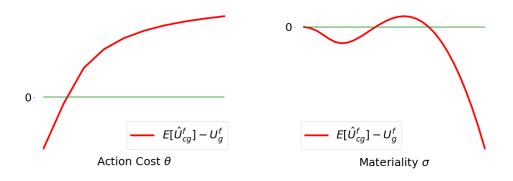
$$\hat{\mathcal{U}}^f = \mathcal{U}^f_{nc} + \max_y \mathcal{I}(y) \left(\mathbb{E}[\hat{\mathcal{U}}^f_{cg}] - \mathcal{U}^f_{nc} \right)$$
(34)

where $\mathcal{U}_{nc}^{f} = \max{\{\mathcal{U}_{v}^{f}, \mathcal{U}_{g}^{f}\}}$ is defined as before and $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}]$ is the firm's expected utility upon issuance of the contingent green contract when manipulation is possible. The firm's issuance choice is formally captured by the sign of the term $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}] - \mathcal{U}_{nc}^{f}$: a positive value associated with this term indicates a preference for the contingent green contract while a negative value indicates a preference for a non-contingent contract. When discussing comparative statics of the firm's contract choice, we focus on the more interesting case in which the alternative to a contingent contract is a noncontingent green debt contract rather than a plain vanilla debt contract, that is when $\mathcal{U}_{nc}^f = \mathcal{U}_g^f$. We will introduce the case where vanilla contracts are optimal when we consider firm types.

Disregarding for a moment of the verification cost α , there are two competing forces which drive the firm's preference for a contingent contract relative to a non-contingent green contract: the op*portunity cost* of committing to projects ex-ante associated with the non-contingent contract, and the distortion discount generated by the fact that reported outcomes can be manipulated that is associated with the contingent contract. If the opportunity cost of committing to projects ex-ante is lower than the *distortion discount* generated by manipulation, then the firm should opt for the non-contingent green security, whereas if the opposite is true than the firm should opt for the contingent green security. For a given distortion cost $\psi \in (0, \infty)$, Figure 2 shows how the expected value of the term $\hat{\mathcal{U}}_{cg}^f - \mathcal{U}_g^f$ varies with the investment cost θ (left-hand plot) as well as with the materiality of the project outcome σ (right-hand plot). The left-hand plot shows that preferences for the contingent contract decrease monotonically for lower values of the action cost θ . This is because, everything else equal, the opportunity cost of foregoing information about the green outcome and committing exante to the project decreases as the expected level of the green outcome (i.e. the inverse of the cost of action) increases. As we will see later when introducing firm types, this feature is relevant in generating equilibrium results that match observed variation in issuance choices within industry. On the other hand, the right-hand plot shows that the firm's preference for a contingent contract is highly non-linear as a function of the materiality parameter σ . This is because a varying σ alters both forces that drive the firm's financing choice. Note that for fully material activities (i.e. $\sigma = 0$) the firm is always indifferent between a contingent and a non-contingent contract because the opportunity cost of committing to a project ex-ante is equal to the ex-post distortion discount, and both are equal to zero. On the other hand, when σ increases, then both the opportunity cost as well as the distortion discount increase. For small values of $\sigma \approx 0$, the firm will opt for a non-contingent contract as the opportunity cost of commitment is very low. On the other hand when σ becomes distinctively larger than zero, the ex-post distortion discount is convex in σ , while the opportunity cost of ex-ante commitment is concave in σ . This means that the firm tends to prefer the contingent green contract for intermediate values of σ , whereas it eventually opts for the non-contingent green contracts when σ is sufficiently large.²⁷ As we will see later in the empirical section, this interesting non-linearity in firm's issuance preferences across project types help explaining observed issuance patterns across industries.

Figure 2. Comparative Statics - Single Firm

The plots show the firm's expected utility upon issuance of an outcome-contingent green debt cg net of the utility from issuing a non-contingent green debt g as a function of the parameter θ (left plot) and σ (right plot) respectively. Other model parameters are $\alpha = 0.0$, $\sigma = 0.5$ (left plot) and $\theta = 1.6$ (right plot) respectively.



As formalized in the proposition below, for extremely low and high values of the distortion cost ψ , the firm has strict preferences for non-contingent contracts and contingent contracts respectively, in the sense that it is a strictly dominant strategy for the firm to finance the green project via one or the other type of contract independently of the other model parameters.

Proposition 2. Let \hat{y} denote the optimal contract choice that maximizes the firm problem in (31). When the measurement system can be manipulated, for each couple of parameters (σ, θ) and verification cost $\alpha > 0$, it always exists a pair $(\psi, \overline{\psi})$ such that:

- if the distortion cost $\psi > \overline{\psi}$, then $\hat{y} = cg$ and the firm always issues a contingent green contract.
- if the distortion cost ψ < ψ, then ŷ ≠ cg and the firm never issues a contingent green contract. In such a case, if 2αθ > 1, then ŷ = v and the firm prefers to issue a non-contingent vanilla contract. On the other hand, if 2αθ ≤ 1, then ŷ = g and the firm prefers to issue a non-contingent vanilla

²⁷The convexity of the distortion cost comes from the fact that the expected level of distortion is quadratic in σ , whereas the concavity of the opportunity cost of commitment comes from the fact that, given that distortion benefits are large when σ is large, the firm chooses to implement the green project independently of the realized state z as expected benefits from manipulation make the green project unambiguously appealing.

contingent green contract.

The proof of the proposition follows from the properties of the truncated normal distribution and is reported in Appendix B. It is worth recalling that, in all of those cases, the project and action choices strictly differ from the first-best benchmark in (5).

6 Multiple Firm Types

So far we have focused on optimal security issuance from the point of view of a single firm, deriving predictions in a general setting which depends on three independent state variables: the cost of action, the cost of distortion, and the materiality of the project outcome. In this section we aim to impose restrictions on the firm's action and distortion technology so as to reduce the number of state variables at play and derive more refined, testable predictions from the model.

We assume that there is a continuum of firm types described by a state variable $k \in [0, 1]$. The firm type k is related with the cost of action and the cost of distortion parameters as follows

$$\theta_k = \phi \frac{1}{k}, \quad \psi_k = \psi \frac{1}{1-k} \tag{35}$$

meaning that the highest type firm, k = 1, has infinite distortion cost and action cost equal to θ , while the lowest type firm, k = 0, has infinite action cost and distortion cost equal to ψ . The pair (θ_k, ψ_k) identifies the firm type and is independent of the parameter σ , which now uniquely identifies the project type in terms of green outcome materiality.

Condition (35) states that the ability to distort the green outcome is negatively correlated with the ability to produce the outcome in the first place. Intuitively, the assumption implies that it is often companies that do not have systems in place to measure negative externalities/green outcomes that both: 1) have leeway to misreport or manipulate, i.e. have low cost of distortion; and 2) do not take action to reduce negative externalities/deliver green outcomes, i.e. have high cost of action. This assumption is also supported by definition that the Environmental Protection Agency (EPA) gives to an Environmental Management System (EMS), namely "[..] a framework that helps an organization achieve its environmental goals through consistent review, evaluation, and improvement of its environmental performance. The assumption is that this consistent review and evaluation will identify opportunities for improving and implementing the environmental performance of the orga*nization*". While the adoption of an EMS stands for commitment to environmental performance, Lyon and Maxwell [2011] also find that corporate adoption of an EMS also makes it more difficult for the firm to manipulate the measurement system which monitors those outcomes. These lend support to the idea that the propensity to take costly action is negatively related to the propensity to manipulate.

In deriving the predictions that follow, we assume that the verification costs α satisfies $0 < 2\alpha\theta < 1$ for a given action cost θ , so that the issuance of the project-based non-contingent green contract has positive (negative) net present value for the highest (lowest) type k = 1 (k = 0).²⁸

6.1 Perfect Information

We first analyse the baseline case where the investor is perfectly informed about the firm type k, that is, the continuum of firm types k can be perfectly observed by the investor.

6.1.1 Optimal security choice

A firm k's contract choice is

$$\hat{\mathcal{U}}^{f}(k) = \mathcal{U}^{f}_{nc}(k) + \max_{y} \mathcal{I}(y) \left(\mathbb{E}[\hat{\mathcal{U}}^{f}_{cg}(k)] - \mathcal{U}^{f}_{nc}(k) \right)$$
(36)

where expected net profits from issuing the contingent contract are defined as

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] - \mathcal{U}_{nc}^{f}(k) = \begin{cases} \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] - \mathcal{U}_{g}^{f}(k) & if \quad k \in (2\alpha\theta, 1] \\ \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] - \mathcal{U}_{v}^{f} & if \quad k \in [0, 2\alpha\theta] \end{cases}$$
(37)

where $\mathcal{U}_{g}^{f}(k) = \mathcal{U}_{v}^{f} + \frac{k}{2\theta} - \alpha$, whereas

$$\hat{\mathcal{U}}_{cg}^{f}(k) = \mathcal{U}_{v}^{f} + \left(\frac{1}{2}\frac{k}{\theta} - \frac{1}{2}\frac{\sigma^{2}(1-k)}{\psi} + \sigma\tilde{z}\right)^{+} - \bar{\rho}_{k}^{cg}$$
(38)

²⁸The condition $0 < 2\alpha\theta < 1$ comes from the firm utility associated with the non-contingent green contract which for the firm type k reads $\mathcal{U}_g^f(k) = \mathcal{U}_v^f + \frac{k}{2\theta} - \alpha$, such that the non-contingent green contract is strictly preferred if and only if $k > 2\alpha\theta$. Thus, there is an internal type $k = 2\alpha\theta \in (0, 1)$ which is indifferent between issuing the plain vanilla and the green non-contingent contract.

with $\bar{\rho}_k^{cg}$ the type-specific interest rate which is obtained by substituting the type-specific cost parameters (35) into (32)

$$\bar{\rho}_{k}^{cg} = \mathbb{E}\left[\frac{\sigma^{2}(1-k)}{\psi}\left(\frac{1}{2}\frac{k}{\theta} + \frac{1}{2}\frac{\sigma^{2}(1-k)}{\psi} + \sigma\tilde{z} > 0\right)\right].$$
(39)

In Appendix B, we introduce a synthetic contract which embeds the manipulation incentive of the contingent contract cq but forces selection of the green project at issuance to show that the net profits in (37) can be decomposed into two separate terms that vary in opposite directions as a function of the firm's type k. The first term is the pure opportunity cost of ex-ante commitment, measured as the difference between the expected utility from issuing the contingent contract cqand the utility from issuing the synthetic contract. This term is always positive and decreases monotonically in k as the action cost θ_k decreases.²⁹ The second term is the pure manipulation discount associated with the contract cg net of the (minimum) cost of moral hazard implicit in the best non-contingent contract nc.³⁰ This term is measured as the differences in expected utility between the synthetic contract and the best non-contingent contract, it can be either positive or negative depending on the relative magnitude of the distortion cost ψ_k and the verification cost α , and *increases* monotonically in the type k as the cost of distortion ψ_k increases. When the two terms are combined together, the result is a piecewise continuous, possibly non-monotonic function of k as reported in Figure 3. One notes that when the distortion cost ψ is low, the second term prevails making net profits from issuance of the contingent contract cg in (37) negative for a wide range of types k. Importantly though, since there is perfect information, the highest type k = 1always issues the contingent contract across all choices of the parameter ψ , in that net profits from issuance are necessarily positive as implied by Proposition 2 and the cost specification choice (35). More formally, we prove in Appendix B the following

Corollary 2. Let \hat{y}_k denote the optimal contract choice that maximizes the firm problem in (36) for a type $k \in [0, 1]$ with action and distortion costs that vary as in (35). Then for a given triple of parameters $(\theta, \sigma, \psi) \in (0, \infty)$ and verification cost α such that $0 < 2\alpha\theta < 1$, there exists two types $k \leq \overline{k}$ such that

- if $k \ge \overline{k}$ then $\hat{y}_k = cg$ the firm issues a contingent green contract.
- if $k \leq \underline{k}$ then $\hat{y}_k = v$ and the firm issues a non-contingent plain vanilla contract.
- if $\underline{k} < k < \overline{k}$ then either $\hat{y}_k = g$ and the firm issues a non-contingent green contract.³¹

²⁹This result goes back to the reason discussed in the left-hand plot of Figure 2 and)This result Provided that the model parameters lie within admissible ranges $\psi \in (0, \infty)$, $\theta \in (0, \infty)$, and $\sigma \in (0, +\infty)$.

³⁰As we show in the appendix, the implied cost of moral hazard is bounded from above by the verification cost α .

³¹We show in the appendix that, for a small region of model parameters, the firm types in $\underline{k} < k < \overline{k}$ may be

Figure 3. Comparative Statics - Multiple Firm Types

The plots show the firm's net expected profits from issuance of a contingent contract as a function of the firm type k for three different values of the distortion cost parameter ψ . Model parameters are $\theta = 0.3$, $\alpha = 1.0$, $\sigma = 2.0$, $\psi = 0.8$ (black line) $\psi = 1.3$ (brown line) and $\psi = 3.2$ (red line) respectively.

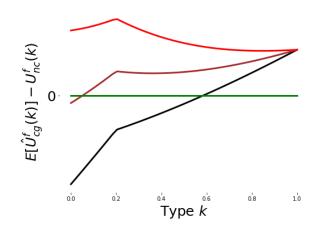


Figure 4 shows how the optimal issuance strategies vary as a function of the distortion cost ψ , the action cost θ , the verification cost α and the project materiality σ . The figure illustrates that on average across possible choices of the parameters, higher types are more likely to issue the contingent green contract (red region), intermediate types are more likely to issue the non-contingent green contract (green region), whereas lower types are more likely to issue the plain vanilla non-contingent contract (grey region).³² It is interesting to note that, as discussed earlier for the single-firm case, preferences for the contingent contract remain on average higher for projects with intermediate level of materiality (bottom right-hand plot in Figure 4).

6.2 Asymmetric Information

In this section we elaborate the model further in that we assume that there is asymmetric information over the firm's type k, meaning that the investor cannot observe the atomistic type k but only knows that it is drawn from a uniform distribution $k \sim \mathcal{U}(0,1)$. As we show below, asymmetric information alters the firm's financing problem to the extent that it modifies the investor's participation constraint associated with the contingent contract, in turn triggering different issuance

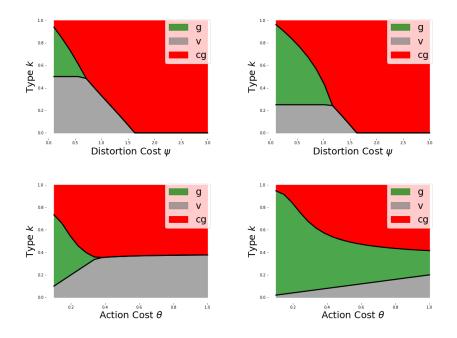
almost in different between choosing a contingent contract cg and the non-contingent contract g, and there exists a cutoff type k' such that if k < k' then $\hat{y}_k = cg$, whereas if $k \ge k'$ then $\hat{y}_k = g$.

³²As we outline in detail in the Appendix B, it may exist a region of the model parameters where the issuance strategy $\hat{y}_k = cg$ is non-monotonic in k. However, such region is very small and not attained for any of the parameters' choices reported in Figure 2.

preferences across types k.

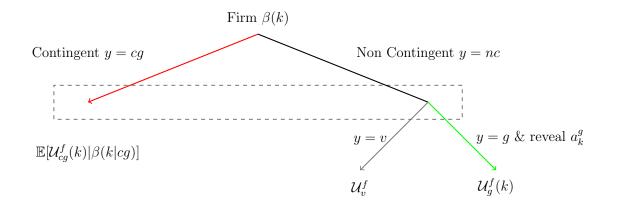
Figure 4. Equilibrium Contract Choice - Perfect Information

The plots show the firm's optimal contract choice as a function of the type k (y-axis) and the action and distortion cost parameters ψ_k (x-axis, top plots) and θ_k (x-axis, bottom plots) respectively, for two distinct values of the verification cost α (left and right-hand plots respectively). Model parameters are $\theta = 0.25, \sigma = 2.0, \alpha = 1.0$ (top left plot), $\theta = 0.25, \sigma = 2.0, \alpha = 0.5$ (top right plot), $\psi = 1.0, \sigma = 2.0, \alpha = 1.0$ (bottom left plot), and $\psi = 1.0, \sigma = 2.0, \alpha = 0.5$ (bottom right plot) respectively.



The game tree below summarizes the signalling game that arises from this framework. The first mover is the firm, which can belong to a continuum of types $k \sim \mathcal{U}(0,1)$ and has two financing strategies, namely to issue a contingent green or a non-contingent debt contract $\hat{y}_k = \{cg, nc\}$. The second mover is the investor, which has prior belief over the firm's type given by the distribution function $\beta(k) \sim \mathcal{U}[0,1]$.³³

³³Note that in principle, the investor has also two strategies, which is to either buy or refuse the proposed contract \hat{y}_k . However, since for the firm is a strictly dominant strategy to issue at least one contract among $\{v, g, cg\}$ (this because $\min\{\mathcal{U}_v^f, \mathcal{U}_g^f(k), \mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k)]\} \geq R > 0$), we can already exclude an equilibrium outcome where the investor refuses the contract and focus on the simplified signalling game described in the graph.



The right branch of the tree shows that if the firm proposes a non-contingent contract nc, then it will either attain the type-independent utility $\mathcal{U}_v^f = R$ (case $\hat{y}_k = v$), or the type-specific utility $\mathcal{U}_g^f(k)$ (case $\hat{y}_k = g$). Importantly, by choosing a non-contingent contract nc, the firm effectively chooses not to retain an informational advantage over the investor, as it will be either not incentivized to manipulate (case $\hat{y}_k = v$), or it will credibly reveal its type at issuance by revealing the optimal action (case $\hat{y}_k = g$). Specifically, through ex-ante commitment to actions a_k^g , the non-contingent green contract g allows the investor to perfectly infer firm's type k at issuance, and therefore to update its prior belief $\beta(k)$ from a distribution function to the atomistic type k. On the other hand, the left branch shows that, as the contingent contract does not require ex-ante commitment to actions, the firm that issues this contract retains an informational advantage over the investor which the firm can exploit to a different extent (depending on its true type) through manipulation. More specifically, denoting the set of types that issue the contingent contract as $\mathcal{K} := \{k \in [0, 1] \ s.t. \ \hat{y}_k = cg\}$, the investor's posterior belief follows the distribution function $\beta(k|cg) \sim \mathcal{U}[\mathcal{K}]$, and each firm $k \in \mathcal{K}$ receives a group-specific interest rate

$$\bar{\rho}_{\mathcal{K}}^{cg} = \int_0^1 \bar{\rho}_k^{cg} \beta(k|cg) dk \tag{40}$$

which differs from the type-specific rate $\bar{\rho}_k^{cg}$ in (39). A firm k's expected utility from issuing the contract cg conditional on the investor's posterior belief is then expressed as

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k)|\beta(k|cg)] = \mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k)] + \bar{\rho}_k^{cg} - \bar{\rho}_{\mathcal{K}}^{cg}$$
(41)

with $\hat{\mathcal{U}}_{cg}^{f}(k)$ as in (38). From the expression in (41), one can intuitively anticipate that asymmetric information skews the firm's preferences for issuing contingent contracts towards lower types k. This is because the minimum required interest rate increases with expected distortion, and the lat-

ter decreases with firm type k. Consequently, lower types (those below the average type in group \mathcal{K}) are receiving a lower rate than the benchmark case with perfect information, i.e. $\bar{\rho}_{\mathcal{K}}^{cg} < \bar{\rho}_{k}^{cg}$ such that $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] > \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)]$, whereas higher types (those k above the average type in group \mathcal{K}) are receiving a higher rate than the benchmark case with perfect information, i.e. $\bar{\rho}_{\mathcal{K}}^{cg} > \bar{\rho}_{k}^{cg}$ such that $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] < \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)]$. Effectively, by issuing the contingent green contract, higher types contribute to lowering the average group-specific rate and thus end up subsidising lower types. In an equilibrium setting, issuing a contingent contract would then be viewed by the investor as a signal of being a lower type. On the other hand, issuing a non-contingent green contract would allow the good types to differentiate themselves from the group of those that would be better off keeping their types private. We introduce the following

Perfect Bayes Equilibrium (PBE) The pair $(\hat{y}_k, \beta(k|\hat{y}_k))$ is a PBE if for each $k \in [0, 1]$ it exists a \mathcal{K} such that

$$\hat{y}_k : \begin{cases} = cg \quad if \quad k \in \mathcal{K} \\ = nc \quad if \quad k \notin \mathcal{K} \end{cases}$$

$$\tag{42}$$

maximizes the firm's issuance problem in

$$\hat{\mathcal{U}}^{f}(k) = \mathcal{U}^{f}_{nc}(k) + \max_{y} \mathcal{I}(y) \left(\mathbb{E}[\hat{\mathcal{U}}^{f}_{cg}(k)|\beta(k|cg)] - \mathcal{U}^{f}_{nc}(k) \right)$$
(43)

where the investor's posterior belief $\beta(k|cg) \sim \mathcal{U}[\mathcal{K}]$.

Formalizing the inutition above, one would look for an equilibrium of the form $\mathcal{K} = [0, e)$ for some cutoff type $e \in [0, 1]$ which verifies the *single-crossing* property³⁴

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{nc}^{f}(k) > 0 \quad for \quad k < e, \quad \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{nc}^{f}(k) < 0 \quad for \quad k > e \quad (44)$$

with $\beta(k|cg) \sim \mathcal{U}[0, k]$. Meaning that types below the cutoff *e* would be strictly better off issuing the contingent contract *cg* whereas types above *e* would prefer issuing the best among non-contingent contracts. Clearly, a sufficient condition for (44) to hold is that firm *k*'s profits from issuance of the contingent contract are strictly *decreasing* in *k*. Such condition is not always verified in our setting, in that firm *k*'s profits from issuance of the contingent contract are strictly *decreasing* in *k*. Such condition is not always verified in our setting, in that firm *k*'s profits from issuance of the contingent contract are increasing in *k* for $k \in [0, 2\alpha\theta)$, though at a weaker rate than in the case of perfect information. Nonetheless, as de-

³⁴As outlined in Mailath [1987], the single-crossing property is necessary and sufficient for the existence of a (semi-) separating PBE in case the first mover has continuum one-dimensional types.

tailed in Appendix B, condition (44) is verified for some admissible range of the model parameters. Importantly, for the semi-separating PBE to exist (i.e. for $e \in (0,1)$), it must necessarily be that the opportunity cost of ex-ante commitment is sufficiently relevant.³⁵ More formally, we prove in Appendix B the following

Proposition 3. Provided the model parameters $(\theta, \psi, \alpha, \sigma)$ are such that the single-crossing condition (44) is verified for $k \in [0, 1]$, the following PBE are possible

- the cutoff type is e = 1, in which case $\hat{y}_k = v$ for $k \in [0, 2\alpha\theta]$, whereas $\hat{y}_k = g$ for $k \in (2\alpha\theta, 1]$.
- the cutoff type $e \in (2\alpha\theta, 1)$, in which case $\hat{y}_k = cg$ for $k \in [0, e]$, whereas $\hat{y}_k = g$ for $k \in (e, 1]$.
- the cutoff type is e = 0, in which case $\hat{y}_k = cg$ for all types $k \in [0, 1]$.

Figure 5 shows how the cutoff types e varies with admissible choices of the distortion cost ψ and the action cost θ for two values of the verification cost α . Note that with asymmetric information, we obtain that across all possible choices of the model parameters that admit an equilibrium, non-contingent green debt contracts are more likely to be issued by higher types, contingent green debt contracts are more likely to be issued by intermediate types, whereas non-contingent vanilla contracts are more likely to be issued by lower types. Such prediction is markedly different from that obtained under perfect information, whereby the best types would always issue the contingent green debt, and motivates the empirical section that follows.³⁶

7 Empirical Testing

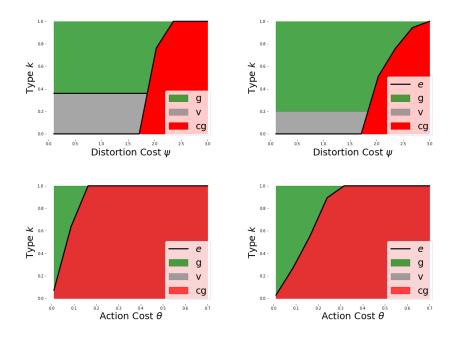
The corporate financing model outlined in the previous section explains broad development trends in the sustainable finance market. Specifically, the model attributes the observed co-existence of noncontingent green debt and contingent green debt to the presence of a measurement friction affecting the investor's ability to observe green outcomes on which contingencies are based. According to the model, non-contingent green debt contracts exist because of the possibility to manipulate reported green outcomes. A rational investor that anticipates a certain level of manipulation associated with

³⁵This because the opportunity cost of ex-ante commitment accounts for the necessary negative correlation between issuance preferences for contingent contract cg and firm's type k, outlined in the single-firm section.

 $^{^{36}}$ In the appendix, we also allow for multiple projects and the possibility to finance each project independently with a different debt contract. Equilibrium results do not alter the baseline model predictions in presence of asymmetric information in that best types should finance at least one project via non-contingent green debt g.

Figure 5. Equilibrium Contract Choice - Asymmetric Information

The plots show the firm's optimal contract choice as a function of the type k (y-axis) and the cost parameters ψ (x-axis, top plots) and θ (x-axis, bottom plots) respectively for two distinct values of the verification cost α (left and right-hand plots respectively). Model parameters are $\theta = 0.2, \sigma = 2.5, \alpha = 1.0$ (top left plot), $\theta = 0.2, \sigma = 2.5, \alpha = 0.5$ (top right plot), $\psi = 3.0, \sigma = 2.5, \alpha = 0.7$ (bottom left plot), and $\psi = 3.0, \sigma = 2.5, \alpha = 0.2$ (bottom right plot) respectively.



the contingent contract will demand an interest rate that is higher relative to that one associated with non-contingent green debt, making the former contract less appealing. Along with this baseline theoretical result, the model predicts that firms' preferences for issuing the contingent and noncontingent debt securities should vary depending on the degree of information available to the investor. The analysis that follows aims to test such predictions combining green securities data with issuers characteristics. Although the model remains generic with respect to the form of the green outcome on which contingencies are based, we focus our analysis on greenhouse gas (GHG) emissions given that they are the most popular metric on which sustainability-linked loans and bonds are written (see Appendix A on securities data) and they are available for a broader set of firms than other pro-environmental and social/sustainable outcomes.

7.1 Issuer Data

We construct the dataset by matching securities issuance data from Bloomberg with issuers' financial and emissions data from Standard & Poor (S&P) Trucost.³⁷ The securities considered, which will collectively be referred to as green debt securities, are sustainability-linked loans and bonds (categorized as contingent debt), and green bonds and loans, social bonds, sustainability bonds (categorized as non-contingent debt). The S&P Trucost database provides quality-checked carbon emissions data differentiating between Scope 1, Scope 2, and Scope 3 emissions as defined by the GHG Protocol Standard.³⁸ We restrict our empirical analysis to the time period between 2017 and 2021, covering the years in which both contingent and non-contingent green debt categories are present in the market.³⁹ We find a total of 661 firms with unique Bloomberg Tickers issuing a total of 1,847 green debt securities between 2017 and 2021, where 334 of those securities are categorised as contingent green debt and the remainder as non-contingent green debt. We also include Environmental, Social, and Governance (ESG) performance ratings in the analysis by matching firms in our dataset with the universe of firms in Sustainalytics.⁴⁰ Sustainalytics is a Morningstar rating company which measure a company's exposure to industry-specific ESG risks and how well a company is managing those risks. As reported in Appendix A, Sustainalytics is the most popular rating provider on which contingent green debt securities are written on. Following this merge, we obtain a total of 476 unique firms with ESG ratings in 2017.

Table 1 reports summary information as of 2017 on the firms in our sample (column Issuers) comparing them with the universe of firms in Trucost (column S&P Trucost Universe). From a financial perspective, the average issuer of green debt securities is larger, has a higher proportion of debt in its capital structure, and is more profitable than the average firm in S&P Trucost. From an environmental perspective, the average issuer is more likely to self-report its emissions (and consistently with its larger size, reports higher emissions levels than the average firm in S&P Trucost), as well as more likely to be tracked by the ESG rating provider. To the extent that

 $^{^{37}\}mathrm{We}$ match issuers in Bloomberg with firms in S&P Trucost using their ticker symbol where possible and using the name for the remainder.

³⁸The GHG Protocol Corporate Accounting and Reporting Standard provides requirements and guidance for companies and other organizations preparing a corporate-level GHG emissions inventory. Scope 1 covers direct emissions from owned or controlled sources. Scope 2 covers indirect emissions from the generation of purchased electricity, steam, heating and cooling consumed by the reporting company. Scope 3 includes all other indirect emissions that occur in a company's value chain. Source: https://ghgprotocol.org/corporate-standard.

³⁹The year 2017 is the start of the market for sustainability-linked loans and bonds.

 $^{^{40}}$ We match the Bloomberg/S&P Trucost dataset with Sustainalytics using the company ticker symbol where possible and using the name for the remainder.

size and the availability of emissions/sustainable performance metrics are barriers to entry in the sustainable finance market (e.g. small firms cannot afford upfront verification costs and/or do not have the technology for writing contingent contracts), these statistics are consistent with the model prediction that the unconditional distribution of debt issuers should feature issuers of vanilla contracts at the lowest end of the type spectrum.⁴¹ Interestingly though, notwithstanding the green financing choice, green issuers receive only marginally better ESG ratings than the universe of ESG-tracked firms in S&P Trucost, suggesting that conditional on issuing green securities, there is still significant variation in the overall quality of the firms' sustainability practices.

Table 1

Summary Statistics

Data are from the Sustainalytics/Bloomberg/Trucost merged dataset. The left column (Issuers) refers to the selected sample of firms that issue at least one green debt security between 2017 and 2021, as identified from the Bloomberg's fixed income database (Appendix A). The right column (S&P Trucost Universe) is the universe of firms in S&P Trucost. Balance-sheet and emissions data are from S&P Trucost and refer to the fiscal year 2017. *All continuous variables are winsorized between the 5^{st} and the 95^{th} percentiles of the pooled distribution. +ESG performance indicators are available for a subset of the sample.

	Issuers		S&P Trucost Universe	
Variable	Mean	(Std.)	Mean	(Std.)
Total Assets [*] (\$ bl)	10.8	(28.8)	5.14	(9.82)
Total Revenues [*] (\$ bl)	9.79	(15.8)	1.91	(3.26)
EBIT to Revenues Ratio [*]	0.37	(0.42)	0.14	(0.21)
Debt to Value Ratio [*]	0.49	(0.19)	0.31	(0.23)
Self-Disclosure of Emissions	0.68	(0.45)	0.25	(0.42)
$Emissions^*$ (ml tCO2e)	5.70	(12.7)	0.86	(1.78)
Tracked by Sustainalytics	0.65	(0.47)	0.27	(0.45)
Sustainalytics ESG Score $^+$	62.3	(11.3)	56.9	(10.4)
Unique Firms	661		14,613	

⁴¹Furthermore, although not directly modelled in our framework, it should be noted that size is also increasing the expected benefits from issuance of a green debt contract, consistent with the view that large firms are more visible and likely face greater level of investor pressure as well as greater exposure to global environmental regulation.

7.2 Issuance by Project Type

Moving to the subset of firms that issue green debt contracts, from the comparative statics section outlined in the single firm model, it emerges that non-contingent green debt contracts are preferred to contingent green debt when the project has either a high level or a low level of materiality. In the model, materiality defines the magnitude of the measurable component represented by costly action relative to the non-measurable uncertain component, meaning how much of the total outcome can be controlled by the firm and credibly verified. Interpreting the green outcome in terms of GHG emissions allows for a neat mapping from the concept of materiality developed in the model to the notion of emissions materiality proposed by the GHG Protocol Standard. The GHG protocol defines a materiality threshold as the maximum percentage difference between the company's reported emissions and the verifier's belief of what the company's emissions would be if all omitted sources were accounted for.⁴² Following this line of reasoning, we define industries as fundamentally material (not material) when their carbon emissions have a high (low) degree of measurement and control, resulting in a low (high) expected discrepancy between the firm's report and the verifier's belief. We order industries according to the degree of materiality of their emissions by making use of the emissions scope breakdown provided by the GHG protocol standard. Scope 1 emissions are those produced by sources directly owned or controlled by the firm, and so they are deemed as most material. Scope 2^+ emissions, which we define as including scope 2 emissions and scope 3 upstream emissions, capture indirect emissions produced by the firm's suppliers or by energy input sources, and so they are deemed as having an intermediate degree of materiality, i.e. intermediate level of control and measurement accuracy. Scope 3 downstream emissions encompass all other indirect emissions produced by the firm's consumers or by its financial investments, and so they are deemed as the least material.

We define an industry-level materiality index as

$$materiality_j = \frac{1}{N_j} \sum_{i=1}^{N_j} m^1 w_{i,j}^1 + m^2 w_{i,j}^2 + m^3 w_{i,j}^3$$
(45)

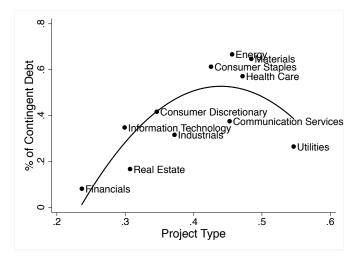
where for each firm *i* in industry *j*, the term $w_{i,j}^1$ is the proportion of scope 1 emissions out of total emissions, $w_{i,j}^2$ is the proportion of scope 2+ emissions out of total emissions, $w_{i,j}^3$ is the proportion of scope 3 emissions out of total emissions, and $m^1 = 1 > m^2 = 0.5 > m^3 = 0$ are decreasing levels

⁴²Information can be found at https://ghgprotocol.org/sites/default/files/standards/ghg-protocol-revised.pdf.

of materiality of each of the emissions scopes. Figure 6 plots the proportion of contingent debt

Figure 6. Issuance Choice by Materiality

The plot shows the proportion of outcome-contingent green debt securities out of the total green debt securities issued between 2017 and 2021 (y-axis) against industry-level materiality index (x-axis). The index is constructed as in (45) using emissions data from S&P Trucost relative to 2017. Industry sectors refer to the Global Industry Classification Standards (GICS) provided by S&P Trucost.



securities relative to all green securities issued between 2017 and 2021 against the industry-level materiality index as of 2017. In line with the model predictions, industries with intermediate levels of materiality are those more likely to issue the contingent green debt. Indeed one observes that both utilities and financial firms, which lie at the end of the materiality spectrum having the lowest and largest share of Scope 1 and Scope 3 downstream emissions respectively, are the most popular issuers of non-contingent green debt.⁴³ The model rationalizes this pattern by showing that the ex-ante commitment to actions associated with non-contingent contracts is less costly when the firm has either very good control of the outcome (such that there is a low opportunity cost of foregoing more profitable investments), or when it has very poor control of the outcome (such that issuing a contingent contract is too costly because of the distortion discount).

7.3 Issuance by Firm Types

The model predicts that when investors have perfect information about firm types, higher types will be the ones issuing contingent green debt as they are the least affected by distortion discounts priced in these contracts, intermediate types will most likely issue non-contingent green debt, whereas lower types will issue vanilla debt or contingent green debt contracts (those who can-

⁴³Figure A.11 in the data Appendix A shows absolute proportions of green issuances by industry.

not afford upfront verification costs). On the other hand, when investors are not informed about firm types, then higher types will be the ones opting for non-contingent green debt contracts to signal their types to the investor, whereas intermediate and lower types will either issue contingent green debt or vanilla debt contracts. Therefore, conditional on the firm issuing a green security, perfect (asymmetric) information would predict a positive or insignificant correlation (significantly negative correlation respectively) between contingent debt issuance and firm types.⁴⁴

Formally, we test for the presence of information frictions by regressing firms' green issuance choice on characteristics that proxy for firms' types. In the model, good types are those that have a better ability to deliver the green outcome (i.e. a *lower* action cost) as well as a worse ability to distort the green outcome in reports (i.e. a *higher* distortion cost). Given that we focus on carbon emission as the green outcome metric, we proxy for the cost of action using the historical emissions intensity of the firm, measured as the logarithm of the firm's total emissions scopes per unit of total assets. The argument is that once controlling for location and industry effects, a higher historical emissions intensity is an endogenous outcome of higher historical abatement costs, in turn predicting lower future abatement capacity, everything else equal. Therefore, we proxy for action cost as

$$actioncost_{i,j} = log(emissions_{i,j}) - log(assets_{i,j})$$

$$(46)$$

where for each firm *i* in industry *j*, emissions_{*i*,*j*} are the sum of scope 1, scope 2+, and scope 3 emissions in kilo tons of carbon dioxide equivalent⁴⁵ (ktCO2e) and $assets_{i,j}$ refers to total assets in million dollars. Proxying distortion costs using realized manipulation is challenging in that one cannot disentangle reported from actual carbon emissions data. To circumvent this challenge, we conceptualize manipulation as greenwashing, defined as selective disclosure of information about a company's environmental or social performance so as to create an overly positive corporate image [Netto et al., 2020]. Following this definition, we measure manipulation propensity as the historical discrepancy between the firm's overall corporate sustainability image, as measured by the aggregate ESG score provided by Sustainalytics, and a credible signal of environmental commitment embedded in these scores, captured the firm's actual adoption of an Environmental Management System (EMS), and whether the adopted EMS is certified by a third party. In defining an EMS, the

⁴⁴This is illustrated in Figures 4 and 5, specifically the variance of the green and red regions across types.

⁴⁵Carbon dioxide equivalent or CO2e is a term for describing different greenhouse gases in a common unit. For any quantity and type of greenhouse gas, CO2e signifies the amount of CO2 which would have the equivalent global warming impact.

Environmental Protection Agency (EPA) explicitly ties the adoption of environmental information systems to a firms' positive environmental performance.⁴⁶ Furthermore, Lyon and Maxwell [2011] provide evidence that corporate adoption of a high-quality EMS reduces incentives for greenwash, in that a well functioning EMS not only increases the firm's information about the green outcome but it also makes it more difficult for the firm to manipulate the measurement system which monitors those outcomes. Therefore, we proxy for distortion cost as

$$distortion cost_{i,j} = ems_{i,j} - esg_{i,j} \tag{47}$$

where $esg_{i,j}$ is the industry-standardized ESG score of firm *i* in industry *j* and $ems_{i,j}$ is the subcomponent of the score that indicates whether the firm has adopted an EMS and whether the EMS has been externally certified. The assumption in our model that the costs of action and distortion are negatively correlated is supported by empirical evidence reported in Table A.6 in Appendix A which confirms a negative correlation between the selected proxies for actions and distortion costs also controlling for industry and location fixed effects.⁴⁷

Table 2 reports linear regressions of firm's issuance choice on the selected proxies for firm types. The dependent variable is a dummy equal to 1 if the firm issues only non-contingent debt securities between 2017 and 2021. The regressors are the firm's action and distortion costs proxies as well as other controls for the firm's financial conditions, all as observed in 2017. The column Regression I refers to the entire sample of firms, while column Regression II refers to the subsample of firms tracked by Sustainalytics. The first thing to note is that the cost of action, as proxied by the firm's historical emissions intensity, is strongly positively correlated with the propensity to issue a contingent green debt contract. Importantly, the correlation remains statistically significant across both the sample choices and when controlling for industry fixed effects, financial characteristics,

 $^{^{46}}$ Specifically, the Environmental Protection Agency (EPA) defines an environmental management system (EMS) as [..] a framework that helps an organization achieve its environmental goals through consistent review, evaluation, and improvement of its environmental performance. The assumption is that this consistent review and evaluation will identify opportunities for improving and implementing the environmental performance of the organization. See https://www.epa.gov/ems/learn-about-environmental-management-systemswhat-is-an-EMS. The most widely used EMS standard is the International Organization for Standardization (ISO) 14001 developed by the Environmental Protection Agency (EPA) and the Eco-Management and Audit Scheme (EMAS) developed by the European Commission.

⁴⁷The regression Table shows that the correlation flips sign and becomes statistically insignificant when controlling for firm's financial characteristics. The reason is primarily related to the fact that firm's revenues, which are strongly negatively correlated with action costs, are also strongly positively correlated with the firm's overall ESG score (without any effect on the EMS-related sub-component of the score), therefore capturing other the relation between action and distortion costs.

Table 2 Security Choice - Linear Regressions

Linear regressions of green debt security choice between 2017 and 2021 on issuers characteristics as of 2017. The dependent variable is a dummy indicator equal to 1 if the firm issues uniquely an outcome-based contingent green debt contract in the observation period, and 0 otherwise. Regressors are collected from Bloomberg/Sustainalytics/S&P Trucost merged dataset. *,**,*** indicate statistical significance at the 10%, 5% and 1% level respectively.

Regressor	Regression I I		tegression II			
Cost of Action	0.06^{***} (0.01)	0.04^{**} (0.02)	0.05^{***} (0.02)	0.07^{***} (0.01)	0.04^{**} (0.02)	0.04^{**} (0.02)
Cost of Distortion	(0.01)	(0.0_)	(0.02)	-0.01^{***} (0.00)	(0.02) -0.01*** (0.00)	(0.02) -0.01** (0.00)
Log Revenues		-0.03^{**} (0.01)	-0.02 (0.01)	~ /	-0.06*** (0.02)	-0.03^{*} (0.02)
EBIT to Revenues Ratio		0.07 (0.05)	0.04 (0.05)		0.03 (0.07)	-0.01 (0.07)
Debt to Value Ratio		-0.16 (0.10)	-0.14 (0.09)		-0.13 (0.12)	-0.09 (0.12)
Self-Disclosure of Emissions		(0.16^{***}) (0.04)	(0.03) (0.04)		(0.01) (0.01)	(0.02) (0.05)
Tracked by Sustainalytics		(0.01) (0.01) (0.04)	(0.02) (0.04)		(0.01)	(0.00)
Intercept	Yes	Yes	Yes	Yes	Yes	Yes
Industry Dummy	No	Yes	Yes	No	Yes	Yes
Location Dummy	No	No	Yes	No	No	Yes
\mathcal{R}^2 Unique Firms	$\begin{array}{c} 0.11 \\ 647 \end{array}$	$\begin{array}{c} 0.18\\ 647 \end{array}$	$0.26 \\ 647$	$\begin{array}{c} 0.14 \\ 476 \end{array}$	$0.21 \\ 476$	$0.28 \\ 476$

as well as for location fixed effects. One notes that firms issuing contingent securities have lower revenues relative to non-contingent green debt issuers in the same sector, which interpreted in light of the recent evidence in De Haas, Martin, Muûls, and Schweiger [2021] that financial constraints inhibit corporate investment in green technologies, provides further support to the model prediction that contingent debt issuers are not the best environmental types. Interestingly, contingent issuers are more likely to self-disclose emissions voluntarily than the remainder of green issuers in the same sector, but the significance seems to be mostly driven by location fixed effects.⁴⁸ On the other hand, as summarized by the regression coefficients on the dummy variable Tracked by

⁴⁸It is worth noting that in the empirical literature on corporate environmental disclosure, there are sharply conflicting results regarding the relationship between the firm environmental performance and its disclosure propensity. For example, Cho and Patten [2007] find that firms with worse environmental records have higher levels of environmental disclosures, while Clarkson, Li, Richardson, and Vasvari [2008] find that firms with better environmental records have higher level of disclosure. In their theoretical study, Lyon and Maxwell [2011] argue that one should expect a non-monotonic relationship between expected environmental performance and disclosure propensity.

Sustainalytics, it seems that being publicly rated at the sustainability level is not a statistically significant determinant of the firm's issuance choice, consistently with the fact that conditional on the issuance of green debt, issuance choices are not driven by a different cost of access to the technology on which contingent securities are written on. Moving on to the subsample of firms tracked by Sustainalytics, one finally notes that issuers of contingent debt contracts also have significantly lower distortion costs relative to the remainder of green issuers in our dataset, as proxied by our metrics of greenwashing, an effect that remains statistically significant when controlling for industry fixed effects, financial characteristics, as well as for location fixed effects.

7.4 Ex-post Debt Performance

To complete the analysis, we look at the ex-post financial performance of contingent and noncontingent green debt securities. We recall that in the model, because of a binding investor participation constraint in equilibrium, all securities issued are expected to yield zero total returns. Specifically, the interest rate on each green debt contract is set to offset – in expectation or across states – the green outcome delivered by the project, in such a way that the monetary and green returns sum to zero. Consequentely, the model predicts that in presence of asymmetric information, contingent green debt contracts issued by lower firm types are expected to yield higher monetary returns so as to compensate the investor for lower green outcomes. To test this implication, we look at differences in the *green bond premia* across the two types of green debt securities, namely contingent and non-contingent debt, where the green bond premium is defined as the negative yield differential between green bonds and the conventional bond counterparts traded in the secondary market. Our empirical estimation follows the methodology in Zerbib [2017], but we are interested in yield differentials across contingent and non-contingent green debt rather than estimating the magnitude of the green bond premium per se.

For this analysis we restrict our attention to the sample of public debt and disregard private green debt securities, namely green loans and sustainability-linked loans. Specifically, we estimate the green premium of green, social, and sustainable bonds (non-contingent green bonds) and compare it with that one of sustainability-linked bonds (contingent green bonds) by using a matching methodology which consists of constructing pairs of securities with the same properties except for the one property whose effects we are interested in. That is, for each green issuer summarized in Table 1, we first collect from Bloomberg the list of conventional bonds issued by that same firm in the same year, finding a total 5,059 of conventional bond issuances against 754 total green issuances (79 contingent and 675 non-contingent bonds respectively). We pair each of the 754 green securities with a conventional bond (or a set of conventional bonds) with similar characteristics from the same issuer, meaning one with the closest maturity, bond type, coupon type, issue year and currency. We disregard differences at the rating level given that only half of the securities are rated. However, in green premium determinants regressions we account for differences in credit ratings at the issuer-level, as well as maturity and coupon biases due to the fact that maturities and coupon rates are not exactly equal. This exercise leaves us with a dataset of 368 pairs of green-conventional bonds (of which 29 contingent green-conventional and 339 non-contingent green-conventional respectively). For each pair of green-conventional bonds, we collect weekly ask yields since the issuance of the green security until the second week of September 2021, and measure the green premium as the average yield differential between each pair of green and conventional bonds.⁴⁹ We use average differentials in bid-ask spreads across green and conventional bonds to control for yield differences related to the liquidity bias (see Beber, Brandt, and Kavajecz [2009]).

Table 3 shows the results of the regression of the green premia for each green-conventional bond pair against a selected set of bonds characteristics. The first column shows that when controlling for currency, bond structure, coupon type and issue year, we find negative yet statistically insignificant green premia of approx -35 basis points between non-contingent green and conventional bonds, and approx -29 basis points between contingent green and conventional bonds. In other words, the green bond premium seems to be 6bp larger for non-contingent green bonds than contingent green bonds, consistent with the evidence summarized in Table 2 which indicates that bad types are more likely to issue contingent green debt, although regression coefficients are not statistically significant. Column two shows that the results do not change when accounting for liquidity effects, residual differences in maturity and coupon rates, as well as differences in amount issued, although the magnitude of the coefficients changes as liquidity and maturity seem to be relevant determinants of yield differentials. Interestingly, the third regression shows that when controlling for rating differences at the issuer level, the positive difference across contingent and non-contingent yield spreads becomes larger (30bp on average) and statistically significant (see also Figure A.12 in the Appendix A). This is in line with evidence that issuer credit rating is one of the strongest

⁴⁹We select ask yields following the methodology in Zerbib [2017]. When more than one conventional bond is available, we take the average across each of the ask yields. If, on a specific week, the green or conventional ask yields are not available, we remove that observation from the dataset. The result is a cross-section of 368 green premia.

Table 3 Debt Performance - Linear Regressions

Linear regressions of the green bond premium on the bonds characteristics. The premium is expressed in average percentage differences in ask yields between green bonds and their conventional bond counterparts. The variable Issue Amount is the amount of green bond issuance in \$ billions. The variables Δ Liquidity, Δ Maturity, and Δ Coupon refer to differences in average bid-ask spreads, maturity, and coupon rates across the pairs of securities. All variables are collected from Bloomberg. *,**,*** indicate statistical significance at the 10%, 5% and 1% level respectively.

Regressor	G	Green Premium			
Constant	-0.35	-0.28	-0.30		
Contingent Debt	(0.94) 0.06 (0.17)	0.08	(0.62) 0.30^{*} (0.18)		
$\log($ Issue Amount $)$	(0.17)	(0.13) -0.01 (0.03)	-0.01		
Δ Liquidity		(0.03) -0.76^{**} (0.32)	-0.49*		
Δ Maturity		(0.02) 0.06^{***} (0.01)	0.06***		
$\Delta Coupon$		0.03 (0.03)	(0.03) (0.02)		
Currency Dummy	Yes	Yes	Yes		
••••••••••••••••••••••••••••••••••••••	Yes Yes	No No	Yes Yes		
Issuer Rating Dummy	No	No	Yes		
\mathcal{R}^2	0.61	0.70	0.72		
Unique Matches	368	368	368		

determinants of cross-sectional variation in green bond premia reported by Zerbib [2017] and more recently by Larcker and Watts [2020]. Taken together, the reduced-form evidence reported in Table 2 and Table 3 supports the presence of information frictions causing adverse selection in the sustainable finance market, implying that financial markets are not yet channelling funds efficiently to sustain the transition to a green economy.

8 Concluding Remarks

This paper takes account of recent market developments, and develops the first theoretical model that formally captures the key features of the two types of debt contracts on the growing market for sustainable finance. The most prevalent type of green debt contract in the sustainable finance market is the green bond, a fixed income debt instrument which earmarks proceeds for specific green projects, but makes no commitment to deliver green outcomes. In contrast, the newly emerging class of sustainability-linked bonds and loans does not impose ex-ante constraints on the use of proceeds, but instead embeds contingencies that ensure commitment to outcomes. These contingent green debt securities should address the limitations inherent in the design of green bonds by eliminating the need to restrict borrower's actions ex-ante and by making outcomes rather than intentions the focus of green projects financing, yet the observed market outcome points to the co-existence of project-based non-contingent contracts and outcome-based contingent contracts, with some firms employing both. We develop a model of firm financing which incorporates an investor with green preferences into an otherwise standard framework of corporate financing with asymmetric information. Firms seek to finance green projects whose outcome embeds an uncertain, non-measurable component that is revealed only to the firm and can be manipulated. We demonstrate that the co-existence of the two green debt contracts is an equilibrium result when green outcomes are manipulable and firm types differ in their ability to manipulate. In presence of asymmetric information about firms' type, green bonds can be used as an expensive screening device, and we find empirically that contingent green debt securities have lower green premium and are issued by more emissions intensive firms.

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A Data Appendix

The issuance of green/social/sustainability bonds as well as sustainability-linked bonds is governed by the principles put forth by the International Capital Market Association (ICMA), summarized in Figure A.7. The so-called Green Bond Principles (GBPs), Social Bond Principles (SBPs) and Sustainability Bond Guidelines (SBGs) outline four key components involved in launching the respective securities, namely: (1) pledging the use of proceeds for environmental/social/sustainable projects; (2) a process for determining project eligibility; (3) management of the proceeds in a transparent fashion that can be tracked and verified; and (4) annual reporting on the use of proceeds. The issuance of sustainability-linked bonds is instead guided by the ICMA Sustainability-Linked Bond Principles (SLBPs), which revolve around five key components: (1) selection of key performance indicators (KPIs); (2) calibration of sustainability performance targets (SPTs); (3) specification of bond characteristics which vary depending on whether the selected KPI(s) reach or not the predefined SPT(s); (4) reporting of performance on the selected KPIs; and (5) verification of KPI performance level against each SPT.

Guidance regarding the issuance of green loans and sustainability-linked loans is provided by the Loan Market Association (LMA), although it is generally less stringent and more customized than that applicable to their public counterpart. For example, the LMA encourages issuers of sustainability-linked loans to publicly report information relating to their SPTs via annual reports or sustainability reports, but also allows them to choose to share this information privately with the lenders. Review of performance reports is negotiated and agreed between the borrower and lenders on a transaction-by-transaction basis, and is only recommended when reporting about SPTs is not made publicly available or otherwise accompanied by an audit/assurance statement. Unlike the case of sustainability-linked bonds, the review of performance can be external or internal. Whereas the external review means that a borrower should have its performance against its SPTs independently verified by a qualified external reviewer, such as an auditor, environmental consultant and/or independent ratings agency, the internal one means that the borrower has developed the internal expertise to validate the calculation of its performance against its SPTs.

We compile the dataset of sustainable corporate debt using Bloomberg's fixed income database. We extract all corporate bonds and loans that are labelled as Green, Social, Sustainability, or

Figure A.7. The Principles





^{*} Under the GBP, SBP and SBG, an amount equal to the net bond proceeds is dedicated to financing eligible projects (Use of Proceeds Bonds) while under the SLBP, proceeds are primarily for the general purposes of an issuer in pursuit of identified KPIs and SPTs (Sustainability-Linked Bonds). A bond that combines SLB and Use of Proceeds features should apply guidance for both types of bonds.

Sustainability-Linked. More precisely, bonds for which the field "Green Instruments Indicator", "Social Instrument Indicator", "Sustainability Instrument Indicator", "Sustainability Linked Bond / Loan Indicator" is "Yes". We exclude securities whose issuer's BICS (Bloomberg Industry Classification System) is "Government".⁵⁰ Bloomberg applies a green/social/sustainability indicator if the issuer self-report (and/or if relevant documentation is available) that 100% of the proceeds of the debt instrument are devoted to predetermined environmental/social/sustainability-oriented activities. Bloomberg's indicator therefore follows loosely the reference guidelines issued by the ICMA corresponding to each of those categories, in that only the first out of the four key requirements is captured by the indicator.⁵¹ In a similar manner, Bloomberg applies a sustainability-linked label

 $^{^{50}}$ Those issuers include development banks and supranational entities which qualify as corporate due to their private status but are not corporations in a traditional sense.

 $^{^{51}}$ See https://www.icmagroup.org/sustainable-finance/the-principles/ for a complete list of the ICMA reference guidelines, and Shurey (2016) for a *Guide to Green Bonds on the Bloomberg Terminal*.

if the issuer self-reports (and/or if relevant documentation is available) that the debt instrument is linked to a sustainability performance metric, which is again only one of the five key requirements of the SLBPs.

Bonds. As Panel A in Table A.4 indicates, our global sample, which runs from January 2013 through April 2021, contains 4,618 "sustainable" bonds (which comprise 3,758 green, 306 social, 391 sustainable bonds, and 149 sustainability-linked bonds) versus 1,055,033 ordinary corporate bonds. The Table shows that relative to ordinary bonds, sustainable bonds are larger in terms of amount issued (\$289 mil versus \$97 mil), a fact that may owe something to the fixed costs of certifying their green/social/sustainable status. On average, sustainable bonds have a lower coupon rate (about 1.8% difference), and more likely to have a fixed coupon rate than ordinary bonds (76% vs 63% have a fixed coupon, respectively). Consistently with early evidence in Baker et al. [2018], they also tend to have longer maturity and higher credit rating. The maturity gap is perhaps not surprising given that green and sustainability-oriented projects tend to have a longer payback horizon than general corporate projects not aimed at helping the company transition to a more sustainable business model. On the other hand as summarized by Figure A.8, differences in credit ratings are uniquely driven by the class of non-contingent green debt, namely green, social, and sustainability-linked bonds have considerably lower ratings than the universe of bonds issued in the same period.

Loans. As Panel B in Table A.4 indicates, our total sample contains 3,971 "sustainable" loans (consisting of 3,251 green loans and 720 sustainability sustainability-linked loans) versus 108,592 ordinary corporate loans. The Table shows that, similarly to their bonds counterpart, sustainable loans are larger in terms of amount issued and longer in maturity than ordinary loans. Interestingly, the difference in maturity seems to be mostly driven by green loans, as the new class of sustainability-linked loans appear to have an average maturity more similar to ordinary loans. The interest rates associated with sustainable loans is lower than that of ordinary loans (this difference is particularly pronounced for sustainability-linked loans), and similarly to ordinary loans, sustainable loans predominantly have a floating interest rate (98% for sustainable loans and 96.5% for ordinary loans). Another interesting fact is that unlike green loans, the majority of sustainability-linked loans is of revolving type. Related to this fact, it is worth mentioning that approximately 20% of the existing sustainability-linked loans were issued as ordinary or green loans, and then later

linked to a metric of sustainability performance. Sustainable loans have a poorer credit coverage compared to ordinary loans but a slightly higher credit rating, and unlike their public counterpart, SLLs have a similar credit rating compared to green loans.

Performance Metrics. We obtain data on the sustainability performance targets (SPTs) underlying sustainability-linked loans and bonds from Bloomberg New Energy Finance (NEF). Table A.5 breaks down the available SPTs⁵² by major categories, namely SPTs based on public Environmental, Social, and Governance (ESG) scores, as well as SPTs based on specific environmental, social, and governance metrics respectively. Worth noting is that 64% of the SPTs are written on environmental metrics, of which 44% are GHGs emissions, a clear evidence of the centrality of climate change with respect to other sustainable issues. In decreasing order, the SPTs based on ESG scores account for roughly 17% of the total sample (which most of those scores being provided by Sustainalytics, the same rating provider that we use in our empirical analysis), whereas social and governance metrics account for roughly 15% and 4% of the remaining SPTs, respectively.

 $^{^{52}}$ One must note that there is not a one-to-one correspondence between SPTs and securities in that one or more SPTs can be associated to the same sustainability-linked bond or loan.

Table A.4 Corporate Sustainable Bonds and Loans

The Table shows summary statistics on corporate bonds (panel A) and loans (panel B) issued between January 2013 and April 2021 as collected from Bloomberg fixed income search. The first column refers to the selected sample of green, social, sustainable, and sustainability-linked securities. The second column refers to the sub-sample of sustainability-linked securities. The third column refers to the entire universe of corporate bonds and loans. The variables Use of Proceeds, Project Selection, Management and Reporting are dummy variables referring to compliance with the four principles issued by ICMA (as observed from ESG reports or other available sources), whether the variable assurance is an indicator equal to 1 if there is third-party assurance of compliance with the principles.

Panel A: Bonds	${\it Green/Social/Sustainable/Sustainability-linked}$	${f Sustainability-linked}$	Ordinary
Variable	Mean	Mean	Mean
Amount Issued (\$ mil)	289	425	97
Coupon Rate (%)	2.5	1.9	4.2
Maturity (years)	8.2	7.7	3.2
Project Selection (%)	96.9	1.3	0.4
Management (%)	95.5	1.3	0.3
Reporting (%)	95.4	1.3	0.3
Assurance (%)	85.1	6.7	0.3
Securities	4,618	149	1,055,033
Panel B: Loans	Green/Sustainability-linked	Sustainability-linked	Ordinary
Variable	Mean	Mean	Mean
Loan Tranche Size (\$ mil)	214	695	326
Is Loan Revolving $(\%)$	18.2	57.2	25.9
Coupon Rate (%)	2.6	1.9	4.5
Maturity (years)	15.7	7.8	8.6
Project Selection (%)	7.9	0.7	0.3
Management (%)	6.6	0.7	0.2
Reporting (%)	5.1	1.1	0.2
Assurance (%)	2.5	1.0	0.1
Securities	3,971	720	108,592

Figure A.8. Credit Ratings by Security Type

The histogram shows the distribution of Standard & Poor (S&P) credit ratings of corporate bond securities issued between January 2013 and April 2021. Grey bars refer to the entire universe of corporate bonds, green bars refer to the subset of corporate bonds which are labelled as Green, Social, or Sustainable, whereas red bars refer to the subset of corporate bonds which are labelled as Sustainability-linked.

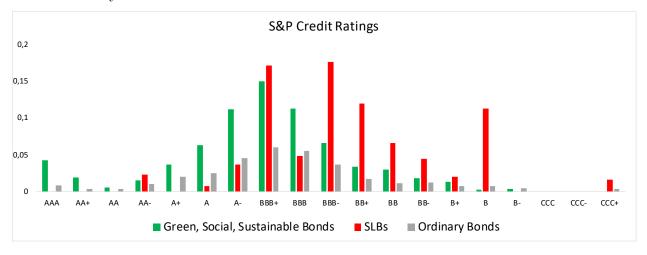


Figure A.9. Holders by Security Type

The histogram shows the distribution of holding shares of corporate bond securities issued between January 2013 and April 2021 by type of investor. Grey bars refer to the entire universe of corporate bonds, green bars refer to the subset of corporate bonds which are labelled as Green, Social, or Sustainable, whereas red bars refer to the subset of corporate bonds which are labelled as Sustainability-linked.

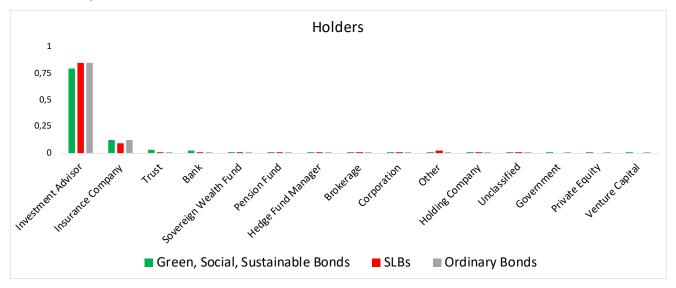


Table A.5

Sustainability Performance Targets (SPTs)

The table breaks down the target performance metrics linked to Sustainability-Linked Loans (SLLs) and Bonds (SLBs) by categories types (general ESG Scores, Environmental metrics, Social metrics, Governance metrics) and sub-categories respectively. Data are collected from Bloomberg NEF and refer to issuance of SLLs as of May 2021.

ESG Score	Environmental Metrics	Social Metrics	Governance Metrics	
143	537	124	38	
 Sustainalytics 31% GRESB 12% EcoVadis 10% Vigeo Eiris 6% Other/Unknown 41% 	 GHGs 44% Renewables 16% Waste 14% Energy Efficiency 7% Water 5% Transport 3% Other/Unknown 11% 	 Work Accidents 21% Labor Rights 11% Female Staff 6% Education 5% Social Returns 3% Disabilities 2% Other/Unknown 51% 	Female Board 26%Other 74%	

Figure A.10. Targets by Industry

The bar plot shows the relative proportion of the four target performance categories (e.g. general ESG Score, Environmental metrics, Social metrics, and Governance metrics respectively) across industry sectors ordered by increasing number of SLLs and SLBs issuances.

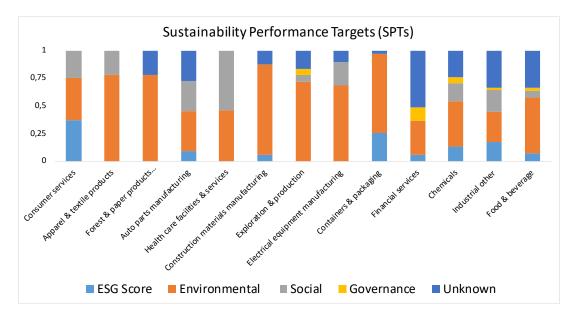


Figure A.11. Issuances by Industry

The top histogram shows the number of "green" issuers in the Bloomberg/S&P Trucost matched dataset by Global Industry Classification (GIC) Sectors. The bottom histogram shows to the conditional proportion of contingent and non-contingent debt in red and green respectively by GIC Sectors.

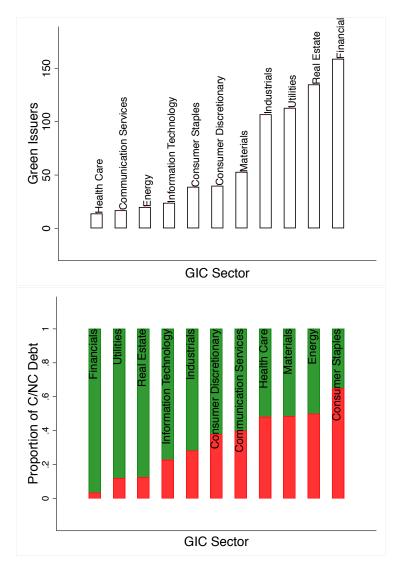


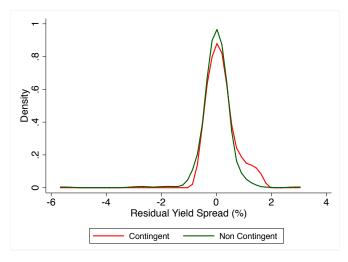
Table A.6 Action and Distortion Cost - Correlations

The table shows correlations (linear regressions) from the firm's distortion cost and action cost as proxied by historical emissions intensity and propensity of greenwashing respectively. Other controls are collected from Bloomberg/Sustainalytics/S&P Trucost merged dataset. *,**,*** indicate statistical significance at the 10%, 5% and 1% level respectively.

	Cost of Distortion		
Cost of Action	- 0.44**	0.00	0.01
Log Revenues	(0.21)	(0.36)	-2.61***
EBIT to Revenues Ratio			(0.34) -3.12** (1.48)
Debt to Value Ratio			(1.40) 1.06 (2.49)
Self-Disclosure of Emissions			(2.45) -0.05^{***} (0.01)
Intercept	Yes	Yes	Yes
Industry Dummy	No	Yes	Yes
Location Dummy	No	No	Yes
_			
\mathcal{R}^2	0.01	0.26	0.41
Unique Firms	476	476	476

Figure A.12. Spread Differentials - Regression Residuals

The plot shows the distribution of residuals in green-conventional bond green premia grouped by type of green security (e.g. contingent green bonds in red and non-contingent green bonds in green respectively). Residuals are obtained from the regression of yield spreads on bond characteristics where the dummy variable Contingent Debt has been excluded.



B Model Appendix

Proposition 1. The financing choice

$$y^* = cg \quad \text{iff} \quad \mathbb{E}[\mathcal{U}_{cg}^f] - \mathcal{U}_{nc}^f > 0 \tag{48}$$

where

$$\mathbb{E}[\mathcal{U}_{cg}^f] = \mathcal{U}_v^f + \mathbb{E}[(\frac{1}{2\theta} + \sigma \tilde{z})^+]$$
(49)

and $\mathcal{U}_{nc}^f = \mathcal{U}_v^f + \max\{0, \frac{1}{2\theta} - \alpha\}$. For any (σ, θ, α) condition (48) reads

$$\mathbb{E}\left[\left(\frac{1}{2}\frac{1}{\theta} + \sigma\tilde{z}\right)^{+}\right] - \max\{0, \frac{1}{2}\frac{1}{\theta} - \alpha\} \ge \mathbb{E}\left[\left(\frac{1}{2}\frac{1}{\theta} + \sigma\tilde{z}\right)^{+}\right] - \frac{1}{2}\frac{1}{\theta} > \frac{1}{2}\frac{1}{\theta} + \sigma\mathbb{E}\left[\tilde{z}\right] - \frac{1}{2}\frac{1}{\theta} = 0$$
(50)

from which the proof follows.

Proposition 2. The financing choice

$$\hat{y} = cg \quad \text{iff} \quad \mathbb{E}[\hat{\mathcal{U}}_{cg}^f] - \mathcal{U}_{nc}^f > 0 \tag{51}$$

where

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^f] = \mathcal{U}_v^f + \mathbb{E}[(\frac{1}{2\theta} - \frac{\sigma^2}{2\psi} + \sigma\tilde{z})\{\frac{1}{2\theta} + \frac{\sigma^2}{2\psi} + \sigma\tilde{z} > 0\}]$$
(52)

For any pair (σ, θ) , it holds that

$$\lim_{\psi \to 0} \mathbb{E}\left[\left(\frac{1}{2\theta} - \frac{\sigma^2}{2\psi} + \sigma\tilde{z}\right) 1\left\{\frac{1}{2\theta} + \frac{\sigma^2}{2\psi} + \sigma\tilde{z} > 0\right\}\right] = -\infty$$
(53)

since the project will be picked with probability one whereas the distortion discount will approach infinite. This implies that for any (σ, θ, α)

$$\lim_{\psi \to 0} \mathbb{E}[\hat{\mathcal{U}}_{cg}^f] - \mathcal{U}_{nc}^f < 0 \tag{54}$$

which by definition of the limit proves the result. On the other hand for any (σ, θ, α) one has

$$\lim_{\psi \to +\infty} \mathbb{E}\left[\left(\frac{1}{2\theta} - \frac{\sigma^2}{2\psi} + \sigma\tilde{z}\right) 1\left\{\frac{1}{2\theta} + \frac{\sigma^2}{2\psi} + \sigma\tilde{z} > 0\right\}\right] = \mathbb{E}\left[\left(\frac{1}{2\theta} + \sigma\tilde{z}\right)^+\right] = \mathcal{U}_{cg}^f \tag{55}$$

which by definition of the limit and the result stated in Proposition 1 proves the result.

The expected green outcome delivered by the non-contingent green debt g reads

$$\mathbb{E}[x^g g(a^g, \tilde{z})] = \mathbb{E}[\frac{1}{\theta} + \sigma \tilde{z}] = \frac{1}{\theta}$$
(56)

whereas the expected green outcome delivered by the outcome-contingent green debt cg reads

$$\mathbb{E}[x^{cg}g(a^{cg},\tilde{z})] = \mathbb{E}[(\frac{1}{\theta} + \sigma\tilde{z})1\{\frac{1}{2\theta} + \sigma\tilde{z} > 0\}]$$
(57)

It is simple to show that (57) is strictly convex in σ , whereas in the proximity of $\sigma = 0$, it holds

$$\lim_{\sigma \to 0} \frac{\partial}{\partial \sigma} \mathbb{E}[x^{cg} g(a^{cg}, \tilde{z})] < 0$$
(58)

Nothing that (57) equates (56) for $\sigma = 0$, it then follows that when σ is small, the expected outcome in (57) is lower in expectation than the one in (56), meaning that the outcome delivered by the contingent security cg is lower in expectation that that one delivered by the non-contingent green debt g, whereas when σ is high, the opposite is true.

The expected green outcome delivered by the contingent security with distortion reads

$$\mathbb{E}[\hat{x}^{cg}g(\hat{a}^{cg},\tilde{z})] = \mathbb{E}[(\frac{1}{\theta} + \sigma\tilde{z})1\{\frac{1}{2\theta} + \frac{\sigma^2}{2\psi} + \sigma\tilde{z} > 0\}]$$
(59)

it is immediate to see that this case lies in between (56) and (57) in that the firm is more likely to implement the green project than in the case without distortion in (57), while still less likely to implement the project than in the non-contingent case (56), meaning

$$\mathbb{E}[x^{cg}] < \mathbb{E}[\hat{x}^{cg}] < x^g = 1.$$
(60)

Corollary 2. Denote the type $k \in [0, 1]$ such that $\theta_k = \theta/k$ and $\psi_k = \psi/(1-k)$, with $0 < \theta < 1/2\alpha$, $\psi > 0$ and $\alpha > 0$.

The utility from issuance of a non-contingent contract reads

$$\mathcal{U}_{nc}^{f}(k) = \mathcal{U}_{v}^{f} + \max\{\mathcal{U}_{g}^{f}(k) - \mathcal{U}_{v}^{f}, 0\} = \mathcal{U}_{v}^{f} + \max\{\frac{1}{2\theta} - \alpha, 0\}$$
(61)

which is a piecewise function of k, whereas

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] = \mathcal{U}_{v}^{f} + \mathbb{E}[(\frac{k}{2\theta} - \frac{1}{2}\frac{\sigma^{2}(1-k)}{\psi} + \sigma\tilde{z})\{\frac{k}{2\theta} + \frac{1}{2}\frac{\sigma^{2}(1-k)}{\psi} + \sigma\tilde{z}\}]$$

$$= \mathcal{U}_{v}^{f} + (\frac{k}{2\theta} - \frac{1}{2}\frac{\sigma^{2}(1-k)}{\psi})F(k,\sigma,\theta,\psi) + \sigma\frac{f(k,\sigma,\theta,\psi)}{F(k,\sigma,\theta,\psi)}$$
(62)

where $F(k, \sigma, \theta, \psi) = \mathcal{N}(\frac{1}{2}\frac{k}{\sigma\theta} + \frac{1}{2}\frac{\sigma(1-k)}{\psi})$ is the cumulative normal distribution and $f(k, \sigma, \theta, \psi) = F'(k, \sigma, \theta, \psi)$ is the density function.

Denote cg' a synthetic contract whose design lies in between cg and nc. Specifically assume the interest rate on cg' varies ex-post as in cg but the project selection is made at issuance as in g. It is simple to show that the utility from this contract reads

$$\mathbb{E}[\hat{\mathcal{U}}_{cg'}^f(k)] = \mathcal{U}_v^f + \frac{k}{2\theta} - \frac{1}{2}\frac{\sigma^2(1-k)}{\psi}$$
(63)

then the net profits $\mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k)] - \mathcal{U}_{nc}^f(k)$ can be decomposed as

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] - \mathcal{U}_{nc}^{f}(k) = \underbrace{\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] - \mathbb{E}[\hat{\mathcal{U}}_{cg'}^{f}(k)]}_{\text{opportunity cost of commitment}} + \underbrace{\mathbb{E}[\hat{\mathcal{U}}_{cg'}^{f}(k)] - \mathcal{U}_{nc}^{f}(k)}_{\text{relative cost of moral hazard}}$$
(64)

allowing to disentangle the two forces driving contingent issuance preferences across types, namely the opportunity to choose projects at an interim date after the uncertain state is realized (term 1) and the relative cost of moral hazard, e.g. distortion discount vs verification cost (term 2). It is simple to show that the first term in (64) is always positive, convex, and monotonically decreasing in the type k. Intuitively, lower types have a poor ability to control green outcomes through action choices, and therefore they face a higher opportunity cost of committing ex-ante to the green project. On the other hand, the second term in (64) can be either positive or negative depending on the contracts' technologies (e.g. distortion cost ψ and verification cost α), is linear and strictly increasing in the type k. The intuition is that higher types have a better ability to implement costly actions as well as a worse ability to manipulate reports, and therefore they benefit more from the introduction of contingencies.

Figure B.13 shows how the net profits in (64) vary across types k for low, intermediate, and high values of the distortion cost ψ , recalling that for extreme values of the distortion cost $\psi = \{0, +\infty\}$ Proposition 2 applies. When the distortion cost is low enough (left-hand plot), the net profits are overall low and strictly increasing in k. This is because the impact of the first term in (64) is negligible as distortion opportunities make the green project choice appealing independently of the realized uncertain state. In such a scenario, it exists a \overline{k} and \underline{k} such that if $k > \overline{k}$ then $\hat{y}_k = cg$, if $k \in [\underline{k}, \overline{k}]$ then $\hat{y}_k = g$, whereas if $k < \underline{k}$ then $\hat{y}_k = v$, which proves the result. When the distortion cost ψ is high enough (right-hand plot), the first term becomes relevant and the net profits in (64) are weakly increasing in k for $k \in [0, 2\alpha\theta)$, whereas they are decreasing in k for $k \in [2\alpha\theta, 1]$. Importantly though, given that manipulation is low, net profits are overall high and such that all firms issue the contingent contract. In such a scenario, $k = \overline{k} = 0$, which again proves the result. On the other hand, when the distortion cost is neither high nor low (mid-plot), there is a small region of other model parameters (where σ is not too high nor too low, and α is sufficiently low) under which preferences for the contingent contract are non-monotonic in k. Specifically while there always exist a $\underline{k} < \overline{k}$ such that if $k < \underline{k}$ then $\hat{y}_k = cg$ whereas if $k > \overline{k}$ then $\hat{y}_k = cg$, it may exist a $k' < \overline{k}$ such that if $k \in [\underline{k}, k']$ then $\hat{y}_k = cg$ whereas if $k \in [k', \overline{k}]$ then $\hat{y}_k = g$. As a matter of fact as the figure reveals, intermediate types are almost indifferent between contingent contracts cq and non-contingent green contracts g in this region of the model parameters.

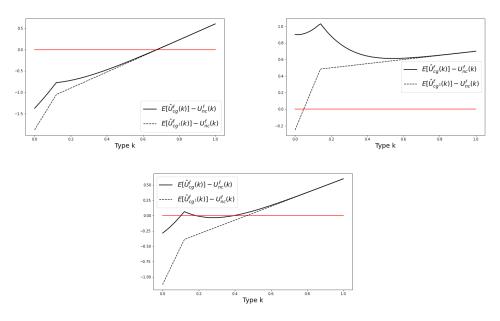
Proposition 3. In presence of asymmetric information, we solve for a semi-separating Perfect Bayes Equilibrium (PBE) of a signalling game where the first mover (the firm) has infinite types $k \sim \mathcal{U}[0,1]$ and two moves (issue a contingent contract or the best of the non-contingent contract) $\hat{y}(k) = \{nc, cg\}$, whereas the second mover (investor) has one type and two moves (accept or refuse the proposed contract) $s = \{1, 0\}$ and belief over the firm's type $\beta(k) \sim \mathcal{U}[0, 1]$. A PBE requires that the firm's issuance strategy is sequentially rational – that is at each information set in which the firm moves, the firm maximizes its expected utility anticipating the investor's beliefs at the information set, and that the investor updates its belief in a Bayesian manner.

A first thing to note is that, independently of the issuance choice, the firm is strictly better off when the investor accepts the proposed contract instead of when it refuses it. This because it holds that $\min\{\mathbb{E}[\mathcal{U}_{cg}^{f}(k)], \mathcal{U}_{nc}^{f}(k)\} \geq R > 0$. Consequentely as in the case of perfect information, the firm will always propose a contract rate so as to satisfy the investor's participation constraint (e.g. the investor always accepts the contract in equilibrium and we can focus on the strategy of the firm).

Following the intuition outlined in the paper, consider the case in which $\mathcal{K} = [0, k)$ is the informa-

Figure B.13. Net Profits From Issuance of Contingent Contract - Perfect Information

The plots show the net profits in (64) (black thick line) and the second component in (64) (black dashed line) as a function of the type k for different values of the model parameters. The red line defines the region below (above) which the firm has strict preference for the contingent contract cg. Parameters $\theta = 0.1$ and $\alpha = 0.6$ and $\sigma = 1.5$, whereas $\psi = 0.6$ (left plot), $\psi = 4.5$ (right plot), and $\psi = 1.6$ (mid plot) respectively.



tion set at which firm k moves when it pursues the strategy $\hat{y}(k) = cg$, and denote $\beta(k|cg) \sim \mathcal{U}[\mathcal{K}]$ the investor's posterior belief about the firm's type after having observed issuance of contract cg. In such a case, the firm's expected utility reads

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] = \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] + \bar{\rho}_{k}^{cg} - \int_{0}^{1} \bar{\rho}_{k}^{cg}\beta(k|cg)dk$$
$$= \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] + \frac{\sigma^{2}(1-k)}{\psi}F(k,\sigma,\theta,\psi) - \frac{1}{k}\int_{0}^{k} \frac{\sigma^{2}(1-k')}{\psi}F(k',\sigma,\theta,\psi)dk' \quad (65)$$
$$\approx \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] - \frac{\sigma^{2}}{\psi}k$$

where the last equality follows immediately once approximating $F(k, \sigma, \theta, \psi) \approx 1$. Plugging the expression in (65) into the net profits in (64) and incorporating asymmetric information, one has

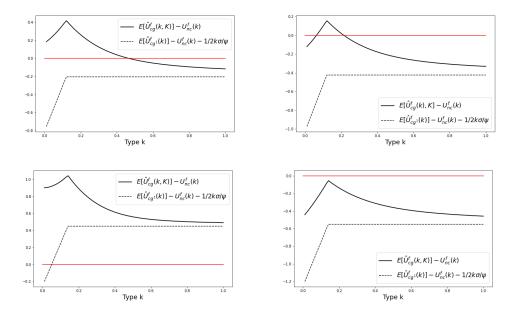
$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{nc}(k) \approx \underbrace{\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)] - \mathbb{E}[\hat{\mathcal{U}}_{cg'}^{f}(k)]}_{\text{opportunity cost of commitment}} + \underbrace{\mathbb{E}[\hat{\mathcal{U}}_{cg'}^{f}(k)] - \mathcal{U}_{nc}^{f}(k) - \frac{\sigma^{2}}{\psi}k}_{\text{relative cost of moral hazard}}$$
(66)

meaning that with respect to the perfect information case in (64), the net profits in (66) have a new (moral hazard-related) term which decreases strictly with the type k, consistently with the

Figure B.14. Net Profits From Issuance of Contingent Contract - Asymmetric Information

The plots show the net profits in (66) (black thick line) and the second component in (66) (black dashed line) as a

function of the type k for different values of the model parameters. The red line defines the region below (above) which the firm has strict preference for the contingent contract cg. Parameters $\theta = 0.1$ and $\alpha = 0.6$ and $\sigma = 1.5$, whereas $\psi = 1.4$ (top left plot), $\psi = 1.1$ (top right plot), $\psi = 4.5$ (bottom left plot), and $\psi = 0.9$ (bottom right plot) respectively.



assumption that $\mathcal{K} = [0, k)$, and in particular that for each type $k \in [2\alpha\theta, 1]$ net profits in (66) are strictly decreasing in k.⁵³ To prove that the case considered is a semi-separating equilibrium, it is sufficient to check for the single-crossing property to be verified (see, for example, the result for one-dimensional signalling games with continuous types in Mailath [1987]). We therefore check for the existence of a unique cutoff type $e \in [0, 1]$ such that

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{nc}^{f}(k) \ge 0 \quad if \quad k \le e, \quad \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{nc}^{f}(k) < 0 \quad if \quad k > e$$
(67)

for any type $k \in [0, 1]$.

Consider the case in which, under perfect information, $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{v}^{f} > 0$ for each $k \in [0, 2\alpha\theta]$. Then $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{v}^{f} > 0$ for each $k \in [0, 2\alpha\theta]$ since $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] = \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(0)]$ and the net profits in (66) increase in k for $k \in [0, 2\alpha\theta]$. This implies that $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] = \mathbb{E}[\hat{\mathcal{U}}_{cg}$

⁵³This follows from the previous discussion recalling that the first term in (66) is strictly decreasing in k and noticing that the second term in (66) is now independent of k for $k \in [2\alpha\theta, 1]$. This is a necessary condition for the existence of a semi-separating PBE and can be used to show that other equilibria of the form $\mathcal{K} = (e, 1]$ for an internal cutoff type $e \in (0, 1)$ cannot exist.

 $\mathcal{U}_{g}^{f}(2\alpha\theta) > 0$, which in turn implies that either condition (67) is verified for $e \in (2\alpha\theta, 1]$ (top lefthand plot in Figure B.14), or e = 1 and all firms issue the contingent contract (bottom left-hand plot in Figure B.14). Consider then the case in which, under perfect information, $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{v}^{f} < 0$ for each $k \in [0, 2\alpha\theta]$. Then necessarily from (66) $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{v}^{f} < 0$ for each $k \in [0, 2\alpha\theta]$. In such a case recalling that $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{g}^{f}(k) < \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{v}^{f} < 0$ for each $k \in (2\alpha\theta, 1]$, condition (67) is verified for e = 0, meaning no firm issues the contingent contract (bottom right-hand plot in Figure B.14). Finally, consider the case in which it exists a $k^{v} \in (0, 2\alpha\theta)$ such that $\mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(k)|\beta(k|cg)] - \mathcal{U}_{v}^{f} = 0$. In such a case an equilibrium does never exist because if there was a cutoff type e, such type would be $e = k^{v}$. But given that firm's utility in (65) increases in k for $k \in [e, 2\alpha\theta]$, condition (67) would be violated (top right-hand plot in Figure B.14).

Robustness

Risk-neutrality. In what follows we show that introducing risk-aversion does not alter the baseline prediction of the model.⁵⁴ Specifically, assume an otherwise equivalent model with a risk-adverse investor, denote Λ the investor's discount factor, with $\mathbb{E}[\Lambda] = 1$ and $Cov(\Lambda, \tilde{z}) < 0$, then recalling the firm's problem in (6), the new investor participation constraint reads

$$-b_0^y + \mathbb{E}[\Lambda(b_1^y + x^y g(\tilde{z}, a^y))] \ge 0$$

$$-1 + \mathbb{E}[\Lambda(1 + \rho^y + x^y g(\tilde{z}, a^y))] \ge 0$$

$$\mathbb{E}[\Lambda(\rho^y + x^y g(\tilde{z}, a^y))] \ge 0$$

$$\mathbb{E}[\rho^y + x^y g(\tilde{z}, a^y)] + Cov(\Lambda, \rho^y + x^y g(\tilde{z}, a^y)) \ge 0$$
(68)

therefore, taking count of risk-aversion amounts to introducing a covariance term in the participation constraint on the contract-specific rate. Such constrained rate therefore becomes

$$\rho^g \ge -\mathbb{E}[g(\tilde{z}, a^g)] - Cov(\Lambda, g(\tilde{z}, a^g)) \tag{69}$$

for the project-based non-contingent green debt, whereas it becomes

$$\bar{\rho}^{cg} \ge \mathbb{E}[\sigma x^{cg}(\tilde{z})d^{cg}(\tilde{z})] - Cov(\Lambda, \sigma x^{cg}(\tilde{z})d^{cg}(\tilde{z}))$$
(70)

 $^{^{54}}$ Similarly, one can show that under the current model specification, a risk-adverse firm would have the same utility function across all contract choices.

for the outcome-based contingent contract. Now recalling that $Cov(\Lambda, g(\tilde{z}, a^g)) = Cov(\Lambda, \sigma \tilde{z})$ and that $Cov(x^{cg}(\tilde{z})d^{cg}(\tilde{z}), \tilde{z}) \geq 0$, it derives that the new covariance term increases the minimum acceptable rate on both the green debt contracts. Notably though, the magnitude of the covariance term in (70) depends on the level of manipulation in the contract. Specifically in absence of manipulation, the covariance term in (70) disappears and the firm has a further reason to issue the contingent contract, in that by doing so it would avoid the risk-premium required by the investor for holding a contract that delivers an uncertain green outcome. Viceversa if the level of manipulation is high (e.g. the distortion cost ψ is low), then the risk-premium required by the investor for holding the contingent contract would be greater than that required for holding the non-contingent green debt, in turn making this contract less appealing, everything else equal. In summary, introducing risk-aversion does not alter the baseline theoretical prediction outlined in the risk-neutral model.

Certain monetary return and firm capital structure. In the model, we assume that monetary returns are certain and therefore we abstract from any analysis regarding the firm's capital structure and how it relates to the investor's green preferences. We show below that in a simple extension of the model which allows for uncertain monetary returns, equity acts as a perfect substitute to vanilla non-contingent debt, and that high firm types should therefore hold more debt relative to low firm types. Specifically, denote $R(\tilde{\epsilon})$ as the uncertain project cashflow with $\mathbb{E}[R(\tilde{\epsilon})] = \bar{R}$ and $Cov(\tilde{\epsilon}, \tilde{z}) = 0$. Assume that the firm can issue equity at the competitive price $e_0 = \$1 + \bar{R}$ at date t = 0 which delivers $e_1 = \$1 + R(\tilde{\epsilon})$ at date t = 1, and denote w as the equity ratio of the firm. Then the firm's utility for a given financing choice w, y becomes

$$\mathcal{U}_{y,w} = \max_{a,x} \mathcal{C}^{f}_{0,y,w} + \mathcal{C}^{f}_{1,y,w} - xc(a)$$
(71)

where

$$\mathcal{C}_{0,y,w}^{f} = we_{0} + (1-w)b_{0}^{y} - 1 = w\bar{R}$$

$$\mathcal{C}_{1,y,w}^{f} = 1 + R(\tilde{\epsilon}) - we_{1} - (1-w)b_{1}^{y} = (1-w)(R(\tilde{\epsilon}) - \rho^{y})$$
(72)

such that

$$-we_{0} - (1 - w)b_{0}^{y} + \mathbb{E}[(1 - w)(b_{1}^{y} + x^{y}g(\tilde{z}, a^{y})) + w(1 + R(\tilde{\epsilon}))] \ge 0$$
$$-w\bar{R} + \mathbb{E}[(1 - w)(\rho^{y} + x^{y}g(\tilde{z}, a^{y})) + wR(\tilde{\epsilon})] \ge 0$$
$$(1 - w)\mathbb{E}[(\rho^{y} + x^{y}g(\tilde{z}, a^{y}))] \ge 0$$

substituting budget and participation constraints into the firm's problem, one gets that the expected utility reads

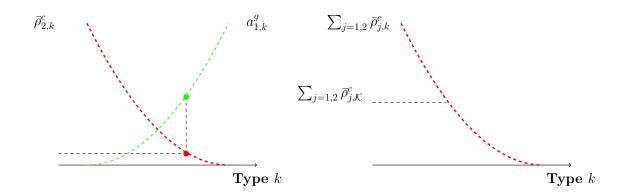
$$\mathbb{E}[\mathcal{U}_{y,w}] = w\bar{R} + (1-w)\mathbb{E}[R(\tilde{\epsilon})] + \mathbb{E}[x^y g(\tilde{z}, a^y)] = \bar{R} + (1-w)\mathbb{E}[x^y g(\tilde{z}, a^y)]$$
(74)

from which derives that the firm is indifferent between debt and equity whenever the expected compensation for the green outcome is zero, whereas has a strict preference for debt when the expected compensation for the green outcome is positive.

Multiple projects. In light of the empirical evidence that some firms issue a combination of contingent and non-contingent green debt contracts, we also look at equilibrium outcomes in the case where the firm has multiple projects and can finance each project independently with a different contract. The additive feature introduced by this model extension, outlined in Appendix B, is that financing one or more projects via non-contingent green debt affects firm k's contract conditions on the remainder of projects financed via contingent green debt. Specifically, issuing a non-contingent green debt contract allows firm k to receive the type-specific interest rate $\bar{\rho}_{k,j}^{cg}$ on each project j financed via the contingent green debt contract. This is because the investor observes firm k's commitment to actions and updates its prior about the level of manipulation (and hence the distortion adjustment embedded in the base rate) prior to the issuance of the contingent security. To clarify the mechanism, consider the simplest case in which the firm has j = 1, 2 independent and identically distributed projects. If implemented, each project j yields an uncertain green outcome

$$g(\tilde{z}_j, a_j) = a_j + \sigma \tilde{z}_j \tag{75}$$

with $\tilde{z}_j \sim \mathcal{N}(0, 1)$, and a certain monetary return \$1 + R. With respect to the baseline case when j = 1, the firm has an additional issuance option in that it can propose a *mixed* debt contract issuance. Specifically, it can propose a contract where project j = 1 or 2 is financed via the optimal type of non-contingent contract (i.e. plain vanilla or green), whereas project -j = 2 or 1 respectively is financed via the contingent green debt contract. As the example below shows, when the mixed option is chosen, and more specifically when the project-based non-contingent green debt contract g is chosen to finance project j = 1, the firm can receive a type-specific rate on the project j = -2 financed through the contingent green debt, whereas if all projects j = 1, 2 are financed using contingent debt, then the investor cannot infer the firm's type k, and hence continues to demand an interest rate that embeds the average distortion across all firm types $k \in \mathcal{K}$.



Noting that projects j = 1, 2 are equivalent at date t = 0, one can express firm k's profits from issuance of the contingent contract as

$$2 \mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k,\mathcal{K})] - \max\{2\mathcal{U}_{nc}^f(k), \ \mathcal{U}_{g}^f(k) + \hat{\mathcal{U}}_{c}^f(k)\}$$
(76)

note that the only difference between (76) and the firm's contract choice in the single project case is that firm k takes count of the new alternative given by the combination of the two green debt contracts. Recalling that the highest type does not distort and that it always prefers the projectbased green debt to the vanilla debt contract⁵⁵, it will exist a type $k \leq 1$ above which the expression in (76) simplifies to

$$\mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k,\mathcal{K})] - \mathcal{U}_g^f(k) + (\bar{\rho}_k^c - \bar{\rho}_\mathcal{K}^c) \tag{77}$$

it is then immediate to note that firm k's preferences are the same as in the single project case except for an additional term $(\bar{\rho}_k^c - \bar{\rho}_{\mathcal{K}}^c)$, which is the discrepancy between the type-specific participation rate and the unconditional average. Since the term is decreasing monotonically in k, it derives that with multiple projects, higher types are even less likely to issue contingent contracts cg than in the case with a single project. This because the alternative mixed contingent and noncontingent green debt is even more appealing that the non-contingent green debt alone. Formally, the following can be proved

Proposition. Suppose the firm can access a set j = 1, 2 of identically distributed projects and let each project's parameters $(\theta, \psi, \alpha, \sigma)$ verify the conditions in Proposition 3. Moreover, define k^v as the type indifferent between a plain vanilla debt contract and a mixed contingent and non-

 $[\]overline{[55]{\text{Specifically, it holds that } \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(1)] > \mathcal{U}_{g}^{f}(1)} = \mathcal{U}_{g}^{f}(1) \text{ and } \mathcal{U}_{g}^{f}(1) > \mathcal{U}_{v}^{f}(1), \text{ hence it follows that } \max\{2\mathcal{U}_{nc}^{f}(1), \mathcal{U}_{g}^{f}(1) + \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(1)]\} = \mathcal{U}_{g}^{f}(1) + \mathbb{E}[\hat{\mathcal{U}}_{cg}^{f}(1)].$

contingent green debt contracts, that is

$$\mathcal{U}_g^f(k^v) + \mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k^v)] = 2\mathcal{U}_v^f$$

Then the pair $(\mathcal{K}^e, \hat{y}(k, \mathcal{K}^e))$ with $\mathcal{K}^e = [0, e)$ is a Bayes Nash equilibrium and the following cases are possible

- if $k^v < 2\theta \alpha$ then
 - if $\hat{y}(0, \mathcal{K}^0) \neq cg$, then e = 0. In such a case, $\hat{y}(k, \mathcal{K}^e) = cg + g$ for $k \in (k^v, 1]$ whereas $\hat{y}(k, \mathcal{K}^e) = v$ for $k \in [0, k^v]$.
 - if $\hat{y}(0, \mathcal{K}^0) = cg$, then $e \in (k_v, 1]$. In such a case, $\hat{y}(k, \mathcal{K}^e) = cg + g$ for $k \in (k^e, 1]$ whereas $\hat{y}(k, \mathcal{K}^e) = cg$ for $k \in [0, k^e]$.
- if $k^v > 2\alpha\theta$
 - $\begin{aligned} & if \, \hat{y}(0, \mathcal{K}^0) \neq cg, \, then \, e = 0. \, In \, such \, a \, case, \, \hat{y}(k, \mathcal{K}^e) = cg + g \, for \, k \in (k^g, 1], \, \hat{y}(k, \mathcal{K}^e) = g \\ & for \, k \in (2\alpha\theta, k^g], \, while \, \hat{y}(k, \mathcal{K}^e) = v \, for \, k \in [0, 2\alpha\theta]. \end{aligned}$
 - $\begin{aligned} &-if\ \hat{y}(0,\mathcal{K}^{0})=cg,\ then\ e\in(2\alpha\theta,1].\ In\ such\ a\ case,\ \hat{y}(k,\mathcal{K}^{e})=cg+g\ for\ k\in(k^{g},1],\\ &\hat{y}(k,\mathcal{K}^{e})=g\ for\ k\in(k^{e},k^{g}],\ while\ \hat{y}(k,\mathcal{K}^{e})=c\ for\ k\in[0,k^{e}].\\ &with\ k^{g}\ as \end{aligned}$

$$\mathcal{U}_g^f(k^g) + \mathbb{E}[\hat{\mathcal{U}}_{cg}^f(k^g)] = 2\mathcal{U}_g^f(k^g)$$