Asymmetries and the Market for Put Options*

Adam Farago † Mariana Khapko‡ Chayawat Ornthanalai§

Abstract

We study implications of asymmetries in both preferences and fundamentals for put option demand across investors and the resulting market behavior. A heterogenous-agent model populated by investors with asymmetric preferences alongside standard risk-averse agents rationalizes the size and the dynamics of the put option market, the expensiveness of put options, and the link between put option demand and the stock market in equilibrium. Disappointment-averse investors take long positions in put options, but only if their reference point is lower than the certainty equivalent. In the cross-section of options with multiple strikes, disappointment-averse investors' open interest peaks for the at-the-money contracts.

Keywords: Disappointment Aversion, Options, Equilibrium Asset Demand, Equilibrium Asset Pricing

JEL Classification. D51, D53, G11, G12, G13

*We have greatly benefited from discussions on this research with Ing-Haw Cheng (discussant), Andrea Eisfeldt, and Michael Hasler. We are grateful to conference participants at the Midwest Finance Association (MFA) 2021 meeting, as well as to seminar participants at the University of Calgary, University of Tilburg, and the Virtual Derivatives Workshop for their insightful comments and suggestions. Financial support from the Canadian Derivatives Institute, University of Gothenburg and the University of Toronto is gratefully acknowledged.

†University of Gothenburg, adam.farago@gu.se

‡University of Toronto, mariana.khapko@rotman.utoronto.ca

§University of Toronto and Canadian Derivatives Institute, chay.ornthanalai@rotman.utoronto.ca

1 Introduction

Index options play a vital role in catering to investors' risk-sharing needs over the business cycles and the market for these products has witnessed impressive growth over the years. On average, daily open interest for CBOE's S&P 500 options rose from one million in 1996 to more than tenfold in 2020. In terms of economic magnitude and potential to transfer risk, this means that the index options market controls trillions of dollars worth of the underlying equity market. About two-thirds of this open interest is comprised of put option contracts, among which out-of-the-money (OTM) put positions dominate. This indicates that a large part of the market is driven by demand for insurance against stock market declines. While the expensiveness of OTM puts and their return have been extensively investigated, little research is dedicated to understanding their *quantities* that are supplied and demanded in equilibrium.[1] Studying the driving forces in index put option market is important and can offer insights to recent empirical connections between index put buying pressure and S&P 500 index returns (Chen, Joslin, and Ni, 2019; Chordia, Kurov, Muravyev, and Subrahmanyam, 2021).

The main contribution of this paper is to show that (i) the size of the put option market, (ii) the expensiveness of low-moneyness put options, and (iii) the link between put option demand and the stock market can be rationalized within a heterogenous-agent model with disappointment-averse risk preferences and tail risk. We find that accounting for asymmetries in preferences coupled with the asymmetric nature of dividend shocks helps to explain the observed level and dynamics of the put option market, provides joint predictions of positions and pricing, and allows for a better understanding of the puzzling observations in the recent literature.

Our focus on disappointment aversion is motivated by the experimental evidence suggesting that investors value gains and losses differently, including cases when the decision is made by teams or professional traders and managers.[2] A growing literature, which we review below, recognizes this asymmetric attitude towards gains and losses, and its important implications for asset pricing. Moreover, a disappointment-averse investor, being particularly concerned about losses, is a natural demander of put protection against market

[1] Notable exceptions include the theoretical model of Buraschi and Jiltsov (2006), the empirical investigations of Bollen and Whaley (2004) and Johnson, Liang, and Liu (2016), and the recent work by Constantinides and Lian (2021).

[2] Choi, Fisman, Gale, and Kariv (2007) and Gill and Prowse (2012) provide experimental evidence that individuals have asymmetric attitudes towards gains versus losses. Sutter (2007) and Haigh and List (2005) show that the same phenomenon emerges for team decisions and professional traders, respectively. Olsen (1997) and Willman, Fenton-O'Creevy, Nicholson, and Soane (2002) provide survey evidence that professional investment managers exhibit aversion to losses. Pope and Schweitzer (2011) argue that experience, competition, and large stakes do not mitigate loss aversion.

corrections. It has been documented that writing put options on the S&P 500 offers high expected returns, suggesting that investors should hold significantly negative positions in these contracts.[3] Indeed, in a partial equilibrium setting, standard power utility investors with various degrees of risk aversion hold negative positions in put options (see e.g. Liu and Pan, 2003; Branger and Hansis, 2012). However, in general equilibrium, since options are in zero net supply, some investors have to be willing to hold put options despite the sellers achieving high returns.

To rationalize the stylized facts about the put option market and extract new testable predictions, we develop a dynamic general equilibrium model with rare event risk and two classes of investors. The model features risk-averse investors with standard CRRA preferences and disappointment-averse investors who are especially concerned about the left-tail outcomes. To model the asymmetric preferences towards gains versus losses, we rely on the disappointment aversion (DA) framework of Gul (1991), in which an investor is loss averse around her certainty equivalent. Considering the definition for the reference point, DA preferences have at least two advantages over the prospect theory of Kahneman and Tversky (1979), which is another commonly used framework to model investors' loss aversion. First, unlike in prospect theory, the threshold distinguishing disappointing outcomes from non-disappointing outcomes is endogenous and related to the investor's expectations. Recent evidence suggests that expectations play an important role in the reference point formation.[4] Second, the generalized disappointment aversion (GDA) of Routledge and Zin (2010), a one-parameter extension of DA preferences, explicitly allows for studying the effect of different reference levels. A GDA investor is loss averse around a reference point that can be different from her certainty equivalent.

Our first main result is that there is a sizable option market in the economy in which some agents have asymmetric preferences over gains and losses. This result stands in sharp contrast to the benchmark economy with heterogenous risk-averse investors in which the size of the option market is negligible, even for very high values of risk aversion. We observe that the option market becomes important quickly as the degree of disappointment aversion increases. In normal times, the disappointment-averse investor buys put options from her

[3] Coval and Shumway (2001), Broadie, Chernov, and Johannes (2009), Benzoni, Collin-Dufresne, and Goldstein (2011), Constantinides, Jackwerth, and Savov (2013), and Bondarenko (2014) document that strategies involving writing put options on the S&P 500 index offer high raw returns. These returns are significant even after standard risk adjustments (Broadie et al., 2009; Bondarenko, 2014), and taking into account leverage (Constantinides et al., 2013).

[4] Choi et al. (2007), Arkes, Hirshleifer, Jiang, and Lim (2008), Abeler, Falk, Goette, and Huffman (2011), and Gill and Prowse (2012) provide experimental evidence that expectations are important in the reference point formation, while Pope and Schweitzer (2011) draw a similar conclusion by studying data from real life contests.

risk-averse counterpart to implement a portfolio insurance strategy and protect her portfolio from drops in the stock price. The risk-averse investor can then be interpreted as representing the intermediary firms making option markets who provide insurance to the disappointment-averse group of investors that includes retail and institutional public investors. In times of economic distress, however, a disappointment-averse investor can switch from buying to selling put options.

Our second main result is that the size of the option market crucially depends on the reference point distinguishing disappointing from non-disappointing outcomes. To reconcile positive demand for put options with their equilibrium returns, this reference point, i.e. the disappointment threshold, needs to be below the certainty equivalent of the investment. When the reference point equals the certainty equivalent, any drop in final wealth below the certainty equivalent constitutes a disappointing outcome. Therefore, the disappointment-averse investor chooses not to participate in risky asset markets and invests all her wealth in the risk-free asset. Moreover, the put option demand is hump-shaped in the disappointment threshold. Intuitively, if the threshold of disappointment is high, the investor has to relinquish the whole upside of the investment to gain full protection against disappointing outcomes. The optimal strategy for such investor is to put all her wealth in the risk-free asset. When the threshold of disappointment becomes lower, the investor can build a disappointment-proof portfolio and still enjoy the upside of the stock investment. That is, the optimal strategy for a disappointment-averse investor is to take a long position in the stock and protect the downside of the investment by buying put options. However, for a very low level of disappointment threshold, disappointing outcomes become unlikely and the investor does not try to build a disappointment-proof portfolio.

Our third main result concerns the cross-section of options, i.e. the demand for options with different levels of moneyness. We show that the presence of disappointment-averse investors is able to generate open interest curve that peaks at or close to the at-the-money contract. This is a well-documented stylized fact (see, e. g. Lakonishok, Lee, Pearson, and Poteshman, 2007; Bollen and Whaley, 2004; Buraschi and Jiltsov, 2006) that has been shown to be incompatible with the equilibrium option demand in the risk-averse economy (Judd and Leisen, 2010).

Our fourth main result concerns the dynamics of the put option market. In our model, the size of put option market exhibits cyclical behavior resulting in interesting co-movement between the stock and the options markets. At lower levels of dividend fundamental (intermediate and bad states when the disappointment-averse agent owns most of the economy and is the net buyer of put options), the option market behaves pro-cyclically and is negatively correlated with the equity risk premium. At higher levels of dividend fundamental (good states when the risk-averse agent owns the economy), the option market size becomes counter-cyclical and is positively correlated with the equity premium. Similar to the intuition in Longstaff and Wang (2012), changes in the size of the option market in our model signal shifts in the distribution of wealth between different types of agents. The cyclicality and the co-movement arise because these shifts in the market "demographics" drive the trading activity and determine the behavior of asset prices.

We also investigate how the presence of disappointment-averse investors influences asset prices. Our fifth main result is that, relative to the benchmark risk-averse economy, the presence of disappointment-averse investors amplifies stock volatility generating up to 15–20% of excess volatility, depending on the level of disappointment aversion.[5] This excess volatility result is due to the shape of the stochastic discount factor in our model. In contrast to the representative-agent GDA model in which the pricing kernel is a step function, the heterogenous-agent pricing kernel is a continuous and monotonically decreasing function of the dividend fundamental. In addition to being countercyclical, the pricing kernel in our model features a steep slope in intermediate and bad states of the economy. In these states, shocks to the dividend news process lead to a larger increase in the stochastic discount factor compared to the standard risk-averse model, thus generating excess stock market volatility. When it comes to the valuation of derivative securities, the stochastic discount factor in the economy with disappointment-averse agents delivers larger negative risk premia for out-of-the-money options than the risk-averse benchmark case.

Our model is consistent with several stylized facts about put option demand in the economy, including that (i) public investors are net buyers of put options; (ii) public investors can switch from buyers to sellers in times of economic distress, when the risk-sharing capacity in the economy dries up; (iii) buying pressure in index put options can predict S&P 500 index returns; see Section 5 for a more detailed discussion.

Literature. This paper is related, on the one hand, to the literature on disappointment aversion. DA and GDA preferences have appeared in equilibrium models with consumption, and have been shown to explain equity premium and return predictability better than standard preferences do. DA preferences are used by Epstein and Zin (2001), who argue that they provide a substantial improvement in the empirical performance of a representative agent, intertemporal asset-pricing model. Bekaert, Hodrick, and Marshall (1997) use DA preferences and find that first-order risk aversion substantially increases excess return predictability. Routledge and Zin (2010) introduce generalized DA and show that GDA preferences give rise to counter-cyclical risk aversion along with a large equity premium.

[5] This is in contrast to the benchmark risk-averse economy in which the volatility of the stock is barely higher than the volatility of dividend news.

Bonomo, Garcia, Meddahi, and Tedongap (2011) and Liu and Miao (2015) explore asset pricing implications of GDA preferences in endowment and production economies, respectively. Schreindorfer (2020) shows that, in addition to explaining the risk premium in the stock market, GDA preferences can also help match the pricing moments of equity index options. Ang, Chen, and Xing (2006), Lettau, Maggiori, and Weber (2014), Delikouras (2017), Farago and Tedongap (2018), and Delikouras and Kostakis (2019) investigate the link between the disappointment aversion and the cross-section of asset returns. What sets us apart from the aforementioned works is that they consider a representative-agent economy with disappointment aversion. Our paper, to the best of our knowledge, is the first one to embed GDA preferences in a heterogenous-agent economy, which allows us to derive implications for both asset prices and quantities traded. Disappointment aversion has also been studied in the context of individual decision-making in Dahlquist, Farago, and Tedongap (2016) and Andries and Haddad (2020). Ang, Bekaert, and Liu (2005) provide a formal treatment of both static and dynamic optimal portfolio choice using DA preferences when the investor considers only a risk-free bond and a single stock. As already mentioned, their key result shows that a sufficiently high degree of disappointment aversion leads to optimal non-participation in the stock market. Our analysis complements this earlier work by highlighting that the optimal non-participation result is driven by the specific assumption about the reference point being equal to the certainty equivalent.

On the other hand, the paper contributes to the literature that studies option pricing and demand in partial and general equilibrium. In a partial equilibrium setting, Liu and Pan (2003), Weinbaum (2009), and Branger and Hansis (2012) show that standard power utility investors take short positions in put options to earn the premium provided by jump risk. Therefore, an alternative preference specification may be needed if we would like to understand what type of preferences lead to optimal long positions. To investigate if investors' asymmetric preferences explain the demand for put options, Driessen and Maenhout (2007) study the optimal portfolio choice of loss-averse and disappointment-averse investors in an empirical portfolio allocation setting when options are included in the set of investment opportunities. They find that long positions in out-of-the money put options are never optimal for the investors they consider. The authors conclude that neither disappointment aversion, nor loss aversion is sufficient to explain the demand for these derivatives. In this paper, we demonstrate that the definition of a disappointing outcome is key in understanding the demand for options generated by asymmetric investor preferences. In this sense, our results do not contradict their findings, since we also show that the original DA preferences introduced by Gul (1991) do not lead to long positions in put options.

Despite the size and the importance of the option market in the economy, the question who is buying and selling options in general equilibrium received relatively little attention in the theoretical finance literature. Garleanu, Pedersen, and Poteshman (2009) relate patterns in option markets to demand pressure and show that demand-pressure effects can play a role in explaining various option pricing anomalies. However, the demand for derivatives in their setting is assumed to be exogenous. In contrast, the focus of our paper is on endogenizing the level and the dynamics of option demand in the economy. Bates's (2008) paper, the closest to ours, studies an equilibrium model where market crashes can occur and agents have heterogeneous preferences towards crash risk. The paper shows that less crash-averse agents sell insurance to more crash-averse investors through options markets. The main emphasis, however, is on *prices*. Bates's (2008) investigates the model's ability to explain the pricing puzzles in the option market. In contrast, the main focus of our paper is on the equilibrium *quantities* and the size of the option market. In addition, the utility specification of Bates (2008) is unusual in the literature, as the number of stock market crashes during the investment period directly enters into the investor's preferences. The approach involving heterogeneous preferences seems promising, but it might be desirable to use preferences that have been more extensively studied in other areas of the literature. One possible candidate is disappointment aversion that is featured in our analysis.

In a related and important paper, Buraschi and Jiltsov (2006) show that the buying activity in out-of-the-money put options can be generated by informational heterogeneity between risk-averse agents. In their setting, investors, unable to directly observe the drivers of economic fundamentals, rely on a Bayesian model to update their beliefs but use different initial priors. The idea is that the agent who is more pessimistic about the expected dividend growth demands insurance protection from the more optimistic investor. This economic mechanism is different from ours, as we study the economy where agents agree about the dynamics of economic fundamentals. Another important difference is that, by introducing stochastic jump sizes, we are able to derive predictions for the cross-section of options. This angle of analysis is missing from Buraschi and Jiltsov (2006), as their model only needs one option to complete the market. Can one generate demand for multiple options with jump risk and heterogenous beliefs instead of heterogenous preferences? Yes. However, as Chen, Joslin, and Tran (2012) point out, such heterogenous beliefs model features a tension between the size of the insurance market and the equity premium: Disagreement generates a large demand for insurance when there is significant belief heterogeneity across investors and the equity risk premium is at a low level (due to ample risk sharing capacity in the economy). Similar to Buraschi and Jiltsov (2006) and Chen et al. (2012), our results rely on the role of heterogenous investor groups for risk sharing and risk allocation in the economy. However, our model focuses on a different risk-sharing mechanism between investors with heterogenous attitudes to losses and can generate large disaster insurance market and high equity premium at the same time. This suggests that our results capture a different phenomenon and the two approaches are complementary.

In a recent paper, Constantinides and Lian (2021) study the equilibrium net buy of index puts by public investors. They do so in a static mean-variance setting, in which investors exogenously specialize into buying out-of-the-money or at-the-money puts, but not both. Relaxing the specification assumption in their model leads to a counterfactual implication that public investors buy at-the-money and sell out-of-the-money puts. We show that this is not the case in our setting. Our paper also differs from this work in a number of other ways. Importantly, we consider a fully dynamic framework without restrictions on investors' positions. This enables us to derive predictions about the cyclical behavior of the put market and its co-movement with equity returns. Moreover, the roles of suppliers and demanders of options is not fixed in our model. This is important as empirical research (e.g., Chen et al., 2019) has shown that these roles can switch depending on the economic environment.

Layout. The remainder of the paper is organized as follows. Section 2 documents the main features of the demand for S&P 500-based index options and motivates out focus on put positions, Section 3 describes the economic fundamentals and outlines the equilibrium, Section 4 presents model results for equilibrium asset prices and equilibrium asset demands, Section 5 presents the empirical predictions, and Section 6 concludes. Derivations, figures, and other supplementary materials are provided in the Appendix.

2 Demand for S&P 500 Index Options

This section summarizes the extent and nature of demand for index options. We focus on CBOE S&P 500 index options (SPX and SPXW) which are the most actively traded cash-settled equity index options listed in the U.S. A comparative study of index options markets by Johnson et al. (2016) during 1990–2012 shows that SPX accounts for about 94% of the total market value of all options with their underlying referenced to the S&P 500 index.[6] In our analysis, we include the SPXW (i.e., "Weeklys") which are short-term SPX options that were introduced in 2016 and quickly gained popularity among investors.

Table 1 reports the index option demand on a number of different dates. The sample period is from January 4, 1996 through December 31, 2020. Gross open interest (OI) represents the sum of all open long and short positions. Following Johnson et al. (2016), we report gross open interest in terms of index-equivalent units. Each SPX contract provides

[6]Besides SPX, investors can gain option-like exposure by trading options written on S&P 500 futures and on the S&P 500 ETF (SPDR).

exposure to 100 units of S&P 500 index. We multiply open interest by a contract multiple of 100 in order to measure the amount of options demand in index-equivalent units.

We find that daily open interest has increased by almost 14 times during the sample and the demand for index puts is substantially larger than calls. We observe the highest open interest of 2.27 billions of index-equivalent units on March 19, 2020, which we refer to as the "Peak of the sample." For a comparison, we report the number of daily index shares which is calculated by dividing the total market capitalization of the S&P 500 by the nominal index level. Table 1 shows that the average number of index shares is 8.82 billions and that this value is quite stable through time. At the peak options OI level in March 2020, the demand for SPX options represented about 26% of total index shares. This number is economically large and confirms the relative importance of SPX options market in the economy.

[Insert Table 1 here]

Table 1 also summarizes the size of index option in terms of market capitalization. The market value for each option type on each day t is calculated as

$$\sum_i 100 \cdot OI_i(t) P_i(t),$$

where 100 is the contract multiplier, and $P_i(t)$ is the end-of-day price for each contract i. On average, the market capitalization for SPX puts is slightly less than calls despite having a substantially larger demand in terms of open interest. As we will show below, this reflects the fact that the demand for index puts concentrates in low strike levels (i.e., out-of-the-money).

Figure 1 plots the time series of average daily open interest for call and put options. Each bar represents an average value calculated from daily option quantities over each quarter. Panel A plots open interest in terms of index-equivalent units. Panel B plots open interest as a fraction of index shares in percentage terms, which is calculated by dividing daily gross open interest by the total number of S&P 500 index shares. Figure 1 shows that the demand for SPX options increased most visibly in the first half of the sample. It is also evident that the demand for index puts dominates. In recent years, put open interest comprised about two thirds of the total SPX option demand and represents more than 10% of the underlying index shares on a daily basis. Results in Panels A and B demonstrate an almost identical pattern in historical option demand. This finding is expected as Table 1 shows that the number index shares does not substantially vary over time. Reporting open interest as a fraction of index shares helps account for trends associated with size of the underlying market as well as occasional changes that are due to index reconstitution. The remainder of this paper reports open interest as the fraction of index shares. Nevertheless, we emphasize that our conclusions are qualitatively similar if we were to report empirical results in terms of index-equivalent units.

[Insert Figure 1 here]

Table 2 reports the average daily open interest for put and call options sorted by moneyness and maturity. All quantities are reported as the percentage of index shares. We sort options into short-, medium-, and long-term based on their number of days to expiration τ. Short-term options are those with maturity of 90 days or less and long-term options are those with maturity greater than 270 days. Medium-term options are those with maturity between 90 and 270 days. We define moneyness K/F as the level of strike price K to the underlying forward price.[7] The forward price for each contract is calculated as $F_i(t) = S(t)e^{(r(t,\tau)-d_s(t))\tau}$ where daily interest-rate curve $r(t,\tau)$ and continuous dividend rate $d_s(t)$ are obtained from Optionmetrics.

[Insert Table 2 here]

We consider three moneyness categories $K/F \leq 0.975$, $0.975 < K/F < 1.025$, and $K/F \geq 1.025$. Table 2 shows that the highest demand for index options concentrates in puts with low strike price. This corresponds to out-of-the-money (OTM) puts. Short-term OTM puts appear to be in highest demand. On average, open interest in OTM puts account for almost 5% of equivalent index shares. Looking at call options, we find that the demand concentrates around at-the-money (ATM) and the OTM calls with high strike price. Similar to puts, short-term calls are in highest demand relative to medium- to long-term contracts.

Figure 2 plots the time series of average daily open interest for various option types. We group options into different maturity and moneyness buckets and report the average gross daily open interest as a percentage of index shares. Panels A–D report results for puts while Panels E–H report results for calls. The figure illustrates the time-series pattern in options demand through the sample. Clearly, Figure 2 shows that the growth in SPX options market is largely driven by the demand for short-term OTM puts.

[Insert Figure 2 here]

Figure 3 provides a cross-sectional view in the demand for SPX options across a finer spectrum of moneyness. In this figure, we classify puts and calls into 17 moneyness buckets. Each bar represents the time-series average value of open interest for each option category. For both puts and calls, we find that option demand peaks when the contracts become

[7] Our results are very similar if we define moneyness using option delta instead of K/F.

close to at-the-money, i.e., when K/F is close to one. This finding is consistent with Judd and Leisen (2010) and Bondarenko (2014), among others. For puts, we observe a clear asymmetric demand for OTM puts.

[Insert Figure 3 here]

What groups of investors constitute the demand for index put options? Bollen and Whaley (2004) show that, in the S&P 500 index option market, end users (i.e., non-market makers) are net buyers of out-of-the-money and at-the-money puts. Chen et al. (2019) study the demand for deep-out-of-the-money index puts and confirm that, in their data, non-market makers, namely retail and institutional public investors, are net buyer of these options types during normal times. Interestingly, they also find that public investors can shift from being buyers to sellers and become "liquidity providers" of crash insurance during times of distress. Our theoretical model offers insights to this empirical observation.

Understanding the driving forces in the index put option market is the focus our this paper. For brevity, we focus our modeling attention on OTM index puts, which is the dominant component of equity index option market. The next section introduces a model that allows us to rationalize the observed empirical pattern in the market for index puts and generate new theoretical insights.

3 Model

This section describes the economy and characterizes the equilibrium.

3.1 The Economy

We consider a heterogenous agent pure exchange economy over a finite time interval $[0, T]$. The preferences of agent i can be described by a concave utility function with a kink at the reference point θ_i:

$$U_i(W_{i,T}) = u_i(W_{i,T}) - \ell_i \left(u_i(\theta_i) - u_i(W_{i,T})\right) I\left(W_{i,T} \leq \theta_i\right), \tag{1}$$

where $I(\cdot)$ is the indicator function and $u_i(\cdot)$ has the power utility form:

$$u_i(x) = \begin{cases} \dfrac{x^{1-\gamma_i} - 1}{1 - \gamma_i}, & \gamma_i > 0 \text{ and } \gamma_i \neq 1, \\ \log(x), & \gamma_i = 1. \end{cases} \tag{2}$$

When $\ell_i = 0$, the utility function U_i in (1) reduces to the standard constant relative risk aversion (CRRA) utility, where the parameter governing risk aversion is γ_i. When $\ell_i > 0$, outcomes lower than θ_i are labeled as disappointing and receive a penalty, decreasing the investor's utility compared to the CRRA case. As ℓ_i increases, the penalty for disappointing outcomes becomes larger, hence ℓ_i can be interpreted as the degree of disappointment aversion. Panel A of Figure 4 shows how the shape of the utility function changes with increasing ℓ_i if γ_i and θ_i are kept fixed. For comparison, Panel B of Figure 4 illustrates how the shape of the power utility (when $\ell_i = 0$) changes as the risk aversion parameter increases.

[Insert Figure 4 here]

Investor i is going to choose a portfolio allocation that maximizes $E\left[U_i\left(W_{i,T}\right)\right]$.

The disappointment threshold is defined as a fraction of the certainty equivalent,

$$\theta_i = \kappa_i \mathcal{R}_i, \tag{3}$$

where $\mathcal{R}_i$ is the certainty equivalent of the investor's final wealth, and it is implicitly defined as the solution to

$$U_i\left(\mathcal{R}_i\right) = E\left[U_i(W_{i,T})\right].$$

When $\kappa_i = 1$, the resulting utility specification corresponds to the disappointment aversion preferences of Gul (1991), and we arrive at the generalized disappointment aversion preferences of Routledge and Zin (2010) if we set $\kappa_i < 1$. Panels C and D of Figure 4 illustrate the shape of the utility function when the disappointment threshold is determined endogenously. While changing κ_i moves the disappointment cutoff, the ℓ_i parameter determines the slope of the utility derived from the disappointing outcomes.[8] Our heterogeneous agent economy is populated by two types of agents, $i = \{A, B\}$, having different preferences. We are going to compare two economies throughout the paper.

The DA economy: one disappointment-averse agent and one risk-averse agent. The main focus of the paper is on an economy, where investor A has asymmetric preferences with respect to gains and losses, and investor B has standard CRRA preferences. In particular, we are going to assume that $\ell_A > 0$ and $\ell_B = 0$. In addition, we are setting $\gamma_A = \gamma_B = \gamma$. That is, the two agents in our main economy have the same risk aversion parameter, but agent A is disappointment-averse, while agent B is not. This can also be illustrated using

[8] Alternatively, the disappointment threshold can be defined as a fraction of the investor's initial wealth, $\theta_i = \kappa_i W_{i,0}$. Berkelaar, Kouwenberg, and Post (2004) consider this case, setting $\kappa_i = 1$ in their application, and refer to the resulting utility function as the "kinked power utility." We obtain qualitatively similar results with this utility specification as well.

Panel C of Figure 4: agent B has the utility function represented by the solid line, while agent A has the utility function represented by one of the dashed lines.

The RA economy: two risk-averse agents. To understand the impact of the presence of disappointment-averse agents in the economy, we are going to compare the results to a benchmark economy without disappointment-averse agents. In particular, we assume $\ell_A = \ell_B = 0$ and $\gamma_A > \gamma_B$ in the benchmark economy. That is, none of the agents are disappointment-averse, and the heterogeneity comes from their risk aversion: agent A is more risk averse than agent B. This can be illustrated by Panel B of Figure 4: agent B has the utility function represented by the solid line, while agent A has the utility function represented by one of the dashed lines. Note that agent B is identical in the two economies, so the only difference between the DA and RA economies is the preferences of agent A.

The investment opportunity set consists of three securities: a stock, an option on the stock, and a riskless bond. The risky stock pays a terminal dividend of D_T at time T. The process for publicly observable dividend news D is given by

$$dD_t = D_{t-}\left[\mu_D dt + \sigma_D dw_t - \delta_D dN_t\right],$$

where w is a standard Brownian motion and N is a Poisson process with constant intensity λ. This means that in normal times, when no "disaster" takes place, the dividend news process follows a geometric Brownian motion, but each disastrous jump reduces it by a fraction δ_D. This specification of a deterministic jump size means that only one derivative security is needed to complete the market. We relax this simplifying assumption later by introducing random jumps with multiple outcomes to study the use of multiple derivatives.

The stock is in unit supply: At time 0 investor A is endowed with fraction $\alpha \in [0, 1]$ of the stock and investor B is endowed with fraction $(1 - \alpha)$. The derivative and the risk-free bond are in zero net supply. Since there is no intermediate consumption, we assume, without loss of generality, that the risk-free bond pays zero interest.[9] The stock and derivative prices are to be determined in equilibrium.

We will use the following baseline calibration. The expected dividend news growth rate is $\mu_D = 3\%$ and the fundamental diffusive volatility is $\sigma_D = 15\%$. The jump in dividend news occurs with intensity $\lambda = 5\%$ (once every twenty years) and implies that the level of dividends falls by $\delta_D = 20\%$. These values are largely in line with the numbers used in the literature and the historical properties of corporate dividends. In the *DA economy*, the disappointment-averse agent's preference parameters are $\gamma_A = 1$, $\ell_A = 2$, and $\kappa_A = 0.98$,

[9] This assumption is equivalent to using the risk-free bond as a numeraire. Such normalization is common in models with no intermediate consumption where there is no intertemporal choice that can pin the interest rate down. See, for example, Pastor and Veronesi (2012) and Basak and Pavlova (2013).

while the risk-averse agent has $\gamma_B = 1$. Note that our choice of κ_A is in line with the literature, and our choice of ℓ_A actually falls on the conservative side.[10] In the *RA economy*, the more risk-averse agent has $\gamma_A = 4$, while the less risk averse agent has $\gamma_B = 1$. That is, agent B has logarithmic preferences in both economies. Unless specified otherwise, the total wealth is equally split between the two agents at time 0 ($\alpha = 0.5$), and the investment horizon of the agents is one month ($T = 1/12$).

3.2 The Equilibrium

Equilibrium is defined in a standard way: equilibrium portfolios and asset prices are such that (i) given the set of investment opportunities, both investors optimally choose their portfolio strategies, and (ii) stock, bond, and derivative markets clear. We will make comparisons of the equilibrium in our economy with equilibrium in a benchmark economy populated by two risk averse investors with different levels of risk aversion. The equilibrium results for the benchmark economy are in Appendix B.

Proposition 1 discusses equilibrium terminal wealth profiles and equilibrium state price density at time T.

Proposition 1. *The equilibrium terminal wealth profiles are*

$$\widehat{W}_{A,T} = \begin{cases} \theta_A \underline{D}^{-1} D_T & \text{if } D_T < \underline{D}, \\ \theta_A & \text{if } \underline{D} \leq D_T \leq \overline{D}, \\ \theta_A \overline{D}^{-1} D_T & \text{if } D_T > \overline{D}, \end{cases} \tag{4}$$

and $\widehat{W}_{B,T} = D_T - \widehat{W}_{A,T}$ *and thresholds* $\underline{D}$ *and* $\overline{D}$ *are defined as*

$$\underline{D} \equiv \left(1 + \left(\frac{\lambda_A}{\lambda_B}\frac{1}{1+\ell_A}\right)^{\frac{1}{\gamma}}\right)\theta_A, \qquad \overline{D} \equiv \left(1 + \left(\frac{\lambda_A}{\lambda_B}\right)^{\frac{1}{\gamma}}\right)\theta_A, \tag{5}$$

where λ_A *and* λ_B *solve a system of equations* (A.15)–(A.16), $\theta_A = \kappa_A \mathcal{R}_A$, *and the certainty equivalent* $\mathcal{R}_A$ *determined using equation* (A.14).

[10] Starting with κ_A, studies that calibrate a representative agent model with GDA preferences use κ values between 0.97 and 0.99 (see Routledge and Zin, 2010; Bonomo et al., 2011; Schreindorfer, 2020). Our choice of $\kappa_A = 0.98$ is right in the middle of that range. Regarding the degree of disappointment aversion, a wider range has been used in the literature. Routledge and Zin (2010) and Schreindorfer (2020) use ℓ values around 9, while Bonomo et al. (2011) use 2.33 in their calibrations. Delikouras (2017) tries to estimate the value of ℓ using the cross section of stock returns and arrives at values between 3 and 8. Our choice of $\ell_A = 2$ falls on the conservative side of these values.

The equilibrium state price density at time T is

$$\xi_T = \begin{cases} c_1 D_T^{-\gamma} & \text{if } D_T < \underline{D}, \\ \\ c_2 \left(D_T - \theta_A\right)^{-\gamma} & \text{if } \underline{D} \leq D_T < \overline{D}, \\ \\ c_3 D_T^{-\gamma} & \text{if } D_T \geq \overline{D}, \end{cases} \tag{6}$$

where $c_1 \equiv \left(\left(\frac{\lambda_A}{1+\ell}\right)^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}} \right)^{\gamma}$, $c_2 \equiv \lambda_B^{-1}$, *and* $c_3 \equiv \left(\lambda_A^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}} \right)^{\gamma}$.

Proof. See Appendix A.1.

Let us start by looking at the equilibrium terminal wealth profiles. It is straightforward to see from Proposition 1, that if $\ell_A = 0$ (i.e, if the two agents are identical and not disappointment averse), then $\widehat{W}_{A,T} = \alpha D_T$ and $\widehat{W}_{B,T} = (1-\alpha) D_T$. That is, investors share the terminal dividend realization in proportion to their initial wealth shares. Panel A of Figure 5 shows terminal wealth profiles in the DA economy for our main calibration. Note that the two agents have equal initial wealth shares, and therefore the dotted line in the middle shows what the terminal wealth profiles would be without the disappointment aversion of agent A (i.e., $\widehat{W}_{A,T} = \widehat{W}_{B,T} = D_T/2$).

[Insert Figure 5 here]

It can be seen both from equation (4) and in Figure 5 that when agent A is disappointment-averse, all states of the world are endogenously classified into three subsets: "good states" (where $D_T > \overline{D}$), "intermediate states" (where $\underline{D} \leq D_T \leq \overline{D}$), and "bad states" (where $D_T < \underline{D}$). Note that it is straightforward to see from equation (5) that $\overline{D} > \underline{D}$ whenever $\ell > 0$. In intermediate states, the disappointment-averse investor keeps her wealth constant exactly at her disappointment thresholds, θ_A, in order to avoid disappointing outcomes. This can only be achieved if the disappointment-neutral investor (agent B) is willing to accept a wealth profile that pays $D_T - \theta_A$ in these intermediate states. That is, agent B fully insures agent A in intermediate states. Agent A accepts disappointing outcomes in bad states (i.e., $\widehat{W}_{A,T} < \theta_A$ in bad states), because these are the most expensive states to insure against. However, she maintains some level of insurance to attain higher final wealth than if she was disappointment-neutral (i.e., compared to the dotted line). In return for providing an insurance in the intermediate and bad states, agent B attains a higher final wealth in good states of the world (compared to the case when both agents are disappointment-neutral).

Different panels of Figure 5 show the terminal wealth profiles for various combinations of ℓ_A and κ_A. Comparing panel A to panel B illustrates the effect of increasing the degree of agent A's disappointment aversion (lower degree of disappointment aversion in panel B). As ℓ_A increases, the set of intermediate states expands (i.e., the distance between $\underline{D}$ and $\overline{D}$ increases). Agent A becomes more keen on avoiding disappointing outcomes, so she tries to decrease the set of bad states. This can be achieved by extending the set of intermediate states, where agent A's final wealth is exactly at her disappointment threshold. Agent B is willing to provide the insurance for an extended set of outcomes in exchange for a higher upside in good states of the world.

Comparing the graphs in the upper row of Figure 5 to Panel C illustrates the effect of changing the disappointment threshold. In the upper panels, we have $\kappa_A = 0.98$, which means that agent A becomes disappointed if she looses more than 2% of her certainty equivalent wealth by time T. The important observation is that in this case the upper threshold of the intermediate states ($\overline{D}$) remains very similar as ℓ_A increases from 1 to 2, while the lower threshold ($\underline{D}$) moves to the left. Looking at the density of D_T, we can see that states of the world are classified in such a way that a relatively large probability mass is assigned to the good states, a smaller probability to the intermediate states, and practically zero probability to the bad states. Note that increasing ℓ_A further results in a similar picture, i.e., $\overline{D}$ remains similar and $\underline{D}$ moves further to the left. The intuition is as follows: since agent A has a relatively low disappointment threshold, she can afford to fully insure herself against disappointment with probability very close to one.

Panel C in the lower row of Figure 5 corresponds to $\kappa_A = 1$, which means that agent A becomes disappointed if her final wealth is lower than her certainty equivalent wealth. In this case the thresholds $\underline{D}$ and $\overline{D}$ are determined (endogenously) in such a way that largest probability mass is assigned to the intermediate states. Comparing to Panels A and B, the probabilities of both the good and the bad states in Panel C are relatively small: $\underline{D}$ moves to the left and $\underline{D}$ moves to the right. For likely D_T realizations in panel C, agent A has constant end of period optimal wealth at her disappointment threshold. Intuitively, if the threshold of disappointment is high, the investor has to relinquish the whole upside of the investment to gain full protection against disappointing outcomes. The difference in the wealth profiles between the $\kappa_A = 1$ and $\kappa_A = 0.98$ ($\kappa_A < 1$ in general) cases will have important implications for optimal asset allocations.

Finally, Panel D of Figure 5 shows equilibrium terminal wealth profiles of the benchmark RA economy with only risk-averse agents. Note that panel D corresponds to our main calibration. We see that the more risk averse agent's payoff is higher in the bad states and the terminal payoff profile is slightly concave. The less risk averse agent enjoys a larger payoff in the good states of the economy and her terminal wealth is almost linear (if only slightly convex) in the dividend payoff D_T.

Let us now look at the equilibrium time T state price density. For the heterogeneous agent DA economy, ξ_T is given by equation (6) of Proposition 1, while the following corollary gives the state price density for the limiting representative agent economies.

Corollary 1. *State prices at T in representative agent economies are as follows:*

(i) If there is only agent A in the economy ($\alpha = 1$), her equilibrium terminal wealth is $\widehat{W}_{A,T} = D_T$ and the state price density becomes

$$\xi_T = \begin{cases} (1+\ell)\,\lambda_A^{-1} D_T^{-\gamma} & \text{if } D_T < \theta_A, \\ \\ \lambda_A^{-1} D_T^{-\gamma} & \text{if } D_T \geq \theta_A, \end{cases}$$

where $\lambda_A = E\left[D_T^{-\gamma}\right] + \ell E\left[D_T^{-\gamma} I\left(D_T < \theta\right)\right]$ and $\theta_A = \kappa_A \mathcal{R}_A$ is determined as in Proposition 1.

(ii) If there is only agent B in the economy ($\alpha = 0$), her equilibrium terminal wealth is $\widehat{W}_{B,T} = D_T$ and the state price density becomes

$$\xi_T = \lambda_B^{-1} D_T^{-\gamma},$$

where $\lambda_B = E[D_T^{-\gamma}]$.

Figure 6 shows the state price density for different ℓ_A values. Let us start with the limiting representative agent cases. When there is only the risk-averse agent in the economy ($\alpha = 0$), we get the standard CRRA pricing kernel, whose curvature is determined by the risk aversion parameter, γ. When there is only the disappointment-averse agent in the economy ($\alpha = 1$), there are two important changes to ξ_T. First, there is a jump in the state price density at the disappointment threshold θ_A. The location of the jump is determined mainly by κ_A; decreasing κ_A moves the location of the jump to the left on Figure 6. The size of the jump is mainly determined by ℓ_A; increasing ℓ_A leads to a larger jump in ξ_T. Second, the state price density becomes steeper on the part below θ_A, which is also mainly driven by ℓ_A. The discontinuous jump in ξ_T implied by a representative disappointment-averse agent economy has already been documented in previous literature (see, e.g., Routledge and Zin, 2010; and Delikouras, 2017).

[Insert Figure 6 here]

Our contribution in the current paper is to show how the shape of the state price density in the heterogenous-agent DA economy bridges those from the two limiting representative-agent economies. The state price density in the heterogeneous agent economy is made up of three parts, separated by $\underline{D}$ and $\overline{D}$, as it can bee seen from equation (6). In good states of the economy, above $\overline{D}$, the state price density is close to the $\alpha = 0$ case (CRRA representative agent). As we move to the region between $\underline{D}$ and $\overline{D}$, instead of a discontinuous jump, the state price density increases continuously but at a much steeper slope. Finally, in the states of the economy when the fundamental falls below $\underline{D}$, the state price density lies close to the $\alpha = 1$ case (disappointment-averse representative agent). This implies high prices of assets that pay off in bad states of the economy.

Proposition 2 presents results for the equilibrium time t state price density and the market prices of risk – risk premia per unit of Brownian and jump risk.

Proposition 2. *The equilibrium state price density at time t is given by*

$$\xi_t = c_1 F_{1t}\left(\underline{D}, \gamma\right) + c_2 F_{2t}\left(\underline{D}, \overline{D}, \theta_A, \gamma, 0\right) + c_3 F_{3t}\left(\overline{D}, \gamma\right), \tag{7}$$

and market prices of diffusion and jump risk are φ_t and $\lambda_t^Q / \lambda_t = 1 - \psi_t$, where

$$\varphi_t = -\frac{1}{\xi_t}\Big[c_1 \frac{\partial F_{1t}}{\partial D_t}\left(\underline{D}, \gamma\right) + c_2 \frac{\partial F_{2t}}{\partial D_t}\left(\underline{D}, \overline{D}, \theta_A, \gamma, 0\right) + c_3 \frac{\partial F_{3t}}{\partial D_t}\left(\overline{D}, \gamma\right)\Big] D_t \sigma_D, \tag{8}$$

$$\psi_t = -\frac{1}{\xi_{t-}}\Big[c_1 \Delta F_{1t}\left(\underline{D}, \gamma\right) + c_2 \Delta F_{2t}\left(\underline{D}, \overline{D}, \theta_A, \gamma, 0\right) + c_3 \Delta F_{3t}\left(\overline{D}, \gamma\right)\Big]. \tag{9}$$

The expressions for $\underline{D}$, $\overline{D}$, and θ_A, as well as the constants c_1, c_2, and c_3 are provided in Proposition 1 and $F_{it}(\cdot)$, $\frac{\partial F_{it}}{\partial D_t}(\cdot)$, and $\Delta F_{it}(\cdot)$ stand for $F_i\left(T-t, D_t; \cdot\right)$, $\frac{\partial F_i}{\partial D}\left(T-t, D_t; \cdot\right)$, and $\Delta F_i\left(T-t, D_{t-}; \cdot\right)$ defined in Lemmas A.1 and A.2 in Appendix A.

Proof. See Appendix A.2.

4 Results

In this section, we study the equilibrium asset prices and equilibrium asset demands. We show that the presence of a disappointment-averse agent in a heterogenous-agent model helps to explain the size and the dynamics of the option market.

4.1 Asset Prices in Equilibrium

The next two propositions discuss stock and derivative market valuations. First, we turn our attention to the stock. Proposition 3 discusses the equilibrium stock price in the DA economy.

Proposition 3. *In equilibrium, the stock price is given by*

$$S_t = \frac{1}{\xi_t}\Big[c_1 F_{1t}\left(\underline{D}, \gamma - 1\right) + c_2 F_{2t}\left(\underline{D}, \overline{D}, \theta_A, \gamma, 1\right) + c_3 F_{3t}\left(\overline{D}, \gamma - 1\right)\Big],$$

where the expressions for ξ_t and constants c_1, c_2, c_3 are provided in Proposition 2. Functions $F_{it}(\cdot)$ stand for $F_i\left(T-t, D_t; \cdot\right)$ defined in Lemma A.1.

The equilibrium stock price dynamics is of the form

$$dS_t = S_{t-}\left[\mu_{S,t} dt + \sigma_{S,t} dw_t + \delta_{S,t} dN_t\right],$$

where the expressions for the drift $\mu_{S,t}$, the diffusion volatility $\sigma_{S,t}$, and the jump volatility $\delta_{S,t}$ are provided in equations (A.19), (A.20), and (A.21).

Proof. See Appendix A.3.

To illustrate the results in Proposition 3, Table 3 plots the properties of equilibrium stock returns. The presence of disappointment-averse agents generates a sizable risk premium for the stock and creates excess stock market volatility. Comparing Panels A and B shows that the risk premium in the economy with disappointment-averse agents (DA economy) is similar in magnitude to that generated by the heterogenous risk-averse benchmark model (RA economy).[11]

[Insert Table 3 here]

Turning to Panels C and D of Table 3, we see that disappointment-averse investors make the stock much more volatile than in the economy with only risk-averse agents with heterogenous degrees of risk aversion. Indeed, in the RA economy the volatility of the stock is barely higher than the volatility of dividend news. This is in contrast to up to 15–20% of excess volatility in the DA economy.

[11] Note that our main calibration leads to an equity risk premium of 4.07% in the DA economy and 4.06% in the RA economy. We deliberately choose $\gamma_A = 4$ in the RA economy to match the risk premium of the DA economy. Also note that we could easily have a higher equity premium in the main calibration by increasing γ_B, but we prefer to use the low value of $\gamma_B = 1$ in order to highlight the effect agent A's disappointment aversion on the upcoming results.

The next proposition discusses the equilibrium valuation of the put option on the stock. Prices of other derivative securities, for example a call option, can be deduced using the pull-call parity.

Proposition 4. *The equilibrium price of a put option with strike price K and maturity T is given by*

$$P_t = \begin{cases} \frac{1}{\xi_t} c_1 \left(K \cdot F_{1t}(K,\gamma) - F_{1t}(K,\gamma-1)\right) & \text{if } K \leq \underline{D}, \\ \frac{1}{\xi_t} \Big[c_1 \left(K \cdot F_{1t}(\underline{D},\gamma) - F_{1t}(\underline{D},\gamma-1)\right) & \\ \quad + c_2 \left(K \cdot F_{2t}(\underline{D},K,\gamma,0) - F_{2t}(\underline{D},K,\gamma,1)\right) \Big] & \text{if } \underline{D} < K \leq \overline{D}, \\ \frac{1}{\xi_t} \Big[c_1 \left(K \cdot F_{1t}(\underline{D},\gamma) - F_{1t}(\underline{D},\gamma-1)\right) & \\ \quad + c_2 \left(K \cdot F_{2t}\left(\underline{D},\overline{D},\theta_A,\gamma,0\right) - F_{2t}\left(\underline{D},\overline{D},\theta_A,\gamma,1\right)\right) & \\ \quad + c_3 \left(K \cdot \left(F_{1t}(K,\gamma) - F_{1t}\left(\overline{D},\gamma\right)\right) - \left(F_{1t}(K,\gamma-1) - F_{1t}\left(\overline{D},\gamma-1\right)\right)\right) \Big] & \text{if } K > \overline{D}, \end{cases}$$

where the expressions for ξ_t, constants c_1, c_2, and c_3 are provided in Proposition 2. Functions $F_{it}(\cdot)$ stand for $F_i(T-t, D_t; \cdot)$ as defined in Lemma A.1.

Moreover, the equilibrium price dynamics of the put option is of the form

$$dP_t = P_{t-}\left[\mu_{P,t}dt + \sigma_{P,t}dw_t + \delta_{P,t}dN_t\right],$$

where the expressions for the drift $\mu_{P,t}$, the diffusion volatility $\sigma_{P,t}$, and the jump volatility $\delta_{P,t}$ are provided in Appendix A.4.

Proof. See Appendix A.4.

To illustrate the equilibrium properties of put option returns, Figure 7 shows monthly put option risk premia and the Black-Scholes implied volatility as a function of option moneyness in our main calibrations of the two economies. Figure 7 demonstrates that (with the same equity premium) the presence of disappointment-averse agents leads to lower risk premia for out-of-the-money options and (slightly) higher implied volatilities compared to the benchmark RA economy. Both economies are able to produce the volatility skew, which is mainly driven by the distributional assumptions about the dividend fundamental.

[Insert Figure 7 here]

4.2 Equilibrium Wealth and Risk Factor Exposures

Before we turn our attention to investors' portfolio planning decisions in equilibrium, Proposition 5 studies the equilibrium wealth profiles and the local exposures to the risk factors that support the equilibrium wealth dynamics. Focusing on the equilibrium exposures helps to build intuition about the relative attractiveness of the fundamental risk factors for different types of investors. Given the available securities, investors choose their asset allocations to achieve the optimal exposures to the diffusion and jump risk.

Proposition 5. *The equilibrium investors' wealth profiles, $\widehat{W}_{i,t}$, are given by*

$$\begin{aligned}
\widehat{W}_{A,t} &= \frac{1}{\xi_t}\Big[\left(\frac{\lambda_A}{1+\ell}\right)^{-\frac{1}{\gamma}} c_1^{1-\frac{1}{\gamma}} F_{1t}\left(\underline{D},\gamma-1\right) + \theta_A c_2 F_{2t}\left(\underline{D},\overline{D},\theta_A,\gamma,0\right) + \lambda_A^{-\frac{1}{\gamma}} c_3^{1-\frac{1}{\gamma}} F_{3t}\left(\overline{D},\gamma-1\right)\Big], \\
\widehat{W}_{B,t} &= \frac{1}{\xi_t}\Big[\lambda_B^{-\frac{1}{\gamma}} c_1^{1-\frac{1}{\gamma}} F_{1t}\left(\underline{D},\gamma-1\right) + c_2 F_{2t}\left(\underline{D},\overline{D},\theta_A,\gamma-1,0\right) + \lambda_B^{-\frac{1}{\gamma}} c_3^{1-\frac{1}{\gamma}} F_{3t}\left(\overline{D},\gamma-1\right)\Big],
\end{aligned} \tag{10}$$

where functions $F_{it}(\cdot)$ stand for $F_i\left(T-t, D_t; \cdot\right)$ as defined in Lemma A.1.

The equilibrium wealth dynamics is of the form

$$d\widehat{W}_{i,t} = \widehat{W}_{i,t-}\left[\mu_{W,i,t}dt + \sigma_{W,i,t}dw_t + \delta_{W,i,t}dN_t\right],$$

where $\sigma_{W,i,t}$ and $\delta_{W,i,t}$ capture local exposure to the diffusion and jump risk factors, respectively,

$$\sigma_{W,i,t} = \alpha_{i,t} + \varphi_t, \tag{11}$$

$$\delta_{W,i,t} = \frac{\beta_{i,t}+\psi_t}{1-\psi_t}. \tag{12}$$

Expressions for φ_t and ψ_t are provided in Proposition 2, whereas $\alpha_{i,t}$ and $\beta_{i,t}$ for $i \in \{A, B\}$ are given by (A.23)–(A.24) in Appendix A.6.

Figure 8 illustrates equilibrium wealth and risk factor exposures of the disappointment-averse agent in the DA economy (Panel A) and of the more risk averse agent in the RA economy for our main calibration. The illustration provides a number of insights into the equilibrium mechanism by highlighting the role of the option market in achieving the equilibrium risk exposures. The solid blue line in all panels is the equilibrium wealth of agent A at $t = 0$, $\widehat{W}_{A,0}$. Note how the optimal wealth profiles at $t = 0$ show a close resemblance to the corresponding optimal terminal wealth profiles in Panels A and D of Figure 5. The slope of the red dashed line tangent to the equilibrium wealth profile at $D_0 = 1$ (the point where the horizontal and vertical gray lines are crossing) is equal to the equilibrium exposure to diffusive risk normalized by the dividend news volatility, $\sigma_{W,A,0}\widehat{W}_{A,0}/\sigma_D$. It shows that agent A remains on the optimal wealth profile for an infinitesimally small diffusion shock, dw_t, i.e. her Brownian risk is hedged. The green arrow represents the equilibrium jump risk exposure and shows what happens in case the rare jump occurs. In the top row of Panel A in Figure 8, we see that if the jump in the dividend fundamental occurs in the next instance, agent A's wealth changes by $\delta_{W,A,0}\widehat{W}_{A,0}$ and she remains on the equilibrium wealth profile.

Looking at the slopes of the red and green lines which capture the optimal exposures to the two types for risk in the economy, we see that the disappointment-averse agent A is willing to accept a rather high diffusion risk exposure and only tolerates a relatively low negative exposure to jump risk. Intuitively, the high diffusive risk exposure and very limited jump risk exposure sought by the disappointment-averse investor can be achieved by holding the risky stock and purchasing an insurance derivative that protects from downward stock price movements. This generates the demand for put options in the DA economy. This demand for disappointment protection is fulfilled by agent B who ends up with a high exposure to both types of shocks.

The bottom row of Panel A depicts an alternative scenario demonstrating what would happen if the agent could only invest in the stock and the riskless bond (i.e., no access to the option market). For illustration purposes, we assume that the exposure to Brownian risk remains the same as in the top row (this assumption will pin down the agent's investment in the stock). The allocation depicted in the bottom row of Panel A is clearly suboptimal: in the event of the jump in the dividend fundamental, the agent's wealth falls *below* the optimal level. To hedge this risk, agent A wants to *buy* a put option, which serves as an insurance if the jump occurs. At the same time, she also adjusts her stock position to achieve the equilibrium exposure to both types of risks depicted in the top row of Panel A.

The graphs in Panel B of Figure 8 present corresponding results for agent A, the more risk-averse agent, in the benchmark RA economy. Her optimal wealth profile is concave in the dividend fundamental. In the top row, the slopes of the red and green lines capture equilibrium diffusion and jump risk exposures, respectively. Contrasting these to the figure on the left we note that the more risk-averse agent in the benchmark RA economy is much more concerned with avoiding the diffusive risk and is less concerned about the jump risk exposure, compared to the disappointment-averse agent in the DA economy. The bottom graph of Panel B depicts the "suboptimal" allocation without access to the option market, where the investor maintains her optimal diffusion risk exposure with only the stock and the risk-free bond. The wealth of agent A would end up *above* the optimal level if a jump occurred. Therefore, if the put option is available, she would like to *sell* it to increase her negative exposure to the jump risk. This is in line with results of Dieckmann and Gallmeyer (2005).

To summarize, the disappointment-averse agent in the DA economy is going to buy the put option to achieve the equilibrium exposure to both the diffusive and jump risks, while the more risk-averse agent in the RA economy is going to sell the put option. By comparing the suboptimal exposures (without the derivative) to the corresponding optimal ones in both Panel A and B of Figure 8, we can see that a much bigger adjustment occurs in the DA economy due to the introduction of the option. This implies, as we are going to see in the following section, that the equilibrium size of the put option market is considerably larger in the DA economy.

4.3 Asset Demands in Equilibrium

We now study investors' portfolios in equilibrium. By taking positions in the available assets, agents effectively invest in the diffusive and jump risks according to their desired exposures outlined in Proposition 5. In order to explicitly identify who buys or sells risky assets in response to cash flow news, Proposition 6 discusses investors' portfolios in terms of the absolute number of shares held of a certain security, as well as the relative proportion of investor's wealth invested in the stock and the derivative asset. We assume that the derivative that completes the market is a put option.[12] Market completeness allows us to map the equilibrium risk exposures into the equilibrium asset demands.

Proposition 6. *In equilibrium, the agents' investments (in terms of the number of shares) in the stock and the put option are*

$$\pi_{S,i,t} = \phi_{S,i,t}\frac{\widehat{W}_{i,t}}{S_t}, \quad \pi_{P,i,t} = \phi_{P,i,t}\frac{\widehat{W}_{i,t}}{P_t}, \quad i \in \{A, B\},$$

where the equilibrium investors' wealth profiles, $\widehat{W}_{i,t}$, are given in Proposition 5, stock and put prices are as in Proposition 3, and the fractions of investors' wealth invested in the stock and the put option, $\phi_{S,i,t}$ and $\phi_{P,i,t}$, are given by

$$\phi_{S,i,t} = \frac{\delta_{W,i,t}\sigma_{P,t} - \delta_{P,t}\sigma_{W,i,t}}{\delta_{S,t}\sigma_{P,t} - \delta_{P,t}\sigma_{S,t}},$$
$$\phi_{P,i,t} = \frac{\delta_{W,i,t}\sigma_{S,t} - \delta_{S,t}\sigma_{W,i,t}}{-\delta_{S,t}\sigma_{P,t} + \delta_{P,t}\sigma_{S,t}}.$$

[12] Note that only at this step we need to specify the derivative that completes the market.

$\sigma_{W,i,t}$ and $\delta_{W,i,t}$ for $i \in \{A, B\}$ are given by (11)–(12). Finally, Appendices A.3 and A.4 provide expressions for $\sigma_{j,t}$, $\delta_{j,t}$ for $j \in \{S, P\}$.

Figure 9 plots equilibrium number of stocks and out-of-the-money ($K/S_t = 0.95$) put options demanded by agent A as a function of the initial wealth-share of the disappointment-avers agents in the economy.[13] The DA economy (left panel) is compared to the benchmark RA economy (right panel). For low values of α the DA agent has little wealth and the put option risk premium that can be earned by agent B from selling puts is relatively low. Therefore, the number of put contracts in equilibrium is relatively low. As α increases the DA investor has more demand for the portfolio protection offered by the put and is willing to pay a higher premium for it. Consequently, the size of the option market increases: agent B sells and agent A buys a higher number of put options. For values of α higher then 60-70% the disappointment-averse agent A owns a high fraction of the wealth in the economy and the risk-averse agent B has less initial wealth and thus can offer less risk sharing capacity. This causes the intermediate flat region of the agent A's wealth profile to become smaller. Despite a lucrative risk premium from writing put options, the number of put options sold by agent B (and bought by agent A) goes down.

Finally, we turn to an extreme case when the relative wealth share of the disappointment-averse agent if close to 1. One can also think of this case as capturing the times of severe economic distress when the risk sharing capacity in the economy is very limited. Indeed, as we demonstrate in Section 5 similar effects are observed at lower levels of the dividend fundamental. When the DA agent owns nearly all the wealth in the economy she starts writing put options. Intuitively, when agent B is small enough, the payoff profile of the disappointment-averse agent A becomes very close to that of the more risk-averse agent in the benchmark economy. The set of intermediate states shrinks and to correctly replicate the concave shape of her payoff with the stock and the put option, agent A sells put options. In turn, agent B, instead of providing protection, finds it optimal to be long both the stock and the put option, effectively implementing a synthetic call payoff profile. Both investors are trying to achieve their desired exposures to the two types of risks. While the two risky assets, the stock and the option, offer exposures to these risks in very different proportions. Note that even when the disappointment-averse agent A is selling the put option to agent B, the latter still has higher exposure to diffusive risk and higher exposure to jump risk.

In the high α region, the DA economy is similar to the benchmark RA economy. The results depicted in the right panel are in line with the previous findings in the literature:

[13] The relative wealth share of the disappointment-averse agent going from zero to one in Figure 9 can be also interpreted in terms of the economy moving across different states, from good to bad. This is because the agent A owns a higher fraction of wealth in the economy in bad states (low level of dividend fundamental), while agent B dominates the economy in good states (high level of dividend fundamental).

The more risk-averse investor sells put options to the less risk-averse investor. This asset allocation allows the two agents to implement their optimal risk exposures. The more risk-averse agent A here wants to avoid diffusion risk, but can tolerate more jump risk (compared to the disappointment-averse agent), so she invests considerably less in the stock and sells a small number of put options to bump up the jump risk exposure to the optimal level. The overall risk exposure from this strategy for the more risk-averse agent is still much lower than that of the less risk-averse agent who takes the other side of the trade.

[Insert Figure 9 here]

We further study how the size of the option market behaves for different values of preference parameters. In the benchmark RA economy the size of the option market is negligible, even for very high values of agent A's risk aversion. In this economy, agents invest in the stock and the risk-free asset and largely do not participate in the option market. For example, when endowed with 50% of the stock supply, the more risk averse agent trades with the less risk-averse counterpart to reduce the risky exposure and takes a very small (practically zero) short position in the put option. In contrast to the benchmark RA case, the economy with the disappointment-averse agent delivers considerably larger number of option contracts demanded. The option market becomes important quickly as ℓ_A increases and at some point gets to be largely insensitive to further changes in ℓ_A or changes in risk aversion γ (recall that $\gamma_A = \gamma_B = \gamma$ in the economy with disappointment aversion).

We also investigate the sensitivity of the option positions to changes in disaster parameters δ_D, λ (figure not reported). The option market's size becomes lower as the jump size, δ_D, increases (options become more expensive in the economy with larger disasters). At the same time the size of the option market is not sensitive at all to jump intensity, λ. In equilibrium, as λ increases, the risk-neutral disaster intensity λ^Q and the risk premium for writing put options also increase. However, the market price of jump risk (λ^Q/λ) remains constant and so does investors' optimal exposure to jump risk .

4.4 Disappointment Threshold and Size of the Option Market

Figure 10 shows the risky asset demands in the disappointment-averse model for various values of the disappointment threshold parameter κ_A. The left panel shows the stock demand of agent A, while the right panel focuses on the demand for options. We see that the option market size is hump-shaped in the disappointment threshold. For a relatively low level of κ_A (e.g., $\kappa_A = 0.9$ – meaning that more than a 10% drop in terminal wealth below the certainty equivalent level is labeled as disappointing) disappointment becomes unlikely. In this case, disappointment-averse investor A increases slightly her risky stock position from the initial endowment of half a share and her long put position does not cover the entire stock exposure. With a higher disappointment threshold ($\kappa_A = 0.95$), investor A becomes disappointed more easily. In this case she implements a fully covered portfolio insurance strategy, maintaining a long position in the stock (close to her initial endowment) and fully hedging it by buying the corresponding number of put options. When the disappointment threshold is even higher ($\kappa_A = 0.98$), in order to avoid disappointing outcomes investor A has to give up some of the upside of the stock investment. We observe that agent A further reduces stock exposure and sets the protection level of her insurance strategy even higher. Finally, consider the special case of the DA preferences ($\kappa_A = 1$). As κ_A becomes close to one (i.e the disappointment threshold is the certainty equivalent), the disappointment-averse investor chooses not to participate in risky asset markets (for $\kappa_A = 1$, both $\pi_{S,A,t}$ and $\pi_{P,A,t}$ are zero for $\ell_A > 1$). That is, in this case there is no demand for the option as the disappointment-averse agent chooses to invest all her wealth in the risk-free asset.

[Insert Figure 10 here]

We emphasize here that there is a clear-cut difference between DA and GDA preferences. For the DA investor ($\kappa_A = 1$), we show that the non-participation result of Ang et al. (2005) extends to the case when an option is added to the investment opportunity set. The GDA investor, on the other hand, implements a protective put strategy by fully (for higher levels of $\kappa_A < 1$), or partially (for lower levels of $\kappa_A < 1$) hedging her long position in the stock. Intuitively, in the case of DA preferences, the disappointment threshold is so high that the investor has to relinquish the whole upside of the investment to have full protection against disappointing outcomes. When the threshold of disappointment becomes lower (the GDA case), the investor can protect herself from disappointment while still enjoying some of the upside of the stock investment. As long as $\kappa_A < 1$, even investors with very high disappointment aversion hold sizable positions in the stock and use the option market to protect themselves from disappointing outcomes.

4.5 Multiple Options

In this section we study the size of the option market when options with multiple strikes are traded at the same time. It is a well-documented stylized fact that that the open interest in the data peaks near the at-the-money contracts (see, e. g. Lakonishok et al., 2007; Bollen and Whaley, 2004; Buraschi and Jiltsov, 2006) and this feature has been shown to be incompatible with the risk-sharing option demand in existing equilibrium models (Judd and

Leisen, 2010). In this subsection, we show that it arises naturally in the heterogenous-agent economy with disappointment aversion.

To analyze the open interest curve across strikes, we consider stochastic jumps with two, three, and four possible jump sizes, so that the multiple options are not redundant. To illustrate the model results for the case of stochastic jump size, we estimate the parameters of our dividend news process using earnings data from Robert Shiller's website. The jump size distribution is assumed to be normal for the estimation. For the figure, we approximate it with a two-, three-, and four-point discretizations.

[Insert Figure 11 here]

Across moneyness, Figure 11 plots the number of options traded (open interest on the left axis) alongside the Black-Scholes implied volatility (right axis). First, confirming our earlier results, we observe the volatility skew. Second, turning our attention to the number of option contracts, we observe that the open interest patterns closely match the patterns we see in the data. Who is buying and who is selling each of these contracts? In the two-options case, the DA agent is long both options. With three jumps and three options, the DA investor buys the OTM and ATM puts and shorts the ITM option. With four jumps and four options available, the DA investor buys the two middle moneyness puts and shorts the two puts at the ends in terms of moneyness for both graphs. Comparing the two graphs in the four-options case, we note that when the moneyness of the put option that is just out of the money becomes closer to 1 the peak of the open interest curve shifts to this position. This is in line with what we observe in the data.

Multiple options allow investors to better disentangle simultaneous exposures to diffusion and jump risks in the stock market. Investors' target is to achieve the desired exposure to all risk factors given the risk sensitivities of the traded assets. More precisely, equilibrium positions in the N risky assets (stock and $N-1$ options) solve a system of N equations with N unknowns. To build intuition, it is instructive to discuss the two-options case more closely. The stock provides almost equal exposures to both the diffusive and jump risks in dividend fundamental. Therefore both investors use the cross-section of options to replicate their desired risk exposures. The DA agent is long both options but the amount invested in the ATM option is higher than that for the OTM. The reason for this is that the ATM put's relative exposures to two jump risks closely match those of the DA investor's equilibrium wealth, which makes it an attractive asset for the DA investor. Whereas the OTM put is very effective in providing a separate exposure to the negative jump risk, so a small amount in enough to do the job.

5 Empirical Content and Discussion

In this section, we discuss the empirical relevance and implications of our theoretical results.

5.1 Consistent Evidence

Our theoretical model is consistent with a number of *stylized facts* about the demand for index puts in the economy:

1. **Put (index) option market is economically large**. See Section 2. On an average trade date, the number of shares referenced by S&P 500 put index options is about 10% of the total equity of the market. In terms of notional value, this amounts to more than 3 trillion dollars of underlying index value that is controlled by these positions.

2. **Buying index puts is expensive**. Indeed, index put options have been shown to have highly negative expected returns. See, e.g., Broadie et al. (2009) and Bondarenko (2014).

3. **Public investors are net buyers of put options**. See, e.g., Bollen and Whaley (2004). In our model, the disappointment-averse investors are buyers of put options during the "normal" times. This group of agents represents end-users or public investors (retail and institutional). The risk-averse investors that do not exhibit disappointment aversion are sellers. They stand for broker-dealers and market-makers in option markets.

4. **Public investors can switch from buyers to sellers**. See, e.g., Chen et al. (2019) on how public investors (institutional) provide insurance to financial intermediaries in times of economic distress by selling deep out-of-the-money index put options. In our model, when the risk sharing capacity in the economy dries up the disappointment-averse investors' demand for put options can turn negative.

5. **Across option moneyness, open interest peaks at the money**. See Section 2 and the evidence in Judd and Leisen (2010), Bondarenko (2014).

6. **Buying pressure in index put options can predict S&P 500 returns**. See, Chen et al. (2019) and Chordia et al. (2021). In our model, observable changes in the size of the option market signal shifts in the market "demographics". These shifts in the distribution of wealth between different types of agents (which are not directly observable) drive the trading activity and determine the behavior of asset prices. Paralleling the results of Longstaff and Wang (2012) for the amount of credit in the economy, the

economic mechanism of our model provides a link between the size of option market and asset-pricing moments. In line with our model mechanism, Chordia et al. (2021) argue that the predictability they document is largely driven by investors' demand for insurance.

5.2 New Predictions

The disappointment-averse agent in our model owns a relatively larger wealth share of the economy in bad states (when the dividend fundamental is low), while the risk-averse agent dominates the economy in the good states. This leads to the following *predictions* about the behavior of the put option market (when the disappointment-averse, i.e. public, investors are buying put options):

1. **Cyclicality of the put option market**. At lower levels of dividend fundamental (intermediate states and bad states when the disappointment-averse agent owns a larger part of the economy, but does not start yet selling put options), the option market behaves procyclically: The size of the put option market increases as the economy expands. However, at higher levels of dividend fundamental (good states when the risk-averse agent owns the economy), the option market size becomes countercyclical.

2. **Price-quantity relationship in the put option market**. There is a positive relationship between changes in put option prices and equilibrium quantities of out-of-the-money options demanded in good times. This relationship turns to negative in intermediate and bad states.

3. **Correlation with the equity premium**. Changes in the hedging demand as captured by the open interest in out-of-the money put options are positively (resp., negatively) correlated with the equity risk premium in good (resp., intermediate and bad) states of the economy.

Figure 12 illustrates these predictions. In particular, the graphs show various quantities at time $t = T/2$, as the function of the current value of the dividend fundamental, D_t, in the DA economy. By time $t = T/2$, D_t has moved away from its initial value of $D_0 = 1$ (good or bad news have arrived), and the equilibrium has adjusted accordingly. For reference, all graphs feature the probability density function of D_t. The top two panels show the stock price (left) and the equity risk premium (right). The bottom two panels show the option demand in terms of put option contracts held by the disappointment-averse investor (left) and the price of the option as measured by implied volatility (right).

First of all, we see that the stock price moves in line with the dividend news in the economy. Therefore, all of the aforementioned predictions can also be formulated in terms of the stock market level. Second, the graph showing the option demand tells a similar picture to Figure 9 which varies the importance of the disappointment-averse agent on the horizontal axis. As stated in prediction 1, there is a hump-shaped relation between the size of the option market and the health of the economy. In good states, demand for options decreases as the level of dividend fundamental increases even further. In other words, in this region the size of the option market decreases in cash-flow news. However, as the economy starts to contract we observe the opposite, procyclical behavior: The size of the option market becomes increasing in cash-flow news. Finally, when the level of dividend fundamental is very low and the proportion of the disappointment-averse agents in the economy is sufficiently high we can observe the option market flipping. The disappointment-averse agent, who is normally a net buyer becomes a net seller of out-of-the money puts. The intuition behind this result is discussed in detail in Section 3.3. If the economy is in distress but the proportion of disappointment-averse agents is not very high ($\alpha = 0.25$ in the figure), then the disappointment-averse investors simply stay out of the risky asset market altogether.

Looking at the the implied volatility plot, we also see that in normal times (when the disappointment-averse investors are net buyers of put options) implied volatility is falling and the option price is becoming cheaper as the economy expands. Comparing this result to the behavior of the put option demand confirms prediction 2. Indeed, in bad and intermediate states increases in put option demand coincide with the put option becoming less expensive. For high dividend levels, this pattern reverses and as the option is becoming cheaper the demand for it drops as well.

Finally, we also observe the stock market risk premium being naturally countercyclical. In line with prediction 3, this means that in good states the put option market demand and the equity risk premium move together, both decrease as the economy expands. As the level of dividend fundamental falls and we move into the intermediate and bad states of the economy, the premium for the stock continues to rise while the equilibrium quantity of out-of-the money put options traded falls.

5.3 Intuition: Supply and Demand Effects in Equilibrium

The intuition behind the predictions outlined in the previous subsection can be understood from the lens of demand and supply analysis.

In periods of economic expansion, as the economy moves to states with higher and higher dividend fundamental, we observe a buyer's market for put options. Risk-average agent (supplier of put options) makes up most of the economy, whereas the disappointment-averse agent (demander of put options) makes up a smaller part of the economy. Indeed, the risk-averse agent enters the good states with a profit from selling put options in the previous period, while the disappointment-averse agent enters good states with a loss as put options never become in-the-money. Loosely speaking, there are more people looking to sell crash insurance than there are people looking to buy it. In such economy, we observe that shocks to the demand side dominate and quantity of put options demanded decreases as the economy expands further. If the economy starts to cool off, demand for put options picks up again. The market for put options in these states is countercyclical. Moreover, as quantity demanded changes the price moves in the same direction and we observe a positive price-quantity relationship in the put option market.

In periods of economic downturn, the situation is reversed. In bad states, the risk-averse agent starts experiencing some losses from selling put options in the earlier periods. Some options became in-the-money. On the other hand, the disappointment-averse agent makes money from her portfolio protection strategy. As a result, disappointment-averse agent ends up owning a larger share of the economy. The risk-averse agent, who underwrites put options, becomes more constrained as her wealth deteriorates. In this case we have a seller's market: there are more potential buyers than sellers of crash insurance. The relatively lower fraction of risk-averse agents implies that shocks to the supply side of put option market dominate. If the economy contracts further, equilibrium quantity of put options demanded decreases as well coupled with the increase in price. The market for put options in these states is procyclical and we observe price and quantity moving in opposite directions.

Depending on whether demand or supply shocks dominate, changes in put options open interest can be positively or negatively associated with changes in stock market risk premium. This model implication (prediction 3 in the previous subsection) potentially reconciles the seemingly contradicting findings of Chen et al. (2019) and Chordia et al. (2021). On the one hand, our findings are in line with the evidence presented in Chen et al. (2019). They show that in periods with a negative price-quantity relation in the deep out-of-the-money index put option market, public net-buying demand *negatively* predicts future market excess returns. In our setting, market expected excess returns are countercyclical and can move in the opposite directions with out-of-the-money put quantities demanded when the latter are procyclical. This happens when supply shocks in the option market dominate, i.e. in periods of economic slowdown and recession. On the other hand, the two can be positively related as well, which happens when demand shocks in the option market dominate. This means that the finding of Chordia et al. (2021) that net put buying *positively* predicts market returns can be due to the overrepresentation of expansion periods in their 2006–2017 sample.

6 Conclusion

In this paper, the main focus is on the derivatives markets and the most important results concern equilibrium asset demands. We study the potential of asymmetric preferences to explain the stylized facts about the amount of put option trading. To this end, we analyze a dynamic heterogenous-agent economy with rare event risk and two classes of investors: risk-averse investors and disappointment-averse investors. The latter are especially concerned about the left-tail outcomes of their investments. To model the asymmetries in investor preferences, we rely on the generalized disappointment aversion framework in which investor is loss-averse around a reference point defined in terms of her certainty equivalent.

We show that accounting for asymmetries in preferences coupled with the asymmetric nature of dividend shocks helps to rationalize the observed size and dynamics of the put option market. Risk aversion alone is not enough. In the economy populated by heterogenous risk-averse investors the size of the option market is negligible, even for very high values of risk aversion. In contrast to the risk-averse setting, we demonstrate that the presence of agents with asymmetric preferences over gains and losses gives rise to a sizable option market in the economy. The size of the option market is hump-shaped in the disappointment threshold. For a relatively low level of disappointment threshold disappointing event becomes unlikely and the investor does not try to implement a portfolio insurance strategy. As the disappointment threshold becomes very high, the disappointment-averse investor chooses not to participate in risky asset markets and invests all her wealth in the risk-free asset. In the cross-section of options, we find that asymmetric preferences explain the shape of the observed open interest curve that poses a challenge to existing models. Finally, we derive new predictions about the behavior of put option demand over the business cycle and the resulting co-movement between the stock and the options markets.

References

Abeler, J., A. Falk, L. Goette, and D. Huffman (2011). Reference points and effort provision. *American Economic Review 101*(2), 470 – 492.

Andries, M. and V. Haddad (2020). Information aversion. *Journal of Political Economy 128*(5), 1901–1939.

Ang, A., G. Bekaert, and J. Liu (2005). Why stocks may disappoint. *Journal of Financial Economics 76*(3), 471–508.

Ang, A., J. Chen, and Y. Xing (2006). Downside risk. *The Review of Financial Studies 19*(4), 1191–1239.

Arkes, H. R., D. Hirshleifer, D. Jiang, and S. Lim (2008). Reference point adaptation: Tests in the domain of security trading. *Organizational Behavior and Human Decision Processes 105*(1), 67 – 81.

Basak, S. and A. Pavlova (2013). Asset prices and institutional investors. *The American Economic Review 103*(5), 1728–1758.

Basak, S. and A. Shapiro (2001). Value-at-risk-based risk management: optimal policies and asset prices. *Review of Financial Studies 14*(2), 371–405.

Bates, D. S. (2008). The market for crash risk. *Journal of Economic Dynamics and Control 32*(7), 2291 – 2321.

Bekaert, G., R. J. Hodrick, and D. A. Marshall (1997). The implications of first-order risk aversion for asset market risk premiums. *Journal of Monetary Economics 40*(1), 3 – 39.

Benzoni, L., P. Collin-Dufresne, and R. S. Goldstein (2011). Explaining asset pricing puzzles associated with the 1987 market crash. *Journal of Financial Economics 101*(3), 552 – 573.

Berkelaar, A. B., R. Kouwenberg, and T. Post (2004). Optimal portfolio choice under loss aversion. *Review of Economics & Statistics 86*(4), 973 – 987.

Bhamra, H. S. and R. Uppal (2014). Asset prices with heterogeneity in preferences and beliefs. *The Review of Financial Studies 27*(2), 519–580.

Bollen, N. P. B. and R. E. Whaley (2004). Does net buying pressure affect the shape of implied volatility functions? *Journal of Finance 59*(2), 711–753.

Bondarenko, O. (2014). Why are put options so expensive? *The Quarterly Journal of Finance 4*(3), 145–195.

Bonomo, M., R. Garcia, N. Meddahi, and R. Tedongap (2011). Generalized disappointment aversion, long-run volatility risk, and asset prices. *Review of Financial Studies 24*(1), 82–122.

Branger, N. and A. Hansis (2012). Asset allocation: How much does model choice matter? *Journal of Banking & Finance 36*(7), 1865 – 1882.

Broadie, M., M. Chernov, and M. Johannes (2009). Understanding index option returns. *Review of Financial Studies 22*(11), 4493–4529.

Buraschi, A. and A. Jiltsov (2006). Model uncertainty and option markets with heterogeneous beliefs. *The Journal of Finance 61*(6), 2841–2897.

Chen, H., S. Joslin, and S. X. Ni (2019). Demand for crash insurance, intermediary constraints, and risk premia in financial markets. *The Review of Financial Studies 32*(1), 228–265.

Chen, H., S. Joslin, and N.-K. Tran (2012). Rare disasters and risk sharing with heterogeneous beliefs. *The Review of Financial Studies 25*(7), 2189–2224.

Choi, S., R. Fisman, D. Gale, and S. Kariv (2007). Consistency and heterogeneity of individual behavior under uncertainty. *American Economic Review 97*(5), pp. 1921–1938.

Chordia, T., A. Kurov, D. Muravyev, and A. Subrahmanyam (2021). Index option trading activity and market returns. *Management Science 67*(3), 1758–1778.

Constantinides, G. M., J. C. Jackwerth, and A. Savov (2013). The puzzle of index option returns. *Review of Asset Pricing Studies 1*, 1–29.

Constantinides, G. M. and L. Lian (2021). The supply and demand of S&P 500 put options. *Critical Finance Review 10*(1), 1–20.

Coval, J. D. and T. Shumway (2001). Expected option returns. *Journal of Finance 56*(3), 983–1009.

Dahlquist, M., A. Farago, and R. Tedongap (2016). Asymmetries and portfolio choice. *The Review of Financial Studies 30*(2), 667–702.

Delikouras, S. (2017). Where's the kink? Disappointment events in consumption growth and equilibrium asset prices. *The Review of Financial Studies 30*(8), 2851–2889.

Delikouras, S. and A. Kostakis (2019). A single-factor consumption-based asset pricing model. *Journal of Financial and Quantitative Analysis 54*(2), 789–827.

Dieckmann, S. and M. Gallmeyer (2005). The equilibrium allocation of diffusive and jump risks with heterogeneous agents. *Journal of Economic Dynamics and Control 29*(9), 1547 – 1576.

Driessen, J. and P. Maenhout (2007). An empirical portfolio perspective on option pricing anomalies. *Review of Finance 11*(4), 561–603.

Epstein, L. G. and S. E. Zin (2001). The independence axiom and asset returns. *Journal of Empirical Finance 8*(5), 537 – 572.

Farago, A. and R. Tedongap (2018). Downside risks and the cross-section of asset returns. *Journal of Financial Economics 129*(1), 69 – 86.

Garleanu, N., L. H. Pedersen, and A. M. Poteshman (2009). Demand-based option pricing. *Review of Financial Studies 22*(10), 4259–4299.

Gill, D. and V. Prowse (2012). A structural analysis of disappointment aversion in a real effort competition. *American Economic Review 102*(1), 469–503.

Gul, F. (1991). A theory of disappointment aversion. *Econometrica 59*(3), pp. 667–686.

Haigh, M. S. and J. A. List (2005). Do professional traders exhibit myopic loss aversion? An experimental analysis. *Journal of Finance 60*(1), 523–534.

Johnson, T., M. Liang, and Y. Liu (2016). What drives index options exposures? *Review of Finance 22*(2), 561–593.

Judd, K. L. and D. P. Leisen (2010). Equilibrium open interest. *Journal of Economic Dynamics and Control 34*(12), 2578 – 2600.

Kahneman, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica 47*(2), 263–291.

Lakonishok, J., I. Lee, N. D. Pearson, and A. M. Poteshman (2007). Option market activity. *The Review of Financial Studies 20*(3), 813–857.

Lettau, M., M. Maggiori, and M. Weber (2014). Conditional risk premia in currency markets and other asset classes. *Journal of Financial Economics 114*(2), 197 – 225.

Liu, H. and J. Miao (2015). Growth uncertainty, generalized disappointment aversion and production-based asset pricing. *Journal of Monetary Economics 69*, 70 – 89.

Liu, J. and J. Pan (2003). Dynamic derivative strategies. *Journal of Financial Economics 69*(3), 401 – 430.

Longstaff, F. A. and J. Wang (2012). Asset pricing and the credit market. *The Review of Financial Studies 25*(11), 3169–3215.

Olsen, R. A. (1997). Prospect theory as an explanation of risky choice by professional investors: Some evidence. *Review of Financial Economics 6*(2), 225 – 232.

Pastor, L. and P. Veronesi (2012). Uncertainty about government policy and stock prices. *The Journal of Finance 67*(4), 1219–1264.

Pope, D. G. and M. E. Schweitzer (2011). Is Tiger Woods loss averse? Persistent bias in the face of experience, competition, and high stakes. *American Economic Review 101*(1), 129 – 157.

Routledge, B. R. and S. E. Zin (2010). Generalized disappointment aversion and asset prices. *Journal of Finance 65*(4), 1303–1332.

Schreindorfer, D. (2020). Macroeconomic tail risks and asset prices. *The Review of Financial Studies 33*(8), 3541–3582.

Sutter, M. (2007). Are teams prone to myopic loss aversion? An experimental study on individual versus team investment behavior. *Economics Letters 97*(2), 128 – 132.

Weinbaum, D. (2009). Investor heterogeneity, asset pricing and volatility dynamics. *Journal of Economic Dynamics and Control 33*(7), 1379 – 1397.

Willman, P., M. Fenton-O'Creevy, N. Nicholson, and E. Soane (2002). Traders, managers and loss aversion in investment banking: a field study. *Accounting, Organizations and Society 27*(1-2), 85–98.

Appendix

A Proofs

A.1 Proposition 1

Proof. The portfolio choice problem for the risk averse agent B is solved by maximizing

$$\mathcal{L}_B = E\left[u(W_{B,T})\right] + \lambda_B \left(W_{B,0} - E[\xi_T W_{B,T}]\right),$$

where $u(x) = u_i(x)$, $i \in A, B$ is given in (2). We obtain the agent's optimal terminal wealth as

$$\widehat{W}_{B,T} = (\lambda_B \xi_T)^{-\frac{1}{\gamma}},$$

where λ_B solves

$$W_{B,0} = E[\xi_T \widehat{W}_{B,T}]. \tag{A.1}$$

To solve the portfolio problem of the disappointment-averse agent A we proceed in two steps. First, we maximize the disappointment-averse agent's objective function subject to the static budget constraint for a given disappointment threshold θ_A. Second, we find the (endogenously determined) agent's disappointment threshold. In the first step we rely on the results of Basak and Shapiro (2001) and Berkelaar et al. (2004) on the solution of nonstandard optimization problems. Alternatively, if the

Maximizing

$$\mathcal{L}_A = E\left[u(W_{A,T})\right] - \ell_A E\left[\left(u(\theta_A) - u(W_{A,T})\right) I(W_{A,T} \leq \theta_A)\right] + \lambda_A \left(W_{A,0} - E[\xi_T W_{A,T}]\right)$$

gives the agent's optimal terminal wealth as

$$\widehat{W}_{A,T} = \begin{cases} (\lambda_A \xi_T)^{-\frac{1}{\gamma}} & \text{if } \xi_T \leq \underline{\xi}(\theta_A) \equiv \dfrac{\theta_A^{-\gamma}}{\lambda_A}, \\ \theta_A & \text{if } \underline{\xi} < \xi_T \leq \overline{\xi}, \\ \left(\dfrac{\lambda_A \xi_T}{1+\ell_A}\right)^{-\frac{1}{\gamma}} & \text{if } \xi_T > \overline{\xi}(\theta_A) \equiv \dfrac{(1+\ell_A)\,\theta_A^{-\gamma}}{\lambda_A}, \end{cases} \tag{A.2}$$

where λ_A solves

$$W_{A,0} = E[\xi_T \widehat{W}_{A,T}]. \tag{A.3}$$

Let $\widehat{W}_{A,T}(\theta_A)$ be the optimal wealth profile for a given θ_A. If the disappointment threshold is defined as in (3), we determine the certainty equivalent $\mathcal{R}_A$ as θ_A/κ, where θ_A is the fixed point of

$$u(\kappa^{-1}\theta_A) = E\left[u(\widehat{W}_{A,T}(\theta_A) - \ell_A\left(u(\theta_A) - u\left(\widehat{W}_{A,T}(\theta_A)\right)\right) I\left(\widehat{W}_{A,T}(\theta_A) \leq \theta_A\right)\right],$$

or, equivalently, for $\gamma \neq 1$

$$\begin{aligned}\kappa^{\gamma-1}\theta_A^{1-\gamma} = & E\left[\left(\widehat{W}_{A,T}(\theta_A)\right)^{1-\gamma}\right] - \ell_A\theta_A^{1-\gamma}E\left[I(\widehat{W}_{A,T}(\theta_A) < \theta_A)\right] \\ & + \ell_A E\left[\left(\widehat{W}_{A,T}(\theta_A)\right)^{1-\gamma} I\left(\widehat{W}_{A,T}(\theta_A) < \theta_A\right)\right],\end{aligned} \tag{A.4}$$

and for $\gamma = 1$

$$\begin{aligned}\log\theta_A - \log\kappa = & E\left[\log\left(\widehat{W}_{A,T}(\theta_A)\right)\right] - \ell_A\log\theta_A E\left[I(\widehat{W}_{A,T}(\theta_A) < \theta_A)\right] \\ & + \ell_A E\left[\log\left(\widehat{W}_{A,T}(\theta_A)\right) I\left(\widehat{W}_{A,T}(\theta_A) < \theta_A\right)\right].\end{aligned} \tag{A.5}$$

Imposing the market clearing condition

$$\widehat{W}_{A,T} + \widehat{W}_{B,T} = D_T$$

allows to solve for the equilibrium terminal state price density as follows

$$\xi_T = \begin{cases} c_1 D_T^{-\gamma} & \text{if } D_T < \underline{D} \equiv \left(1 + \left(\frac{\lambda_A}{\lambda_B}\frac{1}{1+\ell_A}\right)^{\frac{1}{\gamma}}\right)\theta_A, \\ c_2\left(D_T - \theta_A\right)^{-\gamma} & \text{if } \underline{D} \leq D_T < \overline{D}, \\ c_3 D_T^{-\gamma} & \text{if } D_T \geq \overline{D} \equiv \left(1 + \left(\frac{\lambda_A}{\lambda_B}\right)^{\frac{1}{\gamma}}\right)\theta_A, \end{cases} \tag{A.6}$$

where $c_1 \equiv \left(\left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma}$, $c_2 \equiv \lambda_B^{-1}$, and $c_3 \equiv \left(\lambda_A^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma}$. The result in (4) then follows from using (A.6) in (A.2).

For $\gamma \neq 1$, the fixed point equation (A.4) determining θ_A becomes

$$\begin{aligned}\kappa^{\gamma-1}\theta_A^{1-\gamma} = & \theta_A^{1-\gamma}E\left[I\left(\underline{D} \leq D_T < \overline{D}\right)\right] + \left(1 + \left(\frac{\lambda_A}{\lambda_B}\right)^{\frac{1}{\gamma}}\right)^{\gamma-1} E\left[D_T^{1-\gamma}I(D_T \geq \overline{D})\right] \\ & - \ell_A\theta_A^{1-\gamma}E\left[I(D_T < \underline{D})\right] + (1+\ell_A)\left(1 + \left(\frac{\lambda_A}{\lambda_B}\frac{1}{1+\ell_A}\right)^{\frac{1}{\gamma}}\right)^{\gamma-1} E\left[D_T^{1-\gamma}I(D_T < \underline{D})\right].\end{aligned} \tag{A.7}$$

For $\gamma = 1$, the equation (A.5) can be further simplified to yield

$$0 = \log(\overline{D}/D_0) - E[\log(D_T/D_0)] - \log(\overline{D}/D_0)E\left[I(D_T < \overline{D})\right] + E\left[\log(D_T/D_0)I(D_T < \overline{D})\right]$$

$$+ (1+\ell_A)\log(\underline{D}/D_0)E\left[I(D_T < \underline{D})\right] - (1+\ell_A)E\left[\log(D_T/D_0)I(D_T < \underline{D}) - \log\kappa\right].$$

Alternatively, if the disappointment threshold is defined as $\theta_i = \kappa_i W_{i,0}$ we have

$$\theta_A = \kappa_A \alpha S_0,$$

where S_0 can be obtained as

$$\begin{aligned} S_0 = {} & c_1 E\left[D_T^{1-\gamma} I(D_T < \underline{D})\right] + c_2 E\left[(D_T - \theta_A)^{-\gamma} D_T I\left(\underline{D} \le D_T < \overline{D}\right)\right] \\ & + c_3 E\left[D_T^{1-\gamma} I(D_T \ge \overline{D})\right]. \end{aligned} \tag{A.8}$$

Moreover, using (A.6) and (4) in the budget constraint of agent (A.3) we obtain the equation for agent A's Lagrange multiplier λ_A

$$\begin{aligned} \alpha S_0 = {} & \left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}} \left(\left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} E\left[D_T^{1-\gamma} I(D_T < \underline{D})\right] \\ & + \theta_A \lambda_B^{-1} E\left[(D_T - \theta_A)^{-\gamma} I\left(\underline{D} \le D_T < \overline{D}\right)\right] \\ & + \lambda_A^{-\frac{1}{\gamma}} \left(\lambda_A^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} E\left[D_T^{1-\gamma} I(D_T \ge \overline{D})\right]. \end{aligned} \tag{A.9}$$

Similarly, to determine λ_B for agent B we can write his budget constraint (A.1) as

$$\begin{aligned} (1-\alpha)\, S_0 = {} & \lambda_B^{-\frac{1}{\gamma}} \left(\left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} E\left[D_T^{1-\gamma} I(D_T < \underline{D})\right] \\ & + \lambda_B^{-1} E\left[(D_T - \theta_A)^{1-\gamma} I\left(\underline{D} \le D_T < \overline{D}\right)\right] \\ & + \lambda_B^{-\frac{1}{\gamma}} \left(\lambda_A^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} E\left[D_T^{1-\gamma} I(D_T \ge \overline{D})\right]. \end{aligned} \tag{A.10}$$

Finally, in order to compute θ_A, λ_A, and λ_B we need explicit expressions for conditional expectations in (A.7), (A.8), (A.9), and (A.10). We make use of the following result.

Lemma A.1.

$$E_t\left[D_T^{-\gamma} I(D_T < \underline{D})\right] = F_1(T-t, D_t; \underline{D}, \gamma), \tag{A.11}$$

$$E_t\left[(D_T - \theta)^{-\gamma} D_T^{\psi} I\left(\underline{D} \le D_T < \overline{D}\right)\right] = F_2(T-t, D_t; \underline{D}, \overline{D}, \theta, \gamma, \psi), \tag{A.12}$$

$$E_t\left[D_T^{-\gamma} I(D_T \ge \overline{D})\right] = F_3(T-t, D_t; \overline{D}, \gamma). \tag{A.13}$$

where functions F_1, F_2, and F_3 are given by

$$F_1(\tau, D; a, \gamma) = D^{-\gamma} e^{-\lambda\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} e^{-\beta(\tau,n,\gamma)} \Phi\left(d_2\left(\frac{a}{D}, \tau, n, \gamma\right)\right),$$

$$F_2(\tau, D; a, b, \theta, \gamma, \psi) = \theta^{\psi-\gamma} \frac{e^{-\lambda\tau}}{\sqrt{2\pi\tau\sigma_D^2}} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} \Psi\left(\tau, \frac{\theta}{D}; a, b, \theta, \gamma, \psi\right),$$

$$F_3(\tau, D; b, \gamma) = D^{-\gamma} e^{-\lambda\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} e^{-\beta(\tau,n,\gamma)} \Phi\left(-d_2\left(\frac{b}{D}, \tau, n, \gamma\right)\right).$$

$\Phi(x)$ denotes the cumulative distribution function of the standard normal distribution and Ψ is defined by

$$\Psi(\tau, x; a, b, \theta, \gamma, \psi) = \int_{\ln a/\theta}^{\ln b/\theta} (e^y - 1)^{-\gamma} e^{\psi y} e^{-\frac{(\alpha(\tau,n) - y - \ln x)^2}{2\tau\sigma_D^2}} dy.$$

Moreover, auxiliary functions α, β, d_1 and d_2 are defined as follows

$$\begin{aligned}
\alpha(\tau, n) &\equiv \tau(\mu_D - \sigma_D^2/2) + n \ln(1 - \delta_D), \\
\beta(\tau, n, \gamma) &\equiv \gamma\alpha(\tau, n) - \gamma^2 \tau \sigma_D^2/2, \\
d_1(x, \tau, n) &\equiv \frac{\ln x - \alpha(\tau, n)}{\sqrt{\tau}\sigma_D}, \\
d_2(x, \tau, n, \gamma) &\equiv d_1(x, \tau, n) + \gamma\sqrt{\tau}\sigma_D.
\end{aligned}$$

Proof. Results in (A.11) and (A.13) follow from the fact that for a random variable X_t defined as $X_t = \mu t + \sigma W_t + \delta N_t$ we have

$$\begin{aligned}
E\left[e^{kX_t} I(X_t \in [a, b])\right] &= \int_a^b e^{kx} \frac{e^{-\lambda t}}{\sqrt{2\pi t\sigma^2}} \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} e^{-\frac{(t\mu + n\delta - x)^2}{2t\sigma^2}} dx \\
&= e^{-\lambda t} \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} e^{\left(\mu'(n)k + (\sqrt{t}\sigma)^2 k^2/2\right)} \\
&\quad \times \left[\Phi\left(\frac{b - \mu'(n)}{\sqrt{t}\sigma} - \sqrt{t}\sigma k\right) - \Phi\left(\frac{a - \mu'(n)}{\sqrt{t}\sigma} - \sqrt{t}\sigma k\right)\right],
\end{aligned}$$

where $\mu'(n) = t\mu + n\delta$.

To compute the expectation in (A.12) we use a change of variable and rewrite it as follows

$$
\begin{aligned}
E_t\left[(D_T-\theta)^{-\gamma} D_T^{\psi} I\left(D_T \in [a,b]\right)\right] &= \theta_A^{\psi-\gamma} \int_{\ln\frac{a}{\theta}+\ln\frac{\theta}{D_t}}^{\ln\frac{b}{\theta}+\ln\frac{\theta}{D_t}} \left(\frac{D_t}{\theta}e^x-1\right)^{-\gamma}\left(\frac{D_t}{\theta}e^x\right)^{\psi} \frac{e^{-\lambda t}}{\sqrt{2\pi t\sigma^2}} \\
&\quad \times \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} e^{-\frac{(t\mu+n\delta-x)^2}{2t\sigma^2}} dx \\
&= \theta^{\psi-\gamma} \frac{e^{-\lambda t}}{\sqrt{2\pi t\sigma^2}} \\
&\quad \times \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} \int_{\ln\frac{a}{\theta}}^{\ln\frac{b}{\theta}} \left(e^y-1\right)^{-\gamma} e^{\psi y} e^{-\frac{\left(t\mu+n\delta-y-\ln\frac{\theta}{D_t}\right)^2}{2t\sigma^2}} dy,
\end{aligned}
$$

where $\mu=\mu_D-\sigma_D^2/2$, $\sigma=\sigma_D$, and $\delta=\ln(1-\delta_D)$. □

Using Lemma A.1 in (A.7), we can compute the certainty equivalent $\mathcal{R}_A=\theta_A/\kappa$, where θ_A solves the following equation for $\gamma\neq 1$

$$
\begin{aligned}
\kappa^{\gamma-1}\theta_A^{1-\gamma} = {} & \theta_A^{1-\gamma} F_2(T,D_0;\underline{D},\overline{D},\theta_A,0,0) + \left(1+\left(\frac{\lambda_A}{\lambda_B}\right)^{\frac{1}{\gamma}}\right)^{\gamma-1} F_3(T,D_0;\overline{D},\gamma-1) \qquad \text{(A.14)} \\
& - \ell_A\theta_A^{1-\gamma} F_1\left(T,D_0;\underline{D},0\right) + (1+\ell_A)\left(1+\left(\frac{\lambda_A}{\lambda_B}\frac{1}{1+\ell_A}\right)^{\frac{1}{\gamma}}\right)^{\gamma-1} F_1\left(T,D_0;\underline{D},\gamma-1\right).
\end{aligned}
$$

and, similarly, for $\gamma=1$

$$
\begin{aligned}
\log\kappa = {} & \log(\overline{D}/D_0) - T(\mu+\lambda\delta) - \log(\overline{D}/D_0)F_1(T,D_0;\overline{D},0) + F_4(T,D_0;\overline{D}) \\
& + (1+\ell_A)\log(\underline{D}/D_0)F_1(T,D_0;\underline{D},,0) - (1+\ell_A)F_4(T,D_0;\underline{D}),
\end{aligned}
$$

where $\mu=\mu_D-\sigma_D^2/2$, $\delta=\ln(1-\delta_D)$ and

$$
F_4(\tau,D;a) = e^{-\lambda\tau}\sum_{n=0}^{\infty}\frac{(\lambda\tau)^n}{n!}\left(\alpha(\tau,n)\Phi\left(d_1\left(\frac{a}{D},\tau,n\right)\right) - \sqrt{\tau}\sigma_D\phi\left(d_1\left(\frac{a}{D},\tau,n\right)\right)\right).
$$

with functions α and d_1 as defined in Lemma A.1.

Moreover, the budget constraint of investor A in equation (A.9) can be written as

$$
\begin{aligned}
\alpha S_0 = {} & \left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}}\left(\left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} F_1(T,D_0;\underline{D},,\gamma-1) \qquad \text{(A.15)} \\
& + \theta_A\lambda_B^{-1}F_2(T,D_0;\underline{D},\overline{D},\theta_A,\gamma,0) + \lambda_A^{-\frac{1}{\gamma}}\left(\lambda_A^{-\frac{1}{\gamma}}+\lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} F_3(T,D_0;\overline{D},\gamma-1).
\end{aligned}
$$

Similarly, for agent B we can write his budget constraint (A.10) as

$$(1-\alpha)\,S_0 = \lambda_B^{-\frac{1}{\gamma}}\left(\left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} F_1(T, D_0; \underline{D}, \gamma-1) \tag{A.16}$$

$$+ \lambda_B^{-1} F_2(T, D_0; \underline{D}, \overline{D}, \theta_A, \gamma-1, 0) + \lambda_B^{-\frac{1}{\gamma}}\left(\lambda_A^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma-1} F_3(T, D_0; \overline{D}, \gamma-1),$$

where

$$S_0 = c_1 F_1\left(0, D_0; \underline{D}, \gamma-1\right) + c_2 F_2\left(0, D_0; \underline{D}, \overline{D}, \theta_A, \gamma, 1\right) + c_3 F_3\left(0, D_0 \overline{D}, \gamma-1\right).$$

□

A.2 Proposition 2

Proof. In our model, since we normalize the riskless rate to zero, the state price density is a martingale,

$$\xi_t = E_t\left[\xi_T\right]. \tag{A.17}$$

Using the expression for the time-T stochastic discount factor from (6), we can rewrite (A.17) as

$$\xi_t = c_1 E_t\left[D_T^{-\gamma} I(D_T < \underline{D})\right] + c_2 E_t\left[(D_T - \theta_A)^{-\gamma} I\left(\underline{D} \le D_T < \overline{D}\right)\right]) + c_3 E_t\left[D_T^{-\gamma} I(D_T \ge \overline{D})\right],$$

where

$$c_1 \equiv \left(\left(\frac{\lambda_A}{1+\ell_A}\right)^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma},$$
$$c_2 \equiv \lambda_B^{-1},$$
$$c_3 \equiv \left(\lambda_A^{-\frac{1}{\gamma}} + \lambda_B^{-\frac{1}{\gamma}}\right)^{\gamma}.$$

Applying Lemma A.1 to compute the conditional expectations immediately gives the result in (7).

From the dynamics of the stochastic discount factor we can recover the market prices of risk, since

$$d\xi_t = \xi_{t-}\left[-\varphi_t dW_t - \psi_t (dN_t - \lambda dt)\right]. \tag{A.18}$$

Applying Ito's lemma to (7) allows to recover expressions for φ_t and ψ_t in (8) and (9). The next Lemma gives expressions for partial derivatives and induced jump sizes of the F_i functions in (7).

$\square$

Lemma A.2. *The partial derivatives of the F_i functions, $i \in 1,2,3$, defined in Lemma A.1 with respect to D are*

$$\frac{\partial F_1(\tau, D; a, \gamma)}{\partial D} = -D^{-\gamma-1} e^{-\lambda\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} e^{-\beta(\tau,n,\gamma)} \Bigg(\gamma \Phi \left(d_2 \left(\frac{a}{D}, \tau, n, \gamma \right) \right) + \frac{\phi \left(d_2 \left(\frac{a}{D}, \tau, n, \gamma \right) \right)}{\sqrt{\tau}\sigma_D} \Bigg),$$

$$\frac{\partial F_3(\tau, D; b, \gamma)}{\partial D} = -D^{-\gamma-1} e^{-\lambda\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} e^{-\beta(\tau,n,\gamma)} \Bigg(\gamma \Phi \left(-d_2 \left(\frac{b}{D}, \tau, n, \gamma \right) \right) - \frac{\phi \left(-d_2 \left(\frac{b}{D}, \tau, n, \gamma \right) \right)}{\sqrt{\tau}\sigma_D} \Bigg) m$$

and

$$\frac{\partial F_2(\tau, D; a, b, \theta, \gamma, \psi)}{\partial D} = \theta^{\psi-\gamma} \frac{e^{-\lambda\tau}}{\sqrt{2\pi\tau\sigma_D^2}} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} \frac{\partial \Psi \left(\tau, \frac{\theta}{D}; \gamma, \psi \right)}{\partial D},$$

where

$$\frac{\partial \Psi \left(\tau, \frac{\theta}{D}; a, b, \gamma, \psi \right)}{\partial D} = \int_{\ln a/\theta_A}^{\ln b/\theta_A} \left(e^y - 1 \right)^{-\gamma} e^{\psi y} \frac{-\alpha(\tau, n) + y + \ln \frac{\theta}{D}}{\tau\sigma_D^2 D} e^{-\frac{\left(\alpha(\tau,n) - y - \ln \frac{\theta}{D}\right)^2}{2\tau\sigma_D^2}} dy.$$

Moreover,

$$\Delta F_i \left(T - t, D_{t-}; \gamma \right) \equiv F_i \left(T - t, D_{t-} + \Delta D_t; \gamma \right) - F_i \left(T - t, D_{t-}; \gamma \right).$$

A.3 Proposition 3

Proof. The expression for the stock price follows from using the results presented in Proposition 1 and Proposition 2 in

$$S_t = \frac{1}{\xi_t} E_t \left[\xi_T D_T \right],$$

where the explicit expression for the expectation can be obtained using Lemma A.1. Applying Ito's lemma to the both sides of $\xi_t S_t = E_t \left[\xi_T D_T \right]$ gives the following expressions for $\mu_{S,t}$, $\sigma_{S,t}$

and $\delta_{S,t}$:

$$\mu_{S,t} = \varphi_t \sigma_{S,t} - \lambda \delta_{S,t} \left(1 - \psi_t\right), \tag{A.19}$$

$$\begin{aligned}\sigma_{S,t} = \varphi_t + \frac{1}{\xi_t S_t} \Big(& c_1 \frac{\partial F_{1t}}{\partial D_t} \left(\underline{D}, \gamma - 1\right) + c_2 \frac{\partial F_{2t}}{\partial D_t} \left(\underline{D}, \overline{D}, \theta_A, \gamma, 1\right) \\ & + c_3 \frac{\partial F_{3t}}{\partial D_t} \left(\overline{D}, \gamma - 1\right) \Big) D_t \sigma_D,\end{aligned} \tag{A.20}$$

$$\begin{aligned}\delta_{S,t} = \frac{1}{1 - \psi_t} \Bigg(1 + \frac{1}{\xi_{t-} S_{t-}} \Big(& c_1 \Delta F_{1t} \left(\underline{D}, \gamma - 1\right) + c_2 \Delta F_{2t} \left(\underline{D}, \overline{D}, \theta_A, \gamma, 1\right) \\ & + c_3 \Delta F_{3t} \left(\overline{D}, \gamma - 1\right) \Big) \Bigg) - 1,\end{aligned} \tag{A.21}$$

where ξ_t, φ_t, ψ_t, constants c_1, c_2, c_3 are given in Proposition 2 and functions $\frac{\partial F_{it}}{\partial D_t}(\cdot)$, and $\Delta F_{it}(\cdot)$ stand for $\frac{\partial F_i}{\partial D}(T - t, D_t; \cdot)$, and $\Delta F_i (T - t, D_{t-}, \cdot)$ defined in Lemma A.2. □

A.4 Proposition 4

Proof. The price of the contingent claim on S_T of the form $\Phi(x)$ can be obtained as

$$F_t = \frac{1}{\xi_t} E_t[\xi_T \Phi(D_T)].$$

The payoff of the put is

$$\max\left(K - D_T, 0\right) = \left(K - D_T\right) I\left(D_T < K\right)$$

Then

$$\begin{aligned}E_t\left[\xi_T \Phi\left(S_T\right)\right] &= E_t\left[\xi_T \left(K - D_T\right) I\left(D_T < K\right)\right] \\ &= K E_t\left[\xi_T I\left(\ln D_T < \ln K\right)\right] - E_t\left[\xi_T D_T I\left(\ln D_T < \ln K\right)\right]\end{aligned} \tag{A.22}$$

Depending on where K is relative to $\underline{D}$ and $\overline{D}$, we have the three cases: (i) $K \leq \underline{D}$, (ii) $\underline{D} < K \leq \overline{D}$, or (iii) $\overline{D} < K$ and the expectations in (A.22) can be computed using the results Lemma A.1. The dynamics of the put is then obtained similarly to the stock. For the put option on the stock the drift $\mu_{P,t}$ is given by

$$\mu_{P,t} = \varphi_t \sigma_{P,t} - \lambda \delta_{P,t} \left(1 - \psi_t\right)$$

and, depending on the level of the strike price, the diffusion volatility $\sigma_{P,t}$ and the jump volatility $\delta_{P,t}$ are as follows:

1. If $K \leq \underline{D}$,

$$\sigma_{P,t} = \varphi_t + \frac{1}{\xi_t P_t} c_1 \left(K \cdot \frac{\partial F_{1t}}{\partial D_t} (K, \gamma) - \frac{\partial F_{1t}}{\partial D_t} (K, \gamma - 1) \right) D_t \sigma_D,$$
$$\delta_{P,t} = \frac{1}{1 - \psi_t} \Big(1 + \frac{1}{\xi_{t-} P_{t-}} c_1 \left(K \cdot \Delta F_{1t} (K, \gamma) - \Delta F_{1t} (K, \gamma - 1) \right) \Big) - 1.$$

2. If $\underline{D} < K \leq \overline{D}$,

$$\begin{aligned} \sigma_{P,t} = \varphi_t + \frac{1}{\xi_t P_t} \Big[& c_1 \left(K \cdot \frac{\partial F_{1t}}{\partial D_t} (\underline{D}, \gamma) - \frac{\partial F_{1t}}{\partial D_t} (\underline{D}, \gamma - 1) \right) \\ & + c_2 \left(K \cdot \frac{\partial F_{2t}}{\partial D_t} (\underline{D}, K, \theta_A, \gamma, 0) - \frac{\partial F_{2t}}{\partial D_t} (\underline{D}, K, \theta_A, \gamma, 1) \right) \Big] D_t \sigma_D, \\ \delta_{P,t} = \frac{1}{1 - \psi_t} \Big(1 + \frac{1}{\xi_{t-} P_{t-}} \Big[& c_1 \left(K \cdot \Delta F_{1t} (\underline{D}, \gamma) - \Delta F_{1t} (\underline{D}, \gamma - 1) \right) \\ & + c_2 \left(K \cdot \Delta F_{2t} (\underline{D}, K, \theta_A, \gamma, 0) - \Delta F_{2t} (\underline{D}, K, \theta_A, \gamma, 1) \right) \Big] \Big) - 1. \end{aligned}$$

3. If $K > \overline{D}$,

$$\begin{aligned} \sigma_{P,t} = \varphi_t + \frac{1}{\xi_t P_t} \Bigg[& c_1 \left(K \cdot \frac{\partial F_{1t}}{\partial D_t} (\underline{D}, \gamma) - \frac{\partial F_{1t}}{\partial D_t} (\underline{D}, \gamma - 1) \right) \\ & + c_2 \left(K \cdot \frac{\partial F_{2t}}{\partial D_t} \left(\underline{D}, \overline{D}, \theta_A, \gamma, 0\right) - \frac{\partial F_{2t}}{\partial D_t} \left(\underline{D}, \overline{D}, \theta_A, \gamma, 1\right) \right) \\ & + c_3 \Bigg(K \cdot \left(\frac{\partial F_{1t}}{\partial D_t} (K, \gamma) - \frac{\partial F_{1t}}{\partial D_t} \left(\overline{D}, \gamma\right) \right) \\ & - \left(\frac{\partial F_{1t}}{\partial D_t} (K, \gamma - 1) - \frac{\partial F_{1t}}{\partial D_t} \left(\overline{D}, \gamma - 1\right) \right) \Bigg) \Bigg] D_t \sigma_D, \\ \delta_{P,t} = \frac{1}{1 - \psi_t} \Big(1 + \frac{1}{\xi_{t-} P_{t-}} \Big[& c_1 \left(K \cdot \Delta F_{1t} (\underline{D}, \gamma) - \Delta F_{1t} (\underline{D}, \gamma - 1) \right) \\ & + \lambda_B^{-1} \left(K \cdot \Delta F_{2t} \left(\underline{D}, \overline{D}, \theta_A, \gamma, 0\right) - \Delta F_{2t} \left(\underline{D}, \overline{D}, \theta_A, \gamma, 1\right) \right) \\ & + c_3 \Big(K \cdot \left(\Delta F_{1t} (K, \gamma) - \Delta F_{1t} \left(\overline{D}, \gamma\right) \right) \\ & - \left(\Delta F_{1t} (K, \gamma - 1) - \Delta F_{1t} \left(\overline{D}, \gamma - 1\right) \right) \Big) \Big] \Big) - 1. \end{aligned}$$

Expressions for ξ_t, φ_t, ψ_t, constants c_1, c_2, c_3 are given in Proposition 2 and functions $\frac{\partial F_{it}}{\partial D_t} (\cdot)$, and $\Delta F_{it} (\cdot)$ stand for $\frac{\partial F_i}{\partial D} (T - t, D_t; \cdot)$, and $\Delta F_i (T - t, D_{t-}, \cdot)$ defined in Lemma A.2.

□

A.5 Proposition 5

Equilibrium time t wealth can be computed from

$$\widehat{W}_{i,t} = \frac{1}{\xi_t} E_t \left[\xi_T \widehat{W}_{i,T} \right]$$

using results of Proposition 1 and Lemma A.1. The equilibrium wealth dynamics of agent i can be written as

$$dW_{i,t} = W_{i,t-} \left[\mu_{W,i,t} dt + \sigma_{W,i,t} dw_t + \delta_{W,i,t} dN_t \right],$$

where $\sigma_{W,i,t}$ and $\delta_{W,i,t}$ capture local exposure to the risk factors: diffusion risk dw_t and the jump risk dN_t. Recalling that the dynamics of the stochastic discount factor is

$$d\xi_t = \xi_{t-} \left[\psi_t \lambda dt - \varphi_t dw_t - \psi_t dN_t \right]$$

we can obtain the dynamics of the discounted wealth process $Z_{i,t} = \xi_t W_{i,t}$ as

$$\begin{aligned} \frac{dZ_{i,t}}{Z_{i,t-}} =& \left(\mu_{W,i,t} + \psi_t \lambda - \varphi_t \sigma_{W,i,t} \right) dt \\ &+ (-\varphi_t + \sigma_{W,i,t}) dw_t + \left((1 - \psi_t) \left(1 + \delta_{W,i,t} \right) - 1 \right) dN_t. \end{aligned}$$

The risk exposures can be determined from the diffusion and jump parts of the Z-dynamics. Indeed, we have

$$\begin{aligned} \sigma_{W,i,t} &= \alpha_{i,t} + \varphi_t, \\ \delta_{W,i,t} &= \frac{\beta_{i,t} + \psi_t}{1 - \psi_t}. \end{aligned}$$

In order to find α_i and β_i we solve for the equilibrium dynamics of $\xi_t \widehat{W}_{it}$

$$\frac{d\xi_t \widehat{W}_{it}}{\xi_{t-} \widehat{W}_{it-}} = (\dots) \, dt + \alpha_{i,t} dw_t + \beta_{i,t} dN_t,$$

where we do not care about the exact form of the dt term. Using results in (7) and (10) we obtain $\alpha_{i,t}$ and $\beta_{i,t}$ as

$$\begin{aligned}\alpha_{A,t} =& \frac{1}{\xi_t \widehat{W}_{A,t}} \left(\left(\frac{\lambda_A}{1+\ell_A} \right)^{-\frac{1}{\gamma}} c_1^{1-\frac{1}{\gamma}} \frac{\partial F_{1t}}{\partial D_t} \left(\underline{D}, \gamma-1\right) + \theta_A c_2 \frac{\partial F_{2t}}{\partial D_t} \left(\underline{D}, \overline{D}, \theta_A \gamma, 0\right) \right. \\ & \left. + \lambda_A^{-\frac{1}{\gamma}} c_1^{1-\frac{1}{\gamma}} \frac{\partial F_{3t}}{\partial D_t} \left(\overline{D}, \gamma-1\right) \right) D_t \sigma_D, \\ \alpha_{B,t} =& \frac{1}{\xi_t \widehat{W}_{B,t}} \left(\lambda_B^{-\frac{1}{\gamma}} c_1^{1-\frac{1}{\gamma}} \frac{\partial F_{1t}}{\partial D_t} \left(\underline{D}, \gamma-1\right) + \lambda_B^{-1} \frac{\partial F_{2t}}{\partial D_t} \left(\underline{D}, \overline{D}, \theta_A, \gamma-1, 0\right) \right. \\ & \left. + \lambda_B^{-\frac{1}{\gamma}} c_3^{1-\frac{1}{\gamma}} \frac{\partial F_{3t}}{\partial D_t} (\overline{D}, \gamma-1) \right) D_t \sigma_D,\end{aligned} \tag{A.23}$$

and

$$\begin{aligned}\beta_{A,t} =& \frac{1}{\xi_{t-} \widehat{W}_{At-}} \left(\left(\frac{\lambda_A}{1+\ell_A} \right)^{-\frac{1}{\gamma}} c_1^{1-\frac{1}{\gamma}} \Delta F_{1t} \left(\underline{D}, \gamma-1\right) + \theta_A c_2 \Delta F_{2t} \left(\underline{D}, \overline{D}, \theta_A, \gamma, 0\right) \right. \\ & \left. + \lambda_A^{-\frac{1}{\gamma}} c_3^{1-\frac{1}{\gamma}} \Delta F_{3t} \left(\overline{D}, \gamma-1\right) \right), \\ \beta_{B,t} =& \frac{1}{\xi_{t-} \widehat{W}_{Bt-}} \left(\lambda_B^{-\frac{1}{\gamma}} c_1^{1-\frac{1}{\gamma}} \Delta F_{1t} \left(\underline{D}, \gamma-1\right) + c_2 \Delta F_{2t} \left(\underline{D}, \overline{D}, \theta_A, \gamma-1\right) \right. \\ & \left. + \lambda_B^{-\frac{1}{\gamma}} c_3^{1-\frac{1}{\gamma}} \Delta F_{3t} \left(\overline{D}, \gamma-1\right) \right),\end{aligned} \tag{A.24}$$

$\frac{\partial F_{it}}{\partial D_t}(\cdot)$ and $\Delta F_{it}(\cdot)$ stand for $\frac{\partial F_i}{\partial D}(T-t, D_t; \cdot)$ and $\Delta F_i(T-t, D_{t-}, \cdot)$ defined in Lemma A.2.

A.6 Proposition 6

Proof. From standard portfolio theory we know that the wealth dynamics is of the form

$$dW_{i,t} = W_{i,t-} \left[\left(\phi_{S,i,t}\mu_{S,t} + \phi_{P,i,t}\mu_{P,t}\right) dt + \left(\phi_{S,i,t}\sigma_{S,t} + \phi_{P,i,t}\sigma_{P,t}\right) dw_t + \left(\phi_{S,i,t}\delta_{S,t} + \phi_{P,i,t}\delta_{P,t}\right) dN_t \right].$$

Equilibrium portfolio shares can then be determined from the diffusion and jump parts of the dynamics of the discounted wealth process $Z_{i,t} = \xi_t W_{i,t}$. Indeed, we have a system of two equations with two unknowns,

$$\begin{aligned}-\varphi_t + \phi_{S,i,t}\sigma_{S,t} + \phi_{P,i,t}\sigma_{P,t} &= \alpha_{i,t} \\ \left(1-\psi_t\right)\left(1 + \phi_{S,i,t}\delta_{S,t} + \phi_{P,i,t}\delta_{P,t}\right) - 1 &= \beta_{i,t}.\end{aligned}$$

This allows us to solve for portfolio shares $\phi_{S,i,t}$ and $\phi_{P,i,t}$ as

$$\phi_{S,i,t} = -\frac{\delta_{P,t}(\alpha_{i,t} + \varphi_t)(\psi_t - 1) + (\beta_{i,t} + \psi_t)\sigma_{P,t}}{(\psi_t - 1)(\delta_{S,t}\sigma_{P,t} - \delta_{P,t}\sigma_{S,t})}$$
$$\phi_{P,i,t} = \frac{\delta_{S,t}(\alpha_{i,t} + \varphi_t)(\psi_t - 1) + (\beta_{i,t} + \psi_t)\sigma_{S,t}}{(\psi_t - 1)(\delta_{S,t}\sigma_{P,t} - \delta_{P,t}\sigma_{S,t})}$$

Alternatively, using the definitions of the exposures to the two risk factors that support the equilibrium wealth dynamics of agent i, we can write the expressions for the equilibrium positions in the risky assets as

$$\phi_{S,i,t} = \frac{\delta_{W,i,t}\sigma_{P,t} - \delta_{P,t}\sigma_{W,i,t}}{\delta_{S,t}\sigma_{P,t} - \delta_{P,t}\sigma_{S,t}},$$
$$\phi_{P,i,t} = \frac{\delta_{W,i,t}\sigma_{S,t} - \delta_{S,t}\sigma_{W,i,t}}{-\delta_{S,t}\sigma_{P,t} + \delta_{P,t}\sigma_{S,t}}.$$

□

B Benchmark Risk-Averse Economy

Using the results in Bhamra and Uppal (2014), we present here equilibrium in a benchmark heterogenous-agent economy, populated by two agents with CRRA preferences. We assume that the risk aversion of agent A is higher than that of agent B: $\gamma_A > \gamma_B$. The rest of assumptions about the economy are the same as in Section 3.

Proposition 7 (Wealth profiles)**.** *The equilibrium terminal wealth profiles are*

$$\widehat{W}_{A,T} = y_T D_T,$$
$$\widehat{W}_{B,T} = (1 - y_T) D_T,$$

where y_T is the terminal wealth share of the more risk averse agent A given by

$$y_T = \begin{cases} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_B}{\lambda_A} D_T^{\gamma_B - \gamma_A} \right)^{n/\gamma_A} \binom{n \frac{\gamma_B}{\gamma_A}}{n-1} & , \quad \frac{\lambda_A}{\lambda_B} D_T^{\gamma_A - \gamma_B} > R, \\ 1 - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_A}{\lambda_B} D_T^{\gamma_A - \gamma_B} \right)^{n/\gamma_B} \binom{n \frac{\gamma_A}{\gamma_B}}{n-1} & , \quad \frac{\lambda_A}{\lambda_B} D_T^{\gamma_A - \gamma_B} < R, \end{cases}$$

with $R = \left(\frac{(\eta-1)^{\eta-1}}{\eta^\eta} \right)^{\gamma_B}$ for $\eta = \gamma_A/\gamma_B$, and $\binom{z}{k} = \frac{\Gamma(z+1)}{\Gamma(z-k+1)\Gamma(k+1)}$, where $\Gamma(z)$ is a Gamma function, and individual Lagrange multipliers, λ_A and λ_B are implicitly defined as the solution to the following system of equations

$$\lambda_A = (\alpha S_0)^{-1} G_A (T, D_0; 1 - \gamma_A, 1 - \gamma_A),$$
$$\lambda_B = ((1-\alpha) S_0)^{-1} G_B (T, D_0; 1 - \gamma_B, 1 - \gamma_B),$$

where expressions for $G_i(\cdot)$, $i = A, B$ and $\overline{R}$ are provided in (B.7) and (B.8).

Proof. Since the securities market is dynamically complete, there exists a unique state price density process, ξ. In our model, ξ is a martingale with dynamics

$$d\xi_t = \xi_{t-} \left[-\varphi_t dw_t - \psi_t (dN_t - \lambda dt) \right],$$

where the market prices of risk process φ and ψ will be endogenously determined in equilibrium. Maximizing the individual investor's expected objective function

$$E_t \left[u_i(W_{i,T}) \right] = E_t \left[\frac{1}{1-\gamma_i} W_{i,T}^{1-\gamma_i} \right], \quad i = A, B$$

subject to the budget constraint

$$E_t \left[\xi_T W_{i,T} \right] = \xi_t W_{i,t} \tag{B.1}$$

evaluated at $t = 0$ leads to the optimal terminal wealth profile:

$$\widehat{W}_{A,T} = (\lambda_A \xi_T)^{-\frac{1}{\gamma_A}}, \tag{B.2}$$

$$\widehat{W}_{B,T} = (\lambda_B \xi_T)^{-\frac{1}{\gamma_B}}, \tag{B.3}$$

where λ_A and λ_B solve (B.1) for $i = A, B$ evaluated at time $t = 0$. Using the market clearing condition

$$\widehat{W}_{A,T} + \widehat{W}_{B,T} = D_T$$

in (B.2) and (B.3) yields

$$\xi_T = \lambda_A^{-1} \widehat{W}_{A,T}^{-\gamma_A} = \lambda_B^{-1} \left(D_T - \widehat{W}_{A,T} \right)^{-\gamma_B}, \tag{B.4}$$

which can be rearranged as follows

$$\left(\frac{\widehat{W}_{A,T}}{D_T} \right) \left(1 - \frac{\widehat{W}_{A,T}}{D_T} \right)^{-\gamma_B/\gamma_A} = \left(\frac{\lambda_B}{\lambda_A} \right)^{1/\gamma_A} D_T^{\gamma_B/\gamma_A - 1}$$

We see that the terminal wealth share of agent A, $y_T \equiv \frac{\widehat{W}_{A,T}}{D_T}$ is implicitly defined by

$$y_T = g^{-1} \left(\left(\frac{\lambda_B}{\lambda_A} \right)^{1/\gamma_A} D_T^{\gamma_B/\gamma_A - 1} \right),$$

where g^{-1} is the inverse of $g(x) = x(1-x)^{-\gamma_B/\gamma_A}$. The explicit expression for y_T follows from Proposition 1 in Bhamra and Uppal (2014).

Lagrange multipliers λ_A and λ_B which solve the agents' budget constraints at $t = 0$,

$$E\left[\lambda_A^{-1} y_T^{1-\gamma_A} D_T^{1-\gamma_A} \right] = \alpha S_0, \tag{B.5}$$

$$E\left[\lambda_B^{-1} (1 - y_T)^{1-\gamma_B} D_T^{1-\gamma_B} \right] = (1-\alpha) S_0. \tag{B.6}$$

Note that y_T depends on the Lagrange multipliers itself, and thus (B.5) and (B.6) for a system of equations that allow to determine λ_A and λ_B. To be able to compute λ_A and λ_B explicitly we make use of the following result.

Lemma B.3. *Conditional expectations of $y_T^a D_T^b$ and $(1-y_T)^a D_T^b$ can be computed as follows*

$$E_t \left[y_T^a D_T^b \right] = G_A(T-t, D_t; a, b),$$

$$E_t \left[(1-y_T)^a D_T^b \right] = G_B(T-t, D_t; a, b),$$

where

$$G_i(\tau, D; a, b) = G_{i1}(\tau, D; a, b, \overline{R}) + G_{i2}(\tau, D; a, b, \overline{R}), \quad i \in A, B \tag{B.7}$$

with

$$\overline{R} = \frac{1}{(\gamma_A - \gamma_B)} \ln\left(R\frac{\lambda_B}{\lambda_A} \right) \tag{B.8}$$

and

$$\begin{aligned}
G_{A1}(\tau, D; a, b, c) =& \int_{-\infty}^{c} (H_1(y))^a \, e^{by} f_{X_{[\tau]}}(y - \ln D) dy, \\
G_{A2}(\tau, D; a, b, c) =& \int_{c}^{\infty} (H_2(y))^a \, e^{by} f_{X_{[\tau]}}(y - \ln D) dy, \\
G_{B1}(\tau, D; a, b, c) =& \int_{-\infty}^{c} (1 - H_1(y))^a \, e^{by} f_{X_{[\tau]}}(y - \ln D) dy, \\
G_{B2}(\tau, D; a, b, c) =& \int_{c}^{\infty} (1 - H_2(y))^a \, e^{by} f_{X_{[\tau]}}(y - \ln D) dy,
\end{aligned} \tag{B.9}$$

with

$$\begin{aligned}
H_1(y) &= 1 - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_A}{\lambda_B} \left(e^y\right)^{\gamma_A - \gamma_B} \right)^{n/\gamma_B} \binom{n\frac{\gamma_A}{\gamma_B}}{n-1}, \\
H_2(y) &= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_B}{\lambda_A} \left(e^y\right)^{\gamma_B - \gamma_A} \right)^{n/\gamma_A} \binom{n\frac{\gamma_B}{\gamma_A}}{n-1}, \\
f_{X_{[\tau]}}(x) &= \frac{e^{-\lambda\tau}}{\sqrt{2\pi\tau\sigma_D^2}} \sum_{k=0}^{\infty} \frac{(\lambda\tau)^k}{k!} e^{-\frac{\left(\tau\left(\mu_D - \sigma_D^2/2\right) + k\ln(1-\delta_D) - x\right)^2}{2\tau\sigma_D^2}}.
\end{aligned}$$

Hence, the expectation in agent's A budget constraint can be computed as

$$E\left[\lambda_A^{-1} y_T^{1-\gamma_A} D_T^{1-\gamma_A}\right] = \lambda_A^{-1} G_A\left(T, D_0; 1-\gamma_A, 1-\gamma_A\right).$$

Similarly, for agent B we can compute the expectation in B's time 0 budget constraint as

$$E\left[\lambda_B^{-1} (1-y_T)^{1-\gamma_B} D_T^{1-\gamma_B}\right] = \lambda_B^{-1} G_B\left(T, D_0; 1-\gamma_B, 1-\gamma_B\right).$$

□

Proposition 8 (State-price density)**.** *The equilibrium state price density at time t is given by*

$$\xi_t = \lambda_A^{-1} G_A\left(T-t, D_t; -\gamma_A, -\gamma_A\right). \tag{B.10}$$

The market prices of diffusion and jump risk are φ_t and $\lambda_t^Q/\lambda_t = 1-\psi_t$, where

$$\varphi_t = -\frac{1}{G_A\left(T-t, D_t; -\gamma_A, -\gamma_A\right)} \frac{\partial G_A}{\partial D}\left(T-t, D_t; -\gamma_A, -\gamma_A\right) D_t \sigma_D, \tag{B.11}$$

$$\psi_t = -\frac{1}{G_A\left(T-t, D_{t-}; -\gamma_A, -\gamma_A\right)} \Delta G_A\left(T-t, D_{t-}; -\gamma_A, -\gamma_A\right). \tag{B.12}$$

The expressions for $G_A(\cdot)$, $\frac{\partial G_A}{\partial D}(\cdot)$, and $\Delta G_A(\cdot)$ are given in (B.7), (B.14), and (B.15), respectively.

Proof. In our model, since we normalize the riskless rate to zero, the state price density is a martingale,

$$\xi_t = E_t\left[\xi_T\right]. \tag{B.13}$$

Using the expression for the time-T stochastic discount factor from (B.4), we can rewrite (B.13) as

$$\xi_t = E_t\left[\lambda_A^{-1} y_T^{-\gamma_A} D_T^{-\gamma_A}\right]$$

and the result for ξ_t follows immediately from Lemma B.3. From the dynamics of the stochastic discount factor in (A.18) we can recover the market prices of risk by applying Ito's lemma to (B.10).

The partial derivative of G_A (or, analogously, for G_B) with respect to D is

$$\frac{\partial G_A}{\partial D}(T-t, D_t; a, b) = \frac{\partial G_{A1}}{\partial D}(T-t, D_t; a, b, \overline{R}) + \frac{\partial G_{A1}}{\partial D}(T-t, D_t; a, b, \overline{R}), \tag{B.14}$$

where

$$\frac{\partial G_{A1}}{\partial D}(T-t, D_t; a, b, c) = \int_{-\infty}^{c} \left(H_1(y)\right)^a e^{by} f_{X_{[T-t]}}(y - \ln D_t) dy,$$
$$\frac{\partial G_{A2}}{\partial D}(T-t, D_t; a, b, c) = \int_{c}^{\infty} \left(H_2(y)\right)^a e^{by} \frac{\partial f_{X_{[T-t]}}(y - \ln D_t)}{\partial D} dy,$$

with $\frac{\partial f_{X_{[T-t]}}(y-\ln D_t)}{\partial D}$ equal to

$$-\frac{e^{-\lambda(T-t)}}{\sqrt{2\pi(T-t)\sigma^2}} \sum_{n=0}^{\infty} \frac{(\lambda(T-t))^n}{n!} e^{-\frac{((T-t)\mu + n\delta - y + \ln D_t)^2}{2(T-t)\sigma^2}} \frac{((T-t)\mu + n\delta - y + \ln D_t)}{(T-t)\sigma^2 D_t}$$

with $\mu = \mu_D - \sigma_D^2/2$, $\sigma = \sigma_D$, $\delta = \ln(1-\delta_D)$. Moreover, we define ΔG_A (ΔG_B can be defined analogously) as

$$\Delta G_A(T-t, D_{t-}; a, b) = \Delta G_{A1}(T-t, D_{t-}; a, b, \overline{R}) + \Delta G_{A2}(T-t, D_{t-}; a, b, \overline{R}), \tag{B.15}$$

where

$$\Delta G_{A1}(T-t, D_{t-}; a, b, c) = G_{A1}(T-t, D_{t-}(1-\delta_D); a, b, c) - G_{A1}(T-t, D_{t-}; a, b, c),$$
$$\Delta G_{A2}(T-t, D_{t-}; a, b, c) = G_{A2}(T-t, D_{t-}(1-\delta_D); a, b, c) - G_{A2}(T-t, D_{t-}; a, b, c).$$

□

Proposition 9 (Stock price and dynamics)**.** *In equilibrium, the stock price is*

$$S_t = \frac{G_A\left(T-t, D_t; -\gamma_A, 1-\gamma_A\right)}{G_A\left(T-t, D_t; -\gamma_A, -\gamma_A\right)}$$

with dynamics

$$dS_t = S_{t-}\left[\mu_{S,t}dt + \sigma_{S,t}dw_t + \delta_{S,t}dN_t\right],$$

where the drift $\mu_{S,t}$, the diffusion volatility $\sigma_{S,t}$, and the jump volatility $\delta_{S,t}$ are defined by

$$\begin{aligned}
\mu_{S,t} &= \varphi_t\sigma_{S,t} - \lambda\delta_{S,t}\left(1-\psi_t\right),\\
\sigma_{S,t} &= \varphi_t + \frac{1}{G_A\left(T-t, D_t; -\gamma_A, 1-\gamma_A\right)}\frac{\partial G_A}{\partial D}\left(T-t, D_t; -\gamma_A, 1-\gamma_A\right)D_t\sigma_D,\\
\delta_{S,t} &= \frac{1}{1-\psi_t}\left(1 + \frac{\Delta G_A\left(T-t, D_{t-}; -\gamma_A, 1-\gamma_A\right)}{G_A\left(T-t, D_{t-}; -\gamma_A, 1-\gamma_A\right)}\right) - 1,
\end{aligned}$$

and φ and ψ are provided in (B.11) *and* (B.12)*. The expressions for $G_A(\cdot)$, $\frac{\partial G_A}{\partial D}(\cdot)$, and $\Delta G_A(\cdot)$ are given in* (B.7), (B.14), *and* (B.15), *respectively.*

Proof. Stock price can be determined from

$$\xi_t S_t = E_t\left[\xi_T D_T\right], \tag{B.16}$$

where, using (B.4) together with Lemma B.3, the right hand side can be written as

$$E_t\left[\xi_T D_T\right] = \lambda_A^{-1}E_t\left[y_T^{-\gamma_A}D_T^{1-\gamma_A}\right] = \lambda_A^{-1}G_A(T-t, D_t; -\gamma_A, 1-\gamma_A)$$

Applying Ito's lemma on both sides of (B.16) allows us to recover the equilibrium stock price dynamics.

The risk premium on the stock is

$$\frac{1}{dt}E_{t-}\left[\frac{dS_t}{S_{t-}}\right] = \mu_{S,t} + \lambda\delta_{S,t} = \varphi_t\sigma_{S,t} + \lambda\delta_{S,t}\psi_t.$$

□

Proposition 10 (Put option price and dynamics)**.** *If the strike price is $K \leq e^{\overline{R}}$, the equi-*

librium price of the put option on the stock market is given by

$$P_t = \frac{1}{\xi_t}\lambda_A^{-1}\Big[K \cdot G_{A1}\left(T-t, D_t; -\gamma_A, -\gamma_A, \ln K\right) - G_{A1}\left(T-t, D_t; -\gamma_A, 1-\gamma_A, \ln K\right)\Big]$$

with dynamics

$$dP_t = P_{t-}\left[\mu_{P,t}dt + \sigma_{P,t}dw_t + \delta_{P,t}dN_t\right]$$

where

$$\begin{aligned}
\mu_{P,t} &= \varphi_t\sigma_{P,t} - \lambda\delta_{P,t}\left(1-\psi_t\right), \\
\sigma_{P,t} &= \varphi_t + \frac{1}{\xi_t P_t}\lambda_A^{-1}\Big[K \cdot \frac{\partial G_{A1}}{\partial D}\left(T-t, D_t; -\gamma_A, -\gamma_A, \ln K\right) \\
&\qquad - \frac{\partial G_{A1}}{\partial D}\left(T-t, D_t; -\gamma_A, 1-\gamma_A, \ln K\right)\Big] D_t\sigma_D, \\
\delta_{P,t} &= \frac{1}{1-\psi_t}\Big(\frac{1}{\xi_{t-}P_{t-}}\lambda_A^{-1}\Big[K \cdot \Delta G_{A1}(T-t, D_{t-}; -\gamma_A, -\gamma_A, \ln K) \\
&\qquad - \Delta G_{A1}(T-t, D_{t-}; -\gamma_A, 1-\gamma_A, \ln K)\Big] + 1\Big) - 1.
\end{aligned}$$

If the strike price is $K > e^{\overline{R}}$, the equilibrium price of the put option on the stock market is given by

$$\begin{aligned}
P_t = \frac{\lambda_A^{-1}}{\xi_t}\Big[&K \cdot \big(G_{A1}(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R}) + G_{A2}^K(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R})\big) \\
&- \big(G_{A1}(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R}) + G_{A2}^K(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R})\big)\Big],
\end{aligned}$$

and

$$\begin{aligned}
\mu_{P,t} &= \varphi_t\sigma_{P,t} - \lambda\delta_{P,t}\left(1-\psi_t\right), \\
\sigma_{P,t} &= \varphi_t + \frac{1}{\xi_t P_t}\lambda_A^{-1}\Big[K \cdot \Big(\frac{\partial G_{A1}}{\partial D}(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R}) + \frac{\partial G_{A2}^K}{\partial D}(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R})\Big) \\
&\qquad - \Big(\frac{\partial G_{A1}}{\partial D}(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R}) + \frac{\partial G_{A2}^K}{\partial D}(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R})\Big)\Big] D_t\sigma_D, \\
\delta_{P,t} &= \frac{1}{1-\psi_t}\Big(\frac{1}{\xi_{t-}F_{t-}}\lambda_A^{-1}\Big[K \cdot \big(\Delta G_{A1}(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R}) + \Delta G_{A2}^K(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R})\big) \\
&\qquad - \big(\Delta G_{A1}(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R}) + \Delta G_{A2}^K(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R})\big)\Big] + 1\Big) - 1.
\end{aligned}$$

Proof. The option price can be determined from

$$\xi_t P_t = E_t \left[\xi_T \Phi(D_T)\right], \tag{B.17}$$

where the payoff function $\Phi(\cdot)$ is defined by $\Phi(x) = \max[K - x, 0]$. Then the right hand side of (B.17) can be written as follows

$$\begin{aligned} E_t\left[\xi_T \Phi\left(D_T\right)\right] &= E_t\left[\xi_T\left(K - D_T\right) I\left(D_T < K\right)\right] \\ &= K E_t\left[\xi_T I\left(\ln D_T < \ln K\right)\right] - E_t\left[\xi_T D_T I\left(\ln D_T < \ln K\right)\right] \end{aligned}$$

Depending on the level of K, we have the following two cases:

1. If $\ln K \leq \overline{R}$,

$$\begin{aligned} E_t\left[\xi_T \Phi\left(D_T\right)\right] = \lambda_A^{-1}\Big[& K \cdot G_{A1}(T-t, D_t; -\gamma_A, -\gamma_A, \ln K) \\ & - G_{A1}(T-t, D_t; -\gamma_A, 1-\gamma_A, \ln K)\Big]. \end{aligned}$$

2. If $\ln K > \overline{R}$,

$$\begin{aligned} E_t\left[\xi_T \Phi\left(D_T\right)\right] = \lambda_A^{-1}\Big[& K \cdot \big(G_{A1}(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R}) + G_{A2}^K(T-t, D_t; -\gamma_A, -\gamma_A, \overline{R})\big) \\ & - \big(G_{A1}(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R}) + G_{A2}^K(T-t, D_t; -\gamma_A, 1-\gamma_A, \overline{R})\big)\Big], \end{aligned}$$

where $G_{A1}(\cdot)$ is defined by (B.9) and

$$G_{A2}^K(T-t, D_t; a, b, c) = \int_c^{\ln K} \left(H_2(y)\right)^a e^{by} f_{X_{[T-t]}}(y - \ln D_t) dy.$$

Applying Ito on both sides of (B.17) allows us to recover the dynamics of P.

□

Proposition 11 (Investors' portfolios in equilibrium)**.** *In equilibrium, the fractions of investors' wealth invested in the stock and the put option are*

$$\begin{aligned} \phi_{i,S,t} &= \frac{\delta_{P,t}(\alpha_{i,t} + \varphi_t)(\psi_t - 1) + (\beta_{i,t} + \psi_t)\sigma_{P,t}}{(\psi_t - 1)(\delta_{P,t}\sigma_{S,t} - \delta_{S,t}\sigma_{P,t})}, \\ \phi_{i,P,t} &= \frac{\delta_{S,t}(\alpha_{i,t} + \varphi_t)(\psi_t - 1) + (\beta_{i,t} + \psi_t)\sigma_{S,t}}{(\psi_t - 1)(\delta_{S,t}\sigma_{P,t} - \delta_{P,t}\sigma_{S,t})}, \quad i \in \{A, B\}, \end{aligned}$$

where $\alpha_{i,t}$ and $\beta_{i,t}$ are given by

$$\alpha_{i,t} = \frac{1}{G_i\left(T-t, D_t; 1-\gamma_i, 1-\gamma_i\right)} \frac{\partial G_i}{\partial D}\left(T-t, D_t; 1-\gamma_i, 1-\gamma_i\right) D_t \sigma_D,$$
$$\beta_{i,t} = \frac{1}{G_i\left(T-t, D_{t-}; 1-\gamma_i, 1-\gamma_i\right)} \Delta G_i\left(T-t, D_{t-}; 1-\gamma_i, 1-\gamma_i\right).$$

Moreover, the investors' portfolios in terms of the number of shares in the stock and the put option are

$$\pi_{i,S,t} = \phi_{i,S,t} \frac{\widehat{W}_{i,t}}{S_t}, \quad \pi_{i,P,t} = \phi_{i,P,t} \frac{\widehat{W}_{i,t}}{P_t},$$

where

$$\widehat{W}_{i,t} = \frac{1}{\xi_t} \lambda_i^{-1} G_i(T-t, D_t; 1-\gamma_i, 1-\gamma_i), \quad i \in \{A, B\}.$$

with the expressions for φ_t, ψ_t, $\sigma_{S,t}$, $\delta_{S,t}$, $\sigma_{P,t}$, $\delta_{P,t}$ provided in Lemmas 8, 9, and 10. Functions $G_i(\cdot)$, $\frac{\partial G_i}{\partial D}(\cdot)$, and $\Delta G_i(\cdot)$ are given by (B.7), (B.14), (B.15), *respectively.*

Proof. The wealth process of agents' self-financing portfolio has the dynamics (recall that in our setting the riskless rate is zero)

$$\begin{aligned} dW_{i,t} &= W_{i,t-}\left[\phi_{i,S,t} \frac{dS_t}{S_t} + \phi_{i,P,t} \frac{dP_t}{P_t}\right] \\ &= W_{i,t-}\left[\left(\phi_{i,S,t}\mu_{S,t} + \phi_{i,P,t}\mu_{P,t}\right) dt + \left(\phi_{i,S,t}\sigma_S + \phi_{i,P,t}\sigma_{P,t}\right) dw_t + \left(\phi_{i,S,t}\delta_{S,t} + \phi_{i,P,t}\delta_{P,t}\right) dN_t\right]. \end{aligned}$$

We also know that

$$\xi_t \widehat{W}_{A,T} = E_t\left[\lambda_A^{-1} y_T^{1-\gamma_A} D_T^{1-\gamma_A}\right] = \lambda_A^{-1} G_A\left(T-t, D_t; 1-\gamma_A, 1-\gamma_A\right), \tag{B.18}$$
$$\xi_t \widehat{W}_{B,T} = E_t\left[\lambda_B^{-1} (1-y_T)^{1-\gamma_B} D_T^{1-\gamma_B}\right] = \lambda_B^{-1} G_B\left(T-t, D_t; 1-\gamma_B, 1-\gamma_B\right). \tag{B.19}$$

To determine the equilibrium portfolio strategy, i.e. the portfolio which generates $\widehat{W}_{i,t}$ we apply Ito's lemma to (B.18) and (B.19) and compare the results with the dynamics of the self-financing portfolio. This gives a system of two equations with two unknowns,

$$-\varphi_t + \phi_{i,S,t}\sigma_{S,t} + \phi_{i,P,t}\sigma_{P,t} = \alpha_{i,t}$$
$$\left(1-\psi_t\right)\left(1 + \phi_{i,S,t}\delta_{S,t} + \phi_{i,P,t}\delta_{P,t}\right) - 1 = \beta_{i,t}.$$

that allow us to solve for $\phi_{i,S,t}$ and $\phi_{i,P,t}$. Finally, we find $\alpha_{i,t}$ and $\beta_{i,t}$ from the equilibrium dynamics of $\xi_t \widehat{W}_{i,t}$ and using the expressions in (B.18) and (B.19). □

C Tables and Figures

Table 1: SPX option quantities
The table reports option quantity measures on different dates: start, middle and end of the sample. The sample starts on January 4, 1996 and ends on December 31, 2020. "Peak of sample" refers to the date with the highest number of open interest. The last row reports that average daily values for the sample. An open interest contract for SPX corresponds to 100 underlying index shares. Open interest (OI) for puts and calls are reported in billions of index-equivalent units. The column titled "Index shares" reports the number of S&P 500 equivalent shares outstanding (in billions) defined as the market capitalization divided by the index level. The market value (MV) of put and call contracts is reported in billions of dollars. The last column provides the market capitalization of S&P 500 in billions of dollars.

	Date	OI Put	OI Call	Index shares	MV Put	MV Call	MV Index
Start of sample	1996-01-04	0.05	0.03	7.35	0.31	1.25	4537.22
Mid of sample	2008-07-07	0.73	0.43	8.96	55.53	9.47	11216.24
End of sample	2020-12-31	0.75	0.36	8.83	26.56	146.93	33161.23
Peak of sample	2020-03-19	1.43	0.84	8.63	519.46	44.36	20794.27
Sample mean		0.55	0.32	8.82	18.80	29.31	13704.96

Table 2: SPX option quantities by type: Sample average
The table reports sample-average values of daily open interest by moneyness and maturity type as a fraction of index-equivalent shares in percentage terms. We report results separately for put and call options. We sort options based on their days to maturity (τ) and moneyness (K/F). Short-term options are those with maturity $\tau \leq 90$ days. Medium-term options are those with maturity $90 < \tau \leq 270$. Long-term options are those with maturity $\tau > 270$. We define moneyness K/F as the level of strike price over the forward price of the underlying.

	Maturity	Low strike	At-the-money	High strike
		K/F $\leq$ 0.975	0.975 $<$ K/F $<$ 1.025	1.025 $\leq$ K/F
Put	Short-term	2.983	0.575	0.295
	Medium-term	1.159	0.123	0.121
	Long-term	0.703	0.059	0.082
Call	Short-term	0.548	0.702	0.956
	Medium-term	0.253	0.144	0.434
	Long-term	0.141	0.066	0.279

Figure 1: Historical open interest in CBOE SPX options

This figure plots the time series of SPX open interest during our sample period. The sample starts on January 4, 1996 and ends on December 31, 2020. Reported values correspond to averaged daily open interest for each year-quarter. An open interest for SPX contract corresponds to 100 underlying index shares. Panel A reports open interest for puts and calls in billions of index-equivalent units. Panel B reports open interest for puts and calls a fraction (in percentage) of the number of S&P 500 equivalent shares outstanding. The number of S&P 500 equivalent shares outstanding (in billions) defined as the market capitalization divided by the index level.

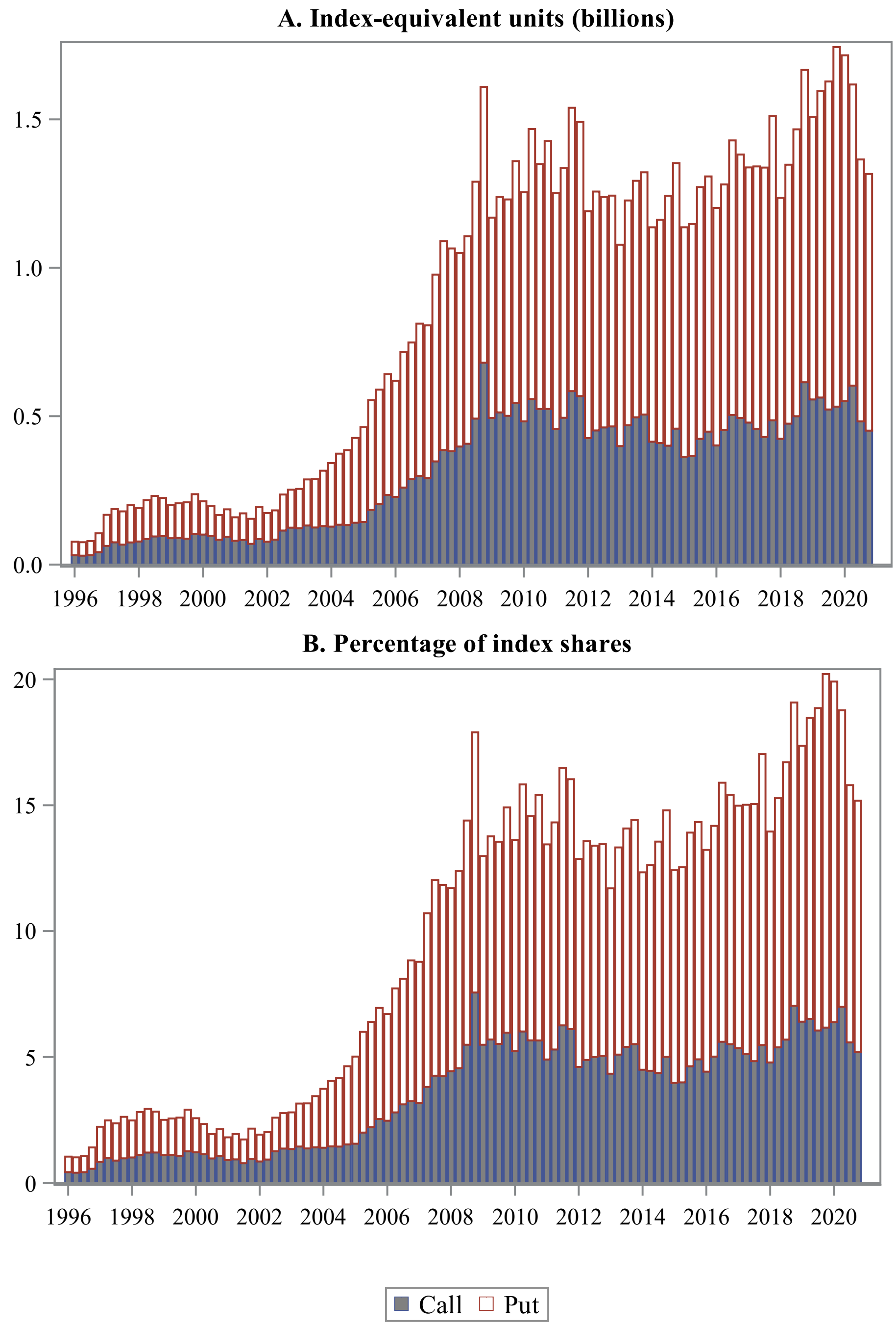

Figure 2: Historical open interest in CBOE SPX options for different maturity and moneyness buckets

We plot the time series of daily open interest as a fraction of index-equivalent shares in percentage terms. Reported values correspond to averaged daily quantities for each year. The sample period is from January 4, 1996 to December 31, 2020. We report results separately for put (top row) and call options (bottom rows). We sort options into four day-to-maturity buckets: 10–50, 70–110, 160–200, and 250–290 days. For each maturity bucket, we label options as in-the-money (ITM), at-the-money (ATM), or out-of-the-money (OTM) based their moneyness (K/F) defined as the level of strike price over the forward price of the underlying. ATM puts and calls are those moneyness $0.975 < K/F < 1.025$. OTM puts and ITM calls are those with $K/F \leq 0.975$. ITM puts and OTM calss are those with $1.025 \leq K/F$.

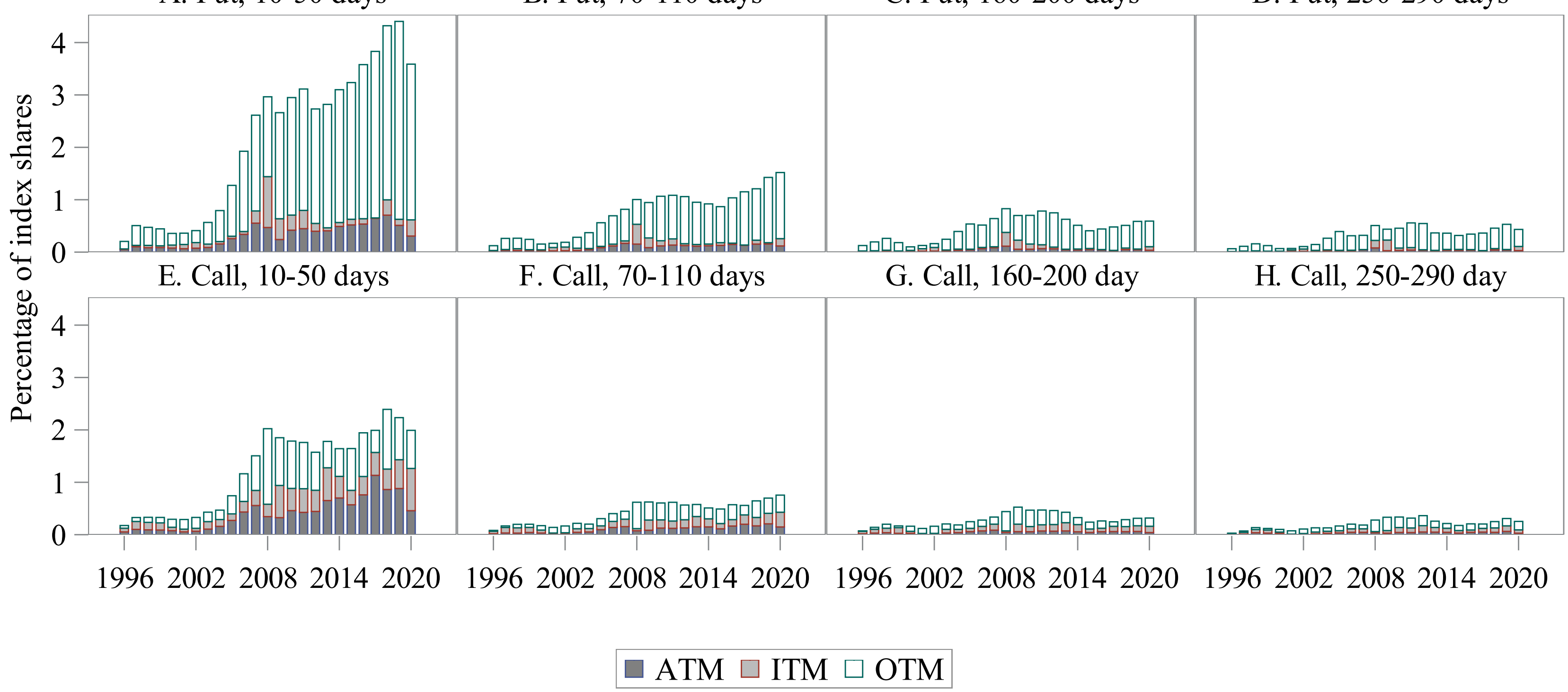

Figure 3: Open interest across moneyness levels

This figure plots the average daily open interest level of SPX puts and calls across option moneyness for different maturity buckets. Average open interest are reported as a fraction of index-equivalent shares in percentage terms. The sample period is from January 4, 1996 to December 31, 2020. We sort options into four day-to-maturity buckets: 10–50 (Panel A), 70–110 (Panel B), 160–200 (Panel C), and 250–290 (Panel D). Each panel plots the average open interest across moneyness for put and call options separately. Option moneyness (K/F) is defined as the level of strike price over the forward price of the underlying. Vertical dashed line indicates the moneyness level close to one.

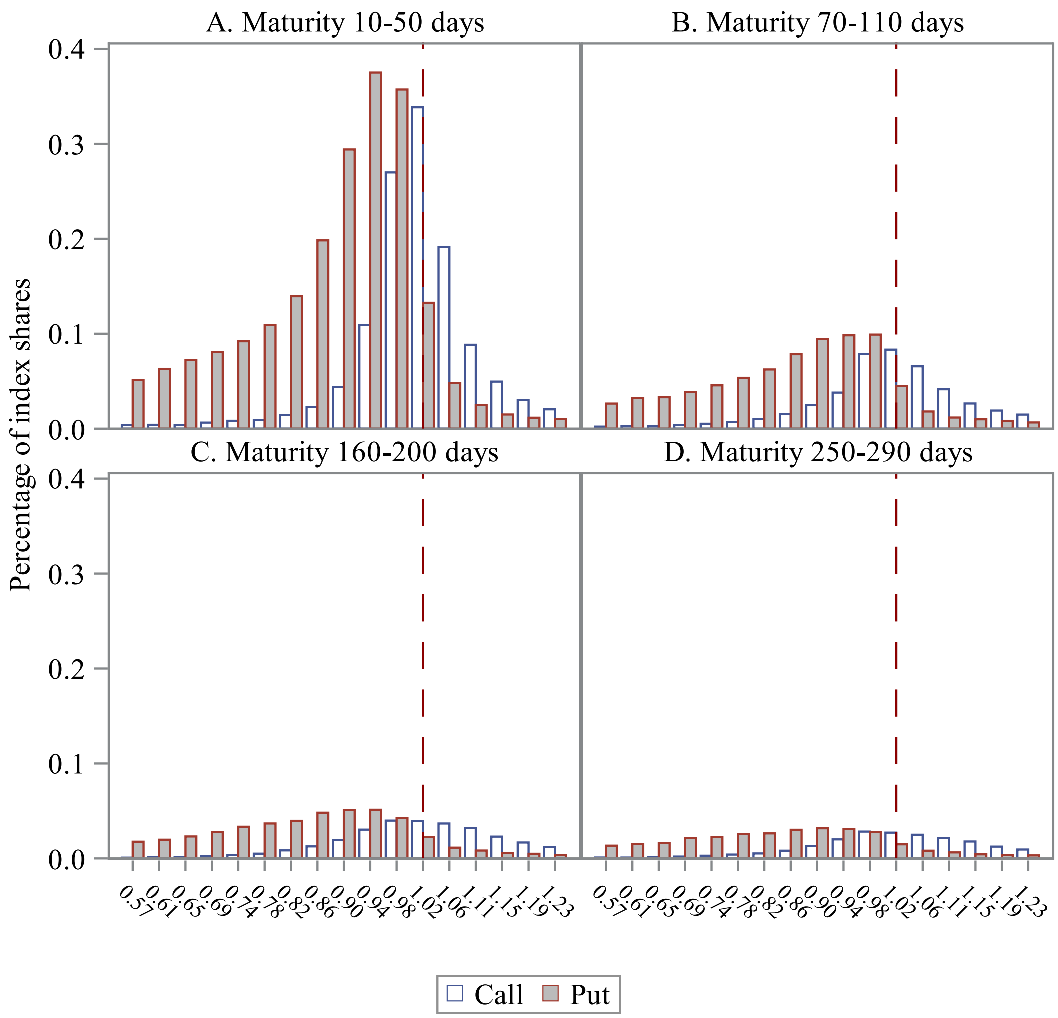

Figure 4: Utility functions

$W_{i,T} \sim N(1.05, 0.1)$ is used to determine the disappointment threshold $\theta_i = \kappa_i \mathcal{R}_i$ for the DA utility in Panels C and D.

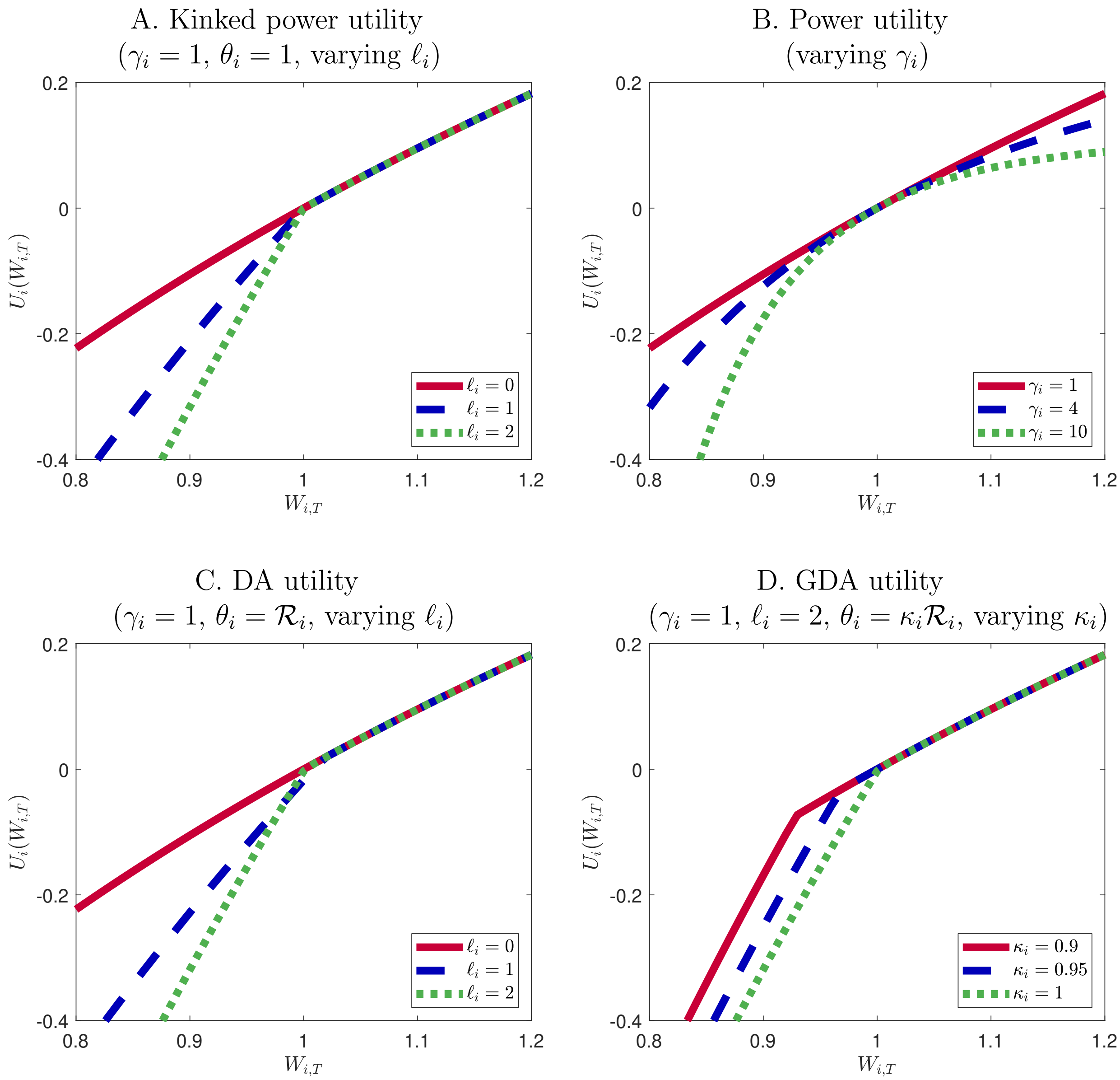

Figure 5: Terminal wealth profiles

Parameters common across figures are $T = 1/12$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$, and $\alpha = 0.5$. In Panels A–C, agent A is disappointment-averse with $\gamma_A = 1$ and disappointment threshold defined in terms of the certainty equivalent as $\theta_A = \kappa_A \mathcal{R}_A$. Disappointment aversion parameters ℓ_A and κ_A are reported in the header of each panel. In Panel D, agent A is risk-averse with $\gamma_A = 4$. For agent B, $\gamma_B = 1$ in all four panels. The shaded area in all the graphs corresponds to the pdf of D_T.

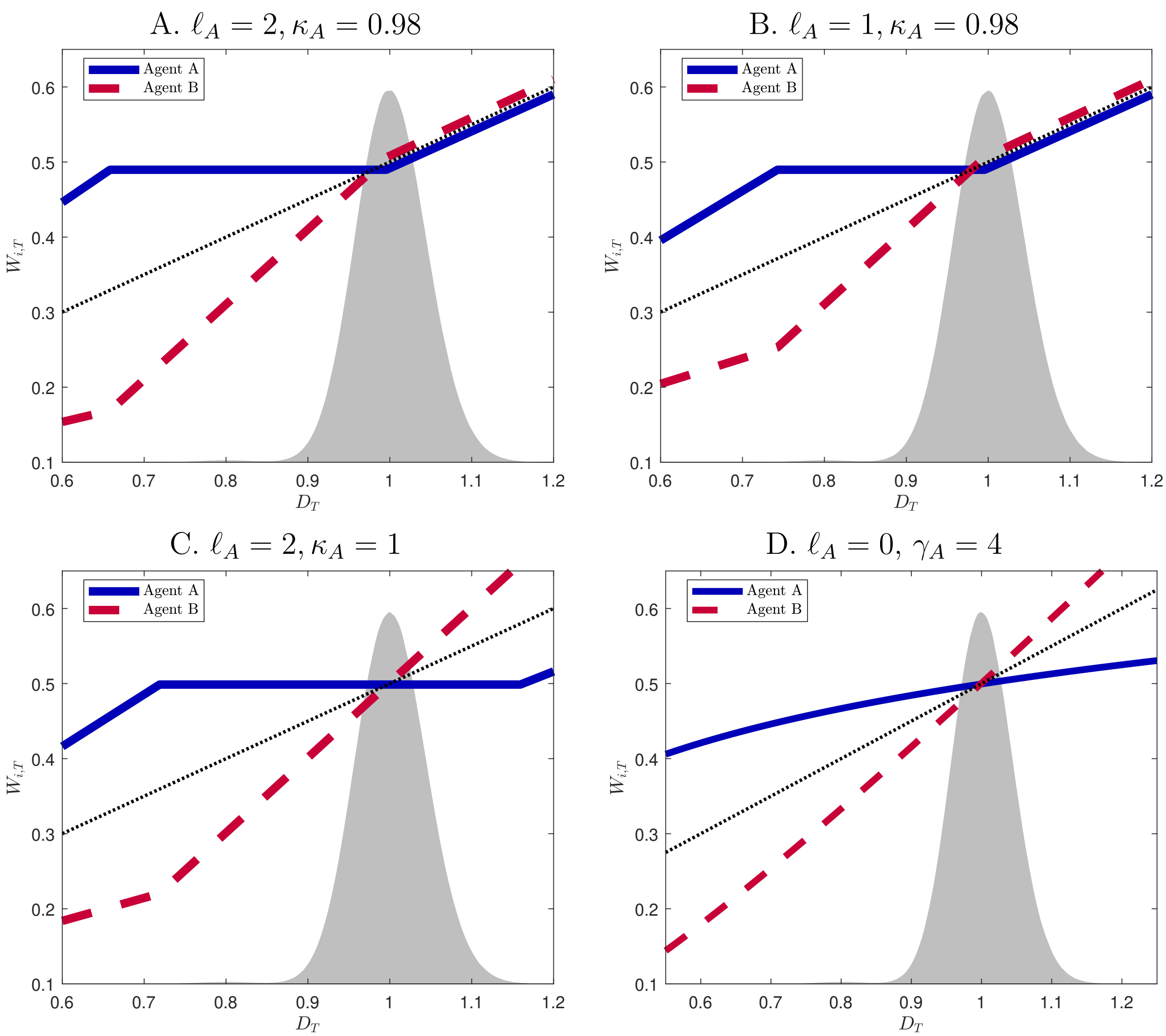

Figure 6: State price density at time T

Parameters common across figures are $T = 1/12$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$, $\gamma_A = \gamma_B = 1$. Disappointment threshold is defined in terms of the certainty equivalent as $\theta_A = \kappa_A \mathcal{R}_A$.

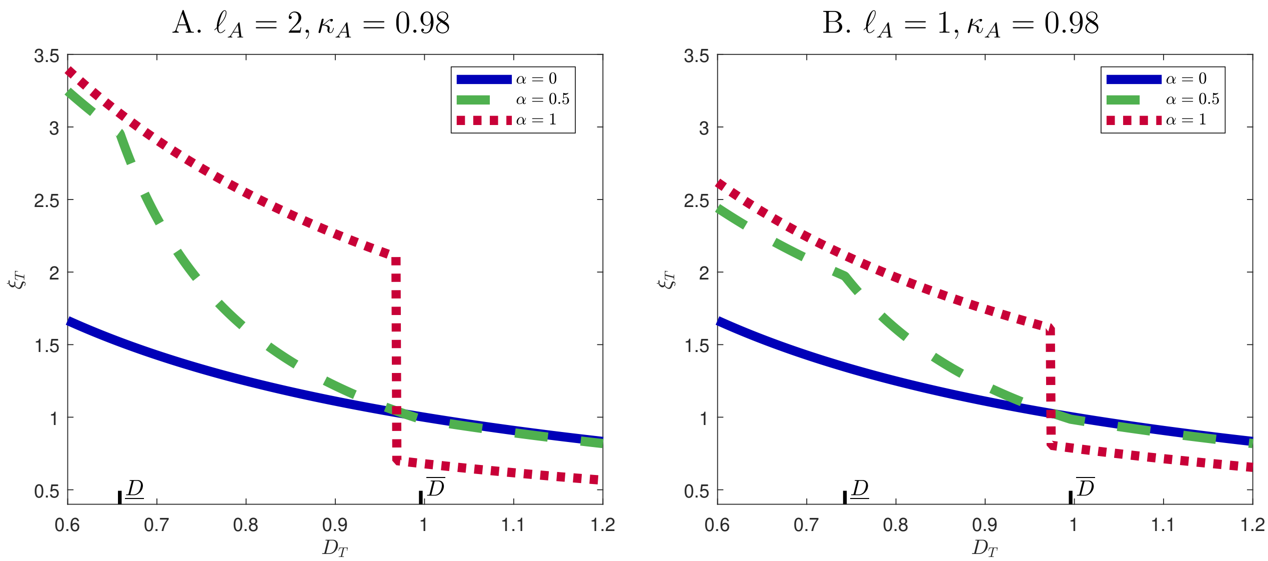

Figure 7: Properties of equilibrium put option returns

The figure shows put option risk premia (monthly) and Black-Scholes implied volatilities (annual) as a function of option moneyness. Parameters common across figures are $\alpha = 0.5$, $T = 1/12$, $t = 0$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$. The preference parameters in the DA economy are $\gamma_A = 1$, $\ell_A = 2$, $\kappa_A = 0.98$, and $\gamma_B = 1$. The preference parameters in the RA economy $\gamma_A = 4$ and $\gamma_B = 1$.

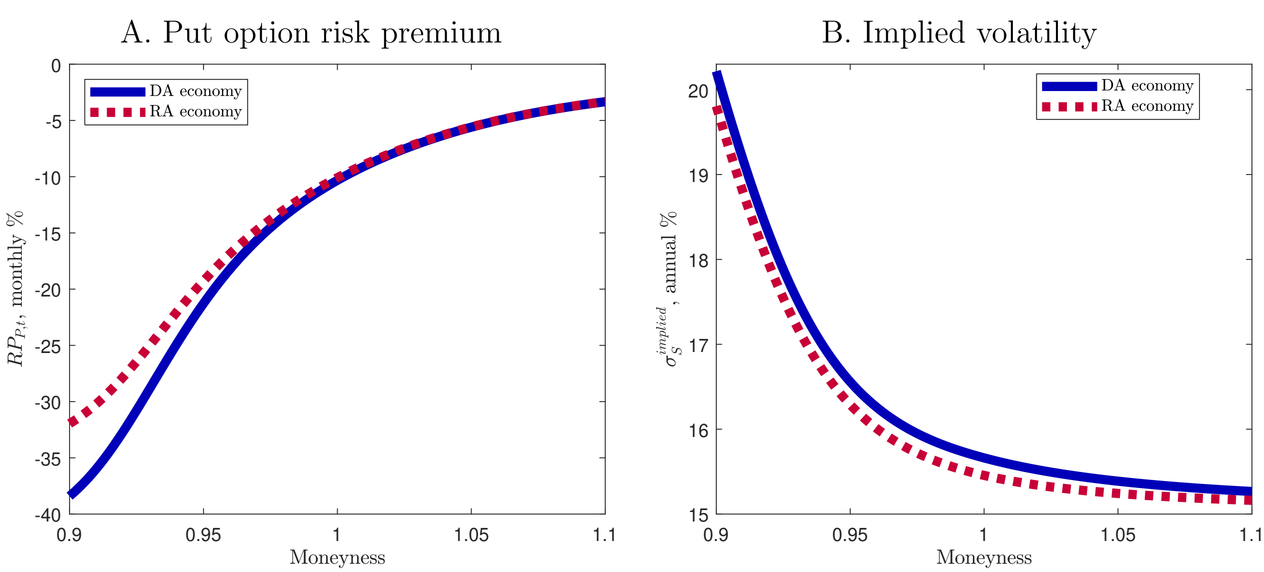

Table 3: Properties of equilibrium stock returns

Parameters common across the table are $T = 1/12$, $t = 0$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$. When agent A is disappointment-averse (DA), her disappointment threshold is defined in terms of the certainty equivalent as $\theta_A = \kappa_A \mathcal{R}_A$, where $\kappa_A = 0.98$, and $\ell_A \in \{1, 2\}$. When agent A is risk-averse (RA), her preference parameter is $\gamma_A \in \{4, 10\}$. For agent B, $\gamma \in \{1, 2\}$ in both cases. Total volatility is defined as $v_{j,t} = \sqrt{\sigma_{j,t}^2 + \delta_{j,t}^2 \lambda}$, $j = S, D$.

Stock risk premium, $RP_{S,t}$ %					
Panel A: DA model					
	$\alpha = 0.05$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.95$
$\gamma_B = \gamma_A = 1, \ell_A = 1$	2.57	2.98	4.06	7.14	16.43
$\gamma_B = \gamma_A = 1, \ell_A = 2$	2.57	2.98	4.07	7.05	25.01
$\gamma_B = \gamma_A = 2, \ell_A = 1$	5.23	6.13	8.16	13.50	19.82
$\gamma_B = \gamma_A = 2, \ell_A = 2$	5.23	6.15	8.63	15.38	29.36
Panel B: RA benchmark					
	$\alpha = 0.05$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.95$
$\gamma_B = 1, \gamma_A = 4$	2.60	3.09	4.06	5.89	9.08
$\gamma_B = 1, \gamma_A = 10$	2.82	3.25	4.67	8.39	21.73
$\gamma_B = 2, \gamma_A = 4$	5.20	5.82	6.83	8.27	9.93
$\gamma_B = 2, \gamma_A = 10$	5.29	6.40	8.74	13.82	25.14

Stock excess volatility, $(v_{S,t} - v_D)/v_D$ %					
Panel C: DA model					
	$\alpha = 0.05$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.95$
$\gamma_B = \gamma_A = 1, \ell_A = 1$	0.10	0.60	1.89	5.41	10.43
$\gamma_B = \gamma_A = 1, \ell_A = 2$	0.10	0.60	1.92	5.69	17.69
$\gamma_B = \gamma_A = 2, \ell_A = 1$	0.19	1.16	3.26	8.53	8.45
$\gamma_B = \gamma_A = 2, \ell_A = 2$	0.19	1.22	3.66	10.54	13.37
Panel D: RA benchmark					
	$\alpha = 0.05$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.95$
$\gamma_B = 1, \gamma_A = 4$	0.01	0.04	0.13	0.28	0.24
$\gamma_B = 1, \gamma_A = 10$	0.03	0.07	0.28	1.17	3.51
$\gamma_B = 2, \gamma_A = 4$	0.01	0.03	0.07	0.09	0.04
$\gamma_B = 2, \gamma_A = 10$	0.02	0.11	0.35	0.99	1.41

Figure 8: Equilibrium wealth and risk factor exposure

The graphs show the equilibrium time t wealth profile of agent A (solid blue line). Parameters common across figures are $t = 0$, $T = 1/12$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$, $K/S_t = 0.95$, and $\alpha = 0.5$. The preference parameters in the DA economy are $\gamma_A = 1$, $\ell_A = 2$, $\kappa_A = 0.98$, and $\gamma_B = 1$. The preference parameters in the RA economy $\gamma_A = 4$ and $\gamma_B = 1$.

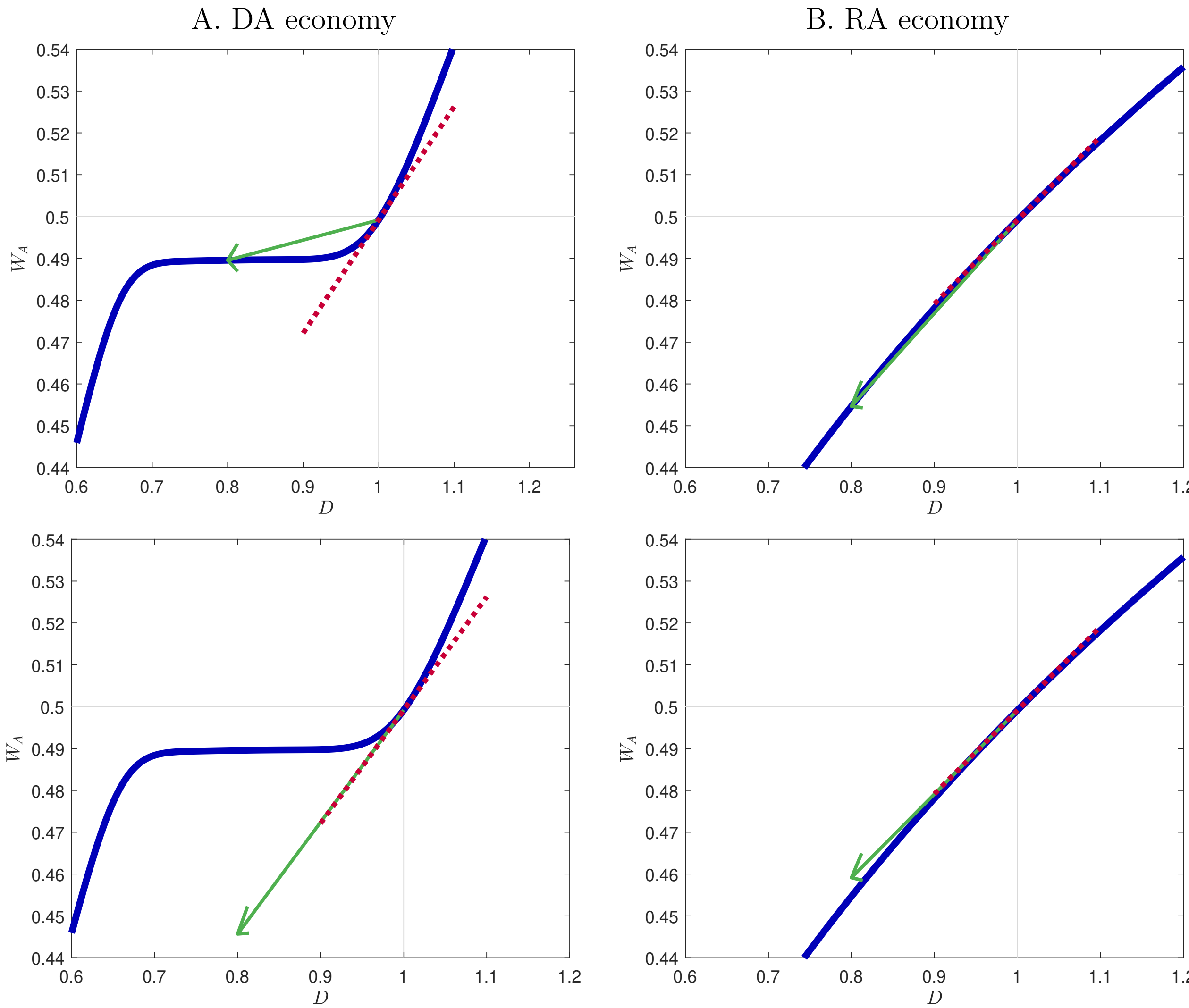

Figure 9: Equilibrium asset positions

The graphs show the number of stocks ($\pi_{S,A,t}$) and the number of options ($\pi_{P,A,t}$) held by agent A. Note that corresponding values for agent B are $\pi_{S,B,t} = 1 - \pi_{S,A,t}$ and $\pi_{P,B,t} = -\pi_{P,A,t}$. Parameters common across figures are $t = 0$, $T = 1/12$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$, $\kappa_A = 0.98$, $K/S_t = 0.95$.

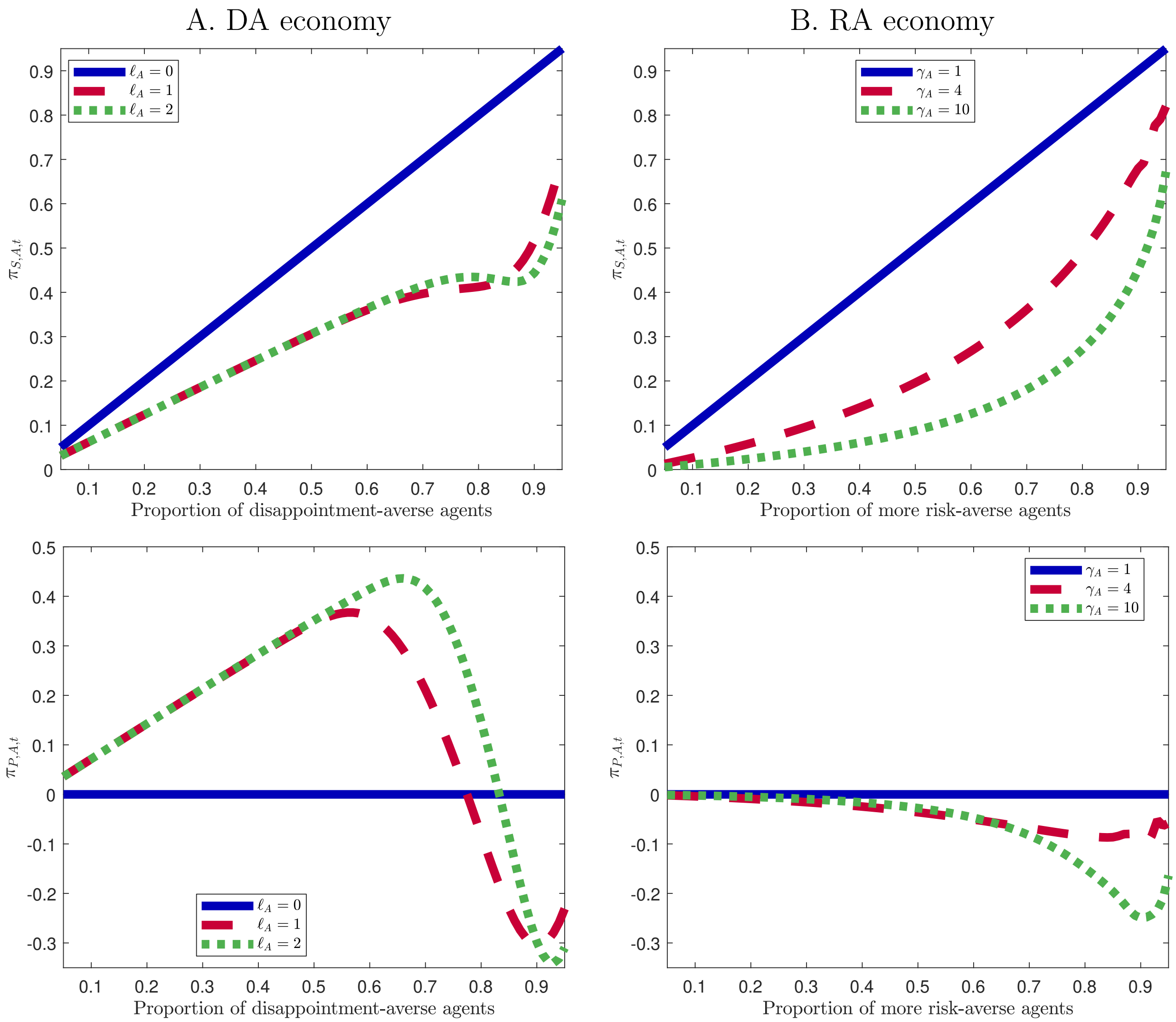

Figure 10: Sensitivity of asset positions to the disappointment threshold

The graphs show the number of stocks ($\pi_{S,A,t}$) and the number of options ($\pi_{P,A,t}$) held by agent A. Note that corresponding values for agent B are $\pi_{S,B,t} = 1 - \pi_{S,A,t}$ and $\pi_{P,B,t} = -\pi_{P,A,t}$. Parameters used are $t = 0$, $T = 1/12$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$, $\alpha = 0.5$, $\gamma_B = 1$, $K/S_t = 0.95$.

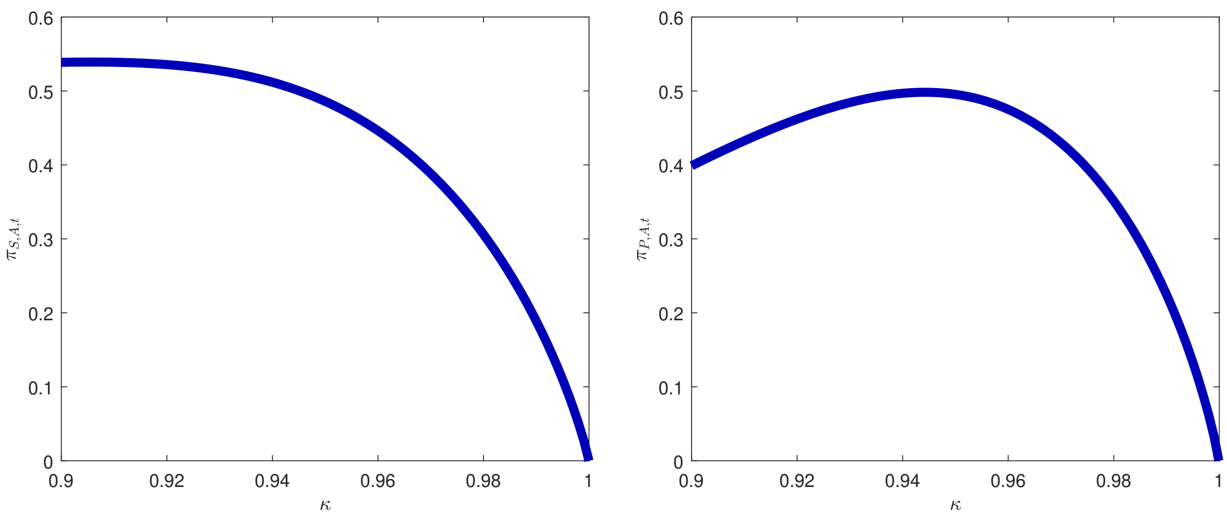

Figure 11: Open interest and implied volatility across moneyness

The graph shows the total number of options traded in the economy. Base values of parameters are $t = 0$, $T = 1/12$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.10$, $\lambda = 0.6$, $\alpha = 0.5$, $\gamma_B = 1$, $\kappa_A = 0.98$, and $\ell_A = 2$. For the two-jump case, the jump sizes take values $\delta_D \in \{0.12, -0.10\}$ with equal probabilities. For the three-jump case, the jump sizes take values $\delta_D \in \{0.18, 0.01, -0.16\}$ with probabilities 0.3, 0.4, and 0.3, respectively. For the four-jump case, the jump sizes take values $\delta_D \in \{0.23, 0.08, -0.06, -0.21\}$ with probabilities 0.15, 0.35, 0.35, and 0.15, respectively.

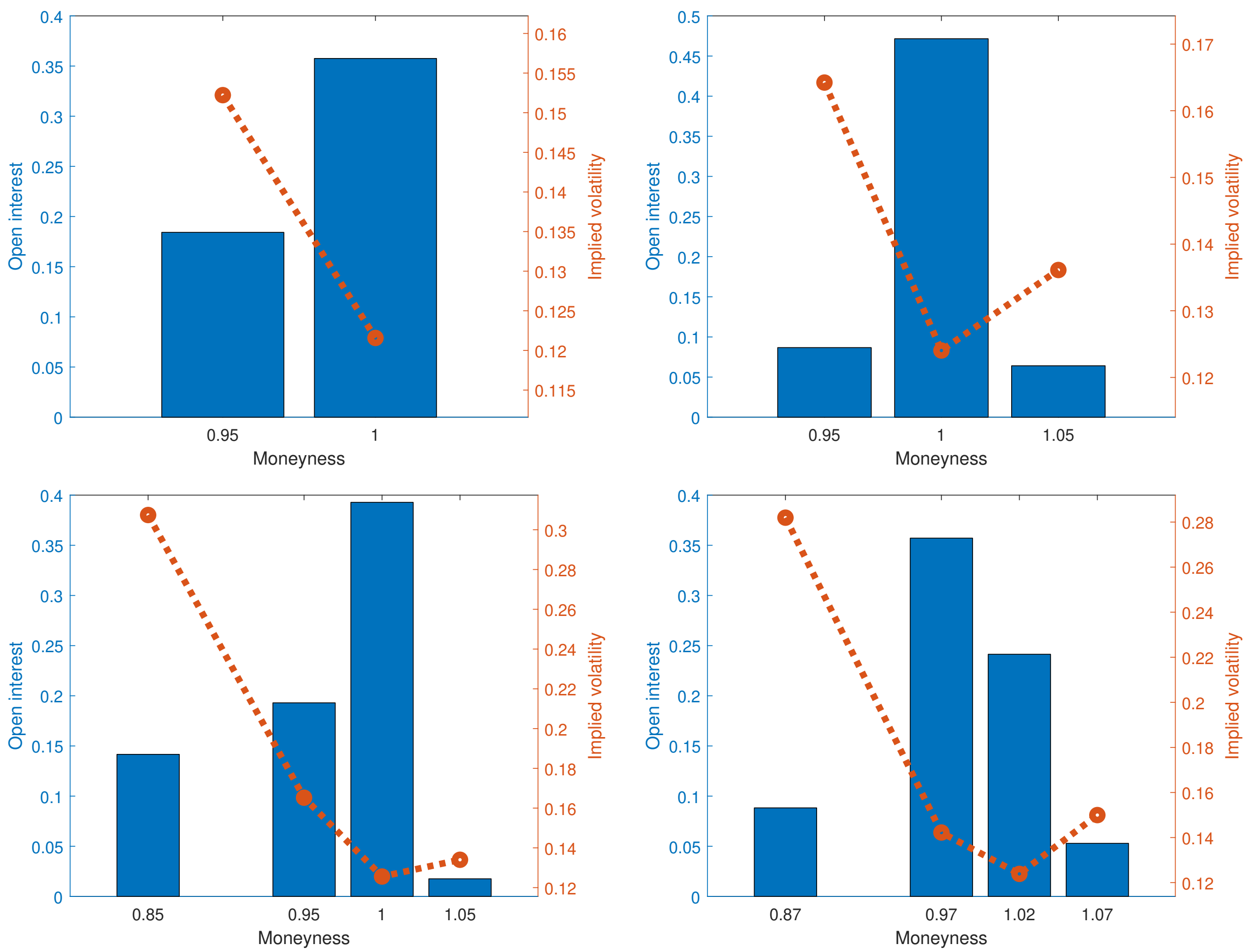

Figure 12: Effect of cash flow news on stock and option markets

Parameters common across figures are $\alpha = 0.5$, $t = T/2$, $T = 1/12$, $D_0 = 1$, $\mu_D = 0.03$, $\sigma_D = 0.15$, $\lambda = 0.05$, $\delta_D = 0.2$, $\kappa_A = 0.98$, $K/S_t = 0.95$.