Out-of-Sample Equity Premium Prediction: The Role of Option-Implied Constraints [†]

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ABSTRACT

We propose a new constrained equity premium forecasting approach that incorporates two option-implied lower bounds for the conditional market risk premium from Martin (2017) and Chabi-Yo and Loudis (2020), respectively. Our constrained approach delivers considerable out-of-sample gains in both statistical and economic criteria relative to the unconstrained predictive regression and the forecast combination method. Even stronger performance is uncovered when the upper bound on the equity premium from Chabi-Yo and Loudis (2020) is incorporated. Our approach also outperforms the prevailing non-negativity constraint, especially at longer forecast horizons. We provide two explanations for the superiority of our method: i) constrained forecasts combine the information provided by conventional predictors and the forward-looking information about the term structure of expected holding period returns implied by option prices and ii) option-implied bounds sharpen unconstrained forecasts and significantly reduce forecast variance at the same time.

***JEL classification*: G11; G12; G13; G17**

***Keywords*: Out-of-sample predictability, forecast constraints, option-implied bounds, term structure, higher-order moments**

1 Introduction

The time-varying equity premium is an important ingredient in many areas of finance, including asset pricing, portfolio management, and capital budgeting. The literature has provided convincing evidence of the in-sample predictability of the equity premium and identified a large list of predictors that can track the equity premium's time variation.[1] As forcefully criticized by Welch and Goyal (2008), however, the out-of-sample (OOS) performance of many well-recognized predictors is quite fragile, and they rarely outperform the historical mean benchmark in a consistent manner. Thus, how to improve the accuracy and empirical reliability of OOS predictions of equity premium remains a challenging but intriguing question for both academics and practitioners.

Perhaps, the most straightforward way is to explore more powerful predictors. Recent studies have discovered many new predictors with remarkable OOS performance.[2] Nonetheless, a growing body of literature recognizes that unreliable OOS predictability may not arise from these fragile predictors alone. First, the data-generating process of equity premium is "highly complex and constantly evolving" (Rapach, Strauss, and Zhou, 2010, p. 845), resulting in model uncertainty or structural breaks (Lettau and Van Nieuwerburgh, 2008; Pesaran and Timmermann, 2002). Second, the information set of market participants that determines the equity premium could be much vaster than that used by researchers (Ludvigsona and Ng, 2007). The preceding problems are exacerbated by the conditional mean of the market return often adopting restricted parametric forms (Harvey, 2001). The interaction of these problems gives rise to model misspecification and hence leads to the unstable OOS performance of standard predictive regressions.

Emerging methods in recent studies tackle some of these issues by condensing information from a multitude of predictors and by allowing for more flexible specifications of the equity premium.[3] A parallel literature, on the other hand, emphasizes that imposing

[1] Prominent predictors include the nominal short rate (Campbell, 1987), stock market volatility (French, Schwert, and Stambaugh, 1987), the dividend–price ratio (Campbell and Shiller, 1988), dividend yield (Fama and French, 1988), term and default spreads (Fama and French, 1989), and net aggregate equity issuance (Boudoukh, Michaely, Richardson, and Roberts, 2007), to name a few. See Rapach and Zhou (2013) for further references.

[2] Some new predictors found in the literature include the variance risk premium (Bollerslev, Tauchen, and Zhou, 2009), the aggregate short interest (Rapach, Ringgenberg, and Zhou, 2016), an aligned investor sentiment index (Huang, Jiang, Tu, and Zhou, 2014), and technical indicators (Neely, Rapach, Tu, and Zhou, 2014).

[3] Recent contributions include the three-pass regression filter (Kelly and Pruitt, 2015), principal compo-

statistical and economic constraints on otherwise standard predictive regressions substantially improves OOS forecasting performance. For instance, Rapach, Strauss, and Zhou (2013) apply shrinkage estimation techniques to obtain more parsimonious predictive models. Alternatively, some researchers impose economically motivated restrictions to prevent the regression coefficient estimates and corresponding forecasts from contradicting rational theory. In particular, Campbell and Thompson (2008, hereafter CT) have pioneered the idea of imposing the non-negativity constraint on equity premium forecasts, which has been widely adopted in subsequent studies (Li and Tsiakas, 2017; Pan, Pettenuzzo, and Wang, 2020; Pettenuzzo, Timmermann, and Valkanov, 2014).

Although the zero lower bound approach is easy to implement, it has two drawbacks. First, given the countercyclical nature of the equity premium (Fama and French, 1989; Henkel, Martin, and Nardari, 2011), the zero bound is uninformative, since it incorporates no conditional information. Second, the expected holding period return on the market grows over the horizon, which makes zero an overly slack bound for the long-term equity premium. Fortunately, two recent studies, by Martin (2017) and Chabi-Yo and Loudis (2020) (CYL hereafter), explore the rich information in the equity index options and derive two tight lower bounds on the conditional market risk premium. The former is fully characterized by conditional risk-neutral market variance, while the latter also considers compensation for exposure to higher-order risk-neutral moment risk. Importantly, these two lower bounds can be calculated in real time, using cross sections of the index option prices in a model-free manner, and are available for various maturities. Compared to zero, these bounds exhibit salient countercyclical dynamics and the shape of the bounds' term structure varies over time, suggesting that they contain useful information about the time variation of the term structure of equity premia.

Despite the rich information in the option-implied lower bounds, it might not provide a complete description of the equity premium. Given numerous predictive variables found in the literature, a combination of the lower bounds with existing predictors is likely to be a promising way to improve the empirical reliability of OOS predictability. To the best of our knowledge, the effect of combining the bounds with standard predictors (univariate or multivariate) in such manner has not been thoroughly studied. Intuitively, one would expect

nent regression (Ludvigsona and Ng, 2007), forecast combination (Rapach, Strauss, and Zhou, 2010), and boosted regression tree (Rossi and Timmermann, 2015).

that the implied lower bounds, being *informative restrictions*, help to remove spurious signals in the predictors and stabilize forecasts by excluding extreme predictions.

In this paper, we study the role of the implied lower bounds of Martin (2017) and CYL in improving the OOS performance of standard predictive regression forecasts. Our method is straightforward: we truncate the regression forecasts whenever they fall below the bounds. This approach is similar to that of CT, the key difference being that the truncation is based on option-implied bounds instead of zero. In our main analyses, we focus on 14 well-known predictors from Welch and Goyal (2008) and evaluate the performance of our methodology at both short and long forecast horizons (monthly, quarterly, semi-annual, and annual). Besides univariate predictive regression, we investigate improvements upon the forecast combination that pools individual forecasts by standard univariate regressions. Rapach, Strauss, and Zhou (2010) show that by diversifying model misspecification risks, forecast combination is a powerful way to mitigate model uncertainty.

Over the OOS evaluation period from February 2001 to June 2019, we find that option-implied lower bounds help to improve the degree of forecast accuracy, as measured by the OOS R^2 (Welch and Goyal, 2008), of unconstrained predictive regression forecasts, especially at quarterly and longer horizons. At the monthly horizon, imposing Martin's and CYL's lower bounds improves the R^2_{OOS} values for 11 and nine of the 14 predictors, and the corresponding increments in R^2_{OOS} are 1% and 0.56%, on average, respectively. Such a magnitude is sizable, since the usual level of monthly R^2_{OOS} is rather small. Martin's lower bound also helps raise the R^2_{OOS} value of the equal-weight combination forecast from -0.12% to 0.10%. Turning to longer horizons, our framework yields more salient improvements. For instance, at the semiannual horizon, the forecasting performance of 11 of the 14 predictors is improved with the aid of CYL's lower bound, and the average increment in R^2_{OOS} reaches 21.06%. Notably, under the unconstrained predictive regression approach, only two predictors produce positive R^2_{OOS} values, and neither is significant, according to the statistic of Clark and West (2007). In contrast, imposing CYL's bound results in eight positive R^2_{OOS} values, the highest being 13.31%, generated by the long-term yield (LTY), and all the R^2_{OOS} values are significant at least at the 10% level. Moreover, our constrained approach enhances the predictive accuracy of the forecast combination method, increasing its R^2_{OOS} value from -5.15% to 10.30%, significant at the 5% level. The effects of Martin's bound are qualitatively similar (albeit slightly weaker). Generally, the benefits derived from imposing option-implied lower bounds increase with the horizon and dominate the effect of the zero lower constraint at the quarterly, semiannual, and annual horizons.

Following CT and Rapach, Strauss, and Zhou (2010), we quantify the economic value of the constrained equity premium forecasts by gains in certainty equivalent return (CER) delivered to a mean–variance investor who can dynamically allocate her wealth between equity and T-bills, based on constrained forecasts. The asset allocation exercise reveals that, compared to the strategy relying on unconstrained forecasts, imposing option-implied bounds leads to extra CER gains in most of the cases, and the mean increment ranges from 33 basis points (bps) to 90 bps for the monthly, quarterly, and semi-annual rebalancing frequencies. Finally, we extend our method by combining the upper bound for the equity premium derived by CYL together with their lower bound. We find that, except for a few cases, the predictive performance of the unconstrained model is uniformly improved, and such improvements are stronger than those from using the lower bounds alone, especially at longer horizons. In addition, our findings are robust to the option-based predictors, including the variance risk premium of Bollerslev, Tauchen, and Zhou (2009), and alternative choices of training period and risk aversion coefficient.

Our study contributes to the vast literature on time series return predictability by introducing the more informative equity premium constraint implied by options into the debate over OOS predictability. Our results demonstrate that the use of option-implied bounds to constrain conventional predictive regression forecasts leads to significant improvements in OOS forecastability, particularly at longer horizons, with sizable utility gains for the mean–variance investor. We attribute these improvements to the bounds' several advantages. First, since option-implied lower bounds incorporate conditional information on the market and vary over time, the restrictions on the equity premium set by the bounds are more informative than a simple zero and stabilize unconstrained forecasts by excluding extreme predictions at the same time. Second, the unique and forward-looking information contained in the bounds expands the information set of conventional economic predictors. Third, the bounds' term structure enables them to serve as tighter lower bounds for the equity premium, especially at longer horizons. By comparing the two implied lower bounds, we find that CYL's lower bound performs better than Martin's at longer forecast horizons, revealing that higher-order risk-neutral moment risk is crucial in driving the long-horizon equity premium, consistent with CYL.

Besides the literature on return predictability, our paper relates to the growing literature exploring the unique role of option-implied information in understanding the dynamics of risk and the risk premium of the underlying asset. There has been a long tradition to recover information from option prices in a model-free manner (Bakshi, Kapadia, and Madan, 2003; Brandt, Koijen, and Van Binsbergen, 2012; Britten-Jones and Neuberger, 2000; Kozhan, Neuberger, and Schneider, 2013). Unlike most of these studies, which focus primarily on the information content of the implied information alone, our paper investigates the combination of this information with that provided by conventional return predictors. In particular, we find novel evidence of synergy derived from combining option-implied bounds with bond yield–based predictors, such as LTY and the default yield spread (DFY). In other words, the predictive power of the constrained model is greater than both the bounds themselves and the unconstrained counterpart. Through decomposition of the mean squared prediction error (MSPE), we explain such success by the reduction in forecast variance. From an economic perspective, the bond yield–based predictors help explain the gap between the bound and the realized excess return when the bound is slack. The synergy thus stems from the information complementarity between the predictors and the bounds. Moreover, subsample analyses suggest that the benefit delivered by the option-implied constraints is stronger when the overall economy is good or the stock market volatility is low.

Overall, our study confirms the distinct value of using option-implied information in forecasting aggregate market returns and reveals the information complementarity between the index options market and the bond markets. We assume that the Standard & Poor's (S&P) 500 option market is informationally efficient and fully integrated with the underlying index. Our approach, however, is subject to potential measurement error and mispricing in the options market. Lastly, other bounds and exact formulas for equity premia are proposed by recent studies[4] that we do not use, and we leave these for future research.

The remainder of this paper proceeds as follows. Section 2 introduces the option-implied bounds and methodology related to forecasting. Section 3 describes the data used in our

[4]Schneider and Trojani (2019) introduce upper and lower bounds of the nth physical moment of market returns. Bakshi, Crosby, Gao, and Zhou (2019) propose an exact formula for the expected market excess return. Martin and Wagner (2019) and Kadan and Tang (2020) extend Martin's (2017) framework to individual stocks and derive a lower bound on expected returns for a single stock.

analysis. Section 4 reports the results on the OOS predictive performance of our approach. Section 5 explores the sources of improvement in forecastability. Section 6 presents the results for extensions and robustness tests. Section 7 concludes the paper.

2 Methodology

This section first introduces a relatively simple equity premium constraint motivated by economic theory. Next, we describe the option-implied bounds on the conditional market risk premium proposed by Martin (2017) and CYL and how we use them to constrain the equity premium forecast.

2.1 Economic constraints on the market premium

Merton (1980) argues that, if the market is not riskless, risk-averse investors require a positive expected excess return on the market. Given this generally reasonable assumption, one would expect a non-negative equity premium when estimating the expected excess return on the market. CT empirically show that imposing a non-negative equity premium constraint increases the OOS forecasting performance of most predictors. Therefore, we use this relatively simple restriction as the first forecast-constrained approach.

2.2 Option-implied constraints on the market premium

As argued in the introduction, zero is an acceptable but naive lower bound for the expected market premium. On the one hand, as a lower bound, zero does not incorporate any conditional information on either the risk of the underlying asset or the overall economic condition. On the other, zero seems too slack to be a meaningful lower bound, especially for long-term market premia. Regarding these issues, we introduce another approach to prescribe a lower and upper bound for the market risk premium.

2.2.1 Martin's lower bound

By imposing mild assumptions on risk aversion, Martin (2017) derives a straightforward lower bound for the conditional market risk premium that exploits the no-arbitrage assumption and forward-looking information from option-implied distributions across different maturities. More specifically, under the so-called negative correlated condition (NCC), Martin shows that the expected market premium should be no less than the risk-neutral variance discounted by the risk-free rate, which can be calculated via the model-free method.

The no-arbitrage assumption implies a positive stochastic discount factor (SDF), $M_{t\to T}$, such that $E_t[M_{t\to T}R_{j,t\to T}] = 1$ for the gross return $R_{j,t\to T}$ from time t to T of every asset j. Let $R_{f,t\to T}$ denote the gross risk-free rate between times t and T satisfying $R_{f,t\to T} = 1/E_t[M_{t\to T}]$. One fundamental implication of the SDF is the link between two probability measures, the risk-neutral measure ($\mathbb{Q}$) and the real-world measure ($\mathbb{P}$, notation omitted for brevity). For instance, the conditional risk-neutral variance of the gross return $R_{j,t\to T}$ for any asset j can be written as

$$Var_t^{\mathbb{Q}}[R_{j,t\to T}] = E_t^{\mathbb{Q}}[R_{j,t\to T}^2] - (E_t^{\mathbb{Q}}[R_{j,t\to T}])^2 = R_{f,t\to T}E_t[M_{t\to T}R_{j,t\to T}^2] - R_{f,t\to T}^2, \qquad (1)$$

where $E_t^{\mathbb{Q}}[\cdot]$ denotes the conditional expectation under the risk-neutral measure. By adding and subtracting $E_t[M_T R_{j,t\to T}^2]$ and applying some elementary properties, Martin demonstrates that the expected excess return of any asset j can be equivalently written as

$$E_t[R_{j,t\to T}] - R_{f,t\to T} = \frac{1}{R_{f,t\to T}}Var_t^{\mathbb{Q}}[R_{j,t\to T}] - Cov_t[M_{t\to T}R_{j,t\to T}, R_{j,t\to T}], \qquad (2)$$

where $Cov_t[\cdot]$ denotes the conditional covariance operator under the physical measure at time t. Accordingly, identity (2) decomposes the expected excess return of any asset j into two components: $Var_t^{\mathbb{Q}}[R_{j,t\to T}]$ and $Cov_t[M_{t\to T}R_{j,t\to T}, R_{j,t\to T}]$.

To see how the decomposition can be linked to a lower bound on the asset's risk premium, Martin further introduces the NCC, which allows the sign of the covariance term to be identified. As termed by Martin, the NCC holds if $Cov_t[M_{t\to T}R_{j,t\to T}, R_{j,t\to T}] \leq 0$. Consequently, the deflated risk-neutral variance serves as a lower bound for the risk premium of asset j, as long as the NCC holds for asset j. Therefore, we can obtain a lower bound on the expected risk premium from time t to T of any asset that obeys the NCC, denoted as $LBM_{t,t\to T}$:

$$E_t[R_{j,t\to T}] - R_{f,t\to T} \geq LBM_{t\to T|t} = \frac{1}{R_{f,t\to T}}Var_t^{\mathbb{Q}}[R_{j,t\to T}]. \qquad (3)$$

In this paper, we focus on the case in which $R_{j,T}$ is the gross return on the market portfolio;[5] from now on, $LBM_{t,t\to T}$ always refers to Martin's lower bound for the expected excess return on the S&P 500 index.

2.2.2 Chabi-Yo and Loudis's lower bound

In addition to Martin's lower bound, we also consider the lower bound for the expected market excess return derived by CYL. Though CYL prescribe an arbitrage-free economy, as Martin (2017) does, their bounds do not rely on the validity of the NCC formalized by Martin. The key to CYL's approach is to express the physical expected excess return of any asset j in terms of the covariance between the return and the inverse of the SDF via application of the Radon–Nikodym theorem:

$$\begin{aligned} E_t[R_{j,t\to T}] - R_{f,t\to T} &= E_t\left(R_{j,t\to T}\frac{M_{t\to T}}{E_t[M_{t\to T}]}\frac{E_t[M_{t\to T}]}{M_{t\to T}}\right) - R_{f,t\to T} \\ &= E_t^{\mathbb{Q}}\left(R_{j,t\to T}\frac{E_t[M_{t\to T}]}{M_{t\to T}}\right) - R_{f,t\to T} \\ &= Cov_t^{\mathbb{Q}}\left(R_{j,t\to T}, \frac{E_t[M_{t\to T}]}{M_{t\to T}}\right), \end{aligned} \tag{4}$$

where the ratio $E_t[M_{t\to T}]/M_{t\to T}$ equals the inverse of Radon-Nikodym derivative.

To derive the lower bound, CYL further consider a one-period economy in which the representative agent with initial wealth W_t has a smooth utility function u satisfying $u'[\cdot] > 0$ and $u''[\cdot] < 0$. The agent maximizes her expected utility on terminal wealth (W_T), and the first-order condition of the utility maximization problem implies that the SDF is proportional to the marginal utility of terminal wealth:

$$\frac{u'[W_T]}{E_t(u'[W_T])} = \frac{M_{t\to T}}{E_t[M_{t\to T}]}, \tag{5}$$

With some simplification, CYL obtain

$$\frac{E_t[M_{t\to T}]}{M_{t\to T}} = \frac{\frac{u'[W_t R_{f,t\to T}]}{u'[W_T]}}{E_t^{\mathbb{Q}}\left(\frac{u'[W_t R_{f,t\to T}]}{u'[W_T]}\right)} \text{ with } W_T = W_t R_{M,t\to T}. \tag{6}$$

[5]For this particular case, Martin shows that the NCC holds in very general environments and is consistent with many leading equilibrium models in which the relative risk aversion is at least one.

After replacing the ratio $E_t[M_{t\to T}]/M_{t\to T}$ in expression (4) by Eq. (6) and performing a Taylor series expansion on the inverse of the marginal utility of wealth, CYL show that expression (4) can be calculated as the sum of an infinite series of risk-neutral moments:

$$E_t[R_{M,t\to T}] - R_{f,t\to T} = Cov_t^{\mathbb{Q}}\left(R_{M,t\to T}, \frac{\frac{u'[W_t R_{f,t\to T}]}{u'[W_T]}}{E_t^{\mathbb{Q}}\left(\frac{u'[W_t R_{f,t\to T}]}{u'[W_T]}\right)}\right) = \frac{\sum_{k=1}^{\infty}\theta_k M_{t\to T}^{\mathbb{Q}(k+1)}}{1+\sum_{k=1}^{\infty}\theta_k M_{t\to T}^{\mathbb{Q}(k)}}, \tag{7}$$

where the authors denote the risk-neutral moments of the market excess return as

$$M_{t\to T}^{\mathbb{Q}(n)} = E_t^{\mathbb{Q}}[(R_{M,t\to T} - R_{f,t\to T})^n] \text{ for } n \geq 1 \tag{8}$$

and θ_k are the preference parameters related to the investor's tolerance for risk, skewness, and kurtosis. By assuming that the odd risk-neutral moments are negative and $\theta_k \leq 0$ if k is even and $\theta_k \geq 0$ if k is odd, CYL argue that the left-hand side of Eq. (7) is no less than the infinite sum truncated at order three. In particular, under a set of restrictions on the preference parameters [6], CYL obtain a lower bound on the expected market excess return:

$$E_t[R_{M,t\to T}] - R_{f,t\to T} \geq LBCL_{t\to T|t} = \frac{\frac{1}{R_{f,t\to T}}M_{t\to T}^{\mathbb{Q}(2)} - \frac{1}{R_{f,t\to T}^2}M_{t\to T}^{\mathbb{Q}(3)} + \frac{1}{R_{f,t\to T}^3}M_{t\to T}^{\mathbb{Q}(4)}}{1 - \frac{1}{R_{f,t\to T}^2}M_{t\to T}^{\mathbb{Q}(2)} + \frac{1}{R_{f,t\to T}^3}M_{t\to T}^{\mathbb{Q}(3)}}, \tag{9}$$

which we refer to as $LBCL_{t\to T|t}$ in our paper.

2.2.3 Chabi-Yo and Loudis's upper bound

CYL further derive an upper bound on the expected market premium. We incorporate this upper bound together with the aforementioned lower bound to constrain return forecasts as an extension of our study and we now briefly introduce it.[7]

CYL decompose the expected market excess return into two parts:

$$E_t[R_{M,t\to T} - R_{f,t\to T}] = E_t[(R_{M,t\to T} - R_{f,t\to T})1_{R_{M,t\to T}>k_0}] + E_t[(R_{M,t\to T} - R_{f,t\to T})1_{R_{M,t\to T}\leq k_0}], \tag{10}$$

where k_0 is a prespecified level of gross return less than $R_{f,t\to T}$. By construction, the second

[6] Specifically, the restrictions are $\theta_1 \geq 1/R_{f,t\to T}$, $\theta_2 \leq -1/R_{f,t\to T}^2$, and $\theta_3 \geq 1/R_{f,t\to T}^3$.

[7] Hansen and Jagannathan (1991) derive a well-known upper bound on the market risk premium that has been widely used in assessing equilibrium asset pricing models. However, it involves an unobservable quantity that can not be computed using options data.

term on the right-hand side of Eq. (10) is always negative, indicating that $E_t[(R_{M,t\to T} - R_{f,t\to T})1_{R_{M,t\to T}>k_0}]$ serves as an upper bound on the expected market excess return. To compute this upper bound, again, CYL use the Radon–Nikodym theorem to transform $E_t[(R_{M,t\to T} - R_{f,t\to T})1_{R_{M,t\to T}>k_0}]$ into a quantity under the risk-neutral measure and apply a Taylor series expansion on the reciprocal of the representative investor's marginal utility in a manner analogous to the derivation of their lower bound. According to Result 6 of CYL, the implied upper bound under certain restrictions on the preference parameters follows

$$E_t[R_{M,t\to T}] - R_{f,t\to T} \leq \frac{(M^{\mathbb{Q}(1)}_{t\to T} - M^{\mathbb{Q}(1)}_{t\to T}[k_0]) + \sum_{k=1}^{3} \frac{(-1)^{k+1}}{R^k_{f,t\to T}}(M^{\mathbb{Q}(k+1)}_{t\to T} - M^{\mathbb{Q}(k+1)}_{t\to T}[k_0])}{1 + \sum_{k=1}^{3} \frac{(-1)^{k+1}}{R^k_{f,t\to T}} M^{\mathbb{Q}(k)}_{t\to T}}, \quad (11)$$

where $M^{\mathbb{Q}(n)}_{t\to T}[k_0]$ is the truncated risk-neutral nth moment of the market excess return defined as

$$M^{\mathbb{Q}(n)}_{t\to T}[k_0] = E^{\mathbb{Q}}_t[(R_{M,t\to T} - R_{f,t\to T})^n 1_{R_{M,t\to T}\leq k_0}] \text{ for } n \geq 1. \quad (12)$$

It is worth noting that the smaller k_0 is, the tighter the upper bound will be. To ensure that there are enough option data to compute the upper bound while maintaining its tightness, we set $k_0 = 0.85$ and denote the right-hand side of Eq. (11) as $UBCL_{t\to T|t}$.[8]

2.3 Univariate prediction model

After introducing the option-implied equity premium bounds, we discuss the predictive regression model framework used in our analysis. Since our paper studies the role of the term structure of option-implied bounds in improving forecasting performance, we focus on the OOS prediction at various horizons. In this section, we first show how we perform a conventional predictive regression over several horizons, and then we describe the implementation of the aforementioned equity premium constraints.

2.3.1 Unconstrained predictive regression

The standard predictive regression approach in the literature assumes that the $h-$month-ahead compound stock market return in excess of the risk-free rate, $r_{t\to t+h}$, is a linear function

[8]CYL use $k_0 = 0.80$. However, due to the limited range of strike prices, we fail to compute the option-implied upper bound for a few of the trading days using $k_0 = 0.80$.

of the lagged predictor variable x_t:

$$r_{t\to t+h} = \alpha + \beta x_t + \epsilon_{t\to t+h},\ t = 1, ..., N - h \text{ and } h = 1, 3, 6, 12. \tag{13}$$

To obtain the out-of-sample forecast, we separate a sample of N observations into two parts: a training period n_1 and a testing period $n_2 = N - n_1$. We start with regressing $\{r_{t\to t+h}\}_{t=1}^{n_1-h}$ on a constant and $\{x_t\}_{t=1}^{n_1-h}$ to obtain the ordinary least squares (OLS) estimates of α and β in Eq. (13), denoted as $\hat{\alpha}_{n_1}$ and $\hat{\beta}_{n_1}$, respectively. Then, the first OOS forecast made at time n_1 is calculated as

$$\hat{r}_{n_1\to n_1+h|n_1} = \hat{\alpha}_{n_1} + \hat{\beta}_{n_1} x_{n_1}. \tag{14}$$

Moving to the next period, we extend the estimation window with the new information at time $t = n_1 + 1$ while fixing the starting point. Similarly, we use the updated OLS estimated coefficients to calculate the second forecast made at time $n_1 + 1$. By repeating this procedure for $t = n_1, ..., N - h$, we can construct a time series of $n_2 - h + 1$ unconstrained forecasts of the equity premium based on x_t, denoted as $\left\{\hat{r}^{UC}_{t\to t+h|t}\right\}_{t=n_1}^{N-h}$.

2.3.2 Constrained predictive regression

Next, we discuss how to impose the aforementioned constraints on equity premium forecasts. Campbell and Thompson (2008) suggest truncating the OOS equity premium forecasts at zero:

$$\hat{r}^{CT0}_{t\to t+h|t} = max\left\{\hat{r}^{UC}_{t\to t+h|t}, 0\right\},\ h = 1, 3, 6, 12 \tag{15}$$

where $\hat{r}^{UC}_{t\to t+h|t}$ is the unconstrained return forecasts generate by Eq. (14). The zero lower bound approach is readily used and widely adopted in the literature (Li and Tsiakas, 2017; Pettenuzzo, Timmermann, and Valkanov, 2014). Empirically, it improves the OOS forecasting performance of unconstrained models. Hence, this constrained approach is used as our benchmark.

In this paper, we make a refinement to CT's methodology that we truncate the predicted equity premium by the option-implied lower and upper bounds described in Section 2.2:

$$\hat{r}^{LBM}_{t\to t+h|t} = max\left\{\hat{r}^{UC}_{t\to t+h|t}, LBM_{t\to t+h|t}\right\}, \tag{16}$$

$$\hat{r}^{LBCL}_{t\to t+h|t} = max\left\{\hat{r}^{UC}_{t\to t+h|t}, LBCL_{t\to t+h|t}\right\}, \tag{17}$$

$$\hat{r}^{L+U}_{t\to t+h|t} = min\{max\left\{\hat{r}^{UC}_{t\to t+h|t}, LBCL_{t\to t+h|t}\right\}, UBCL_{t\to t+h|t}\}, \tag{18}$$

where $LBM_{t\to t+h|t}$ is Martin's lower bound and $LBCL_{t\to t+h|t}$ and $UBCL_{t\to t+h|t}$ are CYL's restricted lower and upper bounds, respectively. While the spirit of our truncation approach is similar to CT's, there remain important differences. First, the option-implied lower bounds we imposed incorporate rich forward-looking information on the underlying asset that is absent from zero. Second, the term structure of the bounds makes them a tighter and more sensible lower constraint for longer-horizon equity premia. Therefore, we expect informative option-implied bounds to improve the forecasting performance of unconstrained regression models and to outperform the zero lower bound constraint.

2.4 Combining multivariate information

As pointed out by Ludvigsona and Ng (2007), Rapach, Strauss, and Zhou (2010), and Pettenuzzo, Timmermann, and Valkanov (2014), due to the limited conditioning information in a single predictor, univariate predictive regressions suffer from model uncertainty and structural breaks, rendering the OOS performance unstable. Thus, we further examine our constrained approach under a multivariate setting by exploiting the rich information in multiple conditioning variables. To do so, we consider the forecast combination method of Rapach, Strauss, and Zhou (2010) to unite the information contained in several predictors. Specifically, our unconstrained forecast combination approach based on an equal-weighted scheme[9] is stated as

$$\hat{r}^{comb,UC}_{t\to t+h|t} = \sum_{i=1}^{M} \omega_i \hat{r}^{UC}_{i,t\to t+h|t},\ h = 1, 3, 6, 12 \tag{19}$$

where M denotes the number of univariate models, $\omega_i = 1/M$, and $t = n_1, ..., N - h$.

Next, we conduct similar procedures as in Section 2.3.2 to obtain the constrained combination forecasts, denoted as $\hat{r}^{comb,CT0}_{t\to t+h|t}$, $\hat{r}^{comb,LBM}_{t\to t+h|t}$, $\hat{r}^{comb,LBCL}_{t\to t+h|t}$, and $\hat{r}^{comb,L+U}_{t\to t+h|t}$, respectively. Note that we combine the unconstrained individual forecasts before imposing equity premi-

[9]In this paper, we only consider the equal-weighted combination method, since we do not seek the best type of combination method but, instead, are concerned about the gains of imposing the option-implied bounds on the combination forecast. Moreover, as pointed out by empirical studies (Li and Tsiakas, 2017; Rapach, Strauss, and Zhou, 2010), the equal-weighted combination method often dominates more sophisticated combination schemes, such as the trimmed mean and the discount MSPE.

um constraints, rather than pooling the constrained individual forecasts. The reason is that we would like to incorporate as much information from various predictors as possible. More importantly, we want to explore the contribution, if any, of the option-implied bounds to the forecast combination.

3 Data and summary statistics

3.1 Predictor variables and options data

The stock returns used in our analysis are compounded on the S&P 500 index with dividends. We subtract the Treasury bill rate from the returns to measure the realized equity premium. We consider a set of widely used conditioning variables studied by Welch and Goyal (2008) and CT as the equity premium predictors. There are 14 predictors in total, including the logarithm of the dividend–price ratio (DP), the logarithm of the dividend yield (DY), the logarithm of the earnings–price ratio (EP), the logarithm of the dividend–payout ratio (DE), stock return volatility (RVOL),[10] the book-to-market ratio (BM), net equity expansion (NTIS), the three-month Treasury bill rate (TBL), the long-term government bond yield (LTY), the return on long-term government bonds (LTR), the term spread (TMS), the default yield spread (DFY), the default return spread (DFR), and inflation (INFL). Appendix A provides detailed descriptions of these predictors. The data of the predictors and the S&P 500 returns are obtained from Amit Goyal's website.[11]

Following Martin (2017) and CYL, we use the S&P 500 index options data from the IvyDB database of OptionMetrics to construct the term structure of option-implied bounds on the market risk premium. The options data run from January 1996 to June 2019, and include the expiration dates, strike prices, open interests, closing bid and ask quotes of all call and put options. Before calculating the implied bounds, we apply some common filters to mitigate the impact of recording errors and option illiquidity. We remove options with zero open interests, zero bid prices, and maturities of less than six days. Options whose prices violate usual non-arbitrage bounds are also excluded. Finally, we retain only

[10] Following Rapach, Ringgenberg, and Zhou (2016), we measure monthly stock market volatility using the 12-month moving standard deviation estimator proposed by Mele (2007). We also examine an alternative variance estimator (SVAR), defined as the sum of the squared daily excess returns within a month (Schwert, 1989). Empirically, RVOL exhibits slightly better predictive performance than SVAR.

[11] See `www.hec.unil.ch/agoyal/`. We thank Amit Goyal for making the data available.

standard index options expiring on the third Friday of each month. In addition, to mitigate the effect of truncation errors[12] when using the discretized versions of the spanning formulas in Appendices B to D, we follow Jiang and Tian (2005) to extrapolate outside the range of available strike prices.[13] Finally, on any trading day, we apply the formulas in Appendices B to D to compute the aforementioned option-implied bounds for all available maturities. We then follow the CBOE's procedure to calculate the bounds for 30, 90, 180, and 365 days by linear interpolation.

3.2 Summary statistics

[**Insert Table 1 here**]

Panel A of Table 1 reports the descriptive statistics for the excess return on the S&P 500 index and the 14 predictors over the full sample period from January 1996 to June 2019. The average monthly market premium is 0.64%, producing an annualized Sharpe ratio of 0.52 (not tabulated). In addition, the distribution of the S&P 500 index returns is negatively skewed and leptokurtic, a fact well established in the literature. Panel B reports the descriptive statistics for the option-implied bounds on the market risk premium used in our paper. Compared to the market excess return, the option-implied lower bounds are right skewed and have fatter tails. Note that $LBM_{t\to t+1m}$ and $LBCL_{t\to t+1m}$ have means of 0.35% and 0.40%, respectively, both lower than the average monthly market premium. In contrast, the mean of $UBCL_{t\to t+1m}$ is higher than the average monthly market premium. Besides, as shown in the eighth column, the estimated $AR(1)$ coefficient of the monthly market excess return is small, nearly zero. This weak persistency is due to temporary shocks contained in the realized returns that are uncorrelated over time. On the other hand, the $AR(1)$ coefficients of option-implied bounds all exceed 0.80, indicating that the bounds are quite persistent, analogous to the expected return.

[**Insert Figure 1 here**]

[12] Jiang and Tian (2005) argue that computing option-implied moments via the model-free approach involves truncation errors due to the limited range of strike prices.

[13] For any maturity in a trading day, we ensure the range of strike prices is at least as wide as $[max\{0, F_0 - 3.5IV_{ATM} \cdot F_0\}, F_0 + 3.5IV_{ATM} \cdot F_0]$, where F_0 is the forward index price and IV_{ATM} is the Black–Scholes implied volatility of the at-the-money option of the given maturity. To do so, we use the implied volatility at the boundary strike prices for those strikes lying outside the available range. Then, we convert the extrapolated implied volatilities to option prices through the Black–Scholes formula.

Across horizons, we find that the average magnitude of the bounds gradually rises. Consistent with CYL's description, the mean, minimum, and maximum of $LBCL$ across all maturities are higher than for LBM. Furthermore, we plot the time series of LBM, $LBCL$, and $UBCL$ at all forecast horizons used in our analysis. Figure 1 shows that all bounds vary substantially over time and increase across maturities, apparently exhibiting a term structure property. We observe that $LBCL$ is tighter than LBM, in accord with the description in Table 1, and both the upper and lower bounds remain low during normal times, while surging during crisis and recession periods.

4 Main results

In this section, we present the OOS performance of the unconstrained predictive regression models and those constrained by the low bounds. We begin with the results on statistical predictability and then move to the evaluation of the economic value of predictability.

4.1 Out-of-sample return forecasts

As mentioned in Section 2.3, we generate the OOS forecasts based on a recursive estimation window. We consider an initial in-sample training period of five years (60 months) so that our OOS period starts in February 2001 and continues through June 2019. After accounting for lags and overlapping data, we obtain totals of 221, 219, 216, and 210 observations at monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$), and annual ($h = 12$) horizons, respectively. An initial training period of 60 months is chosen to balance between a sufficient long OOS period to evaluate the predictive performance and sufficient start-up observations to estimate parameters reliably.

[**Insert Figure 2 and Figure 3 here**]

Before covering the statistical predictability results, we take a brief look at the forecasts constrained by the option-implied lower bounds. Figures 2 and 3 plot the time series of unconstrained and constrained one-step-ahead forecasts over the OOS evaluation period. Regarding space limitations, we pick eight of the 14 predictor variables used in our paper: DP, DE, RVOL, NTIS, TBL, LTY, DFY, and INFL. These predictors are selected as representative variables covering several categories: fundamental-based valuation ratios, bond yields related, stock variance related, corporate finance related, and macroeconomy related (Pettenuzzo, Timmermann, and Valkanov, 2014). We also report the equal-weighted forecast combination ($Econ^{Comb}$), which pools all 14 individual predictive regression forecasts. As shown in Figure 2, all the forecasts produced by the unconstrained predictive regressions are bounded by LBM during the recession periods in 2001 and the 2008 global financial crisis. This is because LBM has a countercyclical pattern and jumps in periods of recession, whereas most predictors signal a decline of the market. In addition, compared with the historical mean benchmark forecast, $\hat{r}^{Bench}_{t+1|t} = 1/t \sum_{\tau=1}^{t} r_\tau$, depicted by the red dash-dot line, both constrained and unconstrained forecasts are more volatile during the sample period. Among the eight representative predictors, the unconstrained forecasts produced by NTIS and INFL continually fall below LBM, resulting in frequent truncation. As depicted by Figure 3, the forecasts constrained by $LBCL$ show similar patterns.

4.2 Statistical predictability analysis

To evaluate the OOS performance of each model, we follow the literature (e.g., Welch and Goyal, 2008) and use OOS R^2 (R^2_{OOS}) as the primary statistical criterion for comparing the OOS forecasts of each model against the historical mean benchmark. The R^2_{OOS} value is calculated as

$$R^2_{OOS} = \frac{MSPE_{Bench} - MSPE_{Model}}{MSPE_{Bench}} = 1 - \frac{\sum_{t=n_1}^{N-h}(r_{t\to t+h} - \hat{r}^{Model}_{t\to t+h|t})^2}{\sum_{t=n_1}^{N-h}(r_{t\to t+h} - \hat{r}^{Bench}_{t\to t+h|t})^2}, \tag{20}$$

where $\hat{r}^{Model}_{t\to t+h|t}$ is the conditional market premium forecast generated by the model of interest and $\hat{r}^{Bench}_{t\to t+h|t}$ is the unconditional forecast of the historical mean benchmark, calculated as the recursive average $\frac{1}{t-h}\sum_{\tau=1}^{t-h} r_{\tau\to\tau+h}$. A positive R^2_{OOS} means that the forecasting model outperforms the benchmark by producing a lower MSPE, that is, $MSPE_{Model} < MSPE_{Bench}$. To test the significance of R^2_{OOS}, we apply the MSPE-adjusted statistic of Clark and West (2007). The null hypothesis of CW's test is that the benchmark MSPE is less than or equal to the predictive regression MSPE against the one-sided alternative hypothesis that the benchmark MSPE is larger than the predictive regression MSPE. To calculate the MSPE-adjusted statistic, we follow CW and Rapach, Strauss, and Zhou (2010) to define the time series $\{f_{t \to t+h}\}_{t=n_1}^{N-h}$, where

$$f_{t \to t+h} = (r_{t \to t+h|t} - \hat{r}^{Bench}_{t \to t+h})^2 - [(r_{t \to t+h} - \hat{r}^{Model}_{t \to t+h|t})^2 - (\hat{r}^{Bench}_{t \to t+h|t} - \hat{r}^{Model}_{t \to t+h|t})^2]. \tag{21}$$

Then, we regress $\{f_{t \to t+h}\}_{t=n_1}^{N-h}$ on a constant and compute the heteroskedasticity- and autocorrelation-consistent standard error (Newey and West, 1987).[14] Eventually, we use the standard normal distribution to obtain the one-sided $p-$value of the null hypothesis.

4.2.1 One-step-ahead predictability

[**Insert Table 2 here**]

Table 2 reports the R^2_{OOS} statistics of the unconstrained models (second column) and the constrained models (third through eighth columns) at a one-month forecast horizon for the 14 predictors and the combination forecast. The second column shows that a majority of the individual predictors, except for DY, fail to beat the historical mean benchmark, consistent with the critique of Welch and Goyal (2008). Even the combination forecast is outperformed by the historical average over our sample period. The results of the seventh column imply that imposing CT's non-negativity restriction ($CT0$) on market premium forecasts improves the OOS performance of most individual predictors, except for DY and INFL, as well as the forecast combination method.

The third and fifth columns of Table 2 display the results for our approach that constrains the return forecasts by LBM and $LBCL$, respectively. From the third column, we can tell that using LBM as a lower constraint on the market premium forecasts increases the R^2_{OOS} values for 11 of the 14 predictors and the combination forecast. On the other hand, using $LBCL$ as the lower constraint increases the R^2_{OOS} values for nine of the 14 predictors, but not for the combination forecast. Overall, the lower constraints by LBM and $LBCL$ improve the average R^2_{OOS} values of the unconstrained models by 0.95% and 0.50%, respectively. Though these improvements seem small, an R^2_{OOS} value of 0.5% for monthly data could create economic value for a mean–variance investor in terms of asset allocation Campbell and Thompson (2008). However, none of the positive R^2_{OOS} values generated by

[14]When $h > 1$, to correct the serial correlation arising from the overlapping return data, we use $2^{*}h - 1$ lags, a relatively large number, as suggested by Bekaert and Hoerova (2014), where h is the forecast horizon.

these two constrained methods is statistically significant at the 10% level, according to the CW statistic.

As shown by the fourth, sixth, and eighth columns of Table 2, 28.2% of the unconstrained forecasts are truncated below by $LBCL$, on average, which is higher than the average percentage of forecasts truncated by LBM (25.8%) and zero (10.7%). This result is to be expected, since $LBCL$ is a tighter lower bound than both LBM and zero, obviously, as described in Section 3. Nevertheless, compared with using the option-implied lower bounds, using zero to constrain the forecasts produced by the predictors used in our analysis leads to more robust improvement in R^2_{OOS} at the monthly horizon. In short, all three types of lower constraint improve the OOS predictive performance of the unconstrained models at the one-month forecast horizon, while $CT0$ performs the best. Since the predictable variation of the monthly expected return is known to be small, the gain from using an informative bound implied by options could be minor. However, this is not the case for longer forecast horizons, whose results are shown in the following.

4.2.2 Multi-step-ahead predictability

[**Insert Table 3 here**]

Panels A to C of Table 3 report the R^2_{OOS} statistic of each model at the quarterly, semi-annual, and annual horizons, respectively. The OOS predictive ability of most unconstrained models deteriorates significantly at longer horizons. The mean of R^2_{OOS} generated by the unconstrained predictive regressions (UC) decreases from -2.27% to -41.87% monotonically as the forecast horizon increases from one month to one year. Neither combination forecast outperforms the historical mean benchmark at any horizon. All three types of lower constraint, LBM, $LBCL$, and $CT0$, continue to improve the predictive performance for a majority of the unconstrained models. For example, at the quarterly horizon, using LBM, $LBCL$, and zero as lower constraints increases the predictive accuracy for 14, 13, and 12 of the 15 cases, and the corresponding average increments in R^2_{OOS} are 7.55%, 7.12%, and 6.19%, respectively. Note that the improvement in R^2_{OOS} increases with the horizon, and, more importantly, the gains in predictive accuracy from imposing the option-implied bounds dominate those of the zero lower constraint at all horizons. For instance, eight of the 15 $LBCL$-constrained models produce positive R^2_{OOS} values that are significant at least at the

10% level, whereas only four of the zero-constrained models outperform the historical mean benchmark for a horizon of one year.

Besides the enhancements in predictive accuracy, we observe that the portion of unconstrained forecasts truncated by the option-implied lower bounds remains high, about 25–30%, on average. In particular, the average percentage of forecasts truncated by $LBCL$ grows with the horizon, reaffirming it as a tighter lower bound. In contrast, the zero lower constraint merely truncates a small portion of the unconstrained forecasts, 8.54%, on average, at the annual horizon, confirming that zero is an overly slack lower bound for the long-term expected return. These findings are consistent with our expectation that the option-implied lower bound should outperform the zero constraint at longer horizons, since the former incorporates rich information about the market and has a time-varying term structure pattern.

Between the two option-implied lower bounds, $LBCL$ is better than LBM as a market premium forecast constraint at the semi-annual and annual horizons. According to the eighth column (h=6) in Table 3, using $LBCL$ to constrain forecasts increases the R^2_{OOS} values for 12 cases, eight of which are statistically significant at the 5% level and one at the 10% level. Such improvements surpass those derived from using LBM as a lower constraint. This is partially because the average percentage of forecasts truncated by LBM decreases over horizons, whereas $LBCL$ behaves just the opposite. Additionally, CYL point out that $LBCL$ considers the effect of high-order risk-neutral moments, which have particular importance in measuring expected returns at longer horizons. This richer set of information contained in $LBCL$ also explains the stronger improvements in predictive performance relative to LBM.

Interestingly, the gains in predictive accuracy from imposing the option-implied bounds constraint vary greatly among different predictors. For example, under the $LBCL$-constrained approach, the increment in R^2_{OOS} ranges from -2.70% (BM) to 91.09% (DFY) at the semiannual horizon. The improvements to the bond yield–based predictors are particularly prominent. Notably, the combination of $LBCL$ with LTY achieves great success, leading to sizable R^2_{OOS} values at the monthly (0.17%), quarterly (3.39%), semiannual (13.31%), and annual (20.20%) horizons, whereas the combination with DP lowers the original R^2_{OOS}. We will discuss such heterogeneity in detail in Section 5. In brief, the forecasting performance of most constrained models is better than that of their unconstrained counterparts at all four horizons, while the benefits of imposing the option-implied bounds dominate the zero lower constraint at the quarterly, semi-annual, and annual horizons.

4.3 Economic value of predictability

The preceding section confirms that imposing the option-implied constraints on return forecasts leads to better performance in terms of statistical predictive accuracy. In this section, we assess the economic value of the market premium predictability generated by each model. Following CT and Rapach, Strauss, and Zhou (2010), we consider a mean–variance investor who can allocate his or her wealth between the S&P 500 index (the risky asset) and Treasury bills (the risk-free asset) based on a constrained or unconstrained return forecast. At the end of month t, the investor optimally allocates a portion

$$\omega_t = \frac{1}{\gamma} \frac{\hat{r}_{t+1|t}}{\hat{\sigma}^2_{t+1|t}} \tag{22}$$

of wealth to the market index, where γ is the risk aversion coefficient, $\hat{r}_{t+1|t}$ is the forecast of the market index excess return at time $t+1$ conditional on the available information at time t, and $\hat{\sigma}^2_{t+1|t}$ is the conditional forecast made at time t of the market index return variance at time $t+1$. In addition, we impose a short-selling constraint as well as a maximum leverage at 50% to restrict ω_t to lie between 0 and 1.5, consistent with the literature.

The nominator of the optimal weight described in Eq. (22) is given by the OOS equity premium forecast, but we also need the denominator, that is, the forecast of the return variance. A typical approach to measure the denominator uses the unconditional sample variance.[15] The conditional variance of the stock market, however, is known to be time-varying and highly persistent. Marquering and Verbeek (2004) empirically verify the significant economic gain derived from employing volatility timing to a strategy relying on timing in returns only. To keep our methodology simple and focus mainly on the economic significance of timing in returns, we apply a parsimonious univariate $AR(1)$ model to predict the market return variance. To this end, we construct a proxy for the realized variance of excess returns on the S&P 500 index by summing squared daily excess returns at a monthly

[15] Specifically, a multitude of studies estimate the return variance forecast by the variance of excess returns over the past 10 years (e.g., Rapach, Strauss, and Zhou, 2010). Given the strong persistence and clustering effect of the stock return volatility, this approach is somewhat inefficient.

sampling frequency (Schwert, 1989):

$$RV_t = \sum_{i=1}^{N_t} (r_{i,t} - \bar{r_t})^2, \tag{23}$$

where N_t denotes the number of trading days in month t, $r_{i,t}$ is the daily excess return on the S&P 500 index, and $\bar{r_t}$ is the mean of the daily returns in month t. Instead of directly forecasting the level of realized variance at $t+1$, we follow Paye (2012) to model the natural logarithm of variance as an $AR(1)$ process:

$$LVAR_{t+1} = \alpha + \beta LVAR_t + \epsilon_{t+1}, \tag{24}$$

where $LVAR_{t+1} \equiv log(RV_{t+1})$ and $LVAR_t$ is the lagged log realized variance. We estimate this $AR(1)$ predictive regression using a 15-year rolling window of historical returns. The conditional variance forecast is then calculated as $\hat{\sigma}_{t+1|t} = exp(\widehat{LVAR}_{t+1} + \frac{1}{2}\sigma_{\epsilon}^2)$.[16]

Given the estimated optimal weight ω_t, the realized return of the investor's market timing strategy is

$$r_{p,t+1} = \omega_t r_{m,t+1} + r_{f,t+1}, \tag{25}$$

where $r_{m,t+1}$ and $r_{f,t+1}$ denote the excess market index return and the risk-free rate between times t and $t+1$, respectively. Over the OOS period, the average realized utility (i.e., CER) for the investor is then measured as

$$CER_p = \hat{\mu}_p - \frac{\gamma}{2}\hat{\sigma}_p^2, \tag{26}$$

where $\hat{u}_p$ and $\hat{\sigma}_p^2$ are the mean and variance of the portfolio return over the OOS evaluation period, respectively. We quantify the economic value of the return predictability afforded by the predictive model by the CER gain, which is defined as the difference between the CER when the investor uses the predictive regression forecast and the CER when the investor uses the historical mean benchmark forecast to construct the trading strategy. Such utility gain can be interpreted as the portfolio management fee that an investor would be willing pay for switching from using the historical mean forecast to the regression forecast. We also

[16]Note that we use the same volatility forecast in Eq. (22) for all portfolios, so that the differences among portfolio weights are determined only by the different return forecasts.

evaluate and compare the performance of different market timing strategies by using the Sharpe ratio. Finally, to assess the statistical significance of the incremental economic value derived from using constrained return forecasts, we use the statistic proposed by Diebold and Mariano (1995) to test the difference between the CER gains of two competing models and the test of Jobson and Korkie (1981) with correction by Memmel (2003) to test the equality of two Sharpe ratios.

Moreover, we assess the economic value of return predictability for horizons longer than one month, by assuming that the investor's portfolio is rebalanced at the same frequency as the forecast horizon, following Rapach, Ringgenberg, and Zhou (2016). Due to limited OOS observation points when using non-overlapping return forecasts, we consider only quarterly and semiannual rebalancing frequencies, in addition to the monthly strategy.

Portfolio performance

Panel A of Table 4 presents the monthly asset allocation results. According to the second column, an investor with a risk aversion coefficient of three using unconstrained return forecasts for trading strategy guidance realizes an annualized CER gain of 52 bps, on average.[17] The third and fourth columns report the difference between the CER gains of the LBM- or $LBCL$-constrained model and the corresponding unconstrained model. Using LBM or $LBCL$ to constrain the forecasts results in higher CER gains for 12 of the 15 cases, where four increments are significant at least at the 10% level. The average improvement in the CER gain by the option-implied lower bounds ranges from 78 to 90 bps.

[**Insert Table 4 here**]

To determine the period in which the utility gains are realized, we follow Eriksen (2017) and plot the time series of the cumulative monthly CER gains. The realized CER gains scaled by the length of the OOS period up to month t is defined as

$$CCERG_{model,t} = \frac{1}{N - n_1} \sum_{\tau=n_1+1}^{t} (CER_{model,\tau} - CER_{bench,\tau}), \; t = n_1 + 1, ..., N, \qquad (27)$$

[17]Note that imposing a short-selling constraint on the portfolio weight is analogous to constraining the equity premium predictions by zero, that is, $CT0$.

where *model* denotes either the constrained or unconstrained predictive regression model, *bench* is the historical mean benchmark model, $N - n_1$ is the number of OOS observations, and $CER_{M,\tau}$ is the realized CER gain at time τ, which can be calculated as

$$CER_{M,\tau} = r_{p,\tau} - \frac{\gamma}{2}(r_{p,\tau} - \bar{\mu}_p)^2, \text{ for } M = \textit{model} \text{ or } \textit{bench} \tag{28}$$

where $r_{p,\tau}$ is the portfolio return at time τ, γ is the investor's risk aversion coefficient, and $\bar{\mu}_p$ is the mean portfolio return over the OOS period. Figure 4 shows that the realized CER gains of both LBM- and $LBCL$-constrained models drop during the 2008 global financial crisis. This is because a series of positive shocks to the expected return during the crisis elevates the option-implied bounds to their all-time high, while the realized market return becomes fairly negative because of the discount rate effect. Therefore, the larger weight on the market index suggested by the constrained return forecast makes the strategy suffer from a bigger loss than its unconstrained counterpart does. Nevertheless, both constrained models shortly thereafter catch the strong market rebound, and the consistently upward trend of the CER gain in some plots indicates that the constrained approach produces substantial economic value during prolonged expansion periods after the crisis.

[**Insert Figure 4 here**]

At a quarterly horizon, Panel B in Table 4 shows that the unconstrained model produces annualized CER gains of 65 bps, on average, while the use of LBM- and $LBCL$-constrained forecasts moderately increases this quantity by 33 bps and 37 bps, respectively. In contrast, our constrained approach leads to more substantial and significant improvements at the semi-annual horizon. We observe an increment in the CER gain for nine of the 14 predictors using the LBM- constrained ($LBCL$-constrained) approach, where seven (six) increments are statistically significant at least at the 10% level. The improvements to the forecast combination method are 91 bps and 111 bps under the LBM- and $LBCL$-constrained approaches, respectively, both significant at the 10% level.

In addition, Table 5 reports the annualized Sharpe ratio of these trading strategies. A majority of the LBM- and $LBCL$-constrained models outperform their unconstrained counterparts in terms of the SR. For instance, at the semi-annual horizon, the $LBCL$-constrained approach improves the SR for 10 of the 15 cases, with seven of them significantly higher than their unconstrained counterparts at the 10% level. The gains in economic value are generally larger for the bond yield–based predictors than the others, consistent with the R^2_{OOS} results. In particular, the CER gain and SR of the trading strategy that employs the constrained forecasts based on LTY are always among the highest over all horizons.

[**Insert Table 5 here**]

We summarize that the economic value derived from imposing the option-implied lower constraints is prominent and presents at the monthly, quarterly, and semi-annual horizons, while the $LBCL$-constrained model tends to deliver greater CER gains than the LBM-constrained model to a mean–variance investor.

5 The role of option-implied lower bounds

Thus far, we have shown that imposing the option-implied lower bounds proposed by Martin (2017) and CYL on return forecasts leads to significant improvements in OOS predictive accuracy and economic value, particularly at longer horizons. In this section, we determine why option-implied constraints improve upon standard predictive regressions. To proceed, we first study the predictive ability of the option-implied bounds themselves. Second, to understand the heterogeneity when combining the bounds with distinct predictors, we link the improvements to the characteristics of the predictors. Finally, we investigate forecasting ability over different economic conditions.

5.1 Option-implied lower bounds as predictors

As argued by Martin and CYL, the lower bounds are tight and thus could be a measure of the equity premium itself. We therefore directly use the conditional option-implied lower bounds as the forecasts for the expected market premium, that is, $\hat{r}_{t\to t+h|t} = LBM_{t\to t+h|t}$ or $LBCL_{t\to t+h|t}$. Similarly, we use R^2_{OOS}, the CER gain, and the Sharpe ratio to measure the statistical and economic significance of the bounds' predictive ability.

[**Insert Table 6 here**]

Table 6 reports the statistical and economic predictability of LBM and $LBCL$ over the same OOS evaluation period. First note that the predictive performance of LBM is slightly better than that of $LBCL$ at the monthly and quarterly horizons, whereas neither generates a significantly positive R^2_{OOS}. Turning to longer horizons (semi-annual and annual), both bounds exhibit prominent forecasting ability, while LBM underperforms $LBCL$, which is consistent with CYL's findings. For instance, at the semi-annual forecast horizon, the R^2_{OOS} values generated by LBM and $LBCL$ are 7.94% and 10.38%, respectively, both significant at the 5% level. The remarkable predictive ability of the bounds partially explains the large improvement to the unconstrained models at longer horizons.

Next, the Sharpe ratios of the trading strategies relying on the implied bounds are higher than those of the historical mean benchmark model at all horizons, as presented in Table 5. Nevertheless, the negative CER gains in Panel B of Table 6 indicate that neither LBM nor $LBCL$ outperforms the historical mean benchmark forecast in terms of the realized utility from the monthly and quarterly asset allocations. This is mainly because the timing strategy based on either bound aggressively weights equity during turbulent times, thereby rendering the portfolio return quite volatile. To address these seemingly contradictory results between the CER gain and the Sharpe ratio, we mainly rely on the former to evaluate a strategy's performance, since the latter does not penalize suboptimal leverage (Kan and Zhou, 2007). For the semi-annual rebalancing frequency, the $LBCL$ model generates a positive CER gain of 61 bps, which is greater than the LBM model (1 bp). Both, however, are inferior to the average CER gain of the unconstrained models.

In sum, the return forecasting ability of LBM and $LBCL$ surpasses most of the predictors of Welch and Goyal (2008) and the term structure feature largely allows the bounds to make fairly good predictions at longer forecast horizons. However, this is not the whole story, since the bounds only truncate about 25–30% of the unconstrained forecasts, on average. Besides, from an asset allocation perspective, the predictive ability fails to deliver any economic value at the monthly and quarterly rebalancing frequencies. Thus, we take a closer look at the improvements among different predictors, as discussed in the following.

5.2 A closer look at the improvements

As indicated in Tables 2 and 3, the increments in R^2_{OOS} resulting from imposing the option-implied bounds vary greatly among the predictors. For instance, we uncover synergy from combining option-implied bounds with some predictors, such as LTY and DFY. In other words, the R^2_{OOS} values of the constrained forecasts are higher than both for the unconstrained counterpart and the bounds themselves. The predictive performance of DP and DY, however, is slightly impaired after imposing either LBM or $LBCL$. Why do some predictors pair well with the lower bounds, while other do not? To address this issue, we explore the relation between $LBCL$ and unconstrained return forecasts in detail.[18]

[**Insert Figure 5 here**]

Figure 5 plots the increment in R^2_{OOS} for the unconstrained forecast against the correlation of unconstrained forecasts based on the 14 predictors with $LBCL$ over the OOS evaluation period. A straight line is fitted via OLS to capture trends. First, we notice that unconstrained forecasts generated by most of the predictors are negatively correlated with $LBCL$ over all horizons. An obviously downward-sloping line presented in each plot implies that, to a certain extent, the more negative the correlation between the unconstrained forecasts and the bound, the larger the increment in R^2_{OOS} derived from imposing the bound. In particular, the forecasts based on DFY or LTY are the most negatively correlated with $LBCL$, and their combinations with $LBCL$ yield the synergy. On the other hand, the correlations between $LBCL$ and forecasts produced by DP or DY are near zero at the monthly horizon and become positive at longer horizons.

In the spirit of the diversification effect in portfolio management, we expect the negative correlation between $LBCL$ and unconstrained forecasts to lead to a decline in forecast variance after the bound is imposed. In addition to forecast variance, forecast bias constitutes part of the MSPE. Therefore, we perform an MSPE decomposition, as Rapach, Strauss, and Zhou (2010), to dissect the change in predictive accuracy.[19] We can attribute the increment in predictive accuracy to a reduction in forecast variance if the constrained forecasts do not have excessive forecast biases compared to their unconstrained counterparts.

[**Insert Figure 6 and Figure 7 here**]

Figures 6 and 7 plot the OOS forecast variance and the squared forecast bias for the $LBCL$-constrained predictive regression model and its unconstrained counterpart at the

[18] We obtain similar results when using LBM. Due to space limitations, these results are not tabulated here and can be found in the Online Appendix.

[19] Rapach, Strauss, and Zhou (2010) show that $MSPE \approx \sigma^2_r + \sigma^2_{\hat{r}} + (\bar{\hat{r}} - \bar{r})^2$, as long as the correlation between the actual and predicted returns is near zero, where σ^2_r is the actual return variance, $\sigma^2_{\hat{r}}$ is the forecast return variance, and $(\bar{\hat{r}} - \bar{r})^2$ is the squared forecast bias.

monthly and semi-annual forecast horizons, respectively. For readability, we only plot the forecast combination and the eight representative predictors selected in Section 4.1, plus BM and TMS. We use a dashed line to split the plot into two parts, so that moving toward the lower-left corner represents a reduction in both the forecast variance and bias. As expected, these two scatter plots show that the forecast variance tends to be lower for those unconstrained models that are weakly or negatively correlated with $LBCL$, except for DP and RVOL. Although our constrained approach increases the forecast bias of some unconstrained models, the decline in variance dominates the change of the MSPE, resulting in considerable increments of R^2_{OOS}, as shown in Section 4.[20] Notably, despite some predictors, such as DFY and NTIS, originally being above the dashed line, their constrained counterparts move toward the origin and are below it.

Besides the diversification effect, we argue that the negative correlation between $LBCL$ and unconstrained forecasts results from the erroneous predictions made by the predictor, provided that $LBCL$ tracks the equity premium. The more negative the correlation is, the more false signals and outliers are involved in unconstrained forecasts, possibly due to the noises and variations of predictors that are unrelated to the equity premium. The bound $LBCL$ being an informative lower bound helps to eliminate those false signals such that the forecasts are well behaved and less volatile. Therefore, we observe an improvement in forecasting performance, especially for those models that are most negatively correlated with the bound.

In addition to the preceding statistical explanations, we offer one plausible interpretation for the heterogeneity in improvement by analyzing the information content of the predictors. Based on the decomposition of Campbell and Shiller (1988), Cochrane (2011) empirically finds that the variation of the DP ratio is largely due to the variation of the changes in the future expected return. Namely, DP is primarily driven by change in the expected return, as are the option-implied bounds. DY and BM behave similarly to DP.[21] Therefore, since the bounds and these predictors share a common component related to the expected return, the assistance of imposing the option-implied constraint is minor. The covariance

[20] The change in the squared forecast bias is far less than the change in forecast variance. For example, in the cases of NTIS and DFY, the reduction in variance accounts for over 62% and 85%, respectively, of the total decline in the MSPE, where the percentages in decline are calculated as $\frac{\Delta Variance}{\Delta MSPE}$ and $\Delta MSPE \approx \Delta Bias + \Delta Variance$.

[21] The variable BM has played a similar role to that of DP in the stock return decomposition (Vuolteenaho, 2002). The correlations of DP with DY and with BM are 0.98 and 0.67, respectively, in this sample.

between them could also lead to additional variance in the constrained forecasts, resulting in a higher MSPE. In contrast, variables related to the bond yields, such as LTY and DFY, contain information on the economic phases spanning several years that are absent from the option-implied bounds. The complementary information of predictors thus contributes to the synergy.

To further determine which predictor provides information complementarity, we use 11 variables (excluding DY, DE, and TMS, for collinearity) of Welch and Goyal (2008) to predict the gap between the bound and the realized excess return. If the bound is always tight, the gap is unpredictable; otherwise, those predictors that can capture the variation of the gap would likely contribute additional information to the bound. The results in Table 7 demonstrate that the 11 predictors together explain a large portion of the gap's variation, producing R^2 values from 12% to 61%. In particular, the bond yield–based predictors are usually among the few significant ones, suggesting that the information from the bond market complements the bound when it is slack. The ability to predict the gap makes the bond yield–based predictors a good "partner" for the bound.

[**Insert Table 7 here**]

Overall, we conclude that the forecasting performance is improved, on average, but with apparent heterogeneity among the predictors. The improvements are mainly due to reductions in forecast variance, while the differences among them could arise from the distinct dynamics and information sources of the predictors.

5.3 Predictability over good and bad times of the economy

The strength of stock return predictability varies over business cycles. We are also interested in the contribution of option-implied bounds to the unconstrained prediction models under different economic conditions. The National Bureau of Economic Research (NBER) business cycle is the standard measure of U.S. business cycle phases in the literature. However, confined by the length of the option data, there are only two recessions over the OOS period from February 2001 to June 2019, with peaks (troughs) in February 2001 (October 2001) and November 2007 (May 2009). Given the limited number of recessions within our sample, we consider the Chicago Fed National Activity Index (CFNAI)[22] as a real-time indicator to split the OOS periods into *good times* and *bad times* of the economy. The CFNAI serves as a summary measure of the overall economic activity at a monthly frequency. In our study, we treat periods with a positive index as good times for the economy and periods with a negative index as bad times, or, to some extent, contraction periods.[23] Following Rapach, Strauss, and Zhou (2010), we compute the monthly R^2_{OOS} over different economic conditions as

$$R^2_{OOS,c} = 1 - \frac{\sum_{t=n_1}^{N-1} I^c_{t+1}(r_{t+1} - \hat{r}^{Model}_{t+1|t})^2}{\sum_{t=n_1}^{N-1} I^c_{t+1}(r_{t+1} - \hat{r}^{Bench}_{t+1|t})^2},\ for\ c = GOOD,\ BAD, \tag{29}$$

where I^{GOOD}_{t+1} (I^{BAD}_{t+1}) is set equal to one whenever the CFNAI is positive (negative) in month $t+1$, and zero otherwise.

[**Insert Table 8 here**]

Panels A and B in Table 8 report the monthly R^2_{OOS} values over bad and good times, respectively. Most unconstrained predictive regression models perform better during bad times than good times, consistent with the literature. Compared with the option-implied lower constraints, $CT0$ produces the largest improvement in predictive accuracy for the unconstrained model during bad economic times. In contrast, during good times, the R^2_{OOS} values of LBM- and $LBCL$-constrained models are uniformly better than for the unconstrained counterparts. The mean R^2_{OOS} of the unconstrained model during good times increases from -2.65% to 2.98% and 4.08% after imposing the LBM and $LBCL$ constraints, respectively. Among the positive R^2_{OOS} values generated by either the LBM- or $LBCL$-constrained models, 10 of the 13 cases are significant at least at the 10% level. Such increments in R^2_{OOS} are substantially larger than those associated with $CT0$.

The results in Table 8 demonstrate that imposing the option-implied constraints leads to markedly large increments in predictive accuracy during good economic times, while performing poorly during bad times. The last row of Table 8 shows similar results for directly using the bounds as forecasts; namely, both LBM and $LBCL$ exhibit strong forecasting

[22]The CFNAI was developed by the Federal Reserve Bank of Chicago based on a series of inflation-adjusted economic indicators. We obtain CFNAI data from the Federal Reserve Bank of St. Louis (the Federal Reserve Economic Data database).

[23]Eriksen (2017) uses the three-month moving average of the CFNAI to measure real economic conditions. The author terms good times an economy with a high CFNAI, and recessions an economy with a low CFNAI.

ability during good times, while neither beats the historical mean benchmark model during bad times. According to Campbell (1991), shocks to the expected returns are negatively correlated with shocks to the realized returns. In bad economic times, especially during the 2008 global financial crisis, we observe a surge in option-implied bounds, potentially driven by a series of positive expected return shocks, whereas realized returns move in the opposite direction. Therefore, the large discrepancy between LBM or $LBCL$ and realized returns creates significant prediction errors.

Moreover, we examine the return predictability during high- and low-VIX periods (Table A.2 in the Online Appendix). Similarly, we find that option-implied constraints generate more substantial improvements during low-VIX periods, while $CT0$ performs better during high-VIX periods. The use of option-implied constraints is optimal when the overall economy is good, as is the use of zero constraints in turbulent market periods.

6 Extensions and robustness analysis

In this section, we consider several extensions to our main results. First, we add the slope constraint recommended by CT to the previously discussed constrained models. Second, we incorporate the upper bound on the equity premium derived by CYL together with their lower bound. Third, we extend our analysis to the option-based predictors. Finally, we examine the robustness of our results with different investor risk aversion coefficients and alternative training periods.

6.1 The impact of slope restriction

[**Insert Table 9 here**]

CT also suggest a slope constraint that sets the regression coefficient to zero whenever its sign contradicts theoretical expectations. In this section, we examine the joint effect of this slope constraint and the lower bound constraint on forecasting performance. We first employ the slope constraint and then truncate the return predictions by the corresponding lower bound. We only consider the monthly and quarterly forecast horizons.[24] Table 9

[24] The short-term relation between returns and certain predictors, such as TMS, can be reversed in the long term (Fama and French, 1989).

reports the OOS results for the constrained model with both restrictions. By comparing the mean R^2_{OOS} in the second to seventh columns with the results in Tables 2 and 3, we find that adding the slope restriction further improves the predictive performance, whereas the improvements are relatively minor. The lower bound constraint plays a dominant role in enhancing the forecasting ability of the unconstrained model.

6.2 Upper bound on market premium forecasts

Our main results emphasize the impact of imposing the option-implied lower bound constraint on return forecasts. In this section, we study the joint effect of using the upper bound, $UBCL$, together with the lower bound, $LBCL$, to constrain the return forecasts, that is, the forecasts generated by Eq. (18). The results in Table 10 show that the R^2_{OOS} values of the constrained models that employ both bounds ($LB + UB$) exceed the corresponding unconstrained models by a large amount at all horizons, except for a few cases at the monthly horizon. More importantly, we observe a greater improvement in predictive performance that surpasses the gains from only employing the lower bound constraint. The additional increments in the average R^2_{OOS} relative to the $LBCL$-constrained model are 1.45%, 4.54%, 12.52%, and 38.66%, respectively, over the four horizons. Intuitively, the more restrictive constraint placed by the lower and upper bounds together removes both negative and extremely positive predictions, further stabilizing the forecast.

[**Insert Table 10 here**]

To provide insight on such improvements, we first examine the bounds themselves. As indicated by Figure 1, the distance between $LBCL$ and $UBCL$ shrinks over the horizons, which confines the forecasts produced by the different predictors to a relatively narrow range. Accordingly, the forecasting performance of the constrained models based on different predictors converges to the bounds at the longer horizon. Second, the expected returns are known to be persistent (Campbell, 1991; Fama and French, 1989).[25] It is thus conceivable that the return forecasts are unlikely to be consistently volatile, being smooth most of the time while fluctuating strongly only during turbulent periods. The noisy return forecasts, either too high or too low, due to the false signals in the predictors deteriorate the predictive

[25]Koijen and Van Binsbergen (2010) empirically show that the annual persistence coefficient of expected returns is greater than 0.9.

precision. Consequently, the additional upper bound constraint further reduces the forecast variance. Through an MSPE decomposition, we also find a decline in the forecast bias after imposing both constraints relative to imposing only the lower one.

As a final point, the upper bound on the equity premium we impose can be linked to an economically motivated constraint on the Sharpe ratio that has a long tradition in asset pricing. Due to quick investor response, an investment opportunity with a high SR, that is, "a good deal" (Cochrane and Saa-Requejo, 2000), is transitory and not likely to persist in the equity market, especially for the market portfolio. In the return predictability study of Pettenuzzo, Timmermann, and Valkanov (2014), the upper limit of the conditional SR for the market portfolio is set at one. Since the expected market excess return equals its conditional SR times the volatility, the SR constraint can avoid improbably high return forecasts. In our case, given the market volatility, $UBCL$ turns into a SR constraint (potentially time-varying) that can rule out near-arbitrage opportunities. In other words, the upper bound on the equity premium eliminates those implausibly large predictions and, consequently, enhances the predictive performance.

6.3 Combining with the option-based predictors

Our main analysis focuses on 14 economic predictors that are widely used in the literature. In addition, the literature has uncovered several strong return predictors from the options market. We are thus interested in whether the bounds or the option-based predictors use the option information more efficiently. In other words, we examine whether the bounds can improve the OOS predictive performance of these predictors. We consider five option-based predictors, the variance risk premium (Bollerslev, Tauchen, and Zhou, 2009, VRP),[26] the forward variance factor (FVF) and the forward skewness factor (FSF) (Andreou, Kagkadis, Philip, and Taamouti, 2019), the squared VIX (VIX^2), all constructed from the S&P 500 options data, and the implied volatility spread (Han and Li, 2020, IVS), constructed from the individual stock options data. Again, we apply an equal-weighted forecast combination to combine individual forecasts.

[**Insert Table 11 here**]

[26] See `sites.google.com/site/haozhoupersonalhomepage/`. We thank Hao Zhou for making the VRP data available.

The results in Table 11 demonstrate that the option-implied lower bound constraint leads to better performance for all option-based predictors, except for a few cases at shorter horizons. The predictive accuracy of the forecast combination method is also improved at semi-annual and annual horizons. The predictor VRP performs well during the financial crisis (Bekaert and Hoerova, 2014). Empirically, we find that VRP of Bollerslev, Tauchen, and Zhou (2009) captures some market downturns at shorter horizons, while this ability is weakened by the lower bound constraint. However, both LBM and $LBCL$ greatly strengthen the predictive performance of VRP at longer horizons, indicating that the bounds incorporate unique information about the long-term equity premium from the options market.

6.4 Alternative risk aversion coefficient

In Section 4.3, we evaluate the economic value of return predictability for a mean–variance investor with a relative risk aversion coefficient (γ) of three. To check the robustness of our results under alternative specifications, we consider the case of $\gamma = 5$ and report the asset allocation results in Table 12.

[**Insert Table 12 here**]

Compared to the results in Table 4, not surprisingly, the magnitude of the CER gain in Table 12 decreases as γ increases. Nevertheless, our constrained approach continues to improve the asset allocation performance of the most unconstrained models and the significance of the improvements barely changes. As shown in Panel A in Table 12, using LBM ($LBCL$) to constrain the monthly market premium forecasts leads to higher CER gains for 12 of the 15 cases, and the average increment in CER gain is 47 bps (54 bps). Similar results can be found at longer horizons (Panels B and C). Therefore, the economic value derived from our method is robust to the investor's risk aversion coefficient.

6.5 Alternative training period

As discussed in Section 4.1, our OOS forecasts begin five years (60 months) after the sample, so that we have sufficient observations to evaluate the OOS forecast. Since the length of the initial in-sample estimation period depends primarily on the underlying sample period, the choice of the initial training period varies in the literature. To explore the sensitivity of our results to this choice, we use the first eight years (96 months) as an alternative initial training period when recursively estimating the market premium forecasts.

[**Insert Table 13 here**]

Table 13 reports the R^2_{OOS} statistics for the constrained and unconstrained models at four different forecast horizons when the initial training period equals 96 months. Similarly, all the unconstrained predictive regression models fail to outperform the historical mean benchmark at the monthly forecast horizon, and the predictive accuracy deteriorates as the forecast horizon increases, except for a few cases. Among the three types of lower constraint, again, $CT0$ generates the largest improvement in R^2_{OOS} for the unconstrained model at the monthly horizon, while LBM and $LBCL$ dominate $CT0$ at longer horizons. Notably, the increment in R^2_{OOS} at longer horizons is more significant in this sample period. For example, the average R^2_{OOS} of the $LBCL$-constrained model becomes positive at the semi-annual and annual horizons.

7 Conclusion

In this study, we propose a new constrained forecasting strategy that employs two option-implied lower bounds on the conditional market risk premium derived by Martin (2017) and CYL, respectively. The empirical results show that our approach significantly improves OOS predictive accuracy of conventional economic and financial predictors, especially for semi-annual and annual forecast horizons, at which most unconstrained predictive regression models perform poorly. The improvements increase with the horizon and exceed the benefits of using the prevailing zero lower constraint. Even stronger performance is uncovered when the upper bound on equity premium from CYL is incorporated. Besides the enhancements in forecasting ability, our constrained approach delivers pronounced economic gains to mean–variance investors.

To gain further insights into our method, we study the differences in combining the bounds with distinct predictors and explain the forecast improvements from two perspectives. On the one hand, compared with using a simple zero as the lower bound, the option-implied lower bound constraint we employ not only rules out negative excess return forecasts, but also sets a term structure of theoretically more reasonable lower constraints on the expected market premium, thereby greatly reducing the false signals of unconstrained forecasts. On the other hand, our method exploits the unique and forward-looking information about the expected returns implied from the index options market. More importantly, this conditional information complements a variety of economic and financial predictors in general, particularly bond yield–based predictors, and contributes to predictive performance that is stronger than that of both the predictors and the bounds themselves. Taken together, our approach can be regarded as a way to accommodate parameter instability and, in the meanwhile, incorporate multiple sources of conditional information.

The aim of our study is to shed light on the role of option-implied bounds in return predictability. Though the aforementioned benefits are substantial and robust, our method of truncating the forecasts from below is relatively simple, since the estimated coefficients of the predictive regression do not learn from the option-implied bounds. A potential extension to our method is to train the regression parameters with the information provided by option-implied bounds through the methods of, for instance, Pettenuzzo, Timmermann, and Valkanov (2014). We stress that this method can be applied to any return predictors besides the ones used in our analysis and could be extended to stock- and portfolio-level return predictability with suitable modifications. Overall, our empirical findings could serve as guideposts for future investigation of OOS stock return predictions.

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Table 1: **Summary Statistics**

This table presents summary statistics for the monthly excess returns on the S&P 500 index (Mkt. Prem.), 14 predictive variables, and option-implied market premium bounds used in our paper. $AR(1)$ denotes the estimated coefficient for an autoregressive model of order one. The variable LBM is the lower bound on the expected market premium (in percent), calculated by the method of Martin (2017), and $LBCL$ and $UBCL$ are the lower and upper bounds, respectively, on the expected market premium (in percent), calculated by the method of CYL. The sample period is from January 1996 to June 2019, containing 282 monthly observations.

(1)	(2) Mean	(3) Std. Dev.	(4) Skewness	(5) Kurtosis	(6) Max.	(7) Min.	(8) $AR(1)$
			Variable				
Mkt. Prem. (%)	0.64	4.29	−0.65	4.02	10.90	−16.79	0.04
DP	−4.01	0.20	−0.09	4.18	−3.28	−4.52	0.98
DY	−4.01	0.20	−0.21	4.03	−3.29	−4.53	0.98
EP	−3.15	0.37	−2.14	9.39	−2.57	−4.84	0.98
DE	−0.86	0.42	3.36	16.03	1.38	−1.24	0.98
RVOL (ann.)	0.14	0.06	0.58	2.87	0.32	0.05	0.97
BM	0.27	0.07	−0.20	2.24	0.44	0.12	0.96
NTIS	0.00	0.02	−0.67	2.92	0.03	−0.06	0.98
TBL (%, ann.)	2.18	2.05	0.44	1.60	6.17	0.01	0.99
LTY (%, ann.)	4.40	1.44	0.03	1.91	7.26	1.75	0.99
LTR (%)	0.58	3.00	0.08	5.23	14.43	−11.24	0.00
TMS (%, ann.)	2.22	1.29	−0.03	1.96	4.53	−0.41	0.98
DFY (%, ann.)	0.99	0.42	3.04	15.18	3.38	0.55	0.96
DFR (%)	0.03	1.78	−0.45	8.90	7.37	−9.75	0.02
INFL (%)	0.18	0.35	−0.90	7.59	1.22	−1.92	0.48
			Option-implied bounds (%)				
Martin (2017)							
$LBM_{t\to t+1m}$	0.35	0.30	3.16	17.13	2.39	0.06	0.81
$LBM_{t\to t+3m}$	1.05	0.76	2.72	13.64	5.72	0.30	0.86
$LBM_{t\to t+6m}$	2.10	1.29	2.31	10.72	9.41	0.65	0.89
$LBM_{t\to t+12m}$	4.19	2.21	1.94	8.51	15.60	0.57	0.91
Chabi-Yo and Loudis (2020)							
$LBCL_{t\to t+1m}$	0.40	0.37	3.55	20.94	3.16	0.07	0.80
$LBCL_{t\to t+3m}$	1.29	1.03	3.17	17.18	8.01	0.34	0.84
$LBCL_{t\to t+6m}$	2.78	1.92	2.70	13.14	14.24	0.76	0.87
$LBCL_{t\to t+12m}$	6.06	3.70	2.37	10.80	26.73	1.62	0.90
$UBCL_{t\to t+1m}$	0.75	0.74	2.97	14.86	5.39	0.09	0.80
$UBCL_{t\to t+3m}$	2.39	1.62	2.11	9.36	11.06	0.35	0.86
$UBCL_{t\to t+6m}$	4.61	2.39	1.86	8.16	16.73	1.37	0.89
$UBCL_{t\to t+12m}$	8.43	3.61	1.70	7.23	25.76	3.59	0.91

Table 2: **Monthly return predictability:** R^2_{OOS}

This table presents the OOS forecasting results at the monthly horizon for the constrained and unconstrained predictive regression models. UC denotes the unconstrained predictive regression, that is, forecasts generated by Eq. (14); LBM denotes the predictive regression constrained by Martin's (2017) lower bound on the market excess return, that is, forecasts generated by Eq. (16); $LBCL$ denotes the predictive regression constrained by CYL's lower bound on the market excess return, that is, forecasts generated by Eq. (17); $CT0$ denotes the constrained predictive regression with Campbell and Thompson (2008) non-negativity restriction on equity premium forecasts, i.e. forecasts generated by Eq. (15). We use the R^2_{OOS} statistic (in percent) calculated by Eq. (20) to evaluate the forecasting performance, where the statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. The heading % bound denotes the percentage of unconstrained forecasts truncated by the corresponding lower bound. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3	(4)	(5)	(6)	(7)	(8)
	UC	LBM		$LBCL$		$CT0$	
Variable	R^2_{OOS} (%)	R^2_{OOS}	% bound	R^2_{OOS}	% bound	R^2_{OOS}	% bound
DP	−0.74	−0.59	10.9	−1.03	12.2	−0.44	0.9
DY	0.30	0.22	10.9	−0.12	12.2	0.29	1.8
EP	−3.12	−0.85	24.9	−1.42	27.6	0.36**	13.6
DE	−6.43	−2.81	24.4	−3.38	27.1	−1.57	11.3
RVOL	−0.37	−0.70	20.4	−1.18	24.4	−0.20	0.9
BM	−3.46	−2.98	18.1	−3.37	20.4	−2.95	7.7
NTIS	−1.74	−1.30	61.1	−1.57	63.3	−0.87	42.5
TBL	−4.15	−2.75	26.2	−3.22	28.1	−2.49	13.6
LTY	−1.39	0.71	29.9	0.17	30.8	0.82*	14.5
LTR	−1.06	−0.30	24.0	−0.76	27.1	−0.54	3.6
TMS	−2.58	−2.55	24.4	−3.11	26.2	−1.91	9.0
DFY	−3.54	−0.18	32.6	−0.59	36.2	0.46	17.6
DFR	−4.63	−3.87	19.5	−4.15	22.6	−3.94	5.9
INFL	−0.95	−1.93	37.1	−2.33	38.9	−1.29	13.1
$Econ^{Comb}$	−0.12	0.10	23.1	−0.36	25.8	0.59	4.5
Mean	−2.27	−1.32	25.8	−1.76	28.2	−0.91	10.7

Table 3: **Return predictability at longer horizons:** R^2_{OOS}

This table presents the OOS forecasting results at the h-month horizon for the constrained and unconstrained predictive regression models. UC denotes the unconstrained predictive regression, that is, forecasts generated by Eq. (14); LBM denotes the predictive regression constrained by Martin's (2017) lower bound on the market excess return, that is, forecasts generated by Eq. (16); $LBCL$ denotes the predictive regression constrained by CYL's lower bound on the market excess return, that is, forecasts generated by Eq. (17); $CT0$ denotes the constrained predictive regression with Campbell and Thompson (2008) non-negativity restriction on equity premium forecasts, that is, forecasts generated by Eq. (15). We use the R^2_{OOS} statistic calculated by Eq. (20) to evaluate the forecasting performance, where the statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. To adjust for the serial correlation, we use 2^*h-1 as the number of Newey–West lags when calculating the Clark and West (2007) t statistic, following Bekaert and Hoerova (2014). The mean (% bound) denotes the average percentage of unconstrained forecasts truncated by the corresponding lower bound. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Panel A: $h=3$				Panel B: $h=6$				Panel C: $h=12$			
Variable	UC	LBM	$LBCL$	$CT0$	UC	LBM	$LBCL$	$CT0$	UC	LBM	$LBCL$	$CT0$
DP	−4.75	−3.76	−3.90	−4.77	−3.38	−4.07	−3.83	−3.38	−6.06*	−6.49*	−7.15*	−6.06*
DY	−1.78	−1.28	−1.97	−1.78	0.61	−0.21	−0.14	0.61	0.73*	−0.34*	−0.82*	0.73*
EP	−15.94	−1.13*	−1.79	−1.26*	−31.47	0.33**	2.16**	−7.21	−44.29	−9.52	−4.39*	−19.44
DE	−38.29	−11.31	−11.91	−12.50	−95.11	−48.21	−45.94	−58.05	−157.72	−151.96	−146.53	−157.57
RVOL	−1.97	0.17	−0.16	−1.47	−4.93	3.77	6.54*	−3.04	−15.19	0.06	6.96*	−10.14
BM	−9.60	−10.85	−11.72	−10.94	−12.22**	−15.57	−14.92	−15.41	−36.17**	−44.34**	−43.84**	−41.97**
NTIS	−7.90	−1.15	−1.54	−1.87	−15.68	5.64**	7.52**	1.68*	−29.30	19.81**	23.18***	13.04*
TBL	−19.39	−10.92	−11.37	−12.90	−48.87	−26.95	−24.39	−35.80	−150.28	−129.59	−124.03	−138.99
LTY	−4.24	3.94*	3.39	1.78*	−11.16	11.61***	13.31***	4.16*	−28.88	16.07***	20.20***	0.34
LTR	−1.38	1.04	0.72	−1.05	−0.41	6.49**	9.28**	0.08	−0.83	6.05*	12.37*	−0.83
TMS	−14.23	−10.04	−10.32	−12.32	−37.89	−27.46	−24.31	−35.92	−78.93	−75.44	−70.14	−78.93
DFY	−24.19	2.45	1.92	2.64	−79.37	10.05***	11.72**	6.85*	−73.74	15.25**	18.51**	6.80
DFR	−5.20	−1.97	−2.13	−3.76	−3.71	3.48	5.98*	−2.18	−4.04	5.13	11.12*	−2.62
INFL	−3.78	0.65	0.45	−3.22	0.19	6.22**	8.45**	0.37*	−1.63	5.04	10.33*	−1.55
$Econ^{Comb}$	−2.69	2.01	1.68	0.87	−5.15	8.10**	10.30**	0.79	−1.65	4.77	9.56*	−1.64
Mean (R^2_{OOS})	−10.36	−2.81	−3.24	−4.17	−23.24	−4.45	−2.55	−9.76	−41.87	−23.03	−18.98	−29.25
Mean (% bound)	-	24.78	28.19	10.14	-	24.14	30.22	10.12	-	22.32	31.78	8.54

Table 4: **Economic value of predictability: CER gains**

This table presents the OOS asset allocation performance measured by the annualized CER gain (in percent) for a mean–variance investor. Each period, the investor optimally allocates a portion $\omega_t = \frac{1}{\gamma}\frac{\hat{r}_{t+1|t}}{\hat{\sigma}^2_{t+1|t}}$ of her wealth to the market index and the remainder of the wealth to the risk-free asset, where γ is the risk aversion coefficient set to three, $\hat{r}_{t+1|t}$ is the forecast of $t+1$ market index excess return conditional on either a constrained predictive regression or an unconstrained predictive regression at time t, and $\hat{\sigma}^2_{t+1|t}$ is the forecast of $t+1$ market index excess return variance made by a simple univariate $AR(1)$ model at time t. The optimal weight ω_t is restricted to the range between zero and 1.5. The forecast horizon h coincides with the investor's rebalancing frequency. The annualized CER gain (in percent) reported in columns (2), (5), and (8) is defined as the difference between the CER for the investor using an unconstrained predictive regression (UC) excess return forecast based on the predictors in the first column and the CER when the investor uses the historical mean benchmark forecast. The difference between two CER gains reported in the columns (3), (4), (6), (7), (9), and (10), is defined as the difference between the CER gain for the investor using the predictive regression forecast constrained by Martin (2017) (CYL) lower bound on the market excess return, LBM ($LBCL$), and the CER gain when she uses an unconstrained predictive regression forecast. The statistical significance of the difference between two CER gains is determined by the upper-tail p-value for the Diebold and Mariano (1995) statistic. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Panel A: $h=1$ $\Delta CER\ gain(\%)$			Panel B: $h=3$ $\Delta CER\ gain(\%)$			Panel C: $h=6$ $\Delta CER\ gain(\%)$		
Variable	UC	$LBM-UC$	$LBCL-UC$	UC	$LBM-UC$	$LBCL-UC$	UC	$LBM-UC$	$LBCL-UC$
$\gamma=3$									
DP	3.63	−0.36	−0.39	1.94	−0.35	−0.40	0.67	−0.19	−0.41
DY	4.11	−0.37	−0.38	2.82	−0.52	−0.76	0.90	−0.28	−0.43
EP	3.49	−0.03	−0.05	2.50	−0.07	−0.11	1.33	0.38	0.54
DE	1.27	0.02	0.03	−0.07	0.14	0.14	−1.76	1.16**	1.46*
RVOL	−0.48	0.66	0.80	−0.88	0.60	0.74	−0.28	0.69*	1.00*
BM	1.20	0.48	0.60	2.56	−0.22	−0.22	5.07	−0.53	−0.54
NTIS	−2.90	2.05**	2.48**	−0.66	0.84	0.97	2.10	−0.18	−0.19
TBL	−0.72	1.30	1.42	−0.68	1.11**	1.20*	−0.81	1.05**	1.28*
LTY	1.37	1.24	1.25	1.83	1.01*	1.01	2.45	0.36	0.37
LTR	−0.40	1.25*	1.42*	0.22	0.36	0.41	−0.55	1.04**	1.42**
TMS	−0.04	0.67	0.68	−1.45	0.80*	0.95	−1.51	1.03**	1.49**
DFY	0.27	0.90	1.12	1.35	0.00	0.08	2.40	−0.51	−0.48
DFR	−1.19	1.24*	1.46*	−0.84	0.62	0.75	−0.55	1.00**	1.55**
INFL	−2.96	1.74**	2.00**	0.09	0.19	0.19	0.08	0.19*	0.32
$Econ^{Comb}$	1.08	0.92	1.07	1.00	0.49	0.64	1.03	0.91**	1.11*
Mean	0.52	0.78	0.90	0.65	0.33	0.37	0.71	0.41	0.57

Table 5: **Economic value of predictability: Sharpe ratios**

This table presents the OOS asset allocation performance measured by the annualized Sharpe ratio for a mean–variance investor who uses a constrained predictive regression excess return forecast or an unconstrained one or the historical average benchmark forecast (HAV) to guide allocations between the market index and risk-free assets. UC denotes the case in which the investor uses the unconstrained predictive regression, that is, forecasts generated by Eq. (14); LBM denotes the case in which the investor uses the predictive regression constrained by Martin's (2017) lower bound on the market excess return; and $LBCL$ denotes the case in which the investor uses the predictive regression constrained by CYL's lower bound on the market excess return. The risk aversion coefficient of the investor is set at three and the market index weight is restricted to range between zero and 1.5. The forecast horizon h coincides with the investor's rebalancing frequency. The annualized SR value reported in the second through seventh columns is defined as the average excess return of a portfolio divided by the standard deviation of the portfolio returns. The statistical significance of whether the SR of the constrained model (LBM and $LBCL$) is higher than the unconstrained model (UC) is assessed by the test of Jobson and Korkie (1981) with the correction of Memmel (2003). The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Panel A: $h = 1$			Panel B: $h = 3$			Panel C: $h = 6$		
Variable	UC	LBM	$LBCL$	UC	LBM	$LBCL$	UC	LBM	$LBCL$
HAV	0.47	-	-	0.45	-	-	0.32	-	-
DP	0.71	0.67	0.67	0.56	0.54	0.53	0.40	0.39	0.38
DY	0.74	0.71	0.70	0.62	0.58	0.57	0.41	0.40	0.39
EP	0.79	0.75	0.73	0.71	0.67	0.66	0.45	0.48	0.48
DE	0.58	0.56	0.56	0.44	0.45	0.44	0.19	0.28**	0.31*
RVOL	0.43	0.48	0.49	0.37	0.42	0.43	0.29	0.36*	0.39*
BM	0.55	0.58	0.58	0.62	0.60	0.60	0.67	0.63	0.63
NTIS	0.19	0.40**	0.43**	0.44	0.50	0.50	0.59	0.52	0.50
TBL	0.40	0.50	0.51	0.39	0.47*	0.48*	0.27	0.35**	0.36*
LTY	0.59	0.67	0.66	0.64	0.72	0.70	0.54	0.56	0.55
LTR	0.43	0.53*	0.54	0.47	0.50	0.50	0.27	0.36**	0.39**
TMS	0.46	0.51	0.51	0.32	0.39*	0.40	0.21	0.29**	0.32**
DFY	0.51	0.57	0.58	0.63	0.59	0.59	0.59	0.50	0.49
DFR	0.36	0.46*	0.48*	0.37	0.42	0.43	0.27	0.36**	0.41**
INFL	0.21	0.36**	0.38**	0.46	0.47	0.47	0.34	0.35	0.36
$Econ^{Comb}$	0.57	0.63	0.63	0.53	0.57	0.58	0.41	0.48**	0.49*
Mean	0.50	0.56	0.56	0.50	0.53	0.52	0.39	0.42	0.43

Table 6: **Forecasting performance of LBM and $LBCL$**

This table presents the OOS forecasting results at the h-month horizon when directly using the option-implied lower bounds derived by Martin (2017) and CYL, respectively, to forecast the expected market premium, that is, $\hat{r}_{t\to t+h|t} = LBM_{t\to t+h|t}$ or $LBCL_{t\to t+h|t}$. Panel A reports the R^2_{OOS} statistic calculated by Eq. (20), where the statistical significance is determined by the upper-tail p-value for the CW statistic. Panel B reports the annualized CER gain (in percent) and the annualized Sharpe ratio for a mean–variance investor with a risk aversion coefficient of three who uses LBM ($LBCL$) to forecast the excess returns on the market index to guide asset allocation. The forecast horizon h coincides with the investor's rebalancing frequency. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Panel A: $R^2_{OOS}(\%)$							
Variable	$h=1$		$h=3$			$h=6$		$h=12$
LBM	0.13		1.95			7.94**		14.35*
$LBCL$	−0.29		1.68			10.38**		20.19**
	Panel B: Asset allocation ($\gamma=3$)							
	$h=1$			$h=3$			$h=6$	
	CER gain(%)	*SR*		*CER gain*	*SR*		*CER gain*	*SR*
LBM	−0.35	0.53		−0.89	0.46		0.01	0.38
$LBCL$	−0.04	0.53		−0.49	0.47		0.61	0.41

Table 7: **Prediction of the gap between $LBCL$ and the realized return**

This table reports the in-sample estimation results of the multivariate predictive regression for the bound gap. Specifically, the gap between the bound and the realized excess return (in percent) is regressed on the 11 predictors of Welch and Goyal (2008):

$$r_{t\to t+h} - LBCL_{t\to t+h} = \alpha + \beta' X_t + \epsilon_{t\to t+h},\ t = 1, ..., N-h \text{ and } h = 1, 3, 6, 12,$$

where X_t denotes the 11 standardized GW predictors (excluding DY, DE, and TMS, for collinearity), $r_{t\to t+h}$ is the realized excess return, and $LBCL_{t\to t+h}$ is the CYL's lower bound. The t-statistic of the $\hat{\beta}$ estimate for testing $H_0 : \beta = 0$ against $H_A : \beta \neq 0$ is obtained, with heteroskedasticity- and autocorrelation-consistent standard errors based on Newey and West (1987). We use $2^*h - 1$ as the number of Newey–West lags. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively, according to two-sided p-values. The sample period is from January 1996 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$h = 1$		$h = 3$		$h = 6$		$h = 12$	
	$\hat{\beta}$	t-Stat	$\hat{\beta}$	t-Stat	$\hat{\beta}$	t-Stat	$\hat{\beta}$	t-Stat
DP	1.98***	4.89	5.16***	5.42	8.55***	5.03	16.28***	9.41
EP	0.39	0.86	−0.45	−0.47	−1.91	−1.42	0.85	0.42
RVOL	0.80**	2.20	1.99**	2.46	2.13	1.47	1.23	0.51
BM	−0.76	−1.27	−0.25	−0.23	1.43	1.06	−2.16	−0.90
NTIS	1.29**	2.30	4.00***	2.98	7.83***	3.71	14.04***	3.73
TBL	1.23**	2.24	4.35***	2.63	7.88**	2.55	8.13**	2.15
LTY	−1.86***	−2.72	−5.36***	−2.81	−8.66***	−2.63	−10.65**	−2.38
LTR	0.32	0.86	−0.71	−1.32	−0.89*	−1.77	−1.42**	−1.97
DFY	−0.82	−1.37	−2.67***	−2.75	−3.50**	−2.41	−1.78	−0.53
DFR	0.59	1.31	0.13	0.21	0.31	0.55	0.77	0.93
INFL	0.58**	2.02	0.10	0.19	−0.57	−0.88	−0.01	−0.01
Adj. R^2(%)	11.64		29.72		49.63		60.66	

Table 8: **Out-of-sample R^2 over good and bad economic times**

This table presents the monthly OOS forecasting results for the constrained and unconstrained predictive regression models over good and bad times of the economy. *UC* denotes the unconstrained predictive regression, that is, forecasts generated by Eq. (14); *LBM* denotes the predictive regression constrained by Martin's (2017) lower bound on the market excess return; *LBCL* denotes the predictive regression constrained by CYL's lower bound on the market excess return; and *CT*0 denotes the constrained predictive regression with Campbell and Thompson (2008) non-negativity restriction on equity premium forecasts. The row labeled *Bound* reports the results for directly using the option-implied lower bounds to forecast the expected market premium. We use the R^2_{OOS} statistic calculated by the following equation to evaluate the forecasting performance over good and bad times:

$$R^2_{OOS,c} = 1 - \frac{\sum_{t=n_1}^{N-1} I^c_{t+1}(r_{t+1} - \hat{r}^{Model}_{t+1|t})^2}{\sum_{t=n_1}^{N-1} I^c_{t+1}(r_{t+1} - \hat{r}^{Bench}_{t+1|t})^2}, \text{ for } c = GOOD,\ BAD,$$

where I^{GOOD}_{t+1} (I^{BAD}_{t+1}) is set equal to one whenever the CFNAI is positive (negative) in month $t+1$, and zero otherwise. The statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Panel A: Bad times				Panel B: Good times			
Variable	*UC*	*LBM*	*LBCL*	*CT*0	*UC*	*LBM*	*LBCL*	*CT*0
DP	−3.04	−3.22	−4.08	−2.63	6.05***	7.18***	7.97***	6.05***
DY	−2.00	−2.55	−3.28	−2.01	7.07***	8.40***	9.18***	7.07***
EP	−2.24	−0.78	−1.89	2.35***	−5.74	−1.06*	−0.03*	−5.49
DE	−7.24	−4.39	−5.54	−0.95	−4.04	1.85	3.00	−3.40
RVOL	−0.49	−2.07	−3.04	−0.26	−0.02	3.33*	4.30*	−0.02
BM	−0.96	−2.13	−3.01	−0.66	−10.83	−5.48	−4.42	−9.70
NTIS	−0.07	−2.25	−3.04	−0.04	−6.65	1.50	2.77	−3.32
TBL	−3.80	−5.05	−6.06	−2.52	−5.19	4.05**	5.18**	−2.42
LTY	−0.62	−0.85	−1.95	1.66**	−3.66	5.29***	6.44**	−1.65
LTR	−1.04	−1.51	−2.54	−0.45	−1.12	3.26*	4.51*	−0.81
TMS	−1.82	−4.02	−5.17	−1.14	−4.84	1.77	2.95	−4.17
DFY	−2.80	−1.84	−2.80	1.12*	−5.73	4.74**	5.90**	−1.49
DFR	−5.86	−6.35	−7.08	−5.02	−1.00	3.47*	4.50*	−0.73
INFL	−0.22	−3.61	−4.54	−1.12	−3.10	3.04*	4.20*	−1.80
$Econ^{Comb}$	0.18	−1.03	−2.08	1.13**	−0.99	3.41*	4.72**	−0.99
Mean	−2.14	−2.78	−3.74	−0.70	−2.65	2.98	4.08	−1.52
Bound	-	−0.67	−1.68	-	-	2.49*	3.82*	-

Table 9: **Return predictability under slope restriction: R^2_{OOS}**

This table presents the OOS forecasting results at the h-month horizon for the constrained predictive regression models. The term $LBM + slp.$ denotes the constrained predictive regression that first introduces the slope constraint following Campbell and Thompson (2008) and then bounds the forecasts from below using Martin's (2017) lower bound on the market premium; $LBCL + slp.$ denotes the constrained predictive regression that first introduces the slope constraint and then bounds the forecasts from below with CYL's lower bound on the market premium; and $CT0 + slp.$ denotes the constrained predictive regression with both slope and non-negativity restriction, as in Campbell and Thompson (2008). We use the R^2_{OOS} statistic calculated by Eq. (20) to evaluate the forecasting performance, where the statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. We use $2^{*}h - 1$ as the number of Newey-West lags. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Panel A: $h = 1$			Panel B: $h = 3$		
Variable	$LBM + slp.$	$LBCL + slp.$	$CT0 + slp.$	$LBM + slp.$	$LBCL + slp.$	$CT0 + slp.$
DP	−0.59	−1.03	−0.40	−3.76	−3.90	−4.77
DY	0.22	−0.12	0.32	−1.28	−1.97	−1.78
EP	−0.85	−1.42	0.43**	−1.19*	−1.84	−0.60**
DE	−2.86	−3.42	−2.26	−11.36	−11.82	−13.43
RVOL	−0.64	−1.12	0.02	0.32	−0.02	−0.42
BM	−2.98	−3.37	−2.81	−10.85	−11.72	−10.94
NTIS	−0.36	−0.86	0.00	1.18	0.96	0.00
TBL	−2.80	−3.27	−2.44	−11.10	−11.39	−12.26
LTY	0.07	−0.42	0.43*	2.19	1.92	1.04**
LTR	−0.17	−0.62	0.14	1.20	0.95	−0.59
TMS	−2.41	−2.99	−1.68	−9.80	−10.06	−10.81
DFY	−0.42	−0.91	−0.06	1.18	0.96	0.00
DFR	−3.72	−4.04	−3.71	1.17	0.86	−0.44
INFL	−0.80	−1.27	−0.50	0.66	0.48	−3.14
$Econ^{Comb}$	−0.08	−0.53	0.12	0.29	0.01	−1.37
Mean	−1.23	−1.69	−0.83	−2.74	−3.11	−3.97

Table 10: **Return predictability with lower and upper constraints: R^2_{OOS}**

This table presents the OOS forecasting results at the h-month horizon for the constrained and unconstrained predictive regression models. UC denotes the unconstrained predictive regression, that is, forecasts generated by Eq. (14); $LB+UB$ denotes the predictive regression constrained by CYL's lower and upper bounds on the market excess return, that is, forecasts generated by Eq. (18). We use the R^2_{OOS} statistic calculated by Eq. (20) to evaluate the forecasting performance, where the statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$h=1$		$h=3$		$h=6$		$h=12$	
Variable	UC	$LB+UB$	UC	$LB+UB$	UC	$LB+UB$	UC	$LB+UB$
DP	−0.74	−0.09	−4.75	0.30	−3.38	8.28*	−6.06*	20.96**
DY	0.30	0.52	−1.78	1.33	0.61	9.32*	0.73*	21.19**
EP	−3.12	0.34	−15.94	2.73	−31.47	11.09**	−44.29	19.59**
DE	−6.43	−0.11	−38.29	1.48	−95.11	9.32**	−157.72	17.73**
RVOL	−0.37	−0.62	−1.97	0.70	−4.93	7.86*	−15.19	16.83**
BM	−3.46	−0.54	−9.60	1.43	−12.22**	11.94**	−36.17**	21.20**
NTIS	−1.74	−0.98	−7.90	0.84	−15.68	10.52**	−29.30	22.27**
TBL	−4.15	−0.47	−19.39	1.45	−48.87	9.50**	−150.28	19.29**
LTY	−1.39	0.29	−4.24	2.87	−11.16	12.43**	−28.88	21.64**
LTR	−1.06	−0.17	−1.38	0.89	−0.41	9.66**	−0.83	18.36**
TMS	−2.58	−0.24	−14.23	0.82	−37.89	8.39**	−78.93	18.59**
DFY	−3.54	−0.34	−24.19	2.04	−79.37	12.11**	−73.74	20.68**
DFR	−4.63	−1.27	−5.20	−0.66	−3.71	8.01*	−4.04	17.71**
INFL	−0.95	−0.77	−3.78	1.85	0.19	10.87**	−1.63	19.68**
$Econ^{Comb}$	−0.12	−0.23	−2.69	1.48	−5.15	10.26**	−1.65	19.54**
Mean	−2.27	−0.31	−10.36	1.30	−23.24	9.97	−41.87	19.68

Table 11: **Combining with the option-based predictors:** R^2_{OOS}

This table presents the OOS forecasting results at the h-month horizon for the constrained and unconstrained predictive regression models. The term UC denotes the unconstrained predictive regression, LBM denotes the predictive regression constrained by Martin's (2017) lower bound on the market excess return, and $LBCL$ denotes the predictive regression constrained by CYL's lower bound on the market excess return. We use the R^2_{OOS} statistic calculated by Eq. (20) to evaluate the forecasting performance, where the statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. We use 2^*h-1 as the number of Newey–West lags. The mean (% bound) denotes the average percentage of unconstrained forecasts truncated by the corresponding lower bound. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Panel A: $h=1$			Panel B: $h=3$		
	UC	LBM	$LBCL$	UC	LBM	$LBCL$
VRP	4.50**	1.28	0.53	6.73**	5.26**	3.48*
VIX^2	−2.91	−1.12	−1.43	−15.69	−7.01	−5.48
IVS	−3.30	−1.98	−2.67	−0.66*	−4.57	−6.08
FVF	−0.16	0.92	0.36	−6.84	0.99	1.09
FSF	−0.78	0.46	0.36	−6.58	0.84	0.85
$Option^{Comb}$	2.69**	0.57	−0.07	7.33**	2.92	2.07
Mean (R^2_{OOS})	0.01	0.02	−0.49	−2.62	−0.26	−0.68
Mean (% bound)	-	38.61	41.40	-	41.17	45.51
	Panel C: $h=6$			Panel D: $h=12$		
	UC	LBM	$LBCL$	UC	LBM	$LBCL$
VRP	3.29**	9.62**	9.86**	−0.61*	8.39*	12.05**
VIX^2	−2.17	1.84	5.43*	−4.92	3.99	10.79*
IVS	−10.91*	8.27**	8.63**	−21.89	9.41**	12.91**
FVF	−5.74	6.79**	9.16**	−6.89	7.04*	12.83**
FSF	−2.79	8.46**	10.66**	−3.78	7.94**	13.71**
$Option^{Comb}$	5.44*	8.45**	9.67**	−1.46	7.26*	12.85**
Mean (R^2_{OOS})	−2.15	7.24	8.90	−6.59	7.34	12.52
Mean (% bound)	-	33.72	42.82	-	26.43	38.81

Table 12: **Alternative risk aversion coefficient: CER gains**

This table presents the OOS asset allocation performance measured by the annualized CER gain (in percent) for a mean–variance investor with a risk aversion coefficient of five. The way to construct the market timing portfolio based on forecasts generated by constrained and unconstrained predictive regressions is described in Table 4. The annualized CER gain (in percent) reported in columns (2), (5), and (8) is defined as the difference between the CER for the investor using an unconstrained predictive regression (UC) excess return forecast based on the predictors in the first column and the CER when the investor uses the historical mean benchmark forecast. The difference between the two CER gains reported in columns (3), (4), (6), (7), (9), and (10) is defined as the difference between the CER gain for the investor using the predictive regression forecast constrained by the lower bound of Martin (2017) (CYL) on the market excess return, LBM ($LBCL$), and the CER gain when the investor uses an unconstrained predictive regression forecast. The statistical significance of the difference between the two CER gains is determined by the upper-tail p-value for the Diebold and Mariano (1995) statistic. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Panel A: $h = 1$ $\Delta CER\ gain(\%)$			Panel B: $h = 3$ $\Delta CER\ gain(\%)$			Panel C: $h = 6$ $\Delta CER\ gain(\%)$		
Variable	UC	$LBM - UC$	$LBCL - UC$	UC	$LBM - UC$	$LBCL - UC$	UC	$LBM - UC$	$LBCL - UC$
$\gamma = 5$									
DP	2.58	−0.21	−0.23	1.07	−0.21	−0.24	−0.98	−0.11	−0.25
DY	2.93	−0.22	−0.22	1.94	−0.32	−0.46	−0.32	−0.17	−0.26
EP	2.66	−0.02	−0.03	2.01	−0.03	−0.05	0.82	0.25	0.35
DE	−0.39	0.02	0.02	−1.91	0.09	0.09	−2.82	0.70**	0.88*
RVOL	−0.78	0.40	0.48	−0.89	0.36	0.44	0.01	0.42*	0.62*
BM	0.46	0.29	0.36	1.74	−0.13	−0.12	3.73	−0.33	−0.33
NTIS	−1.97	1.23**	1.49**	−0.51	0.51	0.59	1.09	−0.10	−0.10
TBL	−1.28	0.78	0.85	−1.26	0.67**	0.73*	−2.20	0.63**	0.77*
LTY	1.39	0.75	0.76	1.66	0.62*	0.62	1.70	0.23	0.24
LTR	−0.32	0.75*	0.85*	0.19	0.22	0.25	−0.32	0.64**	0.87**
TMS	−0.97	0.41	0.41	−2.58	0.48*	0.57	−2.56	0.61**	0.89**
DFY	0.05	0.54	0.67	0.85	0.00	0.05	1.57	−0.30	−0.28
DFR	−1.18	0.75*	0.88*	−0.84	0.38	0.45	−0.59	0.61**	0.94**
INFL	−2.10	1.05**	1.21**	−0.29	0.12	0.12	−0.46	0.12*	0.20
$Econ^{Comb}$	0.61	0.56	0.64	0.75	0.30	0.39	0.85	0.56**	0.69*
Mean	0.11	0.47	0.54	0.13	0.20	0.23	−0.03	0.25	0.35

Table 13: **Alternative sample training period:** R^2_{OOS}

This table presents the OOS forecasting results at the h-month horizon for the constrained and unconstrained predictive regression models. The first OOS forecast starts at 96 months after the sample beginning. The variable UC denotes the unconstrained predictive regression; LBM denotes the predictive regression constrained by Martin's (2017) lower bound on the market excess return; $LBCL$ denotes the predictive regression constrained by CYL's lower bound on the market excess return; and $CT0$ denotes the constrained predictive regression with Campbell and Thompson (2008) non-negativity restriction on equity premium forecasts. We use the R^2_{OOS} statistic calculated by Eq. (20) to evaluate the forecasting performance, where the statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. We use 2^*h-1 as the number of Newey-West lags. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2004 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Panel A: $h=1$				Panel B: $h=3$			
Variable	UC	LBM	$LBCL$	$CT0$	UC	LBM	$LBCL$	$CT0$
DP	−3.57	−2.78	−3.35	−3.16	−12.38	−9.86	−9.84	−12.38
DY	−2.34	−2.00	−2.52	−2.24	−8.53	−7.02	−7.84	−8.53
EP	−8.43	−3.45	−4.16	−2.44	−30.56	−6.43	−7.15	−7.83
DE	−5.71	−0.99	−1.70	0.58	−33.20	1.51	0.53	1.11
RVOL	−0.36	−1.41	−2.10	−0.12	−1.86	−0.85	−1.52	−1.65
BM	−1.24	−1.04	−1.67	−0.78	−2.32	−1.39	−2.43	−2.32
NTIS	−1.57	−1.68	−2.11	−1.09	−7.11	−1.21	−1.73	−2.03
TBL	−0.78	−0.64	−1.31	0.07	−3.01	1.90	1.12	0.69
LTY	−0.61	−0.32	−1.02	0.40	−2.51	2.53	1.72	1.07
LTR	−0.87	−0.94	−1.59	−0.77	−1.69	0.78	0.16	−1.25
TMS	−0.57	−1.03	−1.71	−0.26	−2.05	0.53	−0.06	−0.97
DFY	−4.04	−0.83	−1.43	0.45	−32.69	0.60	−0.19	1.74
DFR	−4.10	−3.58	−4.03	−3.15	−7.55	−3.25	−3.63	−5.39
INFL	−1.03	−2.63	−3.18	−1.77	−3.04	0.56	0.20	−3.04
$Econ^{Comb}$	−0.43	−0.83	−1.49	0.44	−3.81	1.46	0.74	1.05
Mean	−2.38	−1.61	−2.22	−0.92	−10.15	−1.34	−2.00	−2.65

	Panel C: $h=6$				Panel D: $h=12$			
	UC	LBM	$LBCL$	$CT0$	UC	LBM	$LBCL$	$CT0$
DP	−14.64	−14.64	−13.65	−14.64	−32.28	−32.28	−32.11	−32.28
DY	−10.19	−10.19	−9.52	−10.19	−23.99	−23.99	−23.16	−23.99
EP	−57.19	−6.10	−3.23	−17.45	−85.97	−28.90	−21.26	−43.90
DE	−52.61	7.78*	10.46*	−3.56	−0.04	7.69	14.84*	0.19
RVOL	−4.17	2.48	6.00	−4.17	−16.81	−7.50	1.53	−16.81
BM	0.60**	2.82**	4.43**	0.60**	−23.79**	−22.21**	−19.55**	−23.79**
NTIS	−19.89	5.12*	8.13*	−1.43	−61.01	8.98	15.62*	−4.86
TBL	−3.86	9.44**	12.36**	0.60	6.16**	14.46***	20.92**	6.17**
LTY	−5.44	10.59***	12.90**	2.21	−14.39	13.01**	18.37**	−4.55
LTR	−0.69	7.17*	10.53*	−0.24	−0.59	8.57*	16.33*	−0.59
TMS	−2.42	5.90	9.67*	−2.31	5.93**	9.85**	16.76**	5.93**
DFY	−102.18	7.79*	10.14*	4.37	−71.58	12.21*	16.22*	4.22
DFR	−5.78	2.24	5.48	−4.37	−4.27	8.27*	15.46*	−2.18
INFL	1.58***	8.69**	11.51**	1.79***	1.38**	10.43*	17.04*	1.51**
$Econ^{Comb}$	−6.18	9.23**	11.73**	1.74	3.31**	12.03**	18.40**	3.31**
Mean	−18.87	3.22	5.79	−3.14	−21.20	−0.63	5.03	−8.78

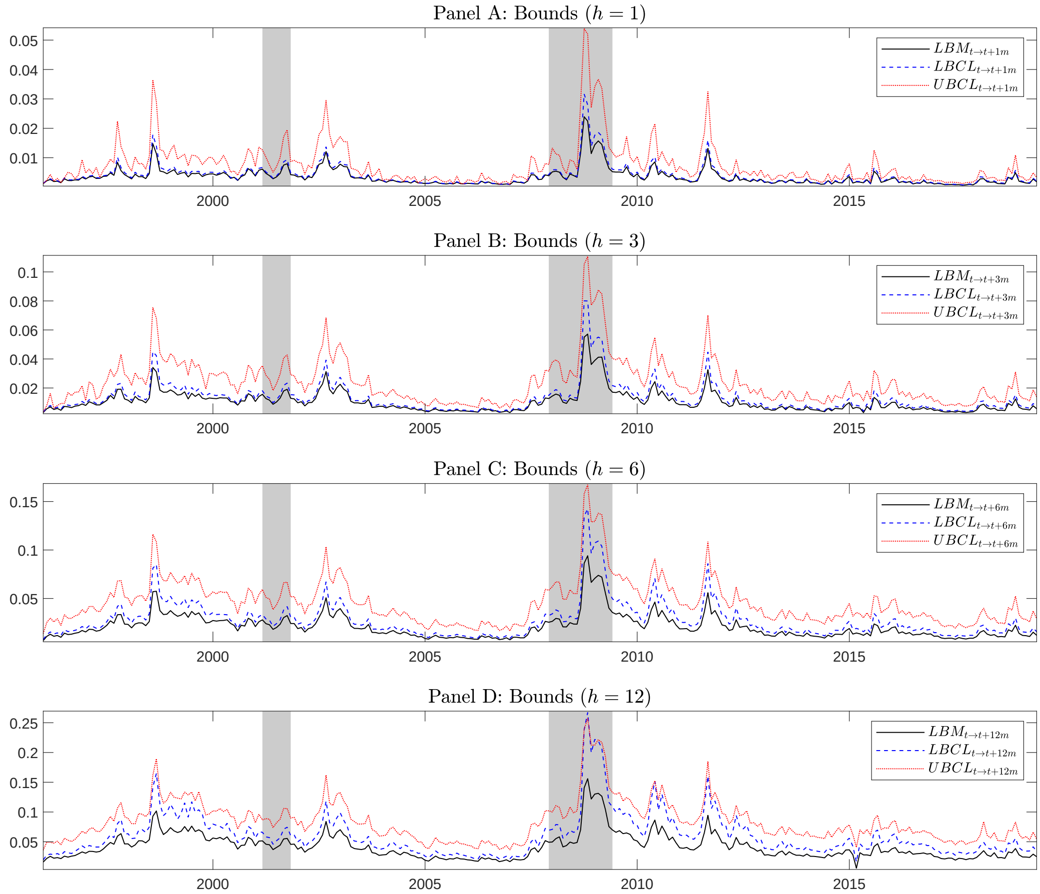

Figure 1: **Option-implied bounds**

Figure 1 plots the term structure of the option-implied bounds. The variable LBM is the lower bound on the expected market premium proposed byMartin (2017), and $LBCL$ and $UBCL$ are the lower and upper bounds, respectively, on the expected market premium proposed by CYL. All observations are taken from month-end data, and the sample period is from January 1996 to June 2019. The shaded area corresponds to the NBER recession period.

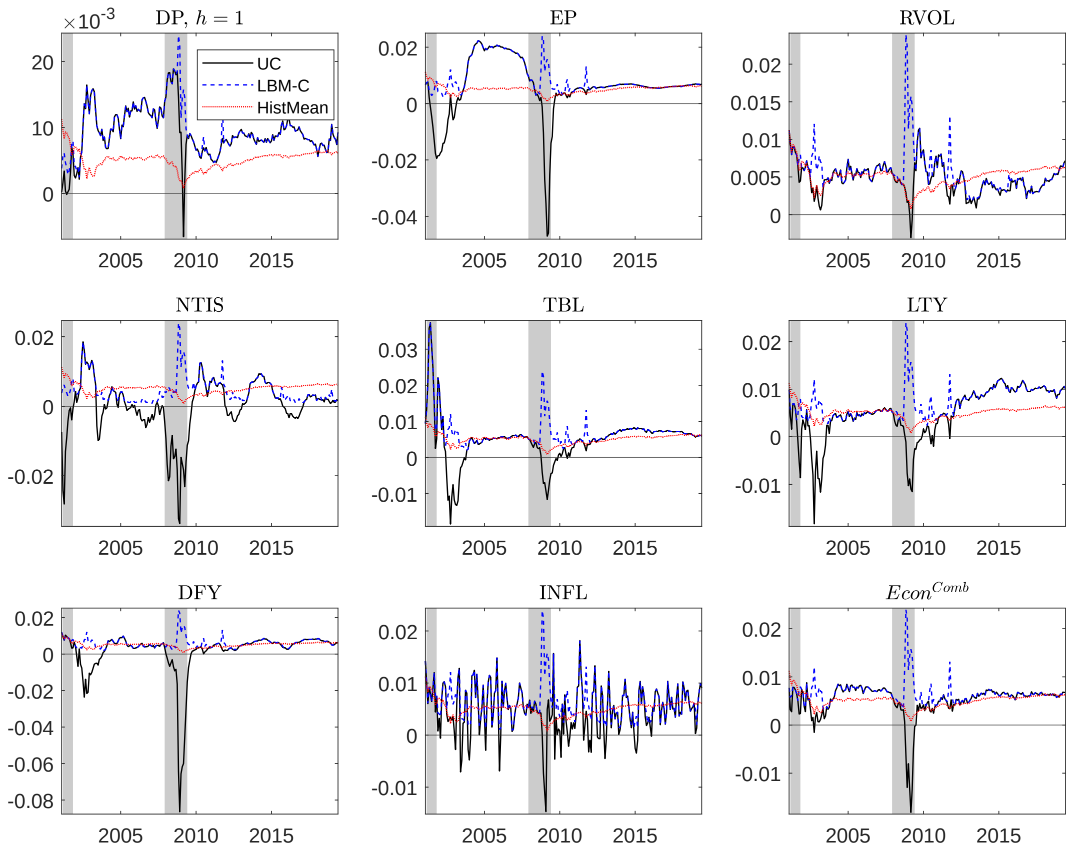

Figure 2: **Monthly OOS forecasts constrained by** LBM

Figure 2 plots the time series of the one-month-ahead ($h = 1$) OOS forecasts based on a set of representative predictors used in the analysis and the equal-weighted forecast combination. The variable UC denotes the unconstrained forecasts, $LBM - C$ denotes the forecasts constrained by Martin's (2017) lower bound on the market risk premium, and $HistMean$ denotes the historical mean benchmark forecasts. The OOS period is from February 2001 to June 2019. The shaded area corresponds to the NBER recession period.

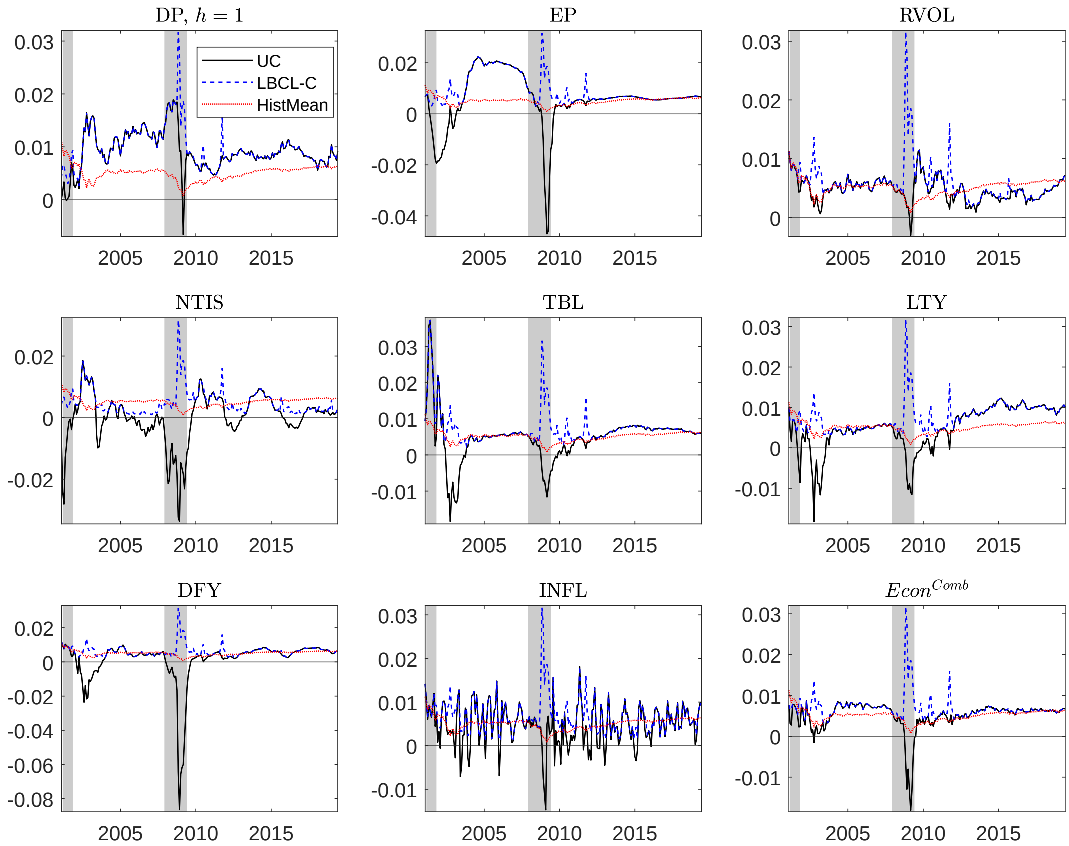

Figure 3: **Monthly OOS forecasts constrained by** $LBCL$

Figure 3 plots the time series of the one-month-ahead ($h = 1$) OOS forecasts based on a set of representative predictors used in the analysis and the equal-weighted forecast combination. The variable UC denotes the unconstrained forecasts, $LBCL - C$ denotes the forecasts constrained by CYL's lower bound on the market risk premium, and $HistMean$ denotes the historical mean benchmark forecasts. The OOS period is from February 2001 to June 2019. The shaded area corresponds to the NBER recession period.

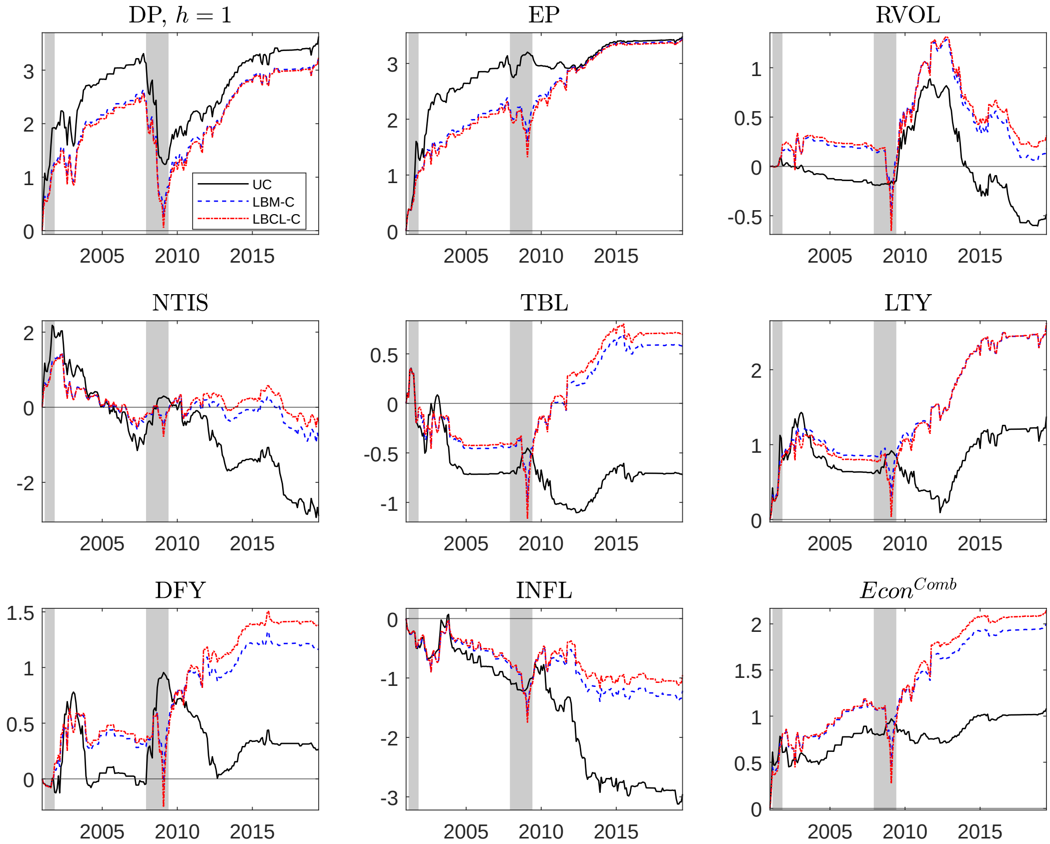

Figure 4: **Cumulative CER gains over monthly asset allocation**

Figure 4 depicts the cumulative CER gain (CCERG) for a mean–variance investor with a risk aversion coefficient of three over the OOS asset allocation period, based on constrained and unconstrained predictive regression forecasts, relative to the asset allocation strategy based on historical mean benchmark forecasts. The realized CER gain scaled by the length of the OOS period up to month t is defined as

$$CCERG_{model,t} = \frac{1}{N-n_1}\sum_{\tau=n_1+1}^{t}(CER_{model,\tau} - CER_{bench,\tau}),\ t = n_1+1, ..., N,$$

where *model* denotes either the constrained or the unconstrained predictive regression model, *bench* is the historical mean benchmark model, $N - n_1$ is the number of OOS observations, and $CER_{M,\tau}$ is the CER gain realized at time τ, which can be calculated as

$$CER_{M,\tau} = r_{p,\tau} - \frac{\gamma}{2}(r_{p,\tau} - \bar{\mu}_p)^2,\ \text{for } M = model \text{ or } bench$$

where $r_{p,\tau}$ is the portfolio return at time τ, γ is the investor's risk aversion coefficient and, $\bar{\mu}_p$ is the mean portfolio return over the OOS period. The variable UC denotes the unconstrained model, and $LBM-C$ and $LBCL-C$ denote the predictive regression forecasts constrained by the lower bounds of Martin (2017) and CYL on the market risk premium, respectively. The OOS period is monthly ($h = 1$), from February 2001 to June 2019. The shaded area corresponds to the NBER recession period.

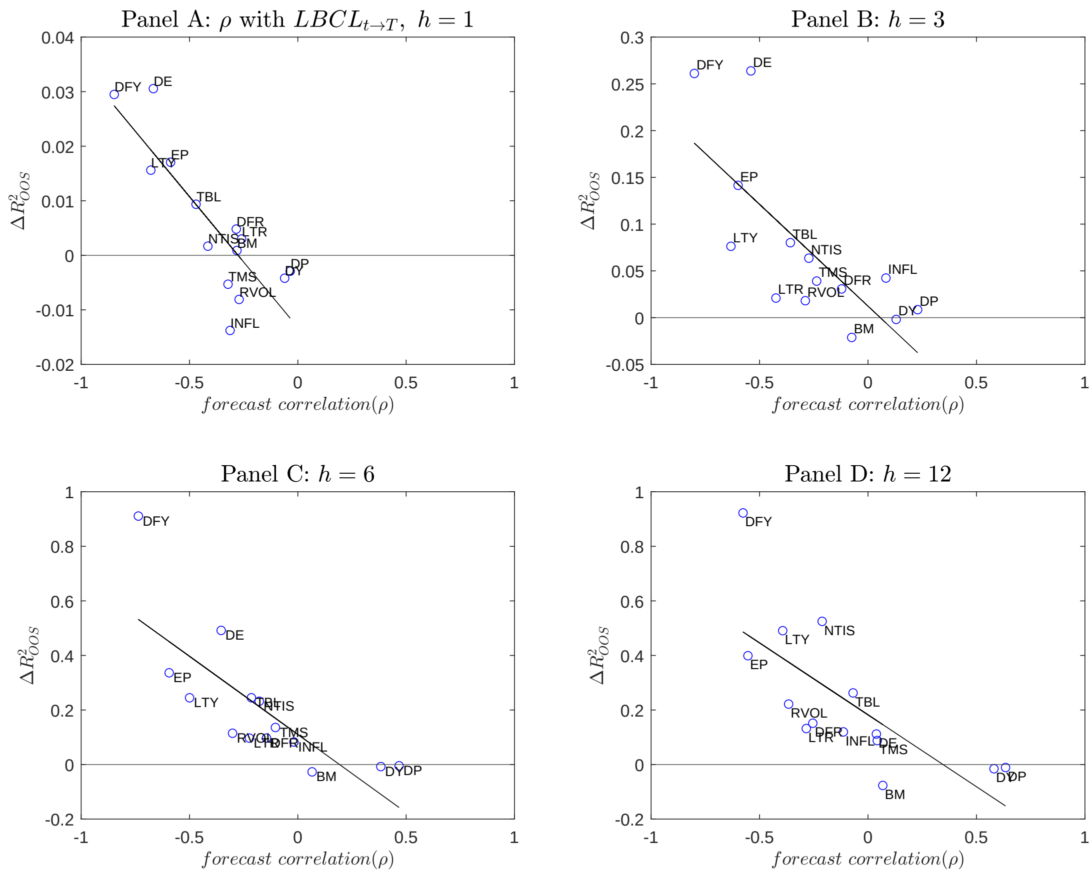

Figure 5: **Forecast correlation with** $LBCL$

Figure 5 plots the increment of R^2_{OOS} (vertical axis) against the correlation (horizontal axis) of the unconstrained predictive regression OOS forecasts, based on the 14 predictor variables from Welch and Goyal (2008) with CYL's lower bound on the market premium, $LBCL$. The straight line is the optimal fit obtained by the OLS method, and h denotes the forecast horizon: monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$), and annual ($h = 12$). The OOS period is from February 2001 to June 2019.

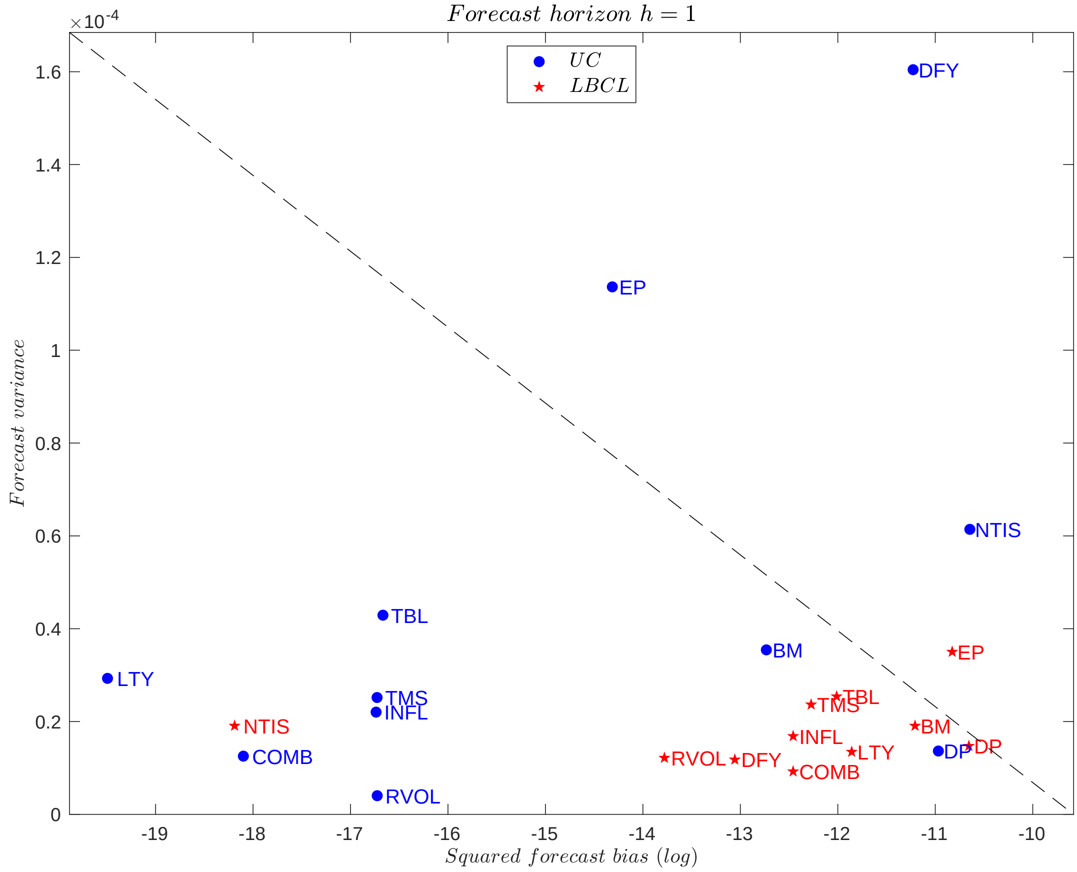

Figure 6: **Monthly forecast variances and squared forecast bias:** $LBCL$

Figure 6 plots the OOS forecast variance (vertical axis) against the logarithm of the squared forecast bias (horizontal axis) at the monthly forecast horizon ($h = 1$). The variable UC corresponds to the unconstrained predictive regression, and $LBCL$ denotes the constrained counterpart using CYL's (2020) lower bound on the market premium. The OOS period is from February 2001 to June 2019.

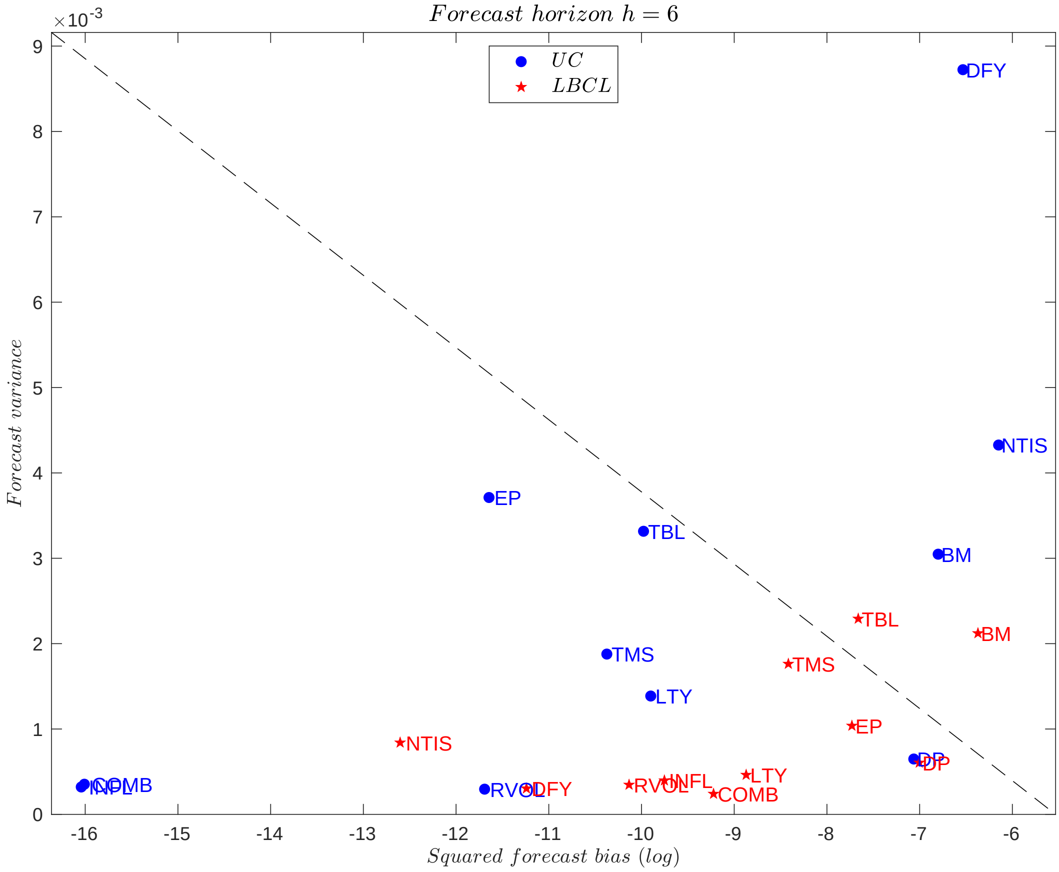

Figure 7: **Semi-annual forecast variances and squared forecast bias:** $LBCL$

Figure 7 plots the OOS forecast variance (vertical axis) against the logarithm of the squared forecast bias (horizontal axis) at the semi-annual forecast horizon ($h = 6$). The variable UC corresponds to the unconstrained predictive regression, and $LBCL$ denotes the constrained counterpart using CYL's lower bound on the market premium. The OOS period is from February 2001 to June 2019.

Appendices

The appendix presents the descriptions of 14 return predictors used in the empirical analyses, as well as the computing procedures of several identities used to construct the option-implied bounds on the market risk premium.

A Return predictors

- Dividend–price ratio (DP): Difference between the logarithm of the one-year moving sum of dividends paid on the S&P 500 index and the logarithm of stock prices (S&P 500 index).
- Dividend yield (DY): Difference between the logarithm of dividends and the logarithm of lagged stock prices.
- Earnings–price ratio (EP): Logarithm of earnings minus the logarithm of prices, where earnings are the 12-month moving sums of earnings on the S&P 500 index.
- Dividend payout ratio (DE): Difference between the logarithm of dividends and the logarithm of earnings.
- Stock variance (RVOL): 12-month moving standard deviation estimator of S&P 500 index returns (Mele, 2007)
- Book-to-market ratio (BM): Ratio of the book value to the market value for the Dow Jones Industrial Average.
- Net equity expansion (NTIS): Ratio of the 12-month moving sums of net issues by stocks listed on the New York Stock Exchange divided by these stocks' total end-of-year market capitalization.
- Treasury bill rate (TBL): Interest rate on a three-month Treasury bill of the secondary market.
- Long-term yield (LTY): Long-term government bond yield.
- Long-term rate of returns (LTR): Return on long-term government bonds.
- Term spread (TMS): Difference between LTY and the Treasury bill rate.
- Default yield spread (DFY): Difference between the yields of corporate bonds rates BAA and AAA.
- Default return spread (DFR): Difference between long-term corporate bond and long-term government bond returns.
- Inflation (INFL): Consumer Price Index (all urban consumers), which is released in the following month. We use the lagged one-month form for inflation, to take its release into account.

B Risk-neutral variance of the market return

The lower bound on the expected market excess return derived by Martin (2017) is fully characterized by the conditional risk-neutral market variance. Therefore, to compute the risk-neutral variance of the simple market return in Eq. (2), we follow the CBOE's procedures to find K_0, defined as the first strike below the forward index level, and we use the spanning formula of Carr and Madan (2001) and Bakshi, Kapadia, and Madan (2003):

$$Var_t^{\mathbb{Q}}[R_{M,t\to T}] = \frac{2R_{f,t\to T}}{S_t^2}\left\{\int_0^{K_0} P_{t,T}(K)dK + \int_{K_0}^{\infty} C_{t,T}(K)dK\right\} - (\frac{K_0}{S_t} - R_{f,t\to T})^2, \quad \text{(A.1)}$$

where S_t is the spot price of the market index, $R_{f,t\to T}$ is the gross risk-free rate, K_0 is the first strike below the forward index level, $C_{t,T}(K)$ is the price of the call option expiring at time T with strike K, and $P_{t,T}(K)$ is the price of the put option expiring at time T with strike K. Then, we utilize a discretized version of Eq. (A.1) to compute the risk-neutral variance:

$$Var_t^{\mathbb{Q}}[R_{M,t\to T}] \approx \frac{2R_{f,t\to T}}{S_t^2}\left\{\sum_{K_i\le K_0} P_{t,T}(K_i)\Delta I(K_i) + \sum_{K_i>K_0} C_{t,T}(K_i)\Delta I(K_i)\right\} + res, \quad \text{(A.2)}$$

where $res = -(\frac{K_0}{S_t} - R_{f,t\to T})^2$ and $\Delta I(K_i)$ is the interval between strike prices, defined as

$$\Delta I(K_i) = \frac{K_{i+1} - K_{i-1}}{2}. \quad \text{(A.3)}$$

Note that $\Delta I(K)$ for the lowest (highest) strike price is the difference between the lowest (highest) strike and the next higher (lower) strike.

C Risk-neutral moments of the market return

The lower bound on the expected market excess return derived by Chabi-Yo and Loudis (2020) depends on the risk-neutral market variance, as well as higher-order moments. Similarly, we use the spanning formula of Carr and Madan (2001) and Bakshi, Kapadia, and

Madan (2003) to compute the risk-neutral nth moment of the market return in Eq. (9):

$$
\begin{aligned}
M_{t\to T}^{\mathbb{Q}(n)} &= E_t^{\mathbb{Q}}[(R_{M,t\to T} - R_{f,t\to T})^n] \\
&= \frac{n(n-1)R_{f,t\to T}}{S_t^2}\left\{\begin{array}{c} \int_0^{K_0}(\frac{K}{S_t} - R_{f,t\to T})^{n-2}P_{t,T}(K)dK \\ +\int_{K_0}^{\infty}(\frac{K}{S_t} - R_{f,t\to T})^{n-2}C_{t,T}(K)dK \end{array}\right\} + \epsilon_n,
\end{aligned} \tag{B.1}
$$

where ϵ_n is the adjustment term corresponding to the risk-neutral nth moment,

$$
\epsilon_n = -(n-1)(\frac{K_0}{S_t} - R_{f,t\to T})^n. \tag{B.2}
$$

Then, we utilize a discretized version of Eq. (B.1) to compute the risk-neutral nth moment:

$$
M_{t\to T}^{\mathbb{Q}(n)} \approx \frac{n(n-1)R_{f,t\to T}}{S_t^2}\left\{\begin{array}{c} \sum_{K_i\le K_0}(\frac{K}{S_t} - R_{f,t\to T})^{n-2}P_{t,T}(K_i)\Delta I(K_i) \\ +\sum_{K_i>K_0}(\frac{K}{S_t} - R_{f,t\to T})^{n-2}C_{t,T}(K_i)\Delta I(K_i) \end{array}\right\} + \epsilon_n, \tag{B.3}
$$

where ϵ_n is defined by Eq. (B.2) and $\Delta I(K_i)$ is defined by Eq. (A.3).

D Truncated risk-neutral moments

The upper bound on the expected market excess return derived by Chabi-Yo and Loudis (2020) is based on the truncated risk-neutral moments of the market excess return. According to Appendix B of Chabi-Yo and Loudis (2020), the truncated risk-neutral nth moment in Eq. (11) is calculated as

$$
\begin{aligned}
M_{t\to T}^{\mathbb{Q}(n)}[k_0] &= E_t^{\mathbb{Q}}[(R_{M,t\to T} - R_{f,t\to T})^n 1_{R_{M,t\to T}\le k_0}] \\
&= (k_0 - R_{f,t\to T})^n Prob_t^{\mathbb{Q}}[S_T \le k_0 S_t] - n(k_0 - R_{f,t\to T})^{n-1}\frac{R_{f,t\to T}}{S_t}P_{t,T}[k_0 S_t] \\
&+ \frac{n(n-1)R_{f,t\to T}}{S_t^2}\int_0^{S_t k_0}(\frac{K}{S_t} - R_{f,t\to T})^{n-2}P_{t,T}(K)dK,
\end{aligned} \tag{C.1}
$$

where k_0 is a prespecified level of the gross return ($k_0 = 0.85$ in our paper), $Prob_t^{\mathbb{Q}}[\cdot]$ is the risk-neutral probability, and $P_{t,T}[k_0 S_t]$ is the price of the put option expiring at time T with strike $k_0 S_t$. We calculate the cumulative risk-neutral probability at $k_0 S_t$ as

$$
Prob_t^{\mathbb{Q}}[S_T \le k_0 S_t] = 1 + R_{f,t\to T}\frac{\partial C}{\partial K}\Big|_{K=k_0 S_t} \approx 1 + \frac{C_{t,T}(k_0 S_t + \Delta_K) - C_{t,T}(k_0 S_t - \Delta_K)}{2\Delta_K}, \tag{C.2}
$$

where Δ_K is the interval between strike prices. Then, we utilize a discretized version of Eq. (C.1) to compute the truncated risk-neutral nth moment:

$$
\begin{aligned}
M^{\mathbb{Q}(n)}_{t\to T}[k_0] \approx{}& (k_0 - R_{f,t\to T})^n Prob^{\mathbb{Q}}_t[S_T \leq k_0 S_t] - n(k_0 - R_{f,t\to T})^{n-1}\frac{R_{f,t\to T}}{S_t}P_{t,T}[k_0 S_t] \\
&+ \frac{n(n-1)R_{f,t\to T}}{S_t^2}\sum_{K_i \leq k_0 S_t}(\frac{K_i}{S_t} - R_{f,t\to T})^{n-2}P_{t,T}(K_i)\Delta I(K_i),
\end{aligned}
\tag{C.3}
$$

where $\Delta I(K_i)$ is defined by Eq. (A.3).

Online Appendices

This online appendix presents the supplementary results for the paper "Out-of-Sample Equity Premium Prediction: The Role of Option-Implied Constraints."

A Supplementary Results

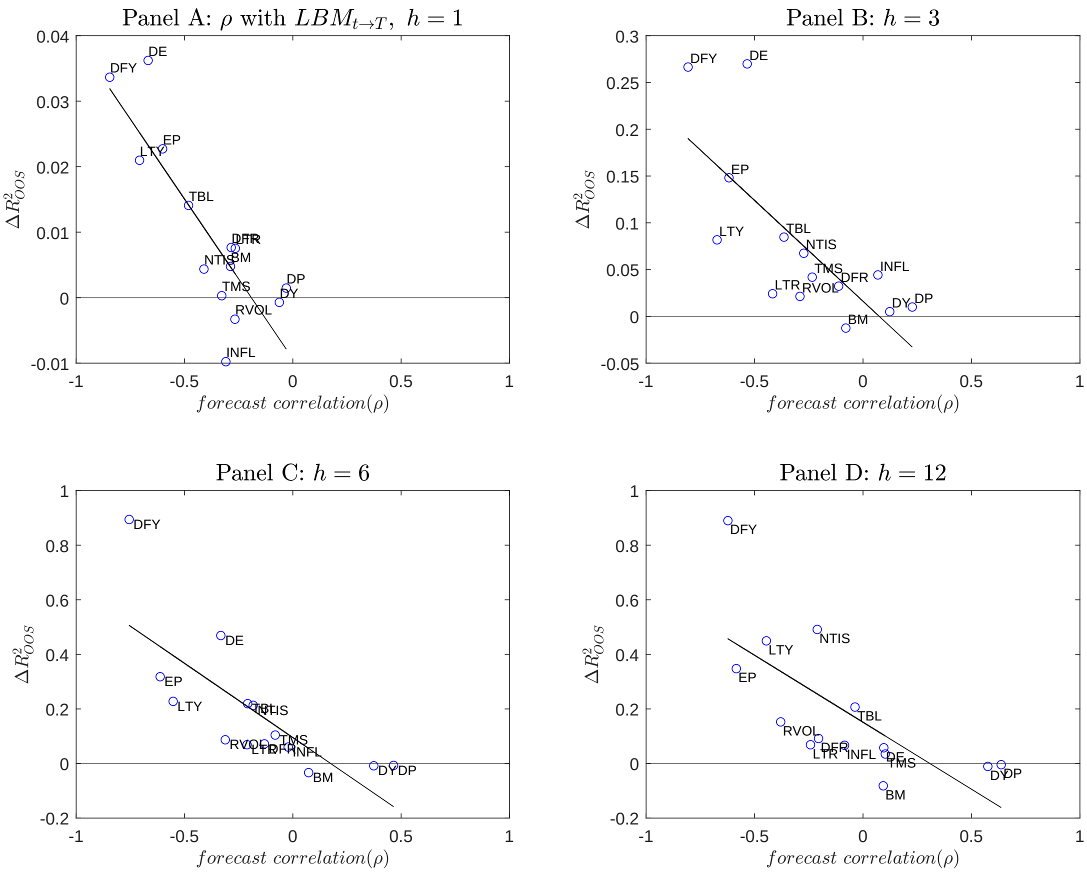

Figure A.1: **Forecast correlation with** LBM

Figure A.1 plots the increment in R^2_{OOS} (vertical axis) against the correlation (horizontal axis) of the unconstrained predictive regression OOS forecasts based on the 14 predictor variables of Welch and Goyal (2008) with Martin's (2017) lower bound on the market premium, LBM. The straight line is the optimal fit obtained by the OLS method, and h denotes the forecast horizon: monthly (h=1), quarterly (h=3), semi-annual (h=6), and annual (h=12). The OOS period is from February 2001 to June 2019.

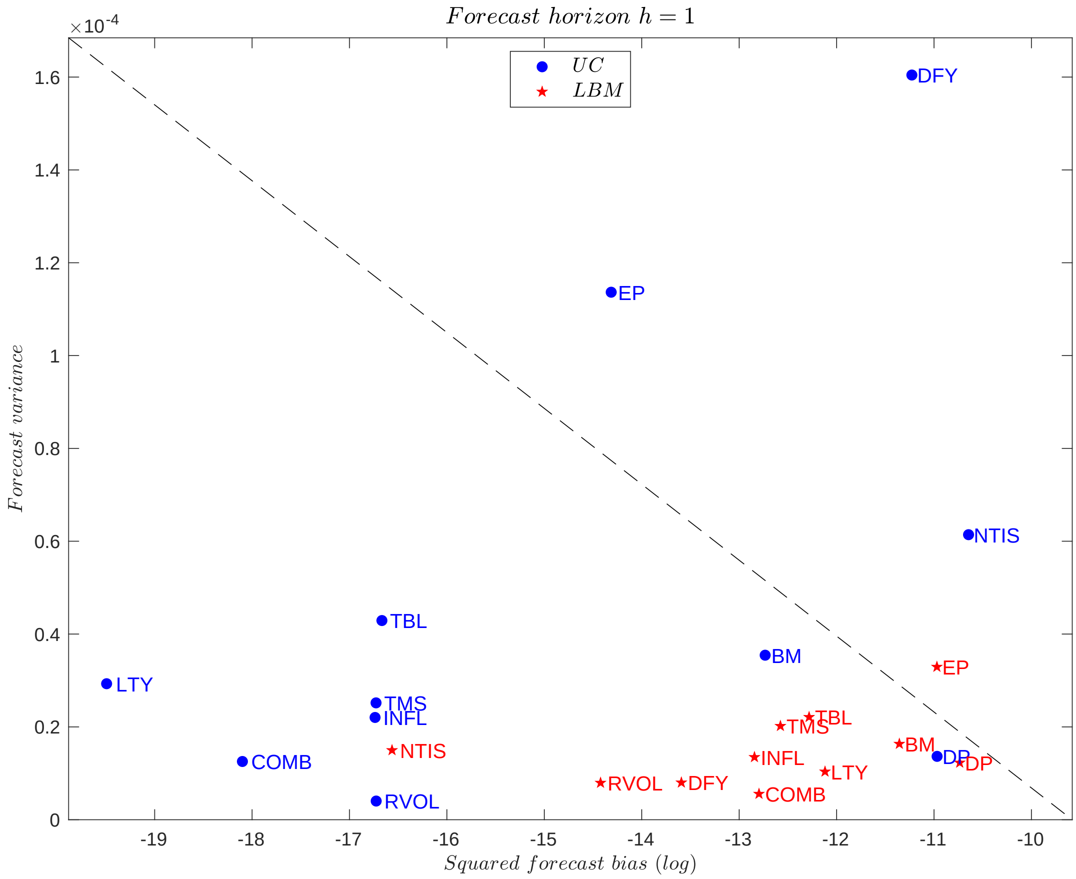

Figure A.2: **Monthly forecast variances and squared forecast biases:** LBM

Figure A.2 plots the OOS forecast variance (vertical axis) against the logarithm of the squared forecast bias (horizontal axis) at the monthly forecast horizon ($h = 1$). The variable UC corresponds to the unconstrained predictive regression, and LBM denotes the constrained counterpart using Martin's (2017) lower bound on the market premium. The OOS period is from February 2001 to June 2019.

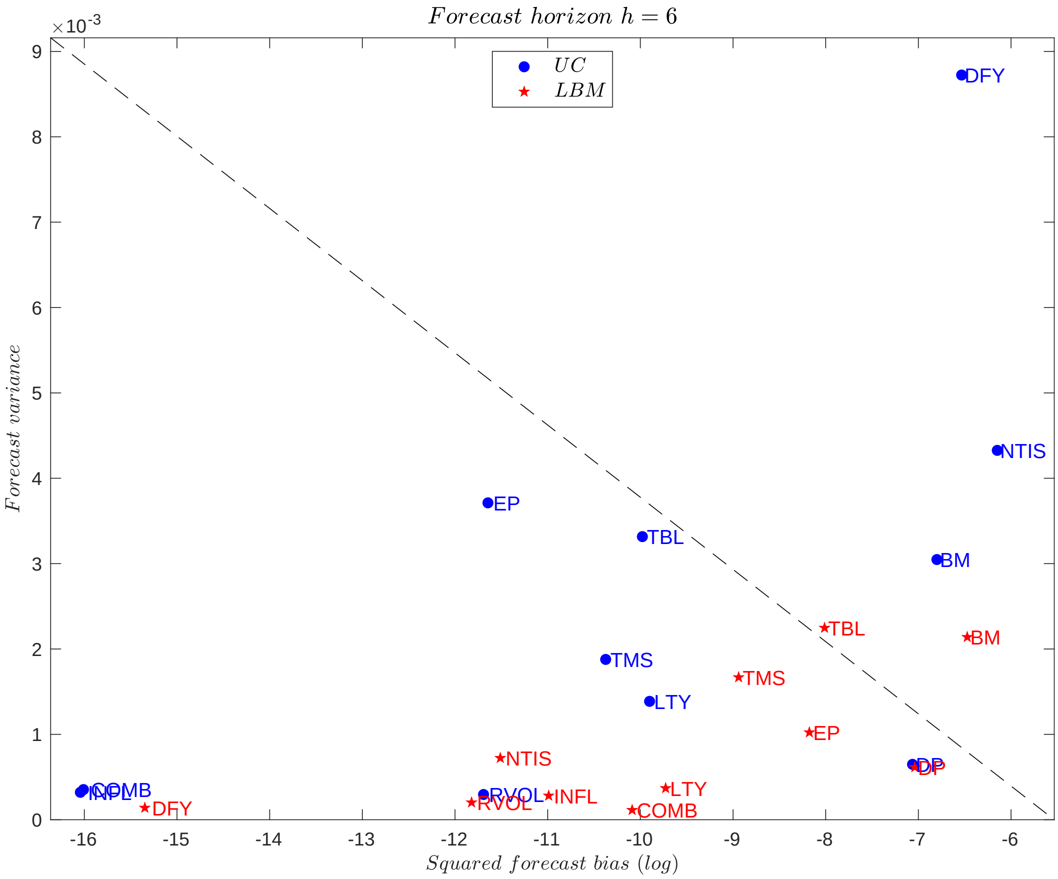

Figure A.3: **Semi-annual forecast variances and squared forecast biases:** LBM

Figure A.3 plots the OOS forecast variance (vertical axis) against the logarithm of the squared forecast bias (horizontal axis) at the semi-annual forecast horizon ($h = 6$). The variable UC corresponds to the unconstrained predictive regression, and LBM denotes the constrained counterpart using Martin's (2017) lower bound on the market premium. The OOS period is from February 2001 to June 2019.

Table A.1: **Predicting the gap between LBM and the realized return**

This table reports the in-sample estimation results of the multivariate predictive regression for bound gap. Specifically, the gap between bound and realized excess return (in percent) is regressed on the 11 predictors from Welch and Goyal (2008):

$$r_{t\to t+h} - LBM_{t\to t+h} = \alpha + \beta' X_t + \epsilon_{t\to t+h},\ t = 1, ..., N-h \text{ and } h = 1, 3, 6, 12,$$

where X_t denotes the 11 standardized GW predictors (DY, DE, and TMS are excluded for collinearity), $r_{t\to t+h}$ is the realized excess return, and $LBM_{t\to t+h}$ is the Martin (2017) lower bound. The t statistic of $\hat{\beta}$ estimate for testing $H_0 : \beta = 0$ against $H_A : \beta \neq 0$ is obtained with the heteroskedasticity and autocorrelation consistent standard error based on Newey and West (1987). We use $2^*h - 1$ as the number of Newey-West lags. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively, according to the two-sided p-values. The sample period is from January 1996 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$h = 1$		$h = 3$		$h = 6$		$h = 12$	
	$\hat{\beta}$	t-stat	$\hat{\beta}$	t-stat	$\hat{\beta}$	t-stat	$\hat{\beta}$	t-stat
DP	1.99***	4.92	5.20***	5.44	8.66***	5.01	16.47***	9.41
EP	0.40	0.88	−0.42	−0.44	−1.81	−1.37	1.13	0.55
RVOL	0.82**	2.25	2.08**	2.55	2.41	1.63	2.02	0.84
BM	−0.77	−1.29	−0.28	−0.25	1.38	1.00	−2.19	−0.94
NTIS	1.30**	2.32	4.06***	3.03	8.00***	3.79	14.37***	3.83
TBL	1.24**	2.27	4.43***	2.68	8.12***	2.61	8.71**	2.26
LTY	−1.86***	−2.73	−5.41***	−2.85	−8.89***	−2.70	−11.28**	−2.52
LTR	0.32	0.86	−0.71	−1.32	−0.88*	−1.74	−1.42*	−1.95
DFY	−0.78	−1.30	−2.47***	−2.58	−3.03**	−2.02	−0.69	−0.20
DFR	0.58	1.27	0.07	0.11	0.19	0.32	0.51	0.59
INFL	0.58**	2.00	0.07	0.15	−0.60	−0.95	−0.09	−0.10
Adj. R^2(%)	11.47		29.63		49.68		61.54	

Table A.2: **Out-of-sample R^2 during high and low VIX periods**

This table presents the monthly OOS R^2 results for the constrained and unconstrained predictive regression models during high and low VIX periods. UC denotes the unconstrained predictive regression; LBM denotes the predictive regression constrained by Martin's (2017) lower bound for the market excess return; $LBCL$ denotes the predictive regression constrained by Chabi-Yo and Loudis's (2020) lower bound for the market excess return; $CT0$ denotes the constrained predictive regression with Campbell and Thompson (2008) non-negativity restriction on equity premium forecasts. We calculate the R^2_{OOS} during high and low VIX periods as:

$$R^2_{OOS,c} = 1 - \frac{\sum_{t=n_1}^{N-1} I^c_{t+1}(r_{t+1} - \hat{r}^{Model}_{t+1|t})^2}{\sum_{t=n_1}^{N-1} I^c_{t+1}(r_{t+1} - \hat{r}^{Bench}_{t+1|t})^2},\ for\ c = HIGH,\ LOW,$$

where I^{HIGH}_{t+1} (I^{LOW}_{t+1}) is set equal to one whenever the month $t+1$ VIX level is above (below) the sample mean and zero otherwise. The statistical significance is determined by the upper-tail p-value for the Clark and West (2007) statistic. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The OOS period is from February 2001 to June 2019.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Panel A: High VIX				Panel B: Low VIX			
Variable	UC	LBM	$LBCL$	$CT0$	UC	LBM	$LBCL$	$CT0$
DP	−3.64	−3.42	−4.04	−3.21	6.27***	6.24***	6.23***	6.27***
DY	−1.96	−2.09	−2.59	−1.97	5.75***	5.81***	5.84***	5.75***
EP	−2.65	0.60	−0.23	2.30***	−4.25	−4.36	−4.29	−4.31
DE	−8.25	−3.25	−4.13	−1.37	−2.04	−1.75	−1.57	−2.04
RVOL	0.45	−0.21	−1.00	0.69*	−2.36	−1.88	−1.61	−2.36
BM	−3.64	−3.41	−4.00	−3.27	−3.03	−1.94	−1.84	−2.15
NTIS	2.09	−0.96	−1.61	1.53	−10.95	−2.13	−1.47	−6.66
TBL	−5.46	−3.89	−4.63	−3.19	−1.00	0.02	0.18	−0.82
LTY	−2.75	−0.16	−0.99	0.36	1.89**	2.80**	2.97**	1.95**
LTR	−1.18	−0.48	−1.19	−0.57	−0.75	0.13	0.29	−0.49
TMS	−2.64	−2.85	−3.71	−1.70	−2.43	−1.84	−1.68	−2.41
DFY	−3.90	0.00	−0.78	1.36*	−2.68	−0.60	−0.15	−1.71
DFR	−5.87	−4.92	−5.43	−4.89	−1.65	−1.32	−1.07	−1.64
INFL	2.04*	−1.30	−2.06	0.72	−8.16	−3.44	−2.98	−6.16
$Econ^{Comb}$	−0.14	0.12	−0.58	0.86	−0.07	0.05	0.16	−0.07
Mean	−2.50	−1.75	−2.46	−0.82	−1.70	−0.28	−0.07	−1.12