Asymmetric Signal and Skewness

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First Version: January 2018
This Version: August 2019

Keywords: Asymmetry; Skewness; Institutional Ownership
JEL Classification Code: G12; G14

[1]Corresponding author. Tel: 86 18810118307. Fang Zhen has been supported by the Research Fund from the Central University of Finance and Economics (Project No. 023063022002/010) and the Program for Innovation Research in Central University of Finance and Economics.

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Abstract

This paper develops a model for analyzing skewness in returns based on a skew-normally distributed signal. The model could generate both positive and negative skewness. The equilibrium third moment increases with the signal's skewness and its magnitude increases with the signal's noisiness. The model also implies that the future absolute asymmetry in returns is negatively correlated with the institutional ownership and the market capitalization. Supportive evidence is found in the Chinese stock market.

1 Introduction

Stock returns are asymmetrically distributed. The sign of the skewness in returns could be positive or negative. To our knowledge, there is no literature theoretically relating this phenomenon with the asymmetry in the public signal's distribution. Given a skew-normally distributed signal, this paper develops a simple model which could generate both positive and negative skewness in returns. The equilibrium third moment has a closed-form expression and its magnitude increases with the signal's noisiness. This property is empirically tested in the Chinese stock market.

The determinants of skewness have been theoretically studied in the literature. Hong and Stein (2003) develop a theory of market crashes based on differences of opinion among investors under short-sales constraints. Their prediction, that returns will be more negatively skewed conditional on high trading volume, is tested and verified by Chen, Hong and Stein (2001). Xu (2007) analyzes skewness in a model where heterogeneous investors observe a normal signal and are prohibited from short selling. He focuses on the relationship between price convexity and skewness, noting an unrealistic assumption of the signal's normality. Albuquerque (2012) reconciles the negative skewness of aggregate stock market returns and the positive skewness of firm stock returns by modelling firm-level heterogeneity in announcement events. Different from previous studies, one distinctive assumption of the model for analyzing skewness in this paper is the asymmetry in the signal's distribution.

Our analysis begins with the derivation of the equilibrium third central moment (i.e., nonstandardized skewness) in a simple one-period rational expectation model, where a representative investor observes a noisy signal regarding a skew-normally distributed payoff. The asymmetry in the signal induces convexity (concavity) in the conditional mean of the true payoff if the signal is positively (negatively) skewed. This convexity (concavity) decreases the magnitude of the third moment, as compared to the linear case. But it does not change the sign of the third moment, which is the same as that of the signal. In other words, the asymmetry in the payoff could counteract the effect of price convexity, which is first discussed in Xu (2007) in a different model setup. Theoretically, this result could explain the sign of skewness in asset returns, which is not determined in Xu's (2007) model.

The noisiness of the signal plays an important role in the equilibrium third moment. Our model predicts that a noisier signal is associated with a higher magnitude of third moment in subsequent returns. By using the institutional ownership and the market capitalization as proxies for the inverse of the signal's noisiness, we empirically test our model predictions in an important emerging market, the Chinese stock market. With the cross-sectional data from the second quarter in 1998 to the second quarter in 2018, we find that the institutional ownership and market capitalization could significantly reduce the magnitude of third moment and both its positive and negative parts. Subsample analysis shows that these results are mainly driven by the recent periods and by the negative part of the third moment.

This paper's contributions to the skewness literature are twofold. First, the equilibrium third moment, whose sign could be either positive or negative, is obtained by building a model which allows the public signal to be asymmetric. The sign of asset returns' skewness is the same as that of the signal's asymmetry. In addition, the returns' third moment increases with the signal's skewness. Second, the magnitude and both the positive and negative parts of the equilibrium third moment decrease with the asset's institutional ownership and market capitalization. This negative relationships are obtained by assuming that institutional investors observe the signal with lesser noise and the signal regarding large firms are more precise. Supportive empirical evidence is provided by using the cross-sectional data from the Chinese stock market.

Our model suggests that it is the magnitude of third moment, rather than the third moment itself, that decreases with the stock's institutional ownership and market capitalization. In other words, the relations between the third moment and the institutional ownership and firm size are not monotonic. This is important because for markets where individual stocks are on average positively skewed (e.g. the US market), our model predicts negative relations between these variables, but for markets which are mainly comprised of negatively skewed stocks, the relations are reversed. In contrast to the significantly negative relations between the skewness and the institutional ownership and the firm size in the US market (see, e.g. Xu (2007)), we find totally reversed relations between these variables in the Chinese stock market, where mean skewness of individual stock returns is negative. Our model could reconcile these two empirical findings.

The remainder of this paper proceeds as follows. Section 2 presents the model, derives the equilibrium third moment and analyzes its properties. Sections 3 empirically tests the relations between the third moment and the institutional ownership and the market capitalization in the Chinese stock market. Section 4 concludes. All proofs are in the Appendix.

2 Model

If investors have symmetric signals, the nonzero skewness could be generated through difference of opinions and short-sale constraints. In Hong and Stein's (2003) model, the investors' signals are uniformly distributed. Heterogeneity of opinions and short-sale constraints could generate both positive and negative asymmetry in the equilibrium returns, where the latter is caused by the revelation of the previously hidden information. In Xu's (2007) model, the investors' signal is normally distributed. Disagreement over the precision of the noise in signal and short-sale constraints induce convexity in prices, which leads to positive asymmetry in returns. In the absence of short sale constraints, both Hong and Stein's (2003) and Xu's (2007) equilibrium price is a linear function of the symmetrically distributed signals, and thus the skewness in equilibrium returns is zero.

However, short-sale constraints are not imposed on all assets. For instance, if an investor is endowed with ample unites of an asset, and determine whether to buy more or sell the asset when he observes a signal. Then short-selling is not restricted on this asset. Without the assumption of short-sale constraints, this paper assumes an asymmetric distribution for the signal and analyzes its impacts on skewness in a rational expectation model.

Consider a one-period model where there exist a risk-free asset with a constant payoff of one, and a risky asset with a skew-normally distributed payoff θ, where $\theta \sim SN(\xi, \omega, \delta)$ and ξ, ω and δ are location, scale and shape parameters, respectively. Specifically, the probability density function of θ is

$$2\phi\Big(\frac{\theta-\xi}{\omega}\Big)\Phi\Big(\frac{\delta}{\sqrt{1-\delta^2}}\frac{\theta-\xi}{\omega}\Big), \tag{1}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal probability density function and cumulative distribution function, respectively. The parameter δ controls for the level of asymmetry in the payoff. If $\delta = 0$, the payoff becomes normally distributed. The nonzero asymmetry in the payoff could be easily understood under extreme market conditions. For example, during the 2007-08 financial crisis, a typical individual stock is more likely to decline in the future, and its payoff corresponds to a negative value of δ. If a listed firm announces a technological breakthrough, the payoff of its stock would have a higher potential to increase in the future, which corresponds to a positive value of δ.

At the beginning of the period, a noisy signal s regarding the payoff of the risky asset is publicly observed. The signal is $s = \theta + \varepsilon$, where ε is normally distributed with mean zero and variance σ_ε^2. Suppose, in this market, there is only one representative investor who cares about the mean and variance of his end-of-period wealth. The investor maximizes his quadratic utility function of wealth $\pi = \pi_0 + X(\theta - p)$, where π_0 is the initial wealth, X is the investor's demand for the risky asset, and p is the risky-asset's price. Solve the following optimization problem

$$\max_X \quad E(\pi|s) - \frac{\gamma}{2} var(\pi|s),$$

where $\gamma > 0$ is the risk-aversion coefficient. We obtain the investor's optimal demand:

$$X = \frac{\hat{\theta} - p}{\gamma \hat{v}},$$

where $\hat{\theta} = E(\theta|s)$, and $\hat{v} = var(\theta|s)$.

A skew-normally distributed variable could be decomposed into two independent parts which are proportional to a normally distributed variable and the absolute value of another normal-distributed variable, respectively. Therefore, with a normally distributed noise, the signal s is still skew-normally distributed, $s \sim SN(\xi, \sqrt{\omega^2 + \sigma_\varepsilon^2}, \frac{\omega\delta}{\sqrt{\omega^2+\sigma_\varepsilon^2}})$. Compared with the true information θ, the signal has a smaller (in magnitude) asymmetry because of the noise, which increases the relative proportion of normal components.

Lemma 1 *Suppose $(z_1, z_2)'$ follows a standardized (i.e., with location 0 and scale 1) bivariate skew-normal distribution. Azzalini and Dalla Valle (1996) show that the moment generating function of z_2 conditional on z_1 is as follows:*

$$M_{z_2|z_1}(t) = \exp\Big[t\rho z_1 + \frac{t^2}{2}(1-\rho^2)\Big]\Phi\Big[\frac{\delta_1 z_1 + t(\delta_2 - \rho\delta_1)}{\sqrt{1-\delta_1^2}}\Big]\Big/\Phi\Big(\frac{\delta_1}{\sqrt{1-\delta_1^2}}z_1\Big), \qquad (2)$$

where δ_1 and δ_2 are the shape parameters of z_1 and z_2, respectively, and $\rho = E(z_1 z_2)$.

Hence, it is easy to obtain the conditional mean and variance of θ as follows:

$$\hat{\theta} = \xi + \frac{\omega^2}{\sqrt{\omega^2 + \sigma_\varepsilon^2}} z + \frac{\alpha \sigma_\varepsilon^2}{\sqrt{\omega^2 + \sigma_\varepsilon^2}} H(-\alpha z), \tag{3}$$

$$\hat{v} = \frac{\omega^2 \sigma_\varepsilon^2}{\omega^2 + \sigma_\varepsilon^2} - \frac{\alpha^2 \sigma_\varepsilon^4}{\omega^2 + \sigma_\varepsilon^2} H(-\alpha z)[\alpha z + H(-\alpha z)], \tag{4}$$

where $\alpha = \frac{\omega\delta}{\sqrt{\omega^2(1-\delta^2)+\sigma_\varepsilon^2}}$, $z = \frac{s-\xi}{\sqrt{\omega^2+\sigma_\varepsilon^2}}$, and $H(x) = \frac{\phi(x)}{\Phi(-x)}$ denotes the hazard function of the standard normal density. The distribution of the standardized signal z is

$$f(z) = 2\phi(z)\Phi(\alpha z).$$

If there is no asymmetry in the payoff (i.e., $\delta = 0$), the conditional mean and variance in Eq. (3)-(4) reduce to those under normal assumption. However, in reality, asymmetry is a salient feature for equity returns (e.g., Harvey and Siddique (2000)). Given the asymmetry setup in our paper, the conditional mean is no longer a linear function of the signal and the conditional variance is no longer a constant, and thus non-normality induces some difficulties when calculating the third moment of the equilibrium return.

The nonlinear component in the conditional mean and variance is related with the hazard function of the standard normal distribution $H(x)$, which is an increasing and convex function as shown in the following lemma.

Lemma 2 *For $x \geq 0$, the following inequalities hold:*

$$\frac{x + \sqrt{x^2 + 4}}{2} > H(x) > \frac{3x + \sqrt{x^2 + 8}}{4}, \tag{5}$$

where the former inequality is proved by Birnbaum (1942) and the latter is given by Sampford (1953). In addition, Sampford (1953) shows that the first order differentiation of $H(x)$ is bounded between 0 and 1, and its second order differentiation is positive; that is

$$0 < H'(x) = H(x)(H(x) - x) < 1, \tag{6}$$

$$H''(x) = H(x)[(H(x) - x)(2H(x) - x) - 1] > 0. \tag{7}$$

With asymmetry in signal ($\alpha \neq 0$), the investor no longer updates his understanding of the true information in a linear fashion. Instead, if the asymmetry is positive (negative), he interprets the information as a convex (concave) function of his signal. As $\delta^2 < 1$, it is easy to show $\alpha^2\sigma_\varepsilon^2 < \omega^2$, which indicates $\hat{\theta}$ is an increasing function of the signal. Hence, compared with the case of no asymmetry ($\alpha = 0$), if the signal is positive (negatively) skewed, the investor would value the true information in a more aggressive (conservative) manner.

Suppose there are u units supply of the risky asset, where u is normally distributed with mean zero and variance σ_u^2 and is independent of the payoff θ and the signal s. Market clearing condition indicates $X = u$. Hence, we obtain the equilibrium price as follows:

$$p = \hat{\theta} - \gamma u \hat{v}. \tag{8}$$

The equilibrium price in Eq. (8) is comparable to Xu's (2007) result if traders in his model were homogeneous. The inclusion of the asymmetry parameter δ makes our result more flexible for analyzing the asymmetry in the equilibrium return. At the end of the period, the payoff θ is realized. Thus, the equilibrium return is given by

$$R = \theta - \hat{\theta} + \gamma u \hat{v}.$$

Noting that $E(R) = 0$, $E(u) = 0$ and $E(u^3) = 0$, the third central moment of R is

$$E[R^3] = E[(\theta - \hat{\theta})^3] + 3\gamma^2\sigma_u^2 E[(\theta - \hat{\theta})\hat{v}^2], \tag{9}$$

where $E[(\theta - \hat{\theta})\hat{v}^2] = E[\hat{v}^2 E(\theta - \hat{\theta}|s)] = 0$. This result indicates that the skewness of the equilibrium return is not affected by the risk-aversion level γ or the conditional variance of the payoff θ, which has greatly simplified the derivation for the third moment of the equilibrium return.

Note that the equilibrium third moment is unrelated with the second term in the righthand side in Eq. (9). We could regard the conditional mean $\hat{\theta}$ as an effective equilibrium price. As discussed before, if the signal is positively (negatively) skewed, $\hat{\theta}$ is an increasing and convex (concave) function of the true information θ. If θ were normally distributed, the convexity (concavity) in price would lead to a negative (positive) equilibrium third moment. However, taking the asymmetry in the signal into consideration, the sign of the equilibrium third moment is actually the same as that of the signal. In other words, the positively (negatively) skewed signal causes the effective price to be higher (lower), but it would not mitigate all the positivity (negativity) in the third moment of subsequent returns. The theoretical value of the equilibrium third moment is given by the following proposition.

Proposition 1 *Noting that $\theta - \hat{\theta} = -\varepsilon + E(\varepsilon|s)$, the third moment of the equilibrium return is as follows:*

$$E[R^3] = -E[skw(\varepsilon|s)] = \frac{\alpha^3\sigma_\varepsilon^6}{(\sqrt{\omega^2+\sigma_\varepsilon^2})^3}E[H''(-\alpha z)] = I_1 + I_2, \tag{10}$$

where $H''(x)$ is the second order differentiation of the hazard function $H(x)$, and

$$I_1 = \frac{\alpha\sigma_\varepsilon^6}{(\sqrt{\omega^2+\sigma_\varepsilon^2})^3}E[(z^2-1)H(-\alpha z)] = -\sqrt{\frac{2}{\pi}}\frac{\delta^3\omega^3\sigma_\varepsilon^6}{(\omega^2+\sigma_\varepsilon^2)^3}, \tag{11}$$

$$I_2 = \frac{\alpha^2(\alpha^2+2)\sigma_\varepsilon^6}{(\sqrt{\omega^2+\sigma_\varepsilon^2})^3}E[-zH^2(-\alpha z)]. \tag{12}$$

The second equality holds in Eq. (10) because given the signal s, the conditional third moment of the noise ε is as follows:

$$skw(\varepsilon|s) = \frac{\alpha^3\sigma_\varepsilon^6}{(\sqrt{\omega^2+\sigma_\varepsilon^2})^3}[(1-\alpha^2z^2)H(-\alpha z) - 3\alpha z H^2(-\alpha z) - 2H^3(-\alpha z)]. \tag{13}$$

In the third moment of the equilibrium return R, the first part I_1 has an explicit expression, but the second part I_2 does not. Numerical integration is needed when calculating I_2. The hazard function of the standard normal distribution $H(x)$ is an increasing and convex function. As shown in Lemma 2, when x goes to infinity, the hazard function $H(x)$ approaches infinity at the same rate as x. Thus, the existence of the third moment of a skew-normally distributed variable ensures the existence of I_2. Moreover, because $H''(x)$ is positive for all x, it could be seen from Eq. (10) that $E(R^3)$ has the same sign as δ.

It is shown in the Appendix that the first order differentiation of $E(R^3)$ with respect to α is positive, that is

$$\frac{\alpha^2\sigma_\varepsilon^6}{(\sqrt{\omega^2+\sigma_\varepsilon^2})^3}E[H''(-\alpha z)(2+z^2)] > 0.$$

Thus, the third central moment is an increasing function of the signal's shape parameter α, which is an increasing function of the signal's skewness. Hence, the higher the skewness of the signal, the higher the third moment of the equilibrium return.

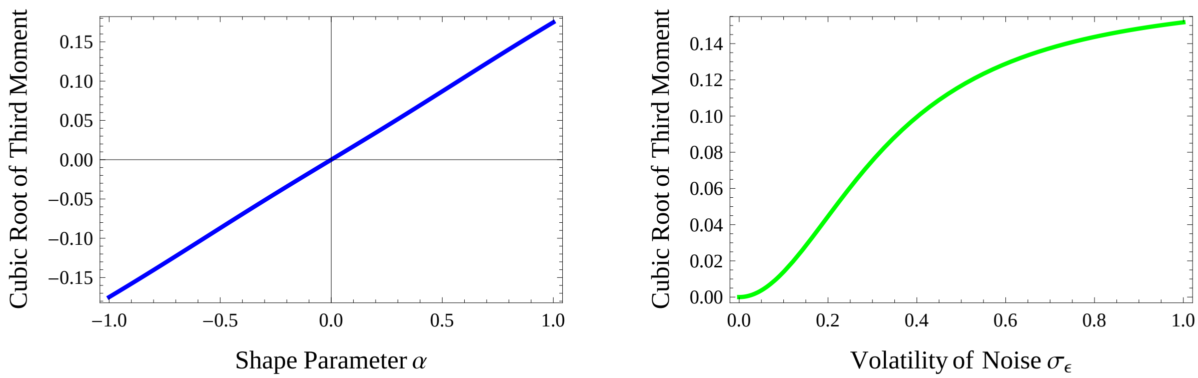
The noise in the signal is crucial for the existence of the third moment of the equilibrium return. As shown in the Appendix, the magnitude of the third moment is increasing with respect to the volatility of noise σ_ε. If σ_ε is zero (i.e., the investor observes the true payoff), the third moment is zero; if σ_ε goes to infinity (i.e., the investor's signal is a pure noise), the third moment of the equilibrium return becomes the third moment of the payoff, and it is equal to $c\omega^3\delta^3$, where $c = (4/\pi - 1)\sqrt{2/\pi}$ is a constant. Intuitively, a lesser noise in the signal prompts the price to be closer to its payoff, and thus reduces the asymmetry in the equilibrium return.

Figure 1 shows the monotonicity of the cubic root of the third moment with respect to α and σ_ε, respectively. Except for the changing variables, the sub-figures are drawn by setting $\omega = 0.4$, $\sigma_\varepsilon = 0.4$, $\delta = 0.7$. Consistent with our theoretical analysis, the third moment of the equilibrium return increases with α, and its magnitude increases with σ_ε.

The third moment measures the possibility of more positive returns and less negative returns. The literature has shown, both theoretical and empirically (see, e.g. Harvey and Siddique (2000)), that investors have a preference for skewness. Suppose that institutional investors in the stock market could obtain a noisy signal regarding the true payoff of a stock,

Figure 1: The Cubic Root of Third Moment

This figure shows the cubic root of the third moment of the equilibrium return as a function of the shape parameter α or of the volatility of noise σ_ε, The left sub-figure is drawn by setting $\omega = 0.4$, $\sigma_\varepsilon = 0.4$, and the right one by $\omega = 0.4$, $\delta = 0.7$.



whereas individual investors do not. That is the institutions' volatility of noise is finite and individuals' is infinite. The model indicates that when the market is bullish (bearish), stocks are attractive to both (neither) individuals and (or) institutions, and stocks are more (less) attractive to individuals than to institutions in terms of third moment. In addition, assume that investors could obtain a less noisy signal for larger firms. The model implies that both individuals and institutions would prefer smaller firms during a bull market for their higher upward potential in returns and prefer larger firms during a bear market for their lower downward possibility.

Using the institutional ownership and market capitalization as a proxy for the signal's precision (i.e., the inverse of the variance of the noise in the signal), the model suggests the following testable hypotheses:

1. *The magnitude of the asymmetry in returns of a stock decreases with the stock's institutional ownership and market capitalization.*

2. *Both the positive part and the negative part of the asymmetry decrease with the stock's*

institutional ownership and market capitalization.[2]

Xu (2007) shows that the US stocks with higher institutional ownership and larger market capitalization are more negatively skewed through price convexity, which is driven by disagreement over information quality and short sales constraints. His model could explain these negative relations because that larger stocks and stocks with higher institutional ownership are easier to sell short, and thus have less convexity in price and less skewness in returns. In contrast, in this paper, the institutional ownership and the market capitalization affect skewness through the level of noisiness in the signal. Our model is consistent with Xu's (2007) model for positive skewness, which is a salient feature for the US stocks. Nevertheless, for negative skewness, our model predicts totally reversed relations between the third moment and the institutional ownership and the market capitalization.

The skewness for individual stock returns in the US market are positive on average (see, e.g. Chen, Hong and Stein (2001); Xu (2007); Jondeau, Zhang, and Zhu (2019)). To empirically test our hypotheses, especially for negative asymmetry, we studied the Chinese stock market in the following section.

3 Empirical Evidence

The Chinese daily stock prices of A-shares, market capitalization and quarterly institutional ownership data are obtained from the Wind financial database. The institutional ownership is the proportion of tradable shares held by institutional investors, including investment funds, security firms, qualified foreign institutional investors (QFIIs), insurance companies, pension funds, trust companies, commercial banks and general legal entities. The ownership data is available at the end of March, June, September and December, and the sample period is from June 1998 to March 2018. The stock price data is from 1 April 1998 to 30 June 2018.

[2]The asymmetry in returns is measured by the cubic root of the third moment of returns' distribution. The positive or negative part is referred to as the absolute value of positive or negative asymmetry, respectively.

Our sample constains 3530 firms. We exclude the days when the magnitude of percentage return exceeds 10% or when the ownership is more than 100%. The quarterly asymmetry for each individual stock is calculated by using the daily log returns within a quarter, defined as:

$$Asym_{i,t} = \Big(\frac{252}{Q_t}\sum_{d=1}^{Q_t}(R_{i,d} - \bar{R}_i)^3\Big)^{1/3}, \quad \bar{R}_i = \frac{1}{Q_t}\sum_{d=1}^{Q_t} R_{i,d}, \tag{14}$$

where $R_{i,d}$ is the log return of stock i on day d, and Q_t is the total number of trading days in quarter t. We also filter out zero asymmetries caused by a trading halt. We use daily returns in a quarter to compute asymmetry for matching the frequency of the ownership data.

The asymmetry define in Eq. (14) is the cubic root of nonstandardized skewness. It is a monotonic transform of the theoretical third moment derived in the previous section. It could also mitigate the impacts of the uncertainty in future realized variance on the following forecasting regressions. As the third moment are closely related with variance, we use historical volatility as a control variable. By using the daily log returns in the most recent quarter, the quarterly historical volatility is defined as:

$$Hvol_{i,t} = \Big(\frac{252}{Q_{t-1}}\sum_{d=1}^{Q_{t-1}}(R_{i,d} - \bar{R}_i)^2\Big)^{1/2}, \quad \bar{R}_i = \frac{1}{Q_{t-1}}\sum_{d=1}^{Q_{t-1}} R_{i,d}, \tag{15}$$

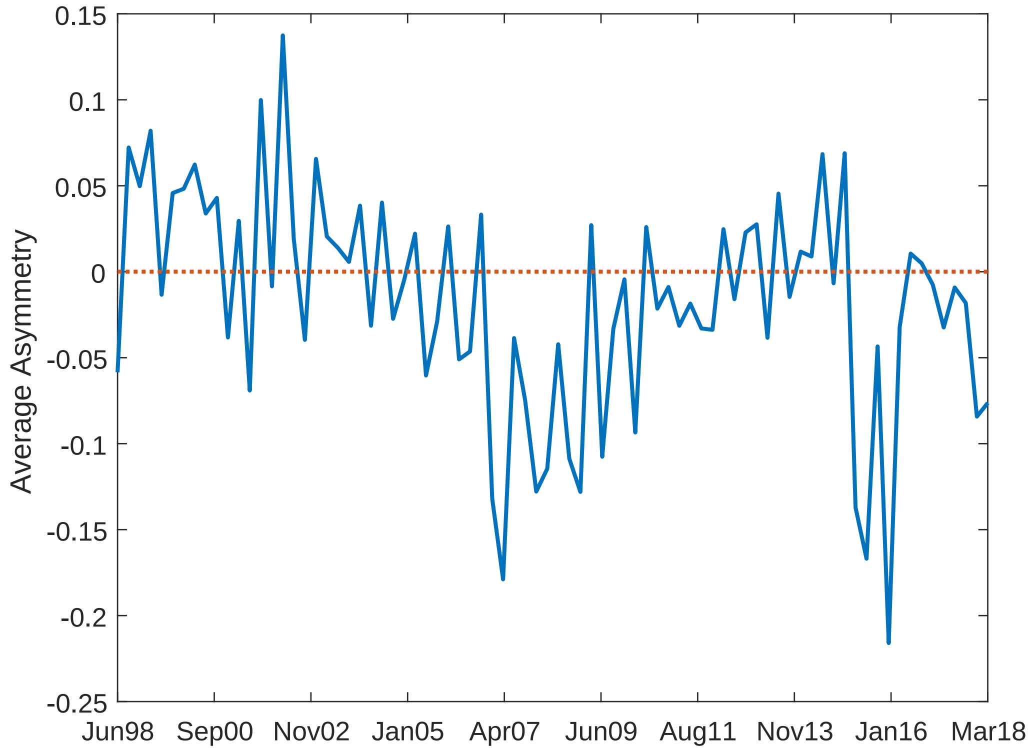
where $R_{i,d}$ is the log return of stock i on day d, and Q_{t-1} is the total number of trading days in quarter $t-1$.

As shown in Figure 2, the cross-sectional averages of asymmetries could either be positive or negative, ranging from -0.216 to 0.1375. During the 2007-08 financial crisis and the 2015-16 Chinese stock market turbulence, the individual stock returns exhibit a negative mean asymmetry, whereas the cross-sectional averages of asymmetry could be positive during bull markets. The sign of returns' asymmetry could be explained by the model developed in this paper, and it is the same as that of the asymmetric signal theoretically.

Institutional investors have been playing a role in the Chinese stock market that is more

Figure 2: Average Asymmetry

This figure shows the cross-sectional averages of stock returns' asymmetry from 1998 to 2018. The asymmetry is calculated as the cubic root of third moment by using the daily log returns in the quarter following the last date of March, June, September and December. Data Source: Wind financial database.

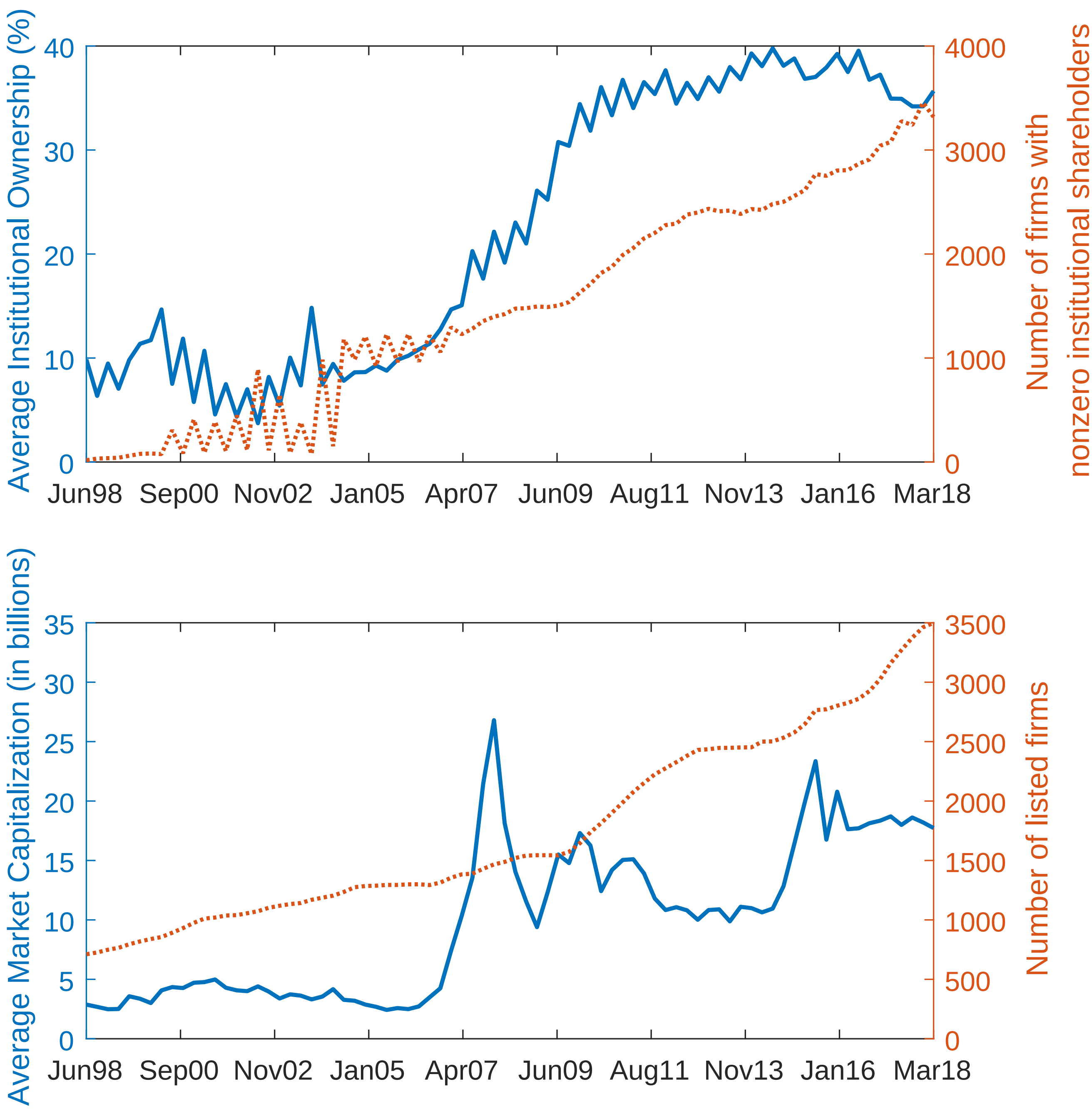


and more important over time. As reflected in Figure 3, there is a upward trend of the cross-sectional averages of institutional ownership over time. During early years from 1998 to 2004, the average institutional shareholding is between 5% and 15%. It grows rapidly from 10% in 2005 to 35% in 2010, and stays relatively stable since 2010 between 30% and 40%. The number of firms with nonzero institutional shareholders increased dramatically over our sample period from 17 in June 1998 to 3318 in March 2018. This number is relatively unstable during the earlier periods.

As shown in Figure 3, before 2007, the average market capitalization is very small around

Figure 3: Average Institutional Ownership and Market Capitalization

The solid lines (left scale) in the figures show the cross-sectional averages of the institutional ownership and the market capitalization observed at the end of each quarter from 1998 to 2018. The number of firms with nonzero institutional shareholders and the number of listed firms are represented by the dashed lines (right scale). Data Source: Wind financial database.



3.5 billion Chinese yuan (CNY). It soars since 2007, reaches the highest value of 26.8 billion CNY on 31 December 2007 and then decreases sharply due to the financial crisis in 2008. The second largest value of average market capitalization is 23.4 billion CNY on 30 June 2015 and it plummets because of the 2015 Chinese stock market selloff. The number of listed firms is increased from 711 in June 1998 to 3499 in March 2018, which reflects the steady growth of the Chinese stock market.

Table 1: Summary Statistics and Correlation Matrix

This table provides the summary statistics and the correlation matrix for the following variables: the absolute value of asymmetry (AbsAsym, LagAsym), the institutional ownership (IO), the market capitalization (in billions), the historical volatility. The sample is from the third quarter in 1998 to the second quarter in 2018 for the cross section of stocks traded in the Shanghai and Shenzhen Stock Exchanges. Data Source: Wind Financial Database.

Panel A: Summary Statistics					
	Mean	First Quartile	Median	Third Quartile	Standard Deviation
AbsAsym	0.1220	0.0818	0.1178	0.1566	0.0542
IO	0.3063	0.0839	0.2773	0.4917	0.2385
MV	12.1454	2.1141	3.9984	8.1344	65.9530
Hvol	0.4377	0.3139	0.4053	0.5283	0.1783
Panel B: Correlation					
	Absasym	IO	MV	Hvol	
IO	-0.0731				
MV	-0.0496	0.1531			
Hvol	0.3327	-0.1074	-0.0476		
LagAsym	0.2493	-0.0925	-0.0446	0.6565	

Table 1 (Panel A) shows that the mean value of absolute asymmetry is 0.1220 with a standard deviation of 0.0542. This indicates that nonzero asymmetry is a salient feature in the Chinese stock market. Among the stock with nonzero institutional ownership, the average share proportion held by institutions is 30.63%. The high proportion of institutional ownership mainly comes from the recent sample when almost all the listed firms have shares held by institutional investors. In each quarter, we use the market capitalization data on

the last trading day. Its average value is 12 billion CNY which exceeds the third quartile of its distribution. This implies that the number of large firms only accounts for a small proportion of the cross section of all stocks over our sample period. The average historical volatility of 43.77% shows that the Chinese exchange-traded firms are very risky.

Consistent with our hypotheses, the correlation between the absolute asymmetry and the institutional ownership (or the market capitalization) is negative as shown in Table 1 (Panel B). The absolute asymmetry and its lagged value are positively correlated with historical volatility (0.3327 for the former and 0.6565 for the latter), indicating that high asymmetry is accompanied by high historical and contemporaneous variance. The absolute asymmetry exhibits a mean-reverting feature as suggested by its auto-correlation coefficient 0.2493. Table 1 also shows that institutional investors tend to hold large and less risky (lower volatility) stocks, and larger firms are less risky in terms of variance.

3.1 Regression Analyses

We empirically test the relations between the absolute value of returns' asymmetry (and its positive and negative parts), the institutional ownership and the market capitalization by using the Fama-Macbeth (1973) approach and a pooled time-series cross-sectional regression. These two types of regressions are also used by Chen, Hong and Stein (2001) and Dennis and Mayhew (2002) for analyzing the physical skewness and risk-neutral skewness, respectively, for the US stock market.

First, we run the following cross-sectional regression for each quarter in our sample period:

$$|Asym_i| = \alpha + \beta_{io} IO_i + \beta_{size} MV_i + \beta_{vol} Hvol_i + \varepsilon_i, \tag{16}$$

where $|Asym_i|$ is the absolute value of asymmetry as defined in Eq. (14) for firm i, IO_i and MV_i are the proportion of shares held by institutional investors and the stock's market capitalization at the end of last quarter, respectively, and $Hvol_i$ is the historical volatility as defined in Eq. (15).

Table 2 (Panel A) shows the time-series averages of each coefficient during the full sample period: from the third quarter (Q3) in 1998 to the second quarter (Q2) in 2018, and four subperiods: Q3 1998-Q2 2003, Q3 2003-Q2 2008, Q3 2008-Q2 2013 and Q3 2013-Q2 2018. In each subperiod (five years), we obtain 20 estimates for each coefficient. The time-series averages of these coefficients are then reported in the table, together with the t-statistics for the null hypothesis that the average is zero. We find a significantly negative relations between the absolute asymmetry and the institutional ownership and the market capitalization in recent periods 2008-2013 and 2013-2018, meaning that the stocks with high institutional ownership and large market value are more stable with less asymmetry in returns. This finding is consistent with our theory that less noisy signal leads to a smaller third moment in magnitude. However, for the earlier periods, we only find a significantly negative relation between the absolute asymmetry and the market capitalization in 2003-2008. The small sample size in 1998-2003 even leads to a result that is inconsistent with our predictions. Overall, we find no supportive evidence in the cross-sectional regressions during the full sample period.

Furthermore, we investigate whether institutions could decrease (increase) the asymmetry for stocks with positive (negative) signals by decomposing the absolute asymmetry into its positive part and negative part. Table 2 (Panel B and C) shows that both the positive part and negative part are significantly negatively correlated with the institutional ownership and the market capitalization in 2013-2018. For periods 2003-2008 and 2008-2013, we find that the negative relations between the absolute asymmetry and the institutional ownership is driven by its negative part. In addition, not surprisingly, the historical volatility are positively related with the absolute asymmetry (and its positive and negative parts) over the full sample and the four subperiods.

Table 2: Fama-Macbeth Approach

This table shows the time-series averages of the coefficients from quarterly cross-sectional regressions of the absolute value of return's asymmetry (and its positive and negative parts) on the institutional ownership (IO), the market capitalization (MV in trillions) and the historical volatility (Hvol). The full sample period is from the third quarter (Q3) in 1998 to the second quarter (Q2) in 2018. The results for subperiods: Q3 1998-Q2 2003, Q3 2003-Q2 2008, Q3 2008-Q2 2013 and Q3 2013-Q2 2018 are also reported. The t-statistics, which are in parentheses, are adjusted for heteroskedasticity and serial correlation.

	Full Sample	1998-2003	2003-2008	2008-2013	2013-2018
Panel A: Absolute Asymmetry					
IO	−0.0033	0.0210	−0.0126	−0.0048	−0.0169
	(−0.6444)	(1.6656)	(−1.4453)	(−2.0342)	(−4.8527)
MV	0.0272	0.2277	−0.0389	−0.0323	−0.0476
	(0.4009)	(0.9676)	(−1.8763)	(−11.8081)	(−5.3727)
Hvol	0.0779	0.0683	0.0992	0.0932	0.0511
	(12.1075)	(4.5662)	(11.3489)	(21.2000)	(4.0317)
Avg. Adj. R^2	0.0576	0.0326	0.0659	0.0645	0.0675
Avg. No. of Obs.	1453	205	1012	1897	2697
Panel B: Positive Part of Asymmetry					
IO	−0.0133	−0.0610	0.0169	0.0014	−0.0106
	(−0.8106)	(−0.9003)	(3.0035)	(0.3029)	(−4.2955)
MV	0.0194	0.2850	−0.0886	−0.0580	−0.0608
	(0.3170)	(1.3813)	(−1.4086)	(−3.8728)	(−4.2871)
Hvol	0.0608	0.0792	0.0674	0.0652	0.0312
	(8.5952)	(4.2711)	(5.1757)	(6.8096)	(3.7713)
Avg. Adj. R^2	0.0414	0.0416	0.0467	0.0431	0.0344
Avg. No. of Obs.	597	125	381	770	1111
Panel C: Negative Part of Asymmetry					
IO	−0.0038	0.0313	−0.0203	−0.0061	−0.0201
	(−0.2215)	(0.4571)	(−2.1965)	(−2.4536)	(−6.4859)
MV	−0.2413	−0.7650	−0.1388	−0.0256	−0.0356
	(−1.9203)	(−1.6208)	(−2.1004)	(−1.4150)	(−1.7557)
Hvol	0.0917	0.0577	0.1264	0.1058	0.0769
	(10.0084)	(2.8554)	(5.5762)	(21.2866)	(4.7497)
Avg. Adj. R^2	0.0795	0.0138	0.0859	0.0867	0.1251
Avg. No. of Obs.	856	80	631	1128	1585

Second, we run the following pooled time-series cross-sectional regression

$$|Asym_{i,t}| = \alpha + \beta_{io} IO_{i,t} + \beta_{size} MV_{i,t} + \beta_{vol} Hvol_{i,t} + \beta_{lag} |Asym_{i,t-1}| + \varepsilon_{i,t} \qquad (17)$$

where the lagged absolute asymmetry $|Asym_{i,t-1}|$ is to capture the persistence of the third moment. We also include dummy variables for each quarter t. This regression is to predict the cross-sectional variation in asymmetry over period t, based on the information available at the end of period $t-1$.

The estimated coefficients for (17) are reported in Table 3 (Panel A). Except for the first subperiod 1998-2003, we find significantly negative relations between the absolute asymmetry and the institutional ownership and the market capitalization. Because of the small sample size in the earlier periods, the significantly negative relations between the absolute asymmetry and the institutional ownership and the market capitalization also hold for the full sample period. The regressions for decomposed asymmetry in Table 3 (Panel B and C) show that both the positive part and negative part of asymmetry are significantly negatively correlated with the institutional ownership and the market capitalization in 2008-2013 and 2013-2018. However, for the second subperiod 2003-2008, the negative relations only hold for the negative part of asymmetry. In addition, the R^2s and the number of observations for the negative parts are higher than those for positive parts, except for the first subperiod 1998-2003.

During the earliest subperiod 1998-2003, we find no supporting evidence of our model's prediction in the pooled regression. On the contrary, we find significantly positive relations between the magnitude of asymmetry and the institutional ownership and the market capitalization. During the second subperiod 2003-2008, the negative relationship does not hold for the positive part of asymmetry. During the recent periods 2008-2013 and 2013-2018, the regression results do support our hypothesis, and the most recent period 2013-2018 provides a stronger evidence. Overall, the subperiod analysis shows an empirical pattern that is more and more consistent with our model over time. In addition, same as the Fama-Macbeth

Table 3: Pooled Regression

This table shows the results of pooled regressions of the absolute value of return's asymmetry (and its positive and negative parts) on the institutional ownership (IO), the market capitalization (MV in trillions), the historical volatility (Hvol) and the lagged value of absolute asymmetry (LagAsym). The full sample period is from the third quarter (Q3) in 1998 to the second quarter (Q2) in 2018. The results for subperiods: Q3 1998-Q2 2003, Q3 2003-Q2 2008, Q3 2008-Q2 2013 and Q3 2013-Q2 2018 are also reported. The t-statistics, which are in parentheses, are adjusted for heteroskedasticity. The regression for the full sample period also contains dummies for each quarter (unreported).

	Full Sample	1998-2003	2003-2008	2008-2013	2013-2018
Panel A: Absolute Asymmetry					
IO	−0.0136	0.0142	−0.0203	−0.0057	−0.0177
	(−21.8439)	(1.7643)	(−9.7296)	(−5.9737)	(−20.3518)
MV	−0.0372	0.0810	−0.0202	−0.0302	−0.0492
	(−15.3067)	(1.6938)	(−4.4759)	(−10.4535)	(−14.8303)
Hvol	0.0564	0.0761	0.0713	0.0880	0.0431
	(40.5707)	(7.7281)	(19.8276)	(34.1152)	(23.8323)
LagAsym	0.0483	0.0352	0.0715	0.0394	0.0412
	(14.6883)	(1.8603)	(9.1329)	(6.9875)	(8.6667)
Adj. R^2	0.3392	0.4008	0.3843	0.1895	0.3661
No. of Obs.	116070	4091	20232	37900	53847
Panel B: Positive Part of Asymmetry					
IO	−0.0069	0.0116	0.0155	−0.0041	−0.0121
	(−7.1846)	(1.2128)	(5.3877)	(−2.6826)	(−8.9527)
MV	−0.0288	0.1260	0.0080	−0.0289	−0.0410
	(−9.7014)	(1.6994)	(1.6248)	(−6.1027)	(−10.5818)
Hvol	0.0389	0.0766	0.0673	0.0676	0.0201
	(17.6495)	(6.5451)	(12.0079)	(16.6643)	(6.8574)
LagAsym	0.0529	0.0351	0.0281	0.0276	0.0711
	(9.9394)	(1.4601)	(2.2011)	(3.0122)	(9.1681)
Adj. R^2	0.1088	0.4541	0.1307	0.0646	0.0844
No. of Obs.	47671	2505	7621	15367	22178
Panel C: Negative Part of Asymmetry					
IO	−0.0184	0.0251	−0.0395	−0.0073	−0.0228
	(−23.6723)	(1.8379)	(−14.5497)	(−6.1890)	(−20.9279)
MV	−0.0390	−0.0028	−0.0332	−0.0268	−0.0511
	(−10.0740)	(−0.0658)	(−3.3990)	(−8.5973)	(−8.9963)
Hvol	0.0700	0.0627	0.0766	0.0999	0.0613
	(40.3466)	(3.5614)	(17.6047)	(30.9386)	(27.3943)
LagAsym	0.0407	0.0454	0.0750	0.0504	0.0187
	(10.1154)	(1.5223)	(8.0804)	(7.4095)	(3.2393)
Adj. R^2	0.4787	0.3594	0.4903	0.3257	0.5141
No. of Obs.	68399	1586	12611	22533	31669

(1973) regressions, all the pooled regressions show that high magnitude of future asymmetry is associated with high historical volatility.

We find evidence that supports our conjecture that institutional investors and firm sizes would help increase negative asymmetry and decrease positive asymmetry. The sub-sample regressions shows that these relations are strongest in the most recent period. For negative asymmetry, which measures the downside possibility in returns, the institutional ownership and firm size decrease its magnitude. For positive asymmetry, which could be regarded as a benefit rather than a risk, the institutional ownership and firm size also reduce it. In a nutshell, we find that in the Chinese market, especially for recent periods, stocks with higher institutional ownership and larger market capitalization are more stable in terms of third central moment with less lottery or crash feature.

3.2 Robustness

In this paper, we analyze the absolute asymmetry, whose value (both positive part and negative part) decreases with the institutional ownership as indicated by our model. In the literature, the skewness itself rather than its absolute value is of interest (e.g., Chen, Hong and Stein (2001); Xu (2007)). Hence, we run the pooled regression for asymmetry as well as for skewness over the full sample period, and report the results in Table 4. In line with the definition of skewness in Chen, Hong and Stein (2001) and Xu (2007), we measure realized skewness as follows:

$$RSkew_{i,t} = \frac{\sqrt{\frac{Q_t}{252}}\sum_{d=1}^{Q_t}(R_{i,d}-\bar{R}_i)^3}{[\sum_{d=1}^{Q_t}(R_{i,d}-\bar{R}_i)^2]^{3/2}}, \quad \bar{R}_i = \frac{1}{Q_t}\sum_{d=1}^{Q_t} R_{i,d}, \tag{18}$$

where $R_{i,d}$ is the log return of stock i on day d, and Q_t is the total number of trading days in quarter t.

As shown in Table 4, the coefficients of the pooled regression of the asymmetry on the institutional ownership and the market capitalization become positive, which are consistent

Table 4: Robustness Check

This table shows the pooled regression results for the logarithmic (Log) absolute asymmetry (LogAbsAsym), the return's asymmetry (Asym) and the return's skewness (RSkew), respectively, on the institutional ownership (IO or LogIO), the market capitalization (MV in trillions or LogMV), the historical volatility (Hvol or LogHvol) and the corresponding lagged value (Lag). The sample period is from the third quarter in 1998 to the second quarter in 2018. The t-statistics, which are in parentheses, are adjusted for heteroskedasticity. The regressions also contain dummies for each quarter (unreported).

	LogAbsAsym	Asym		RSkew	
LogIO	−0.0123		0.0029		0.0007
	(−12.7430)		(11.6782)		(7.1428)
IO		0.0356		0.0114	
		(21.7430)		(15.8536)	
LogMV	−0.0493		0.0093		0.0028
	(−32.8049)		(25.8838)		(17.8128)
MV		0.0398		0.0104	
		(8.4285)		(4.7442)	
LogHvol	0.1512		−0.0175		−0.0018
	(19.9572)		(−14.8364)		(−3.3756)
Hvol		−0.0663		−0.0036	
		(−21.1360)		(−2.5234)	
Lag	0.0244	0.0530	0.0470	0.0250	0.0228
	(6.9705)	(17.6957)	(15.5864)	(5.5310)	(5.0734)
Adj. R^2	0.2427	0.2449	0.2458	0.0944	0.0955
No. of Obs.	116070	116070	116070	116070	116070

with the results for the negative part of asymmetry in Table 3 (Panel C). Compared with the results for the absolute asymmetry in Table 3 (Panel A), the R^2 is decreased from 33.92% to 24.49%. The reduced predictive power is mainly due to the positive part of asymmetry, which ought to be negatively related with the institutional ownership and the market capitalization as shown in Table 3 (Panel B).

Table 4 also shows the pooled regression results for the future realized skewness. We find significantly positive relation between the skewness and the institutional ownership (and the market capitalization), but the significance level and the R^2 are smaller than those for asymmetry. These results imply that it is more reliable to predict asymmetry than to predict skewness, as the latter (though comparable across stocks) also contains the future variance information. When forecasting future realized skewness, the coefficients of skewness on institutional ownership is significantly positive. This result is exactly the opposite to that in Xu (2007), where the skewness of NYSE stocks are being analyzed. Nevertheless, different from the stocks in the Chinese market, where the average skewness is negative (unreported), the individual stocks in the US market are often positively skewed. The mean and standard deviation of the skewness ($RSkew$ multiplied by $\sqrt{252}$) in our sample are -0.0708 and 0.8193, whereas the corresponding values for the US market in Xu (2007) are 0.253 and 1.156 as reported in his Table 2. Our theory could reconcile these two facts if the regression result in the US market is driven by the positive part of skewness, whereas in the Chinese market it is driven by the negative part of skewness.

In addition, we report the results when using logarithms of all the positive values in the full-sample regressions. Table 4 shows that this transform does not change the relations in the baseline regressions, but the R^2 for the absolute asymmetry is reduced from 33.92% to 24.27%.

4 Conclusion

We develop a simple one-period model where a representative investor observes a noisy non-normal signal regarding a risky asset. With the assumption of skew-normality in the risky asset's payoff, we derive the equilibrium return's third moment whose sign is the same as that of the signal. Hence, the equilibrium third moment could explain the observed positive or negative sign of skewness in returns in stock markets. The third moment increases with the signal's skewness and its magnitude increases with the signal's noisiness.

Assuming that institutional investors obtain signals with more precision than individual investors and the signals regarding larger firms are of higher quality, we empirically test the cross-sectional relations between the magnitude (and the positive and negative parts) of third moment and the institutional ownership (and the market capitalization) in the Chinese stock market, and find significantly negative relations during the period 1998-2018 in a pooled regression. These relations are mainly driven by the sample in the recent period 2013-2018, as shown in both the Fama-Macbeth cross-sectional regressions and the pooled regressions. These findings are consistent with the model's prediction and imply a stabilizing role played by the institutional investors and larger firms in terms of third central moment. The empirical findings in the Chinese market are opposite to those in the US market. Our model could reconcile these seemingly contradicting facts by considering positive skewness and negative skewness separately.

Our model also implies a positive relationship between the third moment and the signal's skewness. To test this relationship, an appropriate proxy for the skewness of the signal needs to be obtained. We leave it for future research.

Appendix

A Monotonicity w.r.t. the Shape Parameter

Let $f(\alpha) = \alpha^3 E[H''(-\alpha z)]$. Suppose α is positive, let $y = \alpha z$, then

$$\begin{aligned} f(\alpha) &= \alpha^3 \int_{-\infty}^{\infty} H''(-\alpha z) 2\phi(z)\Phi(\alpha z) dz \\ &= \alpha^2 \int_{-\infty}^{\infty} H''(-y) 2\phi(y/\alpha)\Phi(y) dy. \end{aligned}$$

Its first order differentiation w.r.t. α is as follows:

$$\frac{df(\alpha)}{d\alpha} = \int_{-\infty}^{\infty} \alpha(2 + y^2/\alpha^2) H''(-y) 2\phi(y/\alpha)\Phi(y) dy.$$

Let $z = y/\alpha$, then the above derivative becomes

$$\begin{aligned}\frac{df(\alpha)}{d\alpha} &= \int_{-\infty}^{\infty} \alpha^2(2+z^2)H''(-\alpha z)2\phi(z)\Phi(\alpha z)dz \\ &= \alpha^2 E[(2+z^2)H''(-\alpha z)]\end{aligned}$$

Noting $2+z^2$ and $H''(-\alpha z)$ are both positive, the expectation of their product is positive. Because the change of variables in the integration are conducted twice, the above result also holds when α is negative. Q.E.D.

B Monotonicity w.r.t. the Volatility of Noise.

Differentiating $E(R^3)$ w.r.t. σ_ε gives

$$\frac{\alpha^3\sigma_\varepsilon^5}{(\sqrt{\omega^2+\sigma_\varepsilon^2})^3} E\Big[H''(-\alpha z)\Big(3\frac{\alpha^2(1+\alpha_0^2)}{\alpha_0^2(1+\alpha^2)} + 3 - \Big(1-\frac{\alpha^2}{\alpha_0^2}\Big)(2+z^2)\Big)\Big],$$

where $\alpha_0 = \frac{\delta}{\sqrt{1-\delta^2}}$ and $\alpha_0^2 > \alpha^2$. Let $g(\alpha,\alpha_0) = E[H''(-\alpha z)(3\frac{\alpha^2(1+\alpha_0^2)}{\alpha_0^2(1+\alpha^2)} + 3 - (1-\frac{\alpha^2}{\alpha_0^2})(2+z^2))]$. It is easy to see that $g(\alpha,\alpha_0)$ is a decreasing function of α_0^2. Hence,

$$g(\alpha,\alpha_0) \geq g(\alpha,\infty) = E\Big[H''(-\alpha z)\Big(3\frac{\alpha^2}{(1+\alpha^2)} + 1 - z^2\Big)\Big] > 0.$$

Note that $g(\alpha,\infty)$ is a univariate function of α, and the last inequality above could be verified through numerical integration. Hence, when $\alpha > 0$, the third moment is an increasing function of σ_ε; when $\alpha < 0$, the third moment is a decreasing function of σ_ε. Overall, the magnitude of the third moment is increasing with respect to the volatility of noise. Q.E.D.

References

[1] Albuquerque, Rui, 2012, Skewness in stock returns: reconciling the evidence on firm versus aggregate returns, *Review of Financial Studies* 25(5), 1630-1673.

[2] Azzalini, Adelchi, and Alessandra Dalla Valle, 1996, The multivariate skew-normal distribution, *Biometrika* 83(4), 715-726.

[3] Birnbaum, Zygmunt Wilhelm, 1942, An inequality for Mill's ratio, *Annals of Mathematical Statistics* 13(2), 245-246.

[4] Chen, Joseph, Harrison Hong, and Jeremy C. Stein, 2001, Forecasting crashes: Trading volume, past returns, and conditional skewness in stock prices, *Journal of Financial Economics* 61(3), 345-381.

[5] Dennis, Patrick, and Stewart Mayhew, 2002, Risk-neutral skewness: Evidence from stock options, *Journal of Financial and Quantitative Analysis* 37(3), 471-493.

[6] Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81(3), 607-636.

[7] Harvey, Campbell R., and Akhtar Siddique, 2000, Conditional skewness in asset pricing tests, *Journal of Finance* 55(3), 1263-1295.

[8] Hong, Harrison, and Jeremy C. Stein, 2003, Differences of opinion, short-sales constraints, and market crashes, Review of Financial Studies 16(2), 487-525.

[9] Jondeau, Eric, Qunzi Zhang, and Xiaoneng Zhu, 2019, Average skewness matters, *Journal of Financial Economics*, forthcoming.

[10] Sampford, Michael R, 1953, Some inequalities on Mill's ratio and related functions, *Annals of Mathematical Statistics* 24(1), 130-132.

[11] Xu, Jianguo, 2007, Price convexity and skewness, *Journal of Finance* 62(5), 2521-2552.