Commodity Return Predictability: Evidence from Implied Variance, Skewness and their Risk Premia[⊗]

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Abstract

This paper investigates the role of realized, implied and risk premium moments (variance and skewness) for commodities' future returns. We estimate these moments from high frequency and commodity futures option data that results in forward-looking measures. Risk premium moments are computed as the difference between implied and realized moments. We highlight, from a cross-sectional and time series perspective, the strong positive relation between commodity returns and implied skewness. Moreover, we emphasize the high performance of skewness risk premium. Additionally, we show that their portfolios exhibit the best risk-return tradeoff. Most of our results are robust to other factors such as the momentum and roll yield.

JEL codes: G13; G17.

Keywords: Commodity Forecast; Implied Volatility; Implied Skewness; Risk Premium.

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Abstract

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I. Introduction

Several existing studies support the view that investors are not only concerned about volatility but skewness as well (Kumar, 2007, 2009; Barberis and Huang, 2008; Mitton and Vorkink, 2007). As such, given their preferences for low volatility and positive skewness, investors might require a premium to hold high volatile and negatively skewed portfolios. Moreover, while recent research highlights the positive and negative predictive power of skewness for equity returns, in commodity markets, its evidences are narrow. Our paper contributes to this ongoing debate and sheds light on the role of these variance and skewness moments and their risk premia in commodity markets. Their understanding is of relevance to market participants, policy makers and academics, as well.

In this study, we explore the predictive ability of commodities' realized (physical), implied (risk-neutral) and risk premium moments over their returns. Specifically, we investigate whether the variance, skewness and their risk premia are able to forecast commodity' future returns, from both a time-series and a cross-section perspective. We compute the realized variance and skewness using high frequency data for each of the eight most liquid commodity futures contracts, namely, agricultural (corn, soybean, wheat), metal (copper, silver, gold) and energy (oil, natural gas) commodities. As Amaya et al. (2015) point out: "*... skewness (and kurtosis) measures computed from high-frequency data are likely to contain different information from those computed from daily data or from options*". Using the model-free approach of Bakshi et al. (2003), we then estimate the implied variance and skewness moments and finally, their risk premia as difference between risk-neutral and physical moments.

By taking into account the different information stemming from option and high-frequency data, we uncover several interesting results, especially about skewness. First, we show that from both a time-series and cross-section perspective, generally there is a strong positive and significant relation between implied skewness and future commodity returns. Regarding the skewness risk premium (SRP), while from the time-series perspective there is only a significant positive relation for agricultural commodities except corn, from the cross-section perspective, its portfolio exhibits the highest positive and significant performance. Particularly, a trading strategy entering long on the commodity portfolio with highest implied skewness (SRP) and short on the commodity portfolio with lowest implied skewness (SRP) yields an average annualized return of 17.21% (18.37%) with an annual volatility of 28% (25.9%). These findings empirically support the existence of arbitrage opportunities between commodity and derivative markets. We also find that although the strategy on variance risk premium (VRP) is weaker than that on implied skewness and SRP, it nevertheless provides a reasonable risk-return tradeoff. Second, by constructing two efficient frontiers, namely, with stocks and bonds, and with commodities, we highlight the importance of implied skewness and SRP portfolios from the U.S. investor's viewpoint. We show that these portfolios display the best risk-return tradeoff and relevant weights in the efficient frontier portfolios with equivalent risk as that of bond, equity and commodity markets. Third, from a cross-section perspective, we document a negative, but insignificant, relation between realized skewness and commodity returns. This finding suggests that investors require a premium to hold negatively skewed commodities and is in line Fernandez-Perez et al. (2018). However, from a time-series perspective, there is no predictability for realized skewness. Our results further indicate a positive relation between variance (realized and implied) of both metal and energy commodities and their future returns. Regarding agricultural commodities, our estimates point to a negative but not statistically significant relation.

The economic intuition of our physical moments' results, i.e., the high volatility and low skewness inducing higher expected returns, is given by models of Arditti (1967), Brunnermeier and Parker (2005) and Brunnermeier et al. (2007). The theoretical model of Bollerslev et al. (2009) supports our findings of positive returns for the VRP portfolio. With respect to the implied skewness and skewness risk premium, the insight is that, on the one hand, if risk-neutral skewness is higher than physical skewness, then it is expensive to hedge against negative outcomes relatively to positive outcomes. This is an indication that hedgers fear an asset drop. On the other hand, if risk-neutral skewness is lower than physical one, then it is comparatively expensive to hedge against the positive outcomes. In this case, it is likely that hedgers fear an asset spike.

The positive relation between implied skewness and future commodity returns is related to the informed trading and hedging views as shown in equity markets (Stilger et al., 2017; Xing et al., 2010). Particularly, if there are arbitrage opportunities and information differences between equity and derivative markets, then the positive implied skewness predicts positive future returns. These views are consistent with the demand-based option-pricing model of Gârleanu et al. (2009) where due to the short-selling constraints in equity markets, option market makers are unable to perfectly hedge their positions and, thus, their option demand influences its price. Instead, a negative relation between implied skewness and future returns is supported by skewness preference theory (Bali and Murray, 2013; Conrad et al., 2013). According to this theory, the intuition is that, in absence of arbitrage rules between stock and option markets, the same information should be reflected in both markets. Therefore, the positive implied skewness would predict negative expected returns.

The relation between the SRP and future returns could be explained by the asymmetric information setup where hedgers have superior information about asset outcomes. In this case, information held by hedgers (but not by other agents) about future outcomes is reflected on option prices. Thus, when hedgers fear a price drop they would charge a higher margin for the out-of-the-money puts comparing to out-of-the-money calls. As such, we would have a negative SRP since the left tail of risk-neutral probability distribution would be higher than that of physical distribution. When the negative anticipated outcome is revealed then prices will drop. The other way around, if hedgers fear a price spike then they would charge more for out-of-the-money calls, which would lead to a positive SRP. When the positive outcome is revealed, prices will go up. Therefore, we would have a positive relation between the SRP and future returns. It is worth noting that the asymmetric information setup is important for this relation, and make this explanation different from the model of Bollerslev et al (2009). If there were no superior information by hedgers, the drop or spike in prices would be immediate, and the predictive ability of SRP would not exist.

Our paper contributes to two strands of the literature. First, we add to the literature on predictability of realized and implied moments. Regarding stock markets, Amaya et al. (2015) and Choi and Lee (2015) find support for a theoretical negative relation between physical skewness and expected returns. Instead, the literature on predictability of implied skewness reports contradictory results. While Conrad et al. (2013) and Bali et al. (2011) find a negative relation between implied skewness and future returns, other studies document a positive relation (Rehman and Vilkov, 2012; Cremers and Weinbaum, 2010; Xing et al., 2010). The former studies rely on the existence of arbitrage opportunities between stock and options markets, whereas latter studies assume that both markets are reflecting same information, which leads to lower future returns. As for commodity markets, as far as we know, there is only one study which examines the role of realized skewness for predicting the future returns (Fernandez-Perez et al., 2018) and two studies which focus on the implied variance (Chatrath et al., 2016; Gao, 2017). By studying whether implied variance and skewness could forecast commodity's returns, we extend the literature on their mixed predictability findings for stock markets to a different asset class, where its evidences are either narrow or absent.

Second, we contribute to the literature on predictability of variance and skewness risk premia. Its empirical evidences point out to a positive relation between VRP and future returns in stock markets, namely, higher (lower) variance risk premium predicts higher (lower) future returns (Bollerslev et al., 2014; Bollerslev et al., 2009). Instead, for currency (Ornelas, 2017; Della Corte et al., 2016; Londono and Zhou, 2017) and commodity markets, especially oil market (Chevallier and Sevi, 2014; Triantafyllou et al., 2015; Ornelas and Mauad, 2017) the direction of this relation is controversial. Chevallier and Sevi (2014) document a negative relation between the VRP and crude oil's returns. Similarly, Triantafyllou et al. (2015) confirm the negative relation for agricultural commodities (wheat, maize and soybean). On the contrary, Ornelas and Mauad (2017) find a positive relation for both oil and gold commodities. These studies use as proxy of implied volatility, the Chicago Board Options Exchange (CBOE) volatility index relying on various exchange-traded securities. Our paper contributes to this literature by investigating the predictability of other implied moments using options on commodity futures contracts.

Our study further complements the literature on the role of skewness risk premium in stock markets (Harris and Qiao, 2017; Bali et al., 2016; Lehnert et al., 2014). These studies show that, from a cross-section perspective, there is a positive relation between SRP and equity returns.[1] We add to this literature by being the first paper, to the best of our knowledge, to investigate and provide strong evidence about the predictability of SRP and implied skewness for commodity returns.

The reminder of the paper is structured as follows: Section II describes the data. Section III and IV show the forecasting results from a time-series and cross-section perspective. Section VI presents the robustness tests. Section V concludes.

II. Data

Analysis of commodity return predictability relies on option and high frequency futures data from Thomson Reuters Tick History, and daily futures data from Bloomberg. Our sample consists of eight commodities that cover three main sectors, namely, agricultural (corn, soybeans and wheat), metal (copper, silver and gold) and energy (oil and natural gas), from

[1] It is worth noting that all these studies define differently the skewness risk premium. That is, as the realized skewness minus implied skewness, or physical minus risk-neutral skewness. As their definition is inverse versus ours, the sign of their relation is also inverse.

January 2008 to December 2016. For each commodity, we calculate daily returns and the one-month, two-month and three-month returns of the continuous first nearby futures contracts. We compute the realized variance and skewness by using the sums of 5-minute returns including the overnight returns as in Amaya et al. (2015). Following the well-known model-free approach of Bakshi et al. (2003), we then estimate the implied moments, namely, implied variance and skewness using one-month options on futures contracts. We annualize variance in the traditional way and skewness following Amaya et al. (2015). Finally, we define risk premium moments as the difference between implied and realized moments (Bollerslev et al., 2009).

Table 1 provides summary statistics for the realized (Panel A), implied (Panel B) and risk premium (Panel C) moments. For each of the agricultural, energy and metal commodity sectors, we present the mean and standard deviation, as well as the 25^{th} and 75^{th} quantiles. Considering the variance, we notice that the highest mean and standard deviation values are presented in Panel B showing the implied variance of commodity futures contracts. Regarding realized and implied skewness, as well as the skewness risk premium, from the mean and quantile statistics, we observe that over time these moments are changing their magnitude and sign. Moreover, for both quantiles, the highest (smallest) absolute values belong to the skewness risk premium in Panel C (implied skewness in Panel B). Their standard deviation also presents similar patterns in all three panels.

INSERT TABLE 1 HERE

III. Time-Series Predictability

In this section, we show the univariate outcomes on predictive ability of commodities' realized, implied and risk premium moments (variance and skewness) for their returns. We assess this predictability for the one-month, two-month and three-month returns. We then discuss whether their predictability holds for one-month returns when accounting for the impact of well-known commodity factors, i.e., roll yield and dollar index.

III.1 Univariate Results

This section addresses, from a univariate perspective, the predictive ability of variance and skewness for the commodity returns over one, two and three months. Specifically, we investigate their role by distinguishing between realized and implied moments, and considering their risk premia as well.

The regression specification for the realized moment's predictability is given by:

$$Ret_{i,h,t} = \alpha + \beta_1 * RM_{i,t-1} + \beta_2 * Ret_{i,h,t-h} \tag{1}$$

where $Ret_{i,h,t}$ is the return for each holding period h (one to three months) and commodity i starting at time t. $RM_{i,t}$ is the realized moment, i.e., variance or skewness, of each commodity i for a period of one-month ending at time t using the 5-minute returns.

Regressions cover the period from 2008 to 2016 on a rolling daily basis for our eight commodities, namely, agricultural (corn, soybeans, wheat), metal (copper, silver, gold) and energy (oil, natural gas) sectors. Given our strongly overlapping sample, we use the Hansen-Hodrick *t*-statistics. Table 2 presents the results for Equation (1).

Panel A shows a positive relation between realized variance of both metal and energy commodities and their future returns. However, only certain coefficients are statistically significant. In particular, we observe that copper's and oil's two-month and three-month returns are the easiest to forecast with adjusted R^2 around over 10%. Instead, the agricultural commodity estimates are negative, yet largely insignificant.

Looking at Panel B of Table 2, note that there is no clear relation between realized skewness and future returns. Our one-month return coefficients are mainly negative, whereas the three-month return coefficients are positive. The highlight is copper with two statistically significant coefficients for two-month and three-month returns and highest adjusted R^2.

INSERT TABLE 2 HERE

We next examine the predictability of implied moments using the following specification:

$$Ret_{i,h,t} = \alpha + \beta_1 * IM_{i,t-1} + \beta_2 * Ret_{i,h,t-h} \tag{2}$$

where $IM_{i,t}$ is the implied moment, namely, implied variance or implied skewness, of each commodity *i,* using options with one-month maturity at time *t*. Table 3 presents these findings.

When comparing Table 3's results with those of realized moments from Table 2, we observe that although coefficients have a similar pattern, implied moments are generally better at predicting metals' returns. Panel A shows the existence of a positive and statistically significant relation between implied variance of metals and their three-month returns. Moreover, it emphasizes the positive and strongly significant predictability of copper's implied variance for all return horizons. Nevertheless, Panel A documents an insignificant relation between implied variance of agriculture and energy commodities and their returns.

In Panel B, we highlight the remarkably good predictability of implied skewness for commodity returns. In particular, note that for the one-month returns, six out of the eight commodities have positively significant coefficients. These commodities include corn, soybeans, wheat, gold, oil and natural gas. Additionally, implied skewness' predictability of corn, wheat and natural gas carries on to be significant for their two-month and three-month returns as well.

INSERT TABLE 3 HERE

We further investigate the predictive ability of variance and skewness risk premia as following:

$$Ret_{i,h,t} = \alpha + \beta_1 * MRP_{i,t-1} + \beta_2 * Ret_{i,h,t-h} \tag{3}$$

where $MRP_{i,t}$ is the moment risk premium which is computed using one-month options and 5-minute returns on the previous one month implied moment for each commodity *i,* using options prices at time *t*, and realized moments from one-month before *t*.

Table 4 shows that, generally, risk premium coefficients are positive. However, many of these coefficients are statistically insignificant. These findings indicate that the predictive power of implied and realized moments taken together, namely, of the risk premium moments, is less strong than when separately considering the implied moments, as shown in Table 3. For instance, in Panel A, among agricultural VRP coefficients, only those of corn are statistically significant, although negatively. These outcomes are contrary to Triantafyllou et al. (2015) who document a negative relation between VRP of corn, soybeans and wheat and their two-month returns. We point out that authors define VRP opposite to our study, namely, as difference between realized and implied variance. As such, according to our definition, their negative relation translates into a positive relation in our analyses. The different conclusions might be due to different sample period, e.g., their samples ends in December 2011 and different computation of VRP. That is, authors focus on the two-month VRP and estimate the two-month realized variance using daily prices, whereas our paper considers the one-month VRP and high frequency data for computation of the realized variance. Regarding metal commodities, we observe that their coefficients are positive and only statistically significant for copper. Gold's positive coefficients are in line with Ornelas and Mauad (2017) who document a positive relation between CBOE Gold Volatility Index (GVX) and its returns. However, their relation is statistically significant whereas ours is not. Our silver coefficients are also positive and nearly statistically significant. Coefficients of oil are negative but statistically insignificant. Their sign is in line with Chevallier and Sevi (2014) who use the CBOE Crude Oil Volatility Index (OVX).

Turning to Panel B, we observe that, in general, there is a positive but insignificant relation between commodities' skewness risk premium and their future returns. Exceptions are the positive and statistically significant one-month return coefficients of agricultural commodities (wheat and soybean) and copper's negatively significant two-month return coefficient.

INSERT TABLE 4 HERE

A possible explanation for our findings might be related to computation of skewness risk premium. Specifically, the sampling frequency of realized skewness. Several studies analyzing the skewness risk premium use daily U.S. equity returns for computation of realized skewness (Harris and Qiao, 2017; Bali et al., 2016; Lehnert et al., 2014). Nevertheless, as Amaya et al. (2015) has stated: "*We conclude from these general results for the third and fourth realized moments that we can expect very different estimates of skewness and kurtosis depending on the frequency of data used to estimate these moments. Skewness estimates from moving windows of daily or weekly data are likely to have different averages than skewness measures constructed from intraday data*". As such, it is most likely that, by using different periodicities in estimating the realized skewness, skewness risk premium could provide different results. This conclusion is in contrast with the second moment, i.e., variance, where a higher frequency of returns leads to better and more efficient estimates. In our study, we compute skewness risk premium using high frequency (5-minute) returns. Thus, using another sampling frequency might provide different results. As addressing the best sampling frequency for estimation of skewness risk premium is not the scope of our paper, we let this debate for the future research.

III.2 Multivariate Results

In this section, we add several control variables to the previous time-series analysis. Specifically, we use two known commodity predictors such as the roll yield and dollar index. Roll yield is the yield obtained from the rolling of a short-term futures contract to a long-term futures contract. Hence, the yield an investor receives when its futures contract position converges to spot price. Note that the roll yield is inversely correlated with the slope of the term structure of futures contracts. Following Arnott et al. (2015), we use the one-year roll yield that relies on the first nearby contract and its next-year counterpart, namely:

$$RY_{i,t} = LN\left(\frac{C_0}{C_1}\right) \tag{4}$$

where $RY_{i,t}$ is the roll yield of commodity i at time t, C_0 is the price of first nearby (front-end) contract of commodity i and C_1 is the price of its next-year counterpart. It is worth mentioning that many papers use the slope between nearest two contracts in the curve to calculate the roll yield (e.g. Fernandez-Perez et al., 2018). However, as pointed out by Arnott et al. (2014), this method has several drawbacks, including seasonality and homogeneity problems across commodities' available contracts. Further, it is well-known that commodity prices move in the opposite direction from the U.S. dollar. For this reason, the other control variable is the dollar index measuring the value of the U.S. dollar against a basket of currencies. The higher is this index, the higher is the U.S. dollar value against the basket. We use the one-month return of the dollar index.

We start this analysis by using the realized moment's predictability with the following specification:

$$Ret_{i,h,t} = \alpha + \beta_1 * RM_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,h,t-h} \tag{5}$$

where $RM_{i,t}$ is the realized moment of commodity *i* at time *t* and $Ret_{i,h,t}$ is commodity *i* return for the window t - h to *t*, $RY_{i,t}$ is the roll yield of commodity i at time *t*, $DXY_{i,t}$ is the dollar index return at time *t*. Table 5 shows the results considering the one-month returns.

Panel A reveals similar results to the univariate case, namely, realized variance has no predictability for one-month returns, except those of natural gas. The roll yield coefficients are generally negative and statistically significant, except those of precious metals that are positive. In line with commodity literature (e.g., Chen et al., 2014), the dollar index coefficients are also mainly negative but with a limited statistical significance. Examining Panel B of Table 5, predictability of realized skewness when including commodity factors, we find akin results as in Panel A.

INSERT TABLE 5 HERE

Table 6 presents the implied moment's predictability with the following specification:

$$Ret_{i,h,t} = \alpha + \beta_1 * IM_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,h,t-h} \tag{6}$$

where $IM_{i,t}$ is the implied moment of commodity *i* at time *t* and $Ret_{i,h,t}$ is commodity *i* return for the window t - h to *t*, $RY_{i,t}$ is the roll yield of commodity *i* at time *t*, $DXY_{i,t}$ is the dollar index return at time *t*.

Panel A of Table 6 shows the predictability of implied variance for commodity returns. Copper's coefficient is again positive and significant, as in the univariate approach. Corn's coefficient is now statistically significant, yet negative. Panel B highlights the good predictive power of implied skewness for commodity returns in presence of additional commodity factors. Note the statistical significance for most of our coefficients such as the corn, wheat, gold and natural gas. Exceptions are the coefficients of soybean and oil, which are no longer significant. Surprisingly, copper's coefficient is the only negative and statistically significant coefficient.

INSERT TABLE 6 HERE

We next analyze the predictive role of moment risk premia. The specification is the following:

$$Ret_{i,h,t} = \alpha + \beta_1 * MRP_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,h,t-h} \tag{7}$$

where $MRP_{i,t}$ is the moment risk premium of commodity *i* at time *t* and $Ret_{i,h,t}$ is commodity *i* return for the window t - h to *t*, $RY_{i,t}$ is the roll yield of commodity *i* at time *t*, $DXY_{i,t}$ is the dollar index return at time *t*. Table 7 presents these results.

In Panel A, we find that even after controlling for commodity factors, VRP coefficients of corn and copper are still significant. Instead, looking at Panel B, none of skewness risk premium's coefficients is significant.

INSERT TABLE 7 HERE

Overall, this section's findings emphasize that, from a time-series perspective, implied skewness is our best variable to forecast commodity returns. The roll yield has also a good forecast ability.

IV. Cross-Section Predictability

In this section, we begin by exploring the performance of several trading strategies relying on realized, implied and risk premium moments, i.e., variance and skewness. We then compare their performance with that of other known strategies such as equal weighted portfolio, roll yield and momentum. Finally, we point out the relevance of our portfolios from the viewpoint of an U.S. investor.

IV.1 Portfolio Analysis

This section aims to evaluate the performance of commodity portfolios as investment strategies. These portfolios are build using six types of measures, namely, realized variance, realized skewness, implied variance, implied skewness, variance risk premium and skewness risk premium. Specifically, we build long-short (cash neutral) portfolios by taking a long position on top 25% measures (realized, implied and risk premium moments), and a short position on bottom 25% measures. As such, our portfolio contains two long and two short commodity futures contracts with equal weights. We form these portfolios every day and hold them for one month (21 overlapping business days). Thus, each portfolio has a weight of 1/21. To keep this weight, we assume daily rebalancing.

To benchmark these strategies against other well-known commodity strategies, we build three additional benchmark portfolios. First benchmark portfolio is the simple equally weighted average of the eight commodities in our sample. This is a long-only portfolio. Second benchmark portfolio relies on past one-year performance of our commodities, i.e., the one-year momentum, buying past one-year winners and selling past one-year losers. Finally, last benchmark portfolio uses the one-year roll yield (inverse of slope's futures contract term structure), buying the highest one-year roll yield and selling the lowest one-year roll yield. The momentum and roll yield portfolios are long-short portfolios with same characteristics of the main portfolios in terms of formation rules, rebalancing, etc., except their different sorting criteria. Table 8 presents the portfolio return statistics.

When exploring Table 8, we observe that implied skewness and SRP portfolios exhibit the highest positive and significant performance with their Sharpe ratio over 0.6 and 0.7, respectively. Specifically, these portfolios of implied skewness and SRP yield an average yearly return of 17.21% and 18.37% with an average yearly volatility of 28% and 25.9%, respectively.

We further document an insignificant performance of our other main and benchmark strategies. Although their performance is insignificant, we next discuss whether these strategies conform to commodities' literature. For instance, the negative return of our realized skewness portfolio is consistent with Fernandez-Perez et al. (2018).[2] However, our results are statistically insignificant, whereas previous authors find statistically significant results. Their high number of commodities, different sample period, as well as the use of daily data for estimation of realized skewness might explain this difference in statistical significance of results. The negative mean return of momentum portfolio is consistent with several studies (Daniel and Moskowitz, 2016; Bianchi et al., 2015; Moskowitz et al., 2012). For instance, Bianchi et al. (2015) find a consistent and strong reversal pattern of commodities' momentum profits from 12 to 30 months. Daniel and Moskowitz (2016) show that equity momentum strategies experience negative returns, especially in panic states such as financial crises or market crashes, when volatility is high.

Further, the mean return of roll yield portfolio is negative. Although this finding is not in line with existing literature on commodity markets, the negative return could be due to the fact that most commodities have been in backwardation since mid-2012 (Arnott et al., 2014). Indeed, dividing our sample in two sub-periods, namely, January 2008 to June 2012 and July 2012 to December 2017, we find a positive and negative mean return of the roll yield portfolio during the former and latter sub-period. That is, while during the first sub-period, the mean portfolio return is 3.10% with a *t*-statistic of 0.16, during the second sub-period, it is -23.79% with a *t*-statistic of -2.04.

INSERT TABLE 8 HERE

Figure 1 displays the performance of six commodity strategies using the realized, implied and risk premium moments, namely, variance and skewness. It highlights, starting in 2012, the increasing performance of investment strategies using the implied skewness and SRP. Moreover, although until 2012 the VRP strategy performs better, towards the beginning of 2015 implied skewness and SRP strategies surpass it. Figure 1 also emphasizes that, generally, worst performances are due to strategies relying on variance (realized and implied) and realized skewness. Exception is the Global Financial Crisis when the performance of realized skewness strategy is similar to that of VRP.

[2] Note that authors build the realized skewness portfolio by buying commodities with low skewness and selling those with high skewness. Thus, the positive return of their portfolio means a negative return for our portfolio.

INSERT FIGURE 1 HERE

IV.2 Correlation and Portfolio Optimization

This section analyzes the usefulness of our portfolios from the viewpoint of a diversified U.S. investor. We first present the correlation among commodity strategies and two other asset classes, namely, stocks and bonds. As proxies for equity and bond markets, we use the S&P 500 and U.S. JP Morgan Global Bond indices. Second, using the same portfolios, we build the Markowitz efficient frontier.

Table 9 presents the correlation matrix among portfolios. We observe a high correlation between portfolios of realized variance and, both implied variance and skewness, with coefficients of 78% and 58%, respectively. These implied moments also exhibit a high correlation of 41%. As regards the correlation among risk premium portfolios, notice the existence of a negative correlation between VRP and, both implied skewness and skewness risk premium. SRP portfolio is positively correlated with implied skewness portfolio, with the correlation coefficient being 43%, and it has an extreme negative correlation of -93% with realized skewness portfolio. Our portfolios have very low correlation with equity and bond markets, and equal weighted commodities' portfolio (EW). Thus, this low correlation with our portfolios makes them useful in portfolio construction.

INSERT TABLE 9 HERE

To build the efficient frontiers from the U.S. investor's perspective, we further estimate a Markowitz optimization procedure. Our commodity portfolios are built using long-short futures contract strategies and hence, by construction, they are excess returns. Regarding the U.S. bond and equity markets, we compute their excess returns using as a proxy for the risk free rate the Barclays 3-month T-bill index. In the optimization procedure, we do not allow for leverage or short positions, thus, our weights are within the interval zero and one. Moreover, for adequate treatment of the negative return portfolios, we invert the long and short positions and then to highlight this inversion, we add the minus sign in front of portfolio's name. By doing this adjustment, note that the "minus" momentum portfolio is actually the reversal portfolio, the "minus" roll yield relies on the slope of futures term structure and the "minus" realized skewness indicates buying commodities with low skewness and selling those with high skewness, as in Fernandez-Perez et al. (2018).

Figure 2 shows two efficient frontiers and our portfolios with their annualized risk and return. Specifically, an efficient frontier with the stock and bond markets and one including all our assets. As expected, the latter efficient frontier is far above the former one. Figure 2 points out that best risk-return tradeoff belongs to the SRP and implied skewness portfolios and bond market. The realized and implied variance portfolios present a weaker risk-return tradeoff. Instead, momentum and roll yield portfolios display the highest risk, with similar Sharpe ratio to stock market. We emphasize that SRP portfolio dominates them, that is, exhibits a higher return with lower risk.

INSERT FIGURE 2 HERE

Furthermore, we analyze the weight of various portfolios on the efficient frontier. Table 10 presents the weight of three portfolios having an equivalent risk to that of bond, equity and commodity markets. The first efficient portfolio presents the same risk as the U.S. bond market, i.e., a volatility of 5.05%. Due to low volatility, this portfolio's allocation is 59% in bond market and around 10% in stock market, implied skewness, SRP and VRP. The other two efficient portfolios display the same risk as the U.S. equity and EW commodity markets, i.e., an annual volatility of 22.19% and 21.55%, respectively. In general, their portfolios represent 60% in SRP, 28% in implied skewness and around 7% in the reversal portfolio, namely, the "minus" momentum.

INSERT TABLE 10 HERE

In sum, two of our portfolios – SRP and implied skewness – prove to be beneficial from optimization's viewpoint, with relevant risk-return tradeoff and weights in both low and high volatility portfolios.

V. Robustness

In this section, we examine the portfolio return statistics using a double sorting strategy and considering various holding periods. We then present the portfolio return statistics using single and double sort strategies depending on the innovations for each of the realized, implied and risk premium moments of our commodities. Additionally, we examine their correlation matrix. Finally, we estimate factor regressions using both original variables and their returns; we present the frequency of commodities entering into portfolios and briefly mention other robustness tests done.

Table 11 presents the portfolio return statistics for double sorting strategies with one-month holding period. Given the significant performance of implied skewness and skewness risk premium in Section IV, we construct double sort portfolios considering them together, as well as on each of them and variance risk premium. All three strategies earn positive annual returns with a significant and rather higher performance than strategies in Section IV. Specifically, the skewness risk premium and both variance risk premium and implied skewness double sorting portfolios have the best performance with a Sharpe ratio of 0.75 and 0.65, respectively. In addition, we build double sorting portfolios considering the roll yield and each of the implied and risk premium moments. Their portfolio return statistics are insignificant and thus, we do not report them.

INSERT TABLE 11 HERE

Table 12 shows the portfolios return statistics using single and double sorting strategies for a holding period of two weeks (Panel A) and two months (Panel B), respectively. Panel A's results emphasize the high significant performance of the single and double sorting portfolios as shown in Section IV and previous Table 11. In addition, notice that for two-week holding period, variance risk premium has a significant performance with a Sharpe ratio of 0.73. Instead, the performance of double-sorted portfolio on skewness risk premium and implied skewness is insignificant. Considering a longer holding period, namely, two-month, Panel B shows that except momentum's reversal portfolio, none of the other trading strategies is significant.

INSERT TABLE 12 HERE

In Section IV, we document a high correlation among few commodity portfolio returns. Thus, as an alternative robustness test, following Chang et al. (2013), we also use the commodity innovations for estimation of all our analyses. Particularly, to obtain these innovations, we fit an autoregressive moving average, ARMA (1, 1) model for each of the realized, implied and risk premium moments of commodities, as well as for their roll yield and momentum. As such, we use these ARMA residuals as time-series innovations. Our univariate, multivariate and cross-sectional results using innovations are consistent with those of the realized, implied and risk premium moments. Due to space constraints, we only report the correlation matrix and portfolio return statistics with one-month holding period. All other results are available on request. Table 13 shows the existence of a low correlation among our variables, except for a high negative correlation between implied skewness and skewness risk premium.

INSERT TABLE 13 HERE

Table 14 highlights the strong cross-sectional findings for the implied skewness, variance risk premium together with those for the double sorting strategies. These results confirm the previous ones from Section IV and Table 11.

INSERT TABLE 14 HERE

We next investigate whether the profitability of realized, implied and risk premium moments is solely a compensation due to exposure to benchmark factors such as equally weighted portfolio, momentum, and roll yield factors. Results of these benchmark strategies in Table 8 show that their returns are not statistically significant. Thus, using them in a factor regression may not make sense. Instead, when using their innovations, only the performance of roll yield is significant. Nevertheless, in Panel A and B of Table 15, we present the alpha of these strategies using both original variables and their ARMA (1, 1) innovations. In Panel A, we observe that coefficients of the roll yield are negatively significant and that alpha of implied skewness and SRP is no longer significant. Therefore, we have a surprising situation where, although the roll yield strategy itself is not statistically significant, it helps to explain predictability of our skewness strategies.

INSERT TABLE 15 HERE

A possible explanation for these findings might be related to the backwardation characteristics of its portfolio. Indeed, examining the strategies on the second and third quartiles in the Appendix, notice from Table A. 1 and Table A. 2 that although the roll yield strategy is significant, it is unable to explain our trading strategies. Moreover, Figure A. 1 from the Appendix points out that for implied skewness, skewness risk premium and variance risk premium, the mean of the first and fourth quartile returns is positive and negative, respectively. As such, although the mean of all four quartile returns is insignificant, our trading strategies going long and short on the first and fourth quartile are significant and thus, provide reliable findings. The characteristics of the four quartile portfolios have not been reported due their insignificant performance. When using innovations, Panel B also shows the negatively significant roll yield strategy. However, the alphas of implied skewness and double sorting portfolios are significant and thus, the roll yield does not explain our strategies.

Given that few commodities could consistently enter the long and short portfolios and thus, could drive their returns, Figure 3 displays their frequency. Specifically, it shows the frequency of commodities in the long and short portfolios for each of our realized, implied and risk premium moments, as well as that of the momentum and roll yield. Note that the roll yield portfolio favors commodities with the highest positive roll yield and lowest negative roll yield without giving much attention to other commodities. In particular, most times, we would enter a long position on corn and soybeans commodities and a short position on wheat and natural gas commodities. As such, we once again confirm that after the 2012, backwardation of commodities affects our results and thus, we obtain a negative return for roll yield portfolio in Table 8. In general, commodity frequencies are way below 50% and therefore, over time, various commodities enter the long and short portfolios of our variables.

INSERT FIGURE 3 HERE

We further consider additional multivariate regressions as in Section III that include several other control variables. Taking, for instance, the realized variance's predictability in Equation (1), we also add as control variable the commodity's individual realized skewness and kurtosis. For the realized skewness's predictability, we control for the commodity's individual realized variance and kurtosis. We estimate similar regressions for the implied and risk premium moments shown in Table 6 and Table 7. In addition to the one-month return, we forecast the second-month and third-month return. Moreover, during our analyses we also consider the one-month momentum and roll yield. Furthermore, we control for the Fama and French (1993) factors and the momentum factor. In all cases, results are very similar. Due to space constraints, we do not report these results. They are available on request.

Furthermore, we examine the role of realized and implied kurtosis, as well as that of kurtosis risk premium in predicting commodity returns. Generally, these findings are insignificant and thus, we do not present them, but they are available on request.

VI. Conclusion

This paper provides new empirical evidence on the relation between commodity futures returns and realized, implied and risk premium moments. To estimate the realized and implied variance and skewness, we use high frequency and options on futures data, respectively. Risk premium moments are defined as the difference between the realized and implied moments.

We shed light on the predictive ability of implied skewness and skewness risk premium from both a time series and cross-sectional perspective. Specifically, we highlight the superior predictive power of implied skewness over that of realized variance and skewness as well as the implied variance and variance risk premium from both perspectives. Moreover, we show that a long and short position on the portfolio with highest and respectively lowest implied skewness and risk premium skewness yields an average return of around 17% and 18%, with a volatility of around 28% and 26%. Furthermore, from a U.S. investor's viewpoint, these portfolios exhibit the best risk-return tradeoff and relevant weights in the efficient frontier portfolios with the same risk as that of bond, equity and commodity markets.

Our findings are economically significant and factors such as the momentum and roll yield explain them only partially. All taken together, the time-series and cross-sectional results are robust to other various controls and portfolio selection measures.

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TABLES

Table 1 - Summary Statistics

This Table presents summary statistics for each of the commodities covering the period from January 2008 to December 2016. It provides the mean, standard deviation (Std. Dev.), 25^{th} quantile and 75^{th} quantile. Panels A and B show these statistics for the realized and implied variance and skewness, and Panel C presents them for the variance and skewness risk premia.

Panel A. Realized Moments

	Variance				Skewness			
	Mean	Std. Dev.	25%	75%	Mean	Std. Dev.	25%	75%
Corn	0.007	0.004	0.004	0.010	0.13	1.86	-0.73	1.17
Soybeans	0.006	0.004	0.003	0.009	-0.18	2.16	-1.34	1.16
Wheat	0.013	0.008	0.007	0.018	0.24	1.09	-0.43	0.87
Copper	0.008	0.008	0.004	0.009	-0.46	3.46	-2.57	1.47
Silver	0.008	0.005	0.005	0.010	-0.19	3.42	-2.32	1.72
Gold	0.003	0.003	0.002	0.004	-0.41	3.82	-2.17	1.86
Oil	0.012	0.013	0.004	0.013	-0.42	2.32	-1.79	1.01
Natural Gas	0.014	0.007	0.009	0.020	-0.10	2.70	-1.93	1.75

Panel B. Implied Moments

	Variance				Skewness			
	Mean	Std. Dev.	25%	75%	Mean	Std. Dev.	25%	75%
Corn	0.010	0.008	0.005	0.014	0.32	0.77	-0.16	0.73
Soybeans	0.006	0.005	0.003	0.008	-0.21	0.82	-0.57	0.29
Wheat	0.009	0.006	0.005	0.013	0.49	0.69	0.07	0.96
Copper	0.018	0.019	0.005	0.022	-0.34	1.25	-1.15	0.35
Silver	0.009	0.006	0.005	0.011	-0.11	0.35	-0.33	0.07
Gold	0.004	0.003	0.002	0.004	-0.20	0.43	-0.43	0.07
Oil	0.013	0.011	0.007	0.016	0.01	0.50	-0.29	0.21
Natural Gas	0.014	0.007	0.009	0.018	0.40	0.22	0.27	0.51

Panel C. Moment Risk Premia

	Variance				Skewness			
	Mean	Std. Dev.	25%	75%	Mean	Std. Dev.	25%	75%
Corn	0.003	0.006	0.000	0.004	0.18	2.09	-0.98	1.30
Soybeans	0.000	0.004	-0.002	0.002	-0.03	2.30	-1.42	1.39
Wheat	-0.003	0.008	-0.006	0.001	0.25	1.30	-0.57	1.07
Copper	0.011	0.018	0.001	0.014	0.13	3.82	-2.25	2.54
Silver	0.001	0.004	-0.001	0.002	0.07	3.39	-1.85	2.09
Gold	0.000	0.002	0.000	0.001	0.21	3.79	-1.94	1.87
Oil	0.002	0.006	0.000	0.004	0.43	2.30	-1.03	1.81
Natural Gas	0.000	0.005	-0.003	0.002	0.50	2.74	-1.40	2.39

Table 2 - Realized Moment's Predictability

This Table shows the results of 48 regressions, $\mathrm{Ret}_{i,h,t} = \alpha + \beta_1 * \mathrm{RM}_{i,t-1} + \beta_2 * \mathrm{Ret}_{i,h,t-h}$, where $\mathrm{RM}_{i,t}$ is the realized moment of commodity i at time t and $\mathrm{Ret}_{i,h,t-h}$ is commodity i return for the window t - h to t. The dependent variables are the returns of the commodity futures for h = one, two and three months. Each regression has as independent variables the realized moment of commodity returns using 5-minute returns, the lagged dependent variable and a constant. All the independent variables are lagged. In Panel A, the independent variable is realized variance and in Panel B, is the realized skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The t-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Individual Realized Variance

Dependent Variable	Coefficients			Adjusted R^2		
	Returns			Returns		
	1st Month	2nd Month	3rd Month	1st Month	2nd Month	3rd Month
Corn	-1.13	-1.31	-2.10	0.4%	0.3%	0.6%
	(-0.46)	(-0.31)	(-0.36)			
Soybeans	-1.95	-2.02	-4.77	1.0%	0.6%	3.9%
	(-1.02)	(-0.56)	(-1.08)			
Wheat	-0.92	-0.86	-1.20	1.1%	4.9%	2.1%
	(-0.78)	(-0.60)	(-0.73)			
Copper	0.70	4.20*	7.81***	3.8%	10.2%	12.9%
	(0.49)	(1.71)	(2.96)			
Silver	0.58	3.42	7.87*	0.1%	1.4%	4.8%
	(0.22)	(0.84)	(1.94)			
Gold	0.12	2.53	5.17	2.4%	2.1%	2.9%
	(0.04)	(0.67)	(1.24)			
Oil	1.04	3.61*	5.65**	4.9%	9.0%	11.2%
	(0.94)	(1.73)	(1.99)			
Natural gas	-0.93	0.28	2.96	0.6%	0.0%	1.2%
	(-0.45)	(0.07)	(0.59)			

Panel B. Individual Realized Skewness

Dependent Variable	Coefficients			Adjusted R2		
	Returns			Returns		
	1st Month	2nd Month	3rd Month	1st Month	2nd Month	3rd Month
Corn	0.005	0.004	0.002	0.9%	0.4%	0.4%
	(1.04)	(0.53)	(0.32)			
Soybeans	-0.003	0.003	0.003	0.4%	0.3%	2.1%
	(-0.92)	(0.73)	(0.71)			
Wheat	-0.004	-0.0001	0.002	0.8%	4.6%	1.7%
	(-0.64)	(0.00)	(0.15)			
Copper	-0.0003	0.004**	0.005*	3.4%	5.5%	2.2%
	(-0.22)	(2.04)	(1.73)			
Silver	-0.001	0.0004	0.001	0.1%	0.1%	0.2%
	(-0.53)	(0.17)	(0.21)			
Gold	0.001*	0.0003	0.001	3.3%	1.1%	0.2%
	(1.68)	(0.18)	(0.30)			
Oil	-0.001	0.001	0.006	3.5%	1.7%	1.4%
	(-0.20)	(0.15)	(0.93)			
Natural gas	-0.003	-0.004	-0.001	0.6%	0.3%	0.4%
	(-0.80)	(-0.66)	(-0.18)			

Table 3 - Implied Moment's Predictability

This Table shows the results of 48 regressions, $\mathrm{Ret}_{i,h,t} = \alpha + \beta_1 * \mathrm{IM}_{i,t-1} + \beta_2 * \mathrm{Ret}_{i,h,t-h}$ where $\mathrm{IM}_{i,t}$ is the implied moment of commodity *i* at time *t* and $\mathrm{Ret}_{i,h,t-h}$ is commodity *i* return for the window *t* - *h* to *t*. The dependent variables are the returns of the commodity futures for h = one, two and three months. Each regression has as independent variables the implied moments of our eight commodities, the lagged dependent variable and a constant. All the independent variables are lagged. Implied moments are calculated using one-month options. In Panel A, the independent variable is implied variance and in Panel B, is the implied skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Individual Implied Variance

Dependent Variable	Coefficients			Adjusted R^2		
	Returns			Returns		
	1st Month	2nd Month	3rd Month	1st Month	2nd Month	3rd Month
Corn	-1.75	-2.30	-2.75	2.1%	1.8%	1.9%
	(-1.56)	(-1.42)	(-1.28)			
Soybeans	-1.12	-2.10	-4.16	0.5%	0.9%	3.9%
	(-0.60)	(-0.54)	(-0.78)			
Wheat	-0.55	-1.39	-2.26	0.7%	5.0%	2.5%
	(-0.37)	(-0.80)	(-0.93)			
Copper	1.09***	2.33***	3.51***	9.0%	15.8%	16.5%
	(2.56)	(3.12)	(2.90)			
Silver	1.59	4.38*	7.24***	0.9%	3.3%	6.2%
	(0.89)	(1.65)	(2.49)			
Gold	0.65	3.56	6.40**	2.5%	3.0%	4.5%
	(0.25)	(1.17)	(2.02)			
Oil	1.20	3.91	5.53	4.9%	7.9%	8.2%
	(1.11)	(1.62)	(1.41)			
Natural gas	1.52	2.43	5.24	0.9%	0.8%	2.8%
	(0.47)	(0.50)	(0.82)			

Panel B. Individual Implied Skewness

Dependent Variable	Coefficients			Adjusted R^2		
	Returns			Returns		
	1st Month	2nd Month	3rd Month	1st Month	2nd Month	3rd Month
Corn	0.02***	0.04***	0.06***	3.7%	5.7%	8.8%
	(2.44)	(2.75)	(2.96)			
Soybeans	0.01*	0.02*	0.02	2.0%	1.9%	3.2%
	(1.93)	(1.66)	(1.43)			
Wheat	0.03***	0.04***	0.05***	4.3%	9.3%	6.5%
	(2.78)	(2.97)	(3.10)			
Copper	-0.01	-0.01	-0.0004	4.0%	4.6%	1.2%
	(-1.07)	(-0.59)	(-0.05)			
Silver	0.01	0.02	0.01	0.1%	0.2%	0.2%
	(0.34)	(0.32)	(0.18)			
Gold	0.02*	0.02	0.03	4.3%	2.8%	1.4%
	(1.93)	(1.23)	(1.03)			
Oil	0.04*	0.07	0.10	6.1%	6.1%	6.1%
	(1.81)	(1.52)	(1.36)			
Natural gas	0.14***	0.17**	0.16*	5.1%	4.1%	2.9%
	(2.77)	(2.23)	(1.87)			

Table 4 - Moment Risk Premium's Predictability

This Table shows the results of 48 regressions, $\mathrm{Ret}_{i,h,t} = \alpha + \beta_1 * \mathrm{MRP}_{i,t-1} + \beta_2 * \mathrm{Ret}_{i,h,t-h}$ where $\mathrm{MRP}_{i,t}$ is the moment risk premium of commodity *i* at time *t* and $\mathrm{Ret}_{i,h,t-h}$ is commodity *i* return for the window t - h to *t*. The dependent variables are the returns of the commodity futures for h = one, two and three months. Each regression has as independent variables the moment risk premium of our eight commodities, the lagged dependent variable and a constant. All the independent variables are lagged. Moment risk premia are calculated using one-month options and 5-minute returns on the previous one month. In Panel A, the independent variable is the variance risk premium and in Panel B, is the skewness risk premium. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The t-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Individual Variance Risk Premium

Dependent Variable	Coefficients			Adjusted R^2		
	Returns			Returns		
	1st Month	2nd Month	3rd Month	1st Month	2nd Month	3rd Month
Corn	-2.05*	-2.75*	-3.05*	2.0%	1.8%	1.7%
	(-1.71)	(-1.72)	(-1.94)			
Soybeans	0.46	-0.84	-0.60	0.1%	0.2%	2.0%
	(0.31)	(-0.24)	(-0.16)			
Wheat	0.64	0.05	-0.21	0.8%	4.6%	1.7%
	(0.66)	(0.04)	(-0.17)			
Copper	1.20***	2.17***	3.06***	8.9%	12.0%	9.9%
	(2.87)	(3.04)	(2.58)			
Silver	3.17	5.45	5.74	1.3%	2.0%	1.6%
	(1.64)	(1.63)	(1.48)			
Gold	2.23	4.36	5.61	2.7%	1.8%	0.9%
	(0.78)	(0.88)	(0.77)			
Oil	-0.86	-3.47	-6.05	3.7%	3.2%	3.9%
	(-0.36)	(-0.91)	(-1.28)			
Natural gas	5.31	4.39	4.11	3.5%	1.2%	1.1%
	(1.39)	(0.94)	(0.62)			

Panel B. Individual Skewness Risk Premium

Dependent Variable	Coefficients			Adjusted R^2		
	Returns			Returns		
	1st Month	2nd Month	3rd Month	1st Month	2nd Month	3rd Month
Corn	-0.0002	0.003	0.01	0.2%	0.3%	1.0%
	(-0.04)	(0.41)	(0.90)			
Soybeans	0.004*	0.0003	0.0000	1.3%	0.1%	1.9%
	(1.68)	(0.09)	(0.00)			
Wheat	0.01*	0.01	0.01	2.6%	6.0%	2.7%
	(1.80)	(1.17)	(0.95)			
Copper	-0.0004	-0.004**	-0.004	3.4%	5.7%	2.1%
	(-0.28)	(-2.02)	(-1.53)			
Silver	0.001	-0.0002	-0.001	0.1%	0.1%	0.2%
	(0.61)	(-0.09)	(-0.19)			
Gold	-0.001	0.0000	-0.0003	3.0%	1.1%	0.1%
	(-1.40)	(0.00)	(-0.17)			
Oil	0.003	0.003	-0.002	3.8%	1.9%	0.9%
	(0.90)	(0.67)	(-0.25)			
Natural gas	0.004	0.01	0.003	0.8%	0.5%	0.5%
	(1.07)	(0.84)	(0.31)			

Table 5 - Realized Moment's Predictability with Control Variables

This Table shows the results of 16 regressions, $\text{Ret}_{i,h,t} = \alpha + \beta_1 * \text{RM}_{i,t-1} + \beta_2 * \text{RY}_{i,t-1} + \beta_3 * \text{DXY}_{i,t-1} + \beta_4 * \text{Ret}_{i,h,t-h}$, where $\text{RM}_{i,t}$ is the realized moment of commodity *i* at time *t* and $\text{Ret}_{i,h,t}$ is commodity *i* return for the window *t* - *h* to *t*, $\text{RY}_{i,t}$ is the roll yield of commodity *i* at time *t*, $\text{DXY}_{i,t}$ is the dollar index return at time *t*. The dependent variables are the returns of the commodity futures for h = one month. Each regression has as independent variables the realized moment of our eight commodities, the roll yield of the commodity, the dollar index previous month returns and the one-month lagged dependent variable and a constant. All the independent variables are lagged. Realized moments are calculated using one-month options and 5-minute returns on the previous one month. In Panel A, the main independent variable is the realized variance and in Panel B is the realized skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Realized Variance

Dependent Variable	Coefficients				Adjusted R^2
	Realized Variance	Roll Yield	Dollar Index	Lagged Returns	
Corn	-1.50	-0.12	-0.52*	-0.06	2.9%
	(-0.64)	(-1.49)	(-1.90)	(-0.69)	
Soybeans	-2.24	-0.25***	-0.57***	-0.03	6.5%
	(-1.27)	(-2.37)	(-2.47)	(-0.37)	
Wheat	-1.14	-0.38**	-0.21	-0.01	5.2%
	(-1.04)	(-1.99)	(-0.56)	(-0.16)	
Copper	0.06	-1.26**	-0.32	0.20*	8.5%
	(0.05)	(-2.29)	(-1.39)	(1.84)	
Silver	0.57	3.38**	-0.27	-0.06	5.6%
	(0.24)	(2.14)	(-1.08)	(-0.80)	
Gold	1.32	1.23	-0.06	-0.16*	4.1%
	(0.45)	(1.38)	(-0.40)	(-1.66)	
Oil	-0.82	-0.37*	-0.56*	0.20**	10.4%
	(-0.51)	(-1.88)	(-1.96)	(2.07)	
Natural gas	-4.82***	-0.34***	-0.11	0.03	10.9%
	(-2.95)	(-2.71)	(-0.30)	(0.49)	

Panel B. Realized Skewness

Dependent Variable	Coefficients				Adjusted R^2
	Realized Skewness	Roll Yield	Dollar Index	Lagged Returns	
Corn	0.01	-0.12	-0.58*	-0.08	3.6%
	(1.30)	(-1.52)	(-1.95)	(-1.04)	
Soybeans	0.00	-0.24**	-0.58***	0.03	5.8%
	(-1.06)	(-2.21)	(-2.45)	(0.29)	
Wheat	0.00	-0.36*	-0.24	-0.02	4.4%
	(-0.15)	(-1.92)	(-0.64)	(-0.27)	
Copper	0.00	-1.30***	-0.32	0.22*	8.6%
	(-0.75)	(-2.40)	(-1.40)	(1.75)	
Silver	0.00	3.39**	-0.25	-0.05	5.5%
	(-0.22)	(2.18)	(-0.96)	(-0.71)	
Gold	0.00*	1.01	-0.02	-0.20*	4.7%
	(1.69)	(1.22)	(-0.16)	(-1.90)	
Oil	0.00	-0.30***	-0.64**	0.23**	10.4%
	(-1.04)	(-2.77)	(-2.07)	(2.07)	
Natural gas	0.001	-0.23*	-0.23	-0.03	6.8%
	(0.39)	(-1.89)	(-0.62)	(-0.34)	

Table 6 - Implied Moment's Predictability with Control Variables

This Table shows the results of 16 regressions, $Ret_{i,h,t} = \alpha + \beta_1 * IM_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,h,t-h}$, where $IM_{i,t}$ is the implied moment of commodity *i* at time *t* and $Ret_{i,h,t}$ is commodity *i* return for the window *t* - *h* to *t*, $RY_{i,t}$ is the roll yield of commodity *i* at time *t*, $DXY_{i,t}$ is the dollar index return at time *t*. The dependent variables are the returns of the commodity futures for one month. Each regression has as independent variables the implied moment of our eight commodities, the roll yield of the commodity, the dollar index previous month returns and the one-month lagged dependent variable and a constant. All the independent variables are lagged. Implied moments are calculated using one-month options and 5-minute returns on the previous one month. In Panel A, the main independent variable is the implied variance and in Panel B, is the implied skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Implied Variance

Dependent Variable	Coefficients				Adjusted R^2
	Implied Variance	Roll Yield	Dollar Index	Lagged Returns	
Corn	-1.94*	-0.13*	-0.50*	-0.06	4.79%
	(-1.83)	(-1.67)	(-1.78)	(-0.72)	
Soybeans	-1.04	-0.23**	-0.58***	-0.01	5.54%
	(-0.59)	(-2.16)	(-2.54)	(-0.08)	
Wheat	-1.08	-0.38*	-0.27	-0.02	4.80%
	(-0.76)	(-1.92)	(-0.73)	(-0.17)	
Copper	0.94***	-1.06**	-0.28	0.14	12.48%
	(2.43)	(-2.23)	(-1.31)	(1.07)	
Silver	1.17	3.23**	-0.30	-0.06	5.97%
	(0.73)	(1.97)	(-1.14)	(-0.83)	
Gold	1.94	1.32	-0.08	-0.17*	4.54%
	(0.75)	(1.56)	(-0.59)	(-1.93)	
Oil	-1.26	-0.40**	-0.54*	0.19**	10.68%
	(-0.76)	(-2.05)	(-1.96)	(2.14)	
Natural gas	-3.57	-0.33***	-0.18	0.01	8.35%
	(-1.15)	(-2.80)	(-0.48)	(0.10)	

Panel B. Implied Skewness

Dependent Variable	Coefficients				Adjusted R^2
	Implied Skewness	Roll Yield	Dollar Index	Lagged Returns	
Corn	0.02**	-0.06	-0.53**	-0.03	5.21%
	(2.30)	(-0.81)	(-1.97)	(-0.41)	
Soybeans	0.01	-0.20*	-0.57**	-0.01	5.61%
	(0.92)	(-1.68)	(-2.26)	(-0.11)	
Wheat	0.03***	-0.31*	-0.27	-0.02	7.23%
	(3.01)	(-1.82)	(-0.72)	(-0.18)	
Copper	-0.01*	-1.45***	-0.31	0.17	10.15%
	(-1.82)	(-2.53)	(-1.37)	(1.48)	
Silver	0.02	3.52**	-0.26	-0.07	5.89%
	(0.72)	(2.19)	(-1.01)	(-0.95)	
Gold	0.02**	1.08	-0.06	-0.22**	5.93%
	(2.18)	(1.24)	(-0.46)	(-2.29)	
Oil	0.00	-0.27**	-0.58*	0.20**	10.04%
	(0.17)	(-2.02)	(-1.95)	(2.01)	
Natural gas	0.10***	-0.18	-0.22	-0.01	8.81%
	(2.45)	(-1.60)	(-0.59)	(-0.20)	

Table 7 - Moment Risk Premium's Predictability with Control Variables

This Table shows the results of 16 regressions, $\mathrm{Ret}_{i,h,t} = \alpha + \beta_1 * \mathrm{MRP}_{i,t-1} + \beta_2 * \mathrm{RY}_{i,t-1} + \beta_3 * \mathrm{DXY}_{i,t-1} + \beta_4 * \mathrm{Ret}_{i,h,t-h}$, where $\mathrm{MRP}_{i,t}$ is the moment risk premium of commodity i at time t and $\mathrm{Ret}_{i,h,t}$ is commodity i return for the window t - h to t, $\mathrm{RY}_{i,t}$ is the roll yield of commodity i at time t, $\mathrm{DXY}_{i,t}$ is the dollar index return at time t. The dependent variables are the returns of the commodity futures for one month. Each regression has as independent variables the moment risk premium of our eight commodities, the roll yield of the commodity, the dollar index previous month returns and the one-month lagged dependent variable and a constant. All the independent variables are lagged. Moment risk premia are calculated using one-month options and 5-minute returns on the previous one month. In Panel A, the independent variable is the variance risk premium and in Panel B, is the skewness risk premium. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Variance Risk Premium

Dependent Variable	Coefficients				Adjusted R2
	Variance Risk Premium	Roll Yield	Dollar Index	Lagged Returns	
Corn	-2.13*	-0.11	-0.54*	-0.06	4.47%
	(-1.87)	(-1.51)	(-1.88)	(-0.74)	
Soybeans	0.86	-0.24**	-0.60***	-0.02	5.33%
	(0.55)	(-2.14)	(-2.43)	(-0.18)	
Wheat	0.55	-0.36*	-0.21	-0.02	4.56%
	(0.57)	(-1.91)	(-0.55)	(-0.28)	
Copper	1.12***	-1.17**	-0.28	0.12	13.20%
	(3.00)	(-2.32)	(-1.30)	(1.03)	
Silver	2.07	3.16*	-0.28	-0.06	6.05%
	(1.01)	(1.93)	(-1.03)	(-0.90)	
Gold	2.22	0.99	-0.05	-0.18*	4.04%
	(0.77)	(1.20)	(-0.33)	(-1.75)	
Oil	-0.02	-0.29***	-0.58**	0.20*	10.01%
	(-0.01)	(-2.59)	(-1.96)	(1.74)	
Natural gas	3.97	-0.20*	-0.21	0.00	8.43%
	(1.19)	(-1.90)	(-0.55)	(-0.02)	

Panel B. Skewness Risk Premium

Dependent Variable	Coefficients				Adjusted R2
	Skewness Risk Premium	Roll Yield	Dollar Index	Lagged Returns	
Corn	0.00	-0.11	-0.56*	-0.07	2.67%
	(-0.40)	(-1.40)	(-1.91)	(-0.81)	
Soybeans	0.00	-0.22**	-0.57***	0.03	6.08%
	(1.43)	(-2.09)	(-2.37)	(0.33)	
Wheat	0.01	-0.31*	-0.22	-0.01	5.40%
	(1.60)	(-1.82)	(-0.58)	(-0.09)	
Copper	0.00	-1.26***	-0.32	0.20	8.46%
	(-0.12)	(-2.37)	(-1.42)	(1.62)	
Silver	0.00	3.38**	-0.25	-0.05	5.53%
	(0.37)	(2.19)	(-0.95)	(-0.67)	
Gold	0.00	1.01	-0.02	-0.19*	4.36%
	(-1.39)	(1.22)	(-0.16)	(-1.81)	
Oil	0.00	-0.29***	-0.65**	0.23**	10.44%
	(1.13)	(-2.64)	(-2.10)	(2.14)	
Natural gas	0.00	-0.23*	-0.24	-0.02	6.76%
	(-0.16)	(-1.85)	(-0.64)	(-0.25)	

Table 8 - Portfolio Return Statistics

This Table presents the portfolio return statistics for one-month (21 days) holding period. The mean and standard deviation are annualized and ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-1.77%	32.7%	0.72	8.6	-0.05	-0.17
Realized Skewness	-9.40%	25.2%	-0.43	6.9	-0.37	-1.13
Implied Variance	4.59%	31.5%	0.54	7.2	0.15	0.46
Implied Skewness	17.21%	28.0%	0.84	7.3	0.62	1.66*
Variance Risk Premium	12.30%	25.4%	0.08	5.2	0.48	1.38
Skewness Risk Premium	18.37%	25.9%	0.62	7.7	0.71	1.88**
Equally-Weighted Returns	2.31%	21.7%	-0.03	7.5	0.11	0.31
1 Year Momentum	-15.66%	36.5%	-0.65	6.6	-0.43	-1.58
1 Year Roll Yield	-11.21%	34.9%	-0.79	7.1	-0.32	-1.03

Table 9 - Correlation Matrix

This Table presents the portfolio return correlation among realized, implied and risk premium moments, as well as the equally weighted commodity portfolio, one-year momentum and roll yield. These portfolios are build based on single sorting of previous variables and for one-month holding period. We also show their correlation with the U.S. bond and equity excess returns. The sample period is from 2008 to 2016, on a daily overlapping basis.

	Realized Variance	Realized Skewness	Implied Variance	Implied Skewness	VRP	SRP	EW	Mome ntum	Roll Yield	US Bonds	US Equity
Realized Variance											
Realized Skewness	-14%										
Implied Variance	78%	-9%									
Implied Skewness	58%	-18%	41%								
VRP	-22%	7%	24%	-23%							
SRP	27%	-93%	18%	43%	-15%						
Equally-Weighted	-3%	-2%	-4%	3%	-2%	3%					
Momentum	-42%	35%	-39%	-37%	2%	-43%	2%				
Roll Yield	-62%	27%	-54%	-57%	12%	-40%	1%	62%			
US Bonds	-17%	7%	-23%	-8%	-9%	-10%	-1%	9%	5%		
US Equity	14%	-13%	19%	5%	7%	13%	10%	-10%	-3%	-43%	

Table 10 - Portfolio Weights with Same Risk of Other Assets

This Table presents the weights of a Markowitz portfolio optimization estimation. We build an efficient frontier using 11 portfolios: U.S. bond and U.S. equity portfolios, equally weighted commodity (EW), realized variance (-RV), implied variance (IV), variance risk premium (VRP), realized skewness (-RS), implied skewness (IS), skewness risk premium (SRP), minus one-year momentum (-Momentum) and minus one-year roll yield (-Roll Yield). The estimation period is daily from 2008 to 2016. We do not allow for leverage or short positions. Each column shows the weights of one portfolio on the efficient frontier with a specific volatility. The first column shows the weights of efficient frontier portfolio with the same volatility as that of EW commodities' portfolio. Second column shows the weights of efficient frontier portfolio with same volatility as that of bonds, and the third column, the portfolio with same volatility as that of stocks.

Weights for	**Composition of Efficient Frontier Portfolios with**		
	Annual Volatility of 21.55% (**EW Commodity**)	Annual Volatility of 5.05% (**U.S. Bonds**)	Annual Volatility of 22.19% (**U.S. Stocks**)
Realized Variance	0%	0%	0%
- Realized Skewness	0%	0%	0%
Implied Variance	0.0%	0.0%	0.0%
Implied Skewness	28.0%	9%	28%
Variance Risk Premium	3.7%	11.6%	1.5%
Skewness Risk Premium	61.2%	9.3%	63.1%
Equal-Weighted	0%	0%	0%
- Momentum	7.1%	0.6%	7.4%
- Roll Yield	0%	0%	0%
Bonds	0.0%	59%	0%
Stocks	0.0%	11%	0%

Table 11 - Portfolio Return Statistics using Double Sorting

This Table presents the portfolio return statistics using double sorting and one-month holding period (21 days). The mean and standard deviation are annualized and ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
VRP and Implied Skewness	13.10%	22.1%	0.863	9.1	0.59	1.66*
VRP and SRP	16.44%	22.1%	0.883	10.4	0.75	1.98**
SRP and Implied Skewness	17.94%	27.6%	1.038	9.4	0.65	1.80*

Table 12 - Portfolio Return Statistics using Different Holding Periods

This Table presents the portfolio return statistics based on single and double sorting using various holding periods. Panel A shows the portfolios statistics for two-week holding period (10 days) and Panel B shows them for two-month holding period (42 days). The mean and standard deviation are annualized and ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Two-Week Holding Period

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-1.97%	34.3%	0.646	7.7	-0.06	-0.18
Realized Skewness	-7.49%	28.5%	-0.325	6.1	-0.26	-0.79
Implied Variance	7.81%	32.7%	0.539	7.2	0.24	0.73
Implied Skewness	18.87%	29.6%	0.907	7.0	0.64	1.74*
Variance Risk Premium	20.80%	28.3%	0.076	5.6	0.73	2.12**
Skewness Risk Premium	18.98%	29.1%	0.381	5.8	0.65	1.75*
Equally-Weighted Returns	2.70%	21.6%	-0.030	7.5	0.12	0.36
One-Year Momentum	-15.49%	37.5%	-0.587	6.4	-0.41	-1.46
One-Year Roll Yield	-9.14%	35.5%	-0.757	7.0	-0.26	-0.83
VRP and Implied Skewness	19.88%	24.4%	0.635	6.4	0.81	2.25**
VRP and SRP	23.29%	25.3%	0.955	10.2	0.92	2.50***
SRP and Implied Skewness	15.99%	29.9%	1.036	8.4	0.53	1.52

Panel B. Two-Month Holding Period

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	3.92%	31.3%	0.788	9.0	0.13	0.41
Realized Skewness	-2.34%	21.1%	-0.623	9.6	-0.11	-0.29
Implied Variance	6.03%	30.1%	0.684	8.4	0.20	0.71
Implied Skewness	12.27%	26.2%	0.767	7.3	0.47	1.27
Variance Risk Premium	7.34%	22.4%	0.298	5.9	0.33	0.98
Skewness Risk Premium	9.49%	22.4%	0.890	11.4	0.42	1.11
Equally-Weighted Returns	2.12%	21.7%	-0.026	7.5	0.10	0.27
1 Year Momentum	-17.45%	34.9%	-0.606	6.5	-0.50	-1.92*
1 Year Roll Yield	-15.35%	33.8%	-0.848	7.4	-0.45	-1.57
VRP and Implied Skewness	8.22%	19.9%	0.814	10.6	0.41	1.26
VRP and SRP	8.86%	19.1%	0.880	13.5	0.46	1.24
SRP and Implied Skewness	12.02%	24.9%	1.057	10.5	0.48	1.34

Table 13 - Portfolio Correlation Matrix using Innovations

This Table presents the portfolio return correlations among realized, implied and risk premium moments, as well as the one-year momentum and roll yield. These portfolios are build using single sorting ARMA (1, 1) innovations of previous variables for one-month holding period. The sample period is from 2008 to 2016, on a daily overlapping basis.

	Realized Variance	Realized Skewness	Implied Variance	Implied Skewness	VRP	SRP	Equally-Weighted	Momentum	Roll Yield
Realized Variance									
Realized Skewness	-4%								
Implied Variance	27%	-2%							
Implied Skewness	15%	-10%	5%						
VRP	-34%	8%	51%	-8%					
SRP	8%	-84%	7%	37%	-10%				
Equally-Weighted	3%	-2%	1%	-1%	-1%	-1%			
Momentum	-4%	30%	-16%	-21%	-2%	-36%	0%		
Roll Yield	-9%	27%	-16%	-33%	-1%	-35%	-2%	46%	

Table 14 - Portfolio Return Statistics using Innovations

This Table presents the portfolio return statistics using single and double sorting approaches. These portfolios are build using the ARMA (1, 1) commodity innovations with one-month holding period. The mean and standard deviation are annualized and ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-5.21%	11.5%	0.117	6.5	-0.45	-1.44
Realized Skewness	-0.21%	11.6%	-0.323	6.9	-0.02	-0.06
Implied Variance	2.59%	9.1%	-0.206	10.2	0.29	0.87
Implied Skewness	6.15%	9.7%	1.226	11.4	0.64	1.89*
Variance Risk Premium	5.96%	9.7%	-0.279	10.9	0.61	1.64*
Skewness Risk Premium	1.66%	9.7%	0.362	6.9	0.17	0.55
Equally-Weighted Returns	2.31%	21.7%	-0.028	7.5	0.11	0.31
1 Year Momentum	-1.61%	8.1%	-0.141	9.0	-0.20	-0.59
1 Year Roll Yield	12.00%	9.6%	-1.097	14.0	1.25	3.69***
VRP and Implied Skewness	8.77%	8.9%	0.754	7.1	0.98	2.78***
VRP and SRP	7.40%	9.0%	0.457	6.8	0.82	2.28**
SRP and Implied Skewness	5.43%	9.0%	1.055	10.6	0.60	1.85*

Table 15 - Factor Regressions

This Table presents the performance of commodity portfolios. Panel A shows the performance for the portfolios build using the realized, implied and risk premium moments and in Panel B for the portfolios build using their ARMA (1, 1) innovations. We use one-month holding period. The momentum and roll yield are the one-year momentum and one-year roll yield. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Original Variables

Dependent Variable	Coefficients				Adjusted R^2
	Alfa	Equal-Weighted	Moment	Roll Yield	
Realized Variance	-0.0004	-0.03*	-0.06	-0.54***	38.4%
	(-1.05)	(-1.65)	(-0.54)	(-8.06)	
Realized Skewness	-0.0002	-0.03	0.21***	0.06	12.8%
	(-0.69)	(-1.41)	(3.93)	(1.15)	
Implied Variance	-0.0001	-0.05***	-0.09	-0.43***	30.0%
	(-0.25)	(-2.75)	(-0.95)	(-7.91)	
Implied Skewness	0.0004	0.05	-0.02	-0.45***	33.0%
	(1.23)	(1.62)	(-0.27)	(-6.56)	
Variance Risk Premium	0.0005	-0.03	-0.06	0.13**	2.0%
	(1.50)	(-1.30)	(-0.93)	(2.07)	
Skewness Risk Premium	0.0004	0.05*	-0.21***	-0.16***	21.8%
	(1.49)	(1.78)	(-4.03)	(-2.93)	
VRP and Implied Skewness	0.0004	0.016	-0.033	-0.220***	14.6%
	(1.37)	(0.84)	(-0.57)	(-4.88)	
VRP and SRP	0.0005*	0.016	-0.156***	-0.043	9.3%
	(1.66)	(0.87)	(-3.35)	(-0.89)	
SRP and Implied Skewness	0.0004	0.051*	-0.132**	-0.378***	36.3%
	(1.31)	(1.66)	(-1.98)	(-5.99)	

Panel B. Innovations

Dependent Variable	Coefficients				Adjusted R^2
	Alfa	Equal-Weighted	Moment	Roll Yield	
Realized Variance	-0.0002	0.017	-0.005	-0.104	0.9%
	(-1.14)	(1.16)	(-0.05)	(-1.39)	
Realized Skewness	-0.0001	-0.009	0.330***	0.194**	11.4%
	(-0.60)	(-0.80)	(3.03)	(2.14)	
Implied Variance	0.0001	0.003	-0.125	-0.107	3.7%
	(1.13)	(0.34)	(-1.34)	(-1.30)	
Implied Skewness	0.0004***	-0.006	-0.094	-0.297***	11.4%
	(2.52)	(-0.72)	(-0.87)	(-2.97)	
Variance Risk Premium	0.0002	-0.004	-0.018	-0.001	0.0%
	(1.47)	(-0.49)	(-0.18)	(-0.01)	
Skewness Risk Premium	0.0002	-0.005	-0.303***	-0.241***	17.4%
	(1.52)	(-0.49)	(-3.27)	(-3.72)	
VRP and Implied Skewness	0.0004***	0.006	-0.079	-0.184***	5.7%
	(3.62)	(0.73)	(-1.05)	(-3.29)	
VRP and SRP	0.0003***	-0.001	-0.161**	-0.143***	6.4%
	(2.87)	(-0.08)	(-1.98)	(-2.65)	
SRP and Implied Skewness	0.0003***	-0.009	-0.179*	-0.294***	17.1%
	(2.80)	(-0.94)	(-1.94)	(-4.00)	

FIGURES

Figure 1 - Portfolio Path

This Figure presents the portfolio path of single-sort realized, implied and risk premium moment portfolios (variance and skewness) considering one-month (21 days) holding period. The sample period is from 2008 to 2016, on a daily overlapping basis.

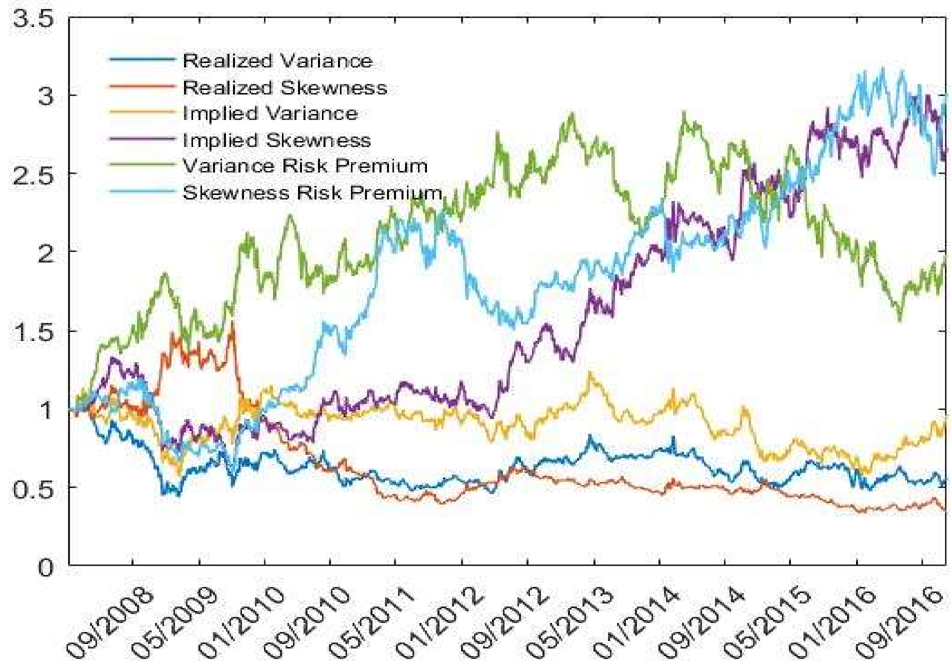


Figure 2 - Portfolio Optimization

This Figure shows the Markowitz portfolio optimization. We consider 11 portfolios: U.S. bond and equity portfolios, equally weighted commodities portfolio (EW), realized variance (-RV), implied variance (IV), variance risk premium (VRP), realized skewness (-RS), implied skewness (IS), skewness risk premium (SRP), minus one-year momentum (-Momentum) and minus one-year roll yield (-Roll Yield). The estimation period is daily from 2008 to 2016. We do not allow for leverage or short positions. The efficient frontier (EF) with a dotted line includes all 11 portfolios, whereas the efficient frontier with continuous line only includes the U.S. equity and bond markets.

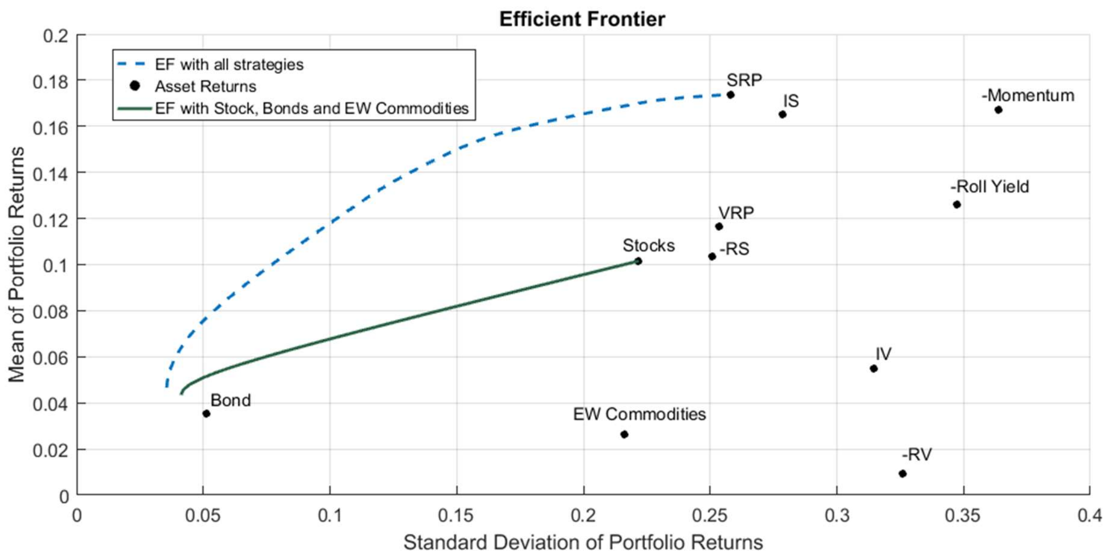
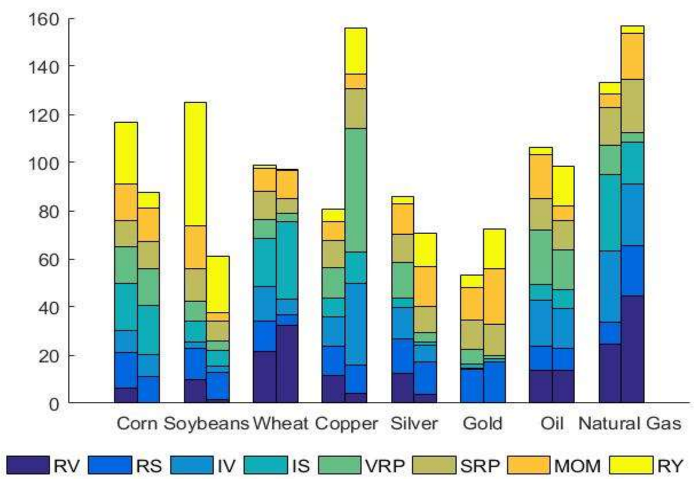


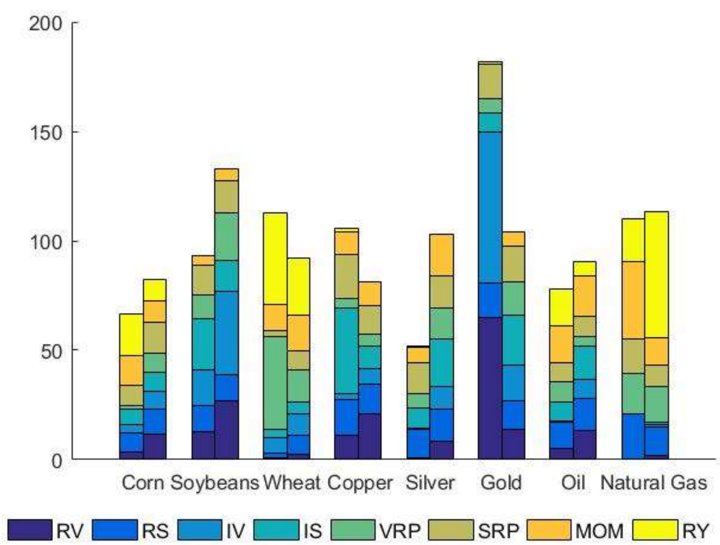
Figure 3 - Frequency of Commodities in Portfolio Analysis

This Figure shows the percentage of days over the entire sample period from January 2008 to December 2016 that each commodity enters the long portfolio (Panel A) and short portfolio (Panel B) for each of the realized and implied moments and risk premium moments, as well as for the one-year momentum and roll yield portfolios. The RV and RS are the realized variance and skewness. IV and IS are the implied variance and skewness. VRP and SRP are the variance and skewness risk premia. MOM and RY are the momentum and roll yield, respectively.

Panel A. Long Portfolios



Panel B. Short Portfolios



Appendix

Table A. 1 - Portfolio Return Statistics – Second and Third Quartiles

This Table presents the portfolio return statists considering one-month holding period (21 days) for the long-short portfolio of the second and third quartiles. That is, we present results for portfolios going long on the second quartile and short on the third quartile. The mean and standard deviation are annualized and ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-0.79%	19.6%	0.359	8.5	-0.04	-0.12
Realized Skewness	-1.25%	15.9%	-0.090	7.6	-0.08	-0.24
Implied Variance	0.63%	16.6%	0.185	6.6	0.04	0.12
Implied Skewness	-4.35%	16.8%	0.646	9.4	-0.26	-0.83
Variance Risk Premium	-0.54%	12.5%	0.272	7.0	-0.04	-0.13
Skewness Risk Premium	0.66%	15.6%	0.234	7.4	0.04	0.15
Equally-Weighted Returns	2.31%	21.7%	-0.028	7.5	0.11	0.31
1 Year Momentum	-4.24%	20.9%	-0.419	7.4	-0.20	-0.69
1 Year Roll Yield	16.64%	23.8%	-0.186	5.4	0.70	1.79*
VRP and Implied Skewness	13.10%	22.1%	0.863	9.1	0.59	1.66*
VRP and SRP	16.44%	22.1%	0.883	10.4	0.75	1.98**
SRP and Implied Skewness	17.94%	27.6%	1.038	9.4	0.65	1.80*

Table A. 2 - Factor Regressions for Second and Third Quartiles

This Table presents the performance of commodity portfolios build using the realized, implied and risk premium moments with one-month holding period. We show performance of the portfolios constructed by going long (short) on the second (third) quartile of realized, implied and risk premium moments. The momentum and roll yield are the one-year momentum and one-year roll yield. The sample period is from 2008 to 2016, on a daily overlapping basis.

Dependent Variable	Coefficients				Adjusted R2
	Alfa	Equal-Weighted	Moment	Roll Yield	
Realized Variance	-0.0001	0.021	-0.083	0.033	0.9%
	(-0.27)	(1.11)	(-1.49)	(0.49)	
Realized Skewness	-0.0001	0.017	0.005	0.001	0.1%
	(-0.25)	(0.69)	(0.14)	(0.02)	
Implied Variance	0.0000	-0.008	-0.030	-0.001	0.2%
	(0.09)	(-0.42)	(-0.55)	(-0.01)	
Implied Skewness	-0.0001	0.013	0.039	-0.071	1.1%
	(-0.62)	(0.93)	(0.66)	(-1.61)	
Variance Risk Premium	0.0000	0.006	-0.002	0.005	0.0%
	(-0.18)	(0.60)	(-0.04)	(0.17)	
Skewness Risk Premium	0.0000	-0.001	0.028	-0.027	0.3%
	(0.28)	(-0.03)	(0.61)	(-0.67)	
VRP and Implied Skewness	0.0005*	0.008	-0.036	-0.040	0.3%
	(1.91)	(0.47)	(-0.58)	(-0.72)	
VRP and SRP	0.0006**	0.010	-0.048	-0.038	0.4%
	(2.01)	(0.58)	(-0.74)	(-0.79)	
SRP and Implied Skewness	0.0008**	0.029	-0.055	-0.187***	3.1%
	(2.07)	(0.82)	(-0.68)	(-2.42)	

Figure A. 1 - Portfolio Mean Return over Quartiles

This Figure shows the portfolio mean return for portfolios based on each quartile using one-month holding period. The means are annualized. RV is the realized variance and RS is the realized skewness. IV is the implied variance and IS is the implied skewness. VRP is the variance risk premium and SRP is the skewness risk premium. MOM is the one-year momentum and RY is the one-year roll yield. The sample period is from 2008 to 2016, on a daily overlapping basis.